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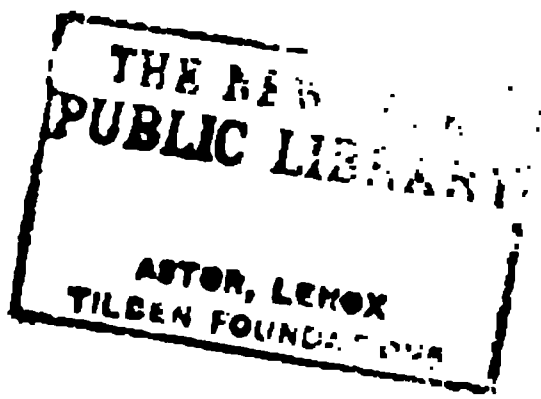
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THE COMPLETE
MATHEMATICAL AND GENERAL
NAVIGATION TABLES,

INCLUDING EVERY TABLE REQUIRED WITH THE NAUTICAL ALMANAC IN FINDING THE

LATITUDE AND LONGITUDE :

WITH AN EXPLANATION OF THEIR CONSTRUCTION, USE, AND APPLICATION TO

NAVIGATION AND NAUTICAL ASTRONOMY,
TRIGONOMETRY, DIALLING, GUNNERY, ETC. ETC.

BY THOMAS KERIGAN, R. N.

AUTHOR OF "THE YOUNG NAVIGATOR'S GUIDE," ETC.

SECOND EDITION ;

CAREFULLY REVISED, AND ADAPTED TO THE IMPROVED ELEMENTS INTRODUCED INTO THE
NAUTICAL ALMANAC.

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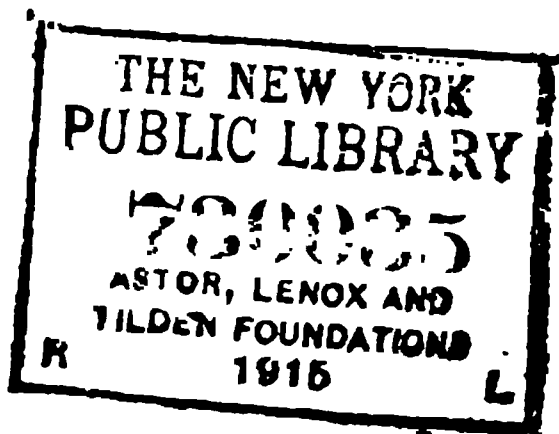
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THE RIGHT HONOURABLE LORD DALMENY, M.P.,

HER MAJESTY'S LORDS COMMISSIONERS FOR EXECUTING THE OFFICE OF LORD HIGH
ADMIRAL OF THE UNITED KINGDOM OF GREAT BRITAIN AND IRELAND,

THIS TREATISE,

DESIGNED TO AFFORD TO THE NAVIGATOR A COMPREHENSIVE
AND PRACTICAL GUIDE

TO THE SCIENCES OF NAVIGATION AND NAUTICAL ASTRONOMY,
AND NOW ADAPTED TO THE IMPROVED ELEMENTS CONTAINED IN
THE NAUTICAL ALMANAC;

IS,

BY THEIR PERMISSION, MOST RESPECTFULLY AND GRATEFULLY

DEDICATED,

BY THEIR LORDSHIPS' MOST HUMBLE

AND MOST OBEDIENT SERVANT,

THE AUTHOR.

NOV 21 1954

PREFACE.

ALTHOUGH the importance and general utility of the subjects treated of in this work are sufficient to recommend it to public attention, without the aid of prefatory matter, yet, since there is an extensive variety of nautical publications now extant, I think it right to say something relative to what I have done, were it for no other purpose than that of satisfying the reader that the present work is widely different from any former treatise on nautical and mathematical subjects. The following observations will develope my motives for commencing so laborious an undertaking.

In perusing the various nautical publications which have appeared for many years past, I observed that they all fell considerably short of the objects at which they professed to aim ;—some, by being too much contracted, and others by not including all the necessary tables, or by being generally defective : and that, therefore, a great deal remained to be done, particularly in the tabular parts, beyond what had yet been brought before the public.

Of the nautical works that came under my notice, some have proved, on examination, to be so inaccurately executed, as to be entirely unfit for the consultation of any person not sufficiently skilled in the mathematics to detect their numerous errors. Many of the works in question are extremely incomplete, through their want of particular tables, and their logarithms not being extended to a sufficient number of decimal places : such as those by Mendoza Rios, where the decimals are only continued to *five places* of figures, and where the logarithmic

tangents are entirely wanting; for, although the addition of a logarithmic sine and a logarithmic secant will always produce a logarithmic tangent, yet there are few mariners so far acquainted with the peculiar properties of the trigonometrical canon, as to be able to find by Rios's tables the arch corresponding to a given logarithmic tangent.* Hence, when the course and the distance between two places are to be deduced from their respective latitudes and longitudes, by logarithmical computation, the mariner is invariably obliged to have recourse to some other work for the necessary table of logarithmic tangents. Besides, since none of the nautical works now in use exhibit the principles upon which the tables contained therein have been constructed, the mariner is left without the means of examining such tables, or of satisfying himself as to their accuracy; though it is to them that he is obliged to make continual reference, and on their correctness that the safety of the ship and stores, with the lives of all on board, so materially depend.

Notwithstanding that Mr. Taylor's Logarithmical Tables are the most extensive, the best arranged, and by far the most useful for astronomical purposes, of any that have ever appeared in print,—yet, since they do not contain the necessary navigation tables, they are but of little use, if of any, to the practical navigator; and, since the same objection is applicable to the very excellent system of tables published by the learned Dr. Hutton, these are, also, ill adapted to nautical purposes, and but rarely consulted by mariners.

Being thus convinced that there was something either deficient or very defective in all the works that had hitherto been published on this subject, I was ultimately led to the conclusion that a *general and complete set of Nautical Tables* was still a desideratum to mariners: with this conviction on my mind, I was at length induced to undertake the laborious task of drawing up the following work; in the prosecution of which I found it necessary to exercise the most determined perseverance and industry, in order to surmount the fatigue and anxiety attendant on such a long series of difficult calculations.

These points premised, it remains to present to the reader a familiar and comparative view of the nature of this work, and of the improvements that have been made in the tables immediately connected with the elements of navigation and nautical astronomy: confining the attention to those that possess the greatest claims to originality, or in which the most useful improvements have been made.

Table VI. contains the parallaxes of the planets in altitude; and

* See Remark, page 98; with diagram and calculations, page 99.

will be found particularly useful in deducing the apparent time from the altitudes of the planets, and, also, in problems relating to the longitude. The hint respecting this was originally taken from the Copenhagen edition of "The Distances of the Planets from the Moon's Centre, for the Year 1823;" but this design has been considerably enlarged and improved upon.

Table VIII. is so arranged that the mean astronomical refraction may be taken out at first sight, without subjecting the mariner to the necessity of making proportion for the odd minutes of altitude. This improvement will have a tendency to facilitate nautical calculations.

Table X.—The arrangement of this table is an improvement of that originally given by the author, in his treatise called "The Young Navigator's Guide to the Sidereal and Planetary Parts of Nautical Astronomy." By this improved table, the correction of the polar star's altitude may be readily taken out, at sight, to the nearest second of a degree, by means of five columns of proportional parts; and, to render the table permanent for at least half a century, the annual variation of that star's correction has been carefully determined to the hundredth part of a second. By means of this table, and that which immediately follows (Table XI.), the latitude may be very correctly inferred at any hour of the night, in the northern hemisphere, to every degree of accuracy desirable for nautical purposes.

Tables XIII. and XIV. contain the equations to equal altitudes of the sun: these have been computed on a new principle, so as to adapt them to proportional logarithms, by means of which they are rendered infinitely more simple than those given under the same denomination in other treatises on nautical subjects; they will be found strictly correct, and, from their simplicity, a hope may be entertained that the truly correct and excellent method of finding the error of a watch or chronometer by equal altitudes of the sun, will be brought into more general use.

Tables XV. and XVI., which are entirely new, contain correct equations for readily reducing the longitudes, right ascensions, declinations, &c. &c., of the sun and moon, as given in the Nautical Almanac, to any given meridian, and to any given time under that meridian.

Table XVII. contains the equation corresponding to the mean second difference of the moon's place in longitude, latitude, right ascension, or declination: this table, besides being newly-arranged, will be found more extensive than those under a similar denomination, usually met with in books on navigation.

Table XVIII. is so arranged as to exhibit the true correction of the moon's apparent altitude corresponding to every second of horizontal parallax, and to every minute of altitude from the horizon to the zenith:

and will prove very serviceable in all problems where the moon's altitude forms one of the arguments either given or required.

Table XIX. is fully adapted to the reduction of the true altitudes of the heavenly bodies, obtained by calculation, to their apparent central altitudes: the reductions of altitude may be very readily taken out to the decimal part of a second. This table will be found of considerable utility in deducing the longitude from the lunar observations, when the distance only has been observed.

Table XX. is new; and by its means the operation of reducing the apparent central distance between the moon and sun, a fixed star, or planet, to the true central distance, is very much abridged, as will appear evident by referring to Method I., vol. i., page 481, where the true central distance is found by the simple addition of five natural versed sines.

Table XXI., which is also new, contains the correction of the auxiliary angle when the moon's distance from a planet is observed: this will be of great use in finding the longitude by the moon's central distance from a planet.

Table XXIV.—The form of this table is entirely original; and though it is comprised in nine pages, yet it is so arranged that the logarithmic difference may be obtained, strictly correct, to the nearest minute of the moon's apparent altitude, and to every second of her horizontal parallax. This table will be found of almost general use in the problem for finding the longitude by the lunar observations.

Table XXVI., which is original, contains the correction of the logarithmic difference when the moon's distance from a planet is observed: this table will be found of great use in computing the lunar observations whenever the moon's distance from the planets appears in the Nautical Almanac; an improvement which, from the advertisement prefixed to the late Almanacs, may be shortly expected to take place.

Table XXVII.; Natural Versed Sines, &c.—The numbers corresponding to the first 90 degrees of this table are expressed by the arithmetical complements of those contained in the Table of Natural Co-sines published by the author in "The Young Navigator's Guide," &c.; the arithmetical complement of the natural co-sine of an arch being the natural versed sine of the same arch. The numbers contained in the remaining 90 degrees of this table are expressed by the natural sines, from the above-mentioned work, augmented by the radius.

This table is so arranged as to render it general for every arch contained in the whole semi-circle, and conversely, whether that arch or its correlative be expressed as a natural versed sine, natural versed sine supplement, natural co-versed sine, natural sine, or natural co-sine.

Table XXVIII. is an extension of that published by the author in

“The Young Navigator’s Guide,” &c.: it is arranged in a familiar manner, and, though concise, contains all the numbers that can be usefully employed in the elements of navigation; for, by means of nine columns of proportional parts, the logarithmic value of any natural number under 1839999 may be obtained nearly at sight, and conversely.

Tables XXX., XXXI., and XXXII., have been carefully drawn up, and proportional parts adapted to them, by means of which the logarithmic half-elapsed time, middle time, and logarithmic rising may be very readily taken out at the first sight, and conversely.

Table XXXV., Logarithmic Secants.—The arrangement of this table is original, as well as its length: the numbers contained therein are expressed by the arithmetical complements of those contained in the table of logarithmic co-sines published by the author in “The Young Navigator’s Guide,” &c.

This table is so drawn up as to be properly adapted to every arch expressed in degrees, minutes, and seconds, in the whole semi-circle, whether that arch or its correlative be considered as a secant or a co-secant; and by means of proportional parts, the absolute value of any arch, and conversely, may be readily obtained at sight.

Table XXXVI., Logarithmic Sines.—This table is rendered general for every degree, minute, and second, in the whole semicircle. The Table of Logarithmic Tangents, which immediately follows, is also rendered general to the same extent; and by means of proportional parts, the true value of any arch, and conversely, may be instantly obtained, without the trouble of either multiplying or dividing: this improvement, to the practical navigator, must be an object of great importance, in reducing the labour attendant on computations in Nautical Astronomy.

Table XXXVIII. has been newly computed to the nearest second of time, so that the mariner may be readily enabled to reduce the time of the moon’s passage over the meridian of Greenwich to that of her passage over any other meridian. This table will be found very useful in determining the apparent time of the moon’s rising or setting, and also in ascertaining the time of high water at any given place by means of Table XXXIX.

Table XLII.—This general Traverse Table, so useful in practical navigation, is arranged in a very different manner from the Traverse Tables given in the generality of nautical books; and although comprised in 38 pages, is more comprehensive than the two combined tables of 61 pages usually found in those books, under the head “Difference of Latitude and Departure.” In this table, every page exhibits all the angles that a ship’s course can possibly make with the

meridian, expressed both in points and degrees ; which does away with the necessity of consulting two tables in finding the difference of latitude and departure corresponding to any given course and distance.

Table XLIV. contains the mean right ascensions and declinations of the principal fixed stars. The eighth column of this table, which is original, and is intended to facilitate the method of finding the latitude by the altitudes of two fixed stars observed at any hour of the night, contains the true spherical distance between the stars therein contained and those preceding or abreast of them on the same horizontal line. The ninth or last column of the page contains the annual variation of that distance, expressed in seconds and decimal parts of a second. Great pains have been taken, in order to find the absolute value of the annual variation of the true spherical distance between the fixed stars ; and the author trusts that he has so far succeeded as to render this part of the table permanent for a *long period of years subsequent to 1824*.

Tables XLV. and XLVI., which are adapted to the reduction of sidereal time into mean solar time, and conversely, have been newly constructed : these will be found considerably more extensive and uniform, than those generally given under the same denomination.

Tables LI. and LII. are entirely new : these will be found exceedingly useful in finding the latitude by the altitude of a celestial object observed at certain intervals from the meridian ; and since they are adapted to proportional logarithms, the operation of finding the latitude thereby becomes extremely simple, and yet far more accurate than that resulting from *double altitudes*, even after *repeating* a troublesome operation, and then applying *correction to correction*.

Table LIV.—This table will be of service to Masters in the Royal Navy, to officers employed in maritime surveys, and to all others who may be desirous of constructing charts agreeably to Mercator's principles of projection.

Table LVI. will be found essentially useful in reducing the French centesimal division of the circle into the English sexagesimal division, and conversely ; and since most of the modern French works on astronomy are now adapted to the centesimal principle, this table will be found of assistance in consulting those works ;—nor will it be of less advantage to the French navigator, in enabling him readily to consult the works of the English astronomers, where the degrees, &c., are expressed agreeably to the original or sexagesimal principle.

Table LVII. is new ; and although it may not immediately affect the interest of the mariner, yet it cannot fail to be useful to officers in charge of Her Majesty's Victualling Stores, in consequence of the Act of Parliament for the establishment of a new general standard or im-

perial gallon measure throughout the United Kingdom.—See Practical Gauging, page 679.

Table LVIII. contains the latitudes and longitudes of all the principal sea-ports, islands, capes, shoals, rocks, &c. &c. in the known world; these are so arranged as to exhibit to the navigator the whole line of coast along which he may have occasion to sail, or on which he may chance to be employed, agreeably to the manner in which it unfolds to his view on a Mercator's chart; a mode of arrangement much better adapted to nautical purposes than the alphabetical. But since the table is not intended for general geographical purposes, the positions of places inland, which do not immediately concern the mariner, have, with a few exceptions, been purposely omitted. The time of high water, at the full and change of the moon, is given at all places where it is known; which will be found considerably more convenient than referring for it to a separate table.

The series of latitudes and longitudes that have been established, astronomically and chronometrically, by Captain William Fitzwilliam Owen, of His Majesty's ship *Eden*, during his extensive survey along the coasts of Africa, Arabia, Madagascar, Brazil, &c., follows the last-mentioned table, have been corrected agreeably to a list of *Errata* obtained from the Hydrographical office.

The sun's declination is not given in this work; nor is it necessary that it should be, since it is contained, in the most ample manner, in the *Nautical Almanac*; a work which is so truly valuable to mariners that few now go to sea without it; the judicious never will.

Having thus taken a survey of the principal part of the Tables, I must briefly notice their *Description and use*;—these will be found at the commencement of the First Volume. The principles and methods of their computation are here fully detailed; and the reader is furnished with the means, in the most simple formulæ, of examining any part of the Tables; which is far more satisfactory than trusting to the author's mere word for their entire accuracy; though, I flatter myself with the hope that, in this extensive mass of figures, very few errors will be found;—at all events, none of *principle*.

My original plan had been to close the work with the description and use of the Tables, but being apprehensive that a series of Tables alone, however well arranged, or clearly illustrated, would not be sufficient to ensure general acceptance, I was induced to show their direct application to the different elements connected with the sciences of navigation and nautical astronomy, as well as to other subjects of a highly interesting nature, such as the art of gunnery, &c. &c. In this part of the work, since my design did not extend beyond an ample illustration of

the various mathematical purposes to which these Tables may be applied, I have restricted myself to the practical parts of the sciences on which I have had occasion to touch ; because those are the points which most concern the mariner, and the commercial interests of this maritime nation. Nevertheless, wherever it has appeared necessary to notice the elementary parts of the sciences, reference has been made to relative problems in “ The Young Navigator’s Guide,” where, it is hoped, the reader will find his inquiries fully satisfied.

The various sciences touched upon commence with a concise system of decimal arithmetic, and complete courses of plane and spherical trigonometry. In the latter, the solution of the quadrantal triangles will be found much simplified.

The practical parts of Navigation begin with parallel sailing ; but, with the view of preventing the work from swelling to an unnecessary size, the cases of plane sailing, usually met with in other nautical books, have been omitted in this ; as these are, in effect, no more than a mere repetition of the cases of right angled plane trigonometry under a different denomination. Middle latitude sailing will be found exceedingly simplified by means of a series of familiar analogies or proportions : and in Mercator’s sailing a series of rational proportions is given ; which, it is hoped, may tend to induce mariners to substitute the rules of reason for the *rules of rote* ; and thus do away with the mistaken system of getting *canons by heart* ; a system which has too long prevailed in the Royal Navy.

The two very useful sailings, oblique and windward, which have been hitherto little noticed by mariners, are also rendered so simple, particularly the latter, that it is to be hoped they will, ere long, be brought into general use.

In current sailing (Example 3), the true principles of steering a vessel in a current, or tideway, are familiarly illustrated. This problem cannot fail of being interesting to every person who is at all curious in the art of navigation.

The solution of a problem in great circle sailing is given, which will be found essentially useful to ships bound from the Cape of Good Hope to New South Wales : comprising a table which exhibits, at sight, all the scientific particulars attendant on the true spherical track between those two places ; by which it will be seen that a saving of 585 miles may be effected by sailing near the arc of a great circle as laid down in that table ; which saving ought to be an object of very high consideration to all ships bound from the Cape of Good Hope to Van Diemen’s Land, or to her Majesty’s colony at New South Wales, with either troops or convicts ; because the length of the voyage on the old track, or that

deduced from the common principles of navigation, generally occasions a great scarcity of fresh water, and this, eventually, adds distress to the many privations under which those on board usually labour. In the same problem, there is a table showing the true spherical route from Port Jackson, in New South Wales, to Valparaiso, on the coast of Chili: in this route there is a saving of 745 miles when compared with that resulting from Mercator's sailing; and this must be of considerable importance to the captain of a ship sailing between these places, who is desirous of making his port in the shortest space of time; particularly since few ships can carry a liberal allowance of fresh water to serve during a passage which measures very nearly one-fourth of the earth's circumference.

The introductory problems to Nautical Astronomy will be found ranged in the most natural order; all of which, except those relating to the altitudes of the objects, are concisely solved by proportional logarithms: the greater part of these will appear entirely new to the navigator. The VIth Problem relating to the latitude exhibits the method of finding the latitude by an altitude of the north polar star taken at any hour of the night, which will be found very useful in all parts of the northern hemisphere.—The VIIth Problem shows the method of finding the latitude by the altitudes of two stars taken at any time of the night, agreeably to the computed spherical distance between them contained in Table XLIV.; this method of ascertaining the latitude is general; it will be found very correct, and far less troublesome than that by *double altitudes* which immediately precedes it.—Problems IX., X., XI., and XII., contain *new* and *accurate methods* of deducing the latitude from the altitudes of the celestial bodies observed at given intervals from the meridian: the operation consists of very little more than the common *addition of three proportional logarithms*, and yet the latitude resulting from it will always be as correct as that deduced from the object's meridional altitudes, provided the watch shows apparent time at the place of observation, and the altitudes be taken within the limits prescribed. These problems will be found highly advantageous to the practical navigator; because, in the event of the sun's, or other celestial object's meridional altitude being neglected to be taken, or of it's being obscured by clouds at the time of transit, he is thus provided with the most safe and ready means of determining his latitude with as much certainty as if the altitude of the object had been observed actually upon the meridian either above or below the pole.

In the methods of computing the altitudes of the heavenly bodies, the solutions to the several problems are rendered exceedingly concise and explicit.

The IIIrd, IVth, Vth, and VIth Problems, relating to the longitude contain the methods of finding the longitude by a chronometer and the respective altitudes of the sun, stars, planets, and the moon ; the three last of which will be found considerably elucidated.

The lunar observations commence with the VIIth Problem on the longitude. In this problem *twelve methods* are given for reducing the apparent central distance between the moon and sun, a fixed star, or planet, to the true central distance ; several of which are entirely original, and all of them adapted to solve this interesting and important problem in the most simple and expeditious manner.

Then follows a series of problems relative to finding the variation of the compass by amplitudes, azimuths, transits of the fixed stars and planets, and by observations of the circumpolar stars. Problems V. and VI. contain the methods of reducing or correcting the true and the magnetic courses, between two places, agreeably to any given variation of the compass.—An improved azimuth compass card is described in this part of the work, which may be applied to the determination of the longitude by the lunar observations.

The series of problems for finding the apparent times of the rising or setting of the celestial bodies, and of the beginning or the end of twilight ;—and that for determining the interval of time between the rising or setting of the sun's upper and lower limbs, it is hoped will prove acceptable to the lovers of the science of Nautical Astronomy ;—likewise the art of Dialling, which, although it may appear foreign or irrelevant to the pursuits of the mariner, cannot fail to be interesting as a branch of science. It is here treated of in a familiar manner.

The IVth Problem in the mensuration of heights and distances, exhibits the method whereby the officers on board two ships of war can readily ascertain their absolute distance from any fort or garrison which they may be directed to cannonade ;—after which follow several problems that will be found exceedingly useful on many military occasions. See Remark at page 617, and also at page 627. Problem XI. showing the method of reducing a base line, measured on any elevated horizontal plane, to its true level at the surface of the sea ; and Problem XIII. exhibiting a new rule for finding the height of a mountain, or other eminence, by means of two barometers and two thermometers, may be of considerable use to engineers, or to others employed in conducting surveys. A problem is also given for finding the direct course steered by a ship seen at a distance ; and being a subject highly interesting to all nautical persons, it is reduced to every desirable degree of simplicity both by geometry and trigonometry.

All the problems in Practical Gunnery are readily solved by logarithms :

labours, under the conviction that, whatever may be the defects in its execution, they will do justice to my motives, in this attempt to lessen the existing obstructions in the way of attaining a practical knowledge of the elements of Navigation and Nautical Astronomy.

THOMAS KERIGAN.

PREFACE

TO THE SECOND EDITION.

As the Edition now offered to the Naval world, and to the public in general, differs essentially from the first impression, which has been for some time back out of print; I therefore think it necessary to make some allusion to the principal points in which the differences occur; for these have been productive of so many important alterations, as to change the features of the First Volume so far as to make it assume the character of being *the Master-Key to the Nautical Almanac*.

The first 296 pages of Volume I. differ but little from those in the preceding Edition; except that between pages 29 and 34, an original article has been introduced, exhibiting a practicable method for deducing the moon's horizontal parallax from her observed meridional altitude; and showing how the distances of the sun and moon from the earth may be determined by the inverse ratio of their parallaxes.

Between page 296 and the end of the Volume, all the subjects relating to Astronomy have been remodelled and newly arranged: this became indispensable, because the new Nautical Almanac differs so very materially from the old, that the number of naval men who are perfect masters of its use, does not amount to many units: and, moreover, because the element called "Sidereal Time," (given in page II. of the Month in the Ephemeris,) affects the principal part of the calculations in such a peculiar manner, that without a competent knowledge thereof, it would be useless to attempt the solution of any problem involving *mean time*; particularly when the moon, a fixed star, or a planet comes under consideration.—Hence, I have been induced to draw up a series of Articles, so as to leave no point unexplained that has any relation to *time*; whether considered as apparent, mean, or sidereal; or that has any affinity to the equation of time. Those Articles are so comprehensive, as to define and elucidate every elementary expression in the new Nautical Almanac, that is of any interest to the Maritime world, from the precession of the equinoxes to the

motion of the imaginary or mean Sun ; and thence down to the golden number. Besides unfolding a considerable portion of the Ephemeris, they clear up a number of curious and important points relative to the motions of the earth, which have hitherto escaped the notice of mere theoretical writers on Navigation and Nautical Astronomy.

The Introductory Problems, which begin at page 341, are solved in such an explanatory manner as to exhibit the most direct and practicable modes of applying the various elements in the Ephemeris to the important purposes for which they are designed. And, in every problem that seemed to require a specific illustration, a summary of its theory is introduced ; so as to convey to the mind of the reader the most familiar idea of the spherical principles upon which the consequent practical rule is founded.

The problem for finding the longitude by means of a chronometer, manifests the unquestionable reason (page 453) why a timekeeper regulated on shore, or at an observatory, and then taken on board a ship, *never answers the expectations of a navigator at sea* ; and it shows a rational cause why the *rate* of a chronometer should *always* be determined on board the ship to which it belongs, and *never on shore*. Aware of the difficulty of removing an old prejudice, or of doing away with a mistaken custom of long standing, I have therefore entered into all the details of the argument, and intentionally sacrificed conciseness to clearness of expression, so as to demonstrate, in the most ample manner, the easy practicability of establishing the error and the rate of a chronometer on board a ship, after it has been duly fixed in its berth, or resting-place.

In the practical part of the Lunar Observations, the peculiarities attendant on the phases of Venus, accordingly as she may be situated in the heavens as an evening or a morning star, are pointed out in the most perspicuous manner (page 517), for the purpose of preventing those who are not much versed in physical Astronomy from falling into a mistake in observing an angular distance between her enlightened limb and that of the moon.

Certain parts of the original work have been omitted :—as thus, finding that “the Method for determining the Longitude on shore by means of an Altitude of the Moon,”—though strictly correct in principle,—was not likely to be of much practical utility, on account of the extreme degree of precision with which the observation should be made ; I have, therefore, struck it out, and substituted in its stead (but not in the same part of the Volume) a more generally useful and prac-

licable problem; which shows the method of deducing the longitude from the transit of the moon's enlightened limb over the meridian of an observer. And, as the operation is remarkably simple and concise, it will be found useful to maritime surveyors, and to all others who are in possession of transit instruments and timekeepers.

With the view of perfecting the Astronomical part of the work, and of leaving nothing undone that could possibly conduce to the advancement of Navigation; a problem is therefore given (page 536), which shows how the longitude is to be deduced from an occultation of a fixed star or planet by the moon, or from an eclipse of the sun. In this problem the *apparently mysterious doctrine of the occultations* is unveiled and laid open to the view of the reader;—familiar terms are substituted for the mystical abstruseness of analytical expressions; and thus the whole of that hitherto occult and highly interesting science is brought down to the standard of ordinary capacities, and reduced to the comprehension of every person who understands the use of the Trigonometrical Tables. And, as the formula which I have laid down renders the calculation extremely simple and free from restrictive cases, there are good grounds for hoping that *the infallible method of finding the longitude by the occultations of the fixed stars &c.*, will soon become as general throughout the Royal Navy as *the Lunar Observations* are at present.

The Volume in question is concluded with a series of useful Problems; the last of which exposes the absurdity of supposing the earth to be at rest, and the heavenly bodies in motion revolving round it.

The Second Volume did not admit of many alterations, and but very few corrections.—At page 611 three new Tables have been introduced; these are of vast importance in problems relating to the longitude:—the principles upon which those Tables are constructed, and the uses for which they are intended, will be found in the Explanatory Articles, between pages 336 and 340 of the First Volume.—To the Tables is subjoined an Appendix, showing the method of finding the ratio which the circumference of the circle bears to its diameter;—the method of deducing the natural sines from the circle;—and, the method of performing the celebrated operation called “Tetragonism,” or the squaring of the circle.—As this new and highly improved Edition is drawn up in such a manner as to combine theory with an extensive practical knowledge of all nautical subjects,—the latter being the result of thirty years' experience in the Royal Naval Service of Her Majesty,—it is therefore particularly adapted to meet the wants and the wishes of

Naval Men, and of all others whose early entrance into the great Maritime World may have prevented their making much progress in the sublime science of Astronomy. Hence, in presenting this Work to the Sons of the Ocean and to the public in general, I feel a degree of confidence that it will prove, on every occasion, to be a *complete Master-Key*, which unfolds, explains, and turns to direct application every elementary expression that has any relation to the sciences of Navigation and Nautical Astronomy.

THOMAS KERIGAN.

Her Majesty's Ship Pique,
January, 1838.

THE FIRST EDITION of this Work was honoured with the immediate patronage of His late Majesty, who held the office of Lord High Admiral at the period of its publication; and it was dedicated, by permission, to the Right Honourable the Lords Commissioners of the Admiralty, who had executed that high office during its progress through the press.

In addition to this testimony of their approbation, their Lordships were pleased to contribute the sum of *one hundred guineas* for ten copies of the work.

The Elder Brethren of the Honourable Trinity Corporation contributed *one hundred pounds* for five copies, and

The Court of Directors of the Honourable East India Company contributed *one hundred guineas* for ten copies.

The Honourable the Commissioners of the Navy, the Honourable the Commissioners for Victualling the Navy, the Directors of Greenwich Hospital, the Committee of Lloyd's, the Royal Naval Club, the British Library, Jersey, and upwards of two hundred Officers of The Royal Navy, and others, likewise honoured the First Edition of the work with their names as Subscribers.

CONTENTS.

DESCRIPTION AND USE OF THE TABLES; WITH THE PRINCIPLES UPON WHICH THEY ARE ESTABLISHED.

<i>Table.</i>		<i>Page.</i>
I.	To convert longitude, or degrees, into time, and conversely Principles, and mode of computing ditto, in page 2.	1
II.	Depression, or dip of the horizon Principles, and mode of computing ditto, between pages 3 and 5.	3
III.	Dip of the horizon at different distances from the observer Principles, and mode of computing ditto, between pages 6 and 8.	6
IV.	Augmentation of the moon's semidiameter, including the principles upon which it is determined	8
V.	Contraction of the semidiameters of the sun and moon	11
VI.	Parallax of the planets in altitude	12
VII.	Parallax of the sun in altitude, and mode of computation	13
VIII.	Mean astronomical refraction, and mode of computation	13
IX.	Correction of the mean astronomical refraction	15
X.	To find the latitude by an altitude of the north polar star Principles, and mode of computing ditto, between pages 17 and 19.	17
XI.	Correction of the latitude deduced from the preceding table	19
XII.	The <i>mean</i> sun's approximate right ascension	21
XIII.	Equations to equal altitudes, and mode of computation	21
XIV.	Equations to ditto, Part the Second, and ditto	22
XV.	To reduce the sun's longitude, right ascension, and declination, and also the equation of time, as given in the Nautical Almanac, to any given time under a known meridian	24
XVI.	To reduce the moon's longitude, latitude, semidiameter, and horizontal parallax, as given in the Nautical Almanac, to any given time under a known meridian A familiar method of computing the parallax of the moon, and of determining her distance from the earth, between pages 29 and 35.	27
XVII.	Equation of the second difference of the moon's place	35
XVIII.	Correction of the moon's apparent altitude, and mode of computation....	38
XIX.	To reduce the true central altitudes of the sun, moon, stars, and planets, to their apparent altitudes.....	40
XX.	Auxiliary angles, and method of computation	42
XXI.	Correction of the auxiliary angle when the moon's distance from a planet is observed	45
XXII.	Error arising from a deviation of <i>one minute</i> in the parallelism of the surfaces of the central mirror of the circular instrument	46
XXIII.	Error arising from an inclination of the time of collimation to the plane of the sextant, &c. &c.	47

<i>Table.</i>	<i>Page.</i>
XXIV. Logarithmic difference, and how determined.....	49
XXV. Correction of the log. difference for the sun, or a fixed star	51
XXVI. Correction of the log. difference when a planet is observed	52
XXVII. Natural versed sines, and natural sines	53
The principles and method of computing ditto, together with the natural tangents and natural secants, between pages 53 and 56.	
To deduce the natural sines from the circle,—See the Appendix to Volume II. between pages 662 and 668.	
XXVIII. Logarithms of numbers, and how determined	62
XXIX. Proportional logarithms, and mode of computation	75
XXX. Logarithmic half elapsed time, expressed by the log. co-secants	84
XXXI. Logarithmic middle time, and how determined.....	86
XXXII. Logarithmic rising,— <i>modes</i> of computation in page 88.....	87
XXXIII. To reduce points of the compass to degrees, and conversely	89
XXXIV. Logarithmic sines, tangents, &c., to every point &c. of the compass.....	89
XXXV. Logarithmic secants to every second in the semicircle	90
Mode of determining the log. secants and co-secants, page 93.	
XXXVI. Logarithmic sines to every second in the semicircle.....	93
The log. sines deduced from the natural sines, page 96.	
XXXVII. Logarithmic tangents to every second in the semicircle	97
To find the arch corresponding to a given log. tangent by means of a table of <i>log. sines</i> ,—See <i>Remark</i> , page 98.	
To find the arch corresponding to a given log. tangent by means of a Table of <i>Natural Sines</i> ,—See the operation in page 99.	
For the manner of computing the logarithmic tangents;—see the <i>Note</i> , page 99; and the second <i>Example</i> in page 100.	
XXXVIII. To reduce the time of the moon's passage over the meridian of Greenwich, to the time of her passage over any other meridian	100
XXXIX. Correction to be applied to the time of the moon's transit, in finding the time of high water at any given place.....	102
XL. Reduction of the moon's horizontal parallax on account of the spheroidal figure of the earth, and mode of computation.....	104
On this subject, see Article 83, page 339.	
XLI. Reduction of latitude on account of the spheroidal figure of the earth	105
On this subject, see Article 82, page 337.	
XLII. A general traverse table, or difference of latitude and departure.....	106
For the method of computing ditto, see <i>Remark</i> , page 113.	
XLIII. Meridional parts, and method of computation	113
XLIV. Mean right ascensions and declinations of the principal fixed stars	114
To compute the distances between the stars,—See <i>Remark</i> , page 116.	
XLV. Acceleration of the fixed stars, or to reduce sidereal time to mean solar time	117
XLVI. To reduce mean solar time into sidereal time.....	119
See the undernamed Articles relative to mean solar time, and sidereal time, viz., 9, 10, 11, and 12; 31 and 32; 39, 40, and 41, between pages 305 and 306; 309 and 314.	
XLVII. Time from noon when the sun's centre is in the prime vertical	119
The mode of computing ditto, page 120.	
XLVIII. Altitude of a celestial object when its centre is in the prime vertical	120
The method of determining ditto, page 121.	
XLIX. Amplitude of a celestial object, and mode of computation	122
L. To find the times of the rising and setting of a celestial object	123
LL & LII. For computing the meridional altitude of a celestial object.....	138
See the Rule, for determining these Tables, in page 142.	

<i>Table.</i>	<i>Page.</i>
LIII. Number of miles in a degree of longitude at every degree of latitude.....	144
LIV. Proportional miles for constructing Mercator's charts	145
LV. To find the distance of terrestrial objects at sea	147
For the manner of computing ditto, see page 149.	
LVI. To reduce French degrees into English degrees, and conversely	150
Respecting the principles &c., see page 151.	
LVII. To gauge, or find the content of all circular-headed casks	152
LVIII. Latitudes and longitudes of the principal ports &c. in the known world ..	154
A. Equation of second differences, for correcting the approximate mean time at Greenwich; with principles and mode of computation ..,.....	336
B. To reduce the geographical to the geocentrical latitude	337
C. Logarithmical radius of the earth, for reducing the moon's horizontal parallax to the oblate figure of the earth.....	338
A CONCISE SYSTEM OF DECIMAL ARITHMETIC	156
Addition, and subtraction of decimals	157
Multiplication, and division of decimals	158
Reduction of decimals	160
The rule of proportion, or <i>rule of three</i> , by decimals	163
Proportion and properties of numbers.....	165
PLANE TRIGONOMETRY.....	168
Definitions and principles, between pages 168 and 170.	
SOLUTION OF PROBLEMS IN RIGHT ANGLED TRIGONOMETRY.	
I. Given the angles and the hypotenuse, to find the base and the perpendicular	171
II. Given the angles and one side, to find the hypotenuse and the other side	172
III. Given the hypotenuse and one side, to find the angles and the other side	174
To find the side independently of the angles,—See <i>Remark</i> , page 175.	
IV. Given the base and the perpendicular, to find the angles and the hypotenuse	175
To find the hypotenuse independently of the angles,—See <i>Remark</i> , page 176	
SOLUTION OF PROBLEMS IN OBLIQUE ANGLED TRIGONOMETRY.	
I. Given the angles and one side, to find the other two sides	177
II. Given two sides and an angle opposite to one of them, to find the other angles and the third side	178
III. Given two sides and the included angle, to find the other two angles and the third side	179
IV. Given the three sides of a plane triangle, to find the angles	180
SPHERICAL TRIGONOMETRY	181
Definitions and principles, between pages 181 and 183.	
The five circular parts in right angled spherics.....	182
The middle part, and the extremes conjunct and disjunct	183
SOLUTION OF PROBLEMS IN RIGHT ANGLED SPHERICS.	
I. Given the hypotenuse and one leg; to find the angles and the other leg..	184
II. Given the hypotenuse and one angle; to find the other angle and the two legs.....	185
III. Given a leg and its opposite angle; to find the other angle, the other leg, and the hypotenuse	187
IV. Given a leg and its adjacent angle; to find the other angle, the other leg, and the hypotenuse	188
V. Given the two legs; to find the angles and the hypotenuse	190
VI. Given the two angles; to find the hypotenuse and the two legs.....	191

	<i>Page.</i>
SOLUTION OF PROBLEMS IN QUADRANTAL SPHERICS.	
I. Given a quadrantal side, its opposite angle, and an adjacent angle ; to find the remaining angle and the other two sides	193
II. Given the quadrantal side and the other two sides; to find the three angles	195
SOLUTION OF PROBLEMS IN OBLIQUE ANGLED SPHERICS.	
I. Given two sides and an angle opposite to one of them ; to find the other two angles and the third side	198
II. Given two angles and a side opposite to one of them ; to find the remaining angle and the other two sides	200
III. Given two sides and the contained angle ; to find the other two angles and the third side	202
To find the <i>third side</i> independently of the <i>angles</i> ,— See <i>Remark 1</i> , in page 203, and <i>Remark 2</i> , in page 204.	
IV. Given two angles and the comprehended side ; to find the remaining angle and the other two sides	205
To find the <i>remaining angle</i> , independently of the <i>sides</i> ,— See <i>Remark 1</i> , in page 206, and <i>Remark 2</i> , in page 207.	
V. Given the three sides of a spherical triangle; to find the angles	207
Another method of finding ditto,— See <i>Rule</i> , page 208.	
VI. Given the three angles of a spherical triangle; to find the sides	209
Another method of finding ditto,— See <i>Rule</i> , page 210.	
NAVIGATION	211
Definitions, and principles, between pages 211 and 214.	
INTRODUCTORY PROBLEMS IN NAVIGATION.	
I. Given the latitudes of two places ; to find the difference of latitude	214
II. Given the latitude left and the difference of latitude ; to find the latitude in	215
III. Given the longitudes of two places ; to find the difference of longitude	215
IV. Given the longitude left and the difference of longitude ; to find the longitude in	216
See Remarks on the preceding Problems	
217	
SOLUTION OF PROBLEMS IN PARALLEL SAILING	
217	
I. Given the difference of longitude between two places, in the <i>same parallel of latitude</i> ; to find their distance	218
II. Given the distance between two places, in the <i>same parallel of latitude</i> ; to find the difference of longitude between those places	219
III. Given the difference of longitude and the distance between two places, in the <i>same parallel of latitude</i> ; to find the latitude of that parallel	220
SOLUTION OF PROBLEMS IN MIDDLE LATITUDE SAILING	
221	
Definitions, principles, and analogies, between pages 221 and 223.	
I. Given the latitudes and longitudes of two places; to find the course and distance between them	223
II. Given the latitude and longitude of the place sailed from, the course and distance; to find the latitude and longitude of the place come to	224
III. Given both latitudes and the course; to find the distance and the longitude in	226
IV. Given the latitude and longitude of the place sailed from, the course and the departure ; to find the distance sailed, and the latitude and longitude of the place come to	227
V. Given both latitudes and the distance sailed; to find the course and difference of longitude	229

CONTENTS.

XXV

<i>Problem.</i>	<i>Page.</i>
VI. Given one latitude, distance, and departure; to find the other latitude, the course, and the difference of longitude	230
VII. Given both latitudes and the departure; to find the course, distance, and difference of longitude	232
VIII. Given one latitude, departure, and difference of longitude; to find the other latitude, the course, and the distance	233
IX. Given the distance, difference of longitude, and middle latitude; to find the course, and both latitudes	235
MERCATOR'S SAILING	236
Definitions, principles, and analogies, between pages 236 and 238.	
PROBLEMS IN MERCATOR'S SAILING.	
I. Given the latitudes and longitudes of two places; to find the course, and the distance between them	238
II. Given the latitude and longitude of the place sailed from, the course, and distance; to find the latitude and longitude of the place come to	239
III. Given the latitude and longitude of the place sailed from, the course, and the departure; to find the distance sailed, and the latitude and longitude of the place come to	240
IV. Given both latitudes and the course; to find the distance, and the longitude in	242
V. Given both latitudes and the distance; to find the course, and the longitude come to	243
VI. Given one latitude, distance, and departure; to find the other latitude, the course, and the difference of longitude	244
VII. Given both latitudes, and the departure; to find the course, distance, and difference of longitude	246
VIII. Given one latitude, course, and difference of longitude; to find the distance, and the other latitude	248
IX. To find the course, distance, difference of latitude, and difference of longitude, made good upon compound courses; and also the bearing and distance from the ship to her intended port	249
To make out a <i>day's work</i> at sea, by inspection	251
Solution of <i>cases</i> in oblique sailing	255
Solution of <i>cases</i> in windward sailing	262
Solution of <i>cases</i> in current sailing	266
Solution of problems relative to the <i>errors</i> of the log. sine and the half-minute glass	272
Solution of a very useful problem in <i>great circle sailing</i>	276
Spherical route from the Cape of Good Hope to the north point of King's Island, at Bass' Strait	285
Spherical route from Port Jackson, in New South Wales, to Valparaiso, on the coast of Chili	294
A compendium of <i>practical navigation</i> , for the use of such as are unacquainted with the elements of trigonometry	669
To make out a <i>day's work at sea</i> , by calculation	701
Of the log.-book, and particulars relating thereto	706
<i>Correction of the course steered</i> on account of the variation of the compass	708
<i>Correction of the course steered</i> on account of lee-way	708
Allowance for lee-way under particular circumstances	709
Setting and drift of currents, and <i>heave</i> of the sea	710
A general rule for correcting the dead reckoning	711
Form of a log-book	713

	<i>Page.</i>
Of the measure of a knot on the log. line, and the spherical figure of the earth	716
How to find the true length of a <i>knot</i> on the log. line	720
Errors of the log. line and half-minute glass, between pages 272 and 276.	
To find the time of high water at any known place	163
To find the course steered by a ship seen at a distance	636

SOLUTION OF PROBLEMS IN NAUTICAL ASTRONOMY.

General Definitions.

1. The phenomenon of day and night	296
2. The celestial poles	296
3. The equator	296
4. The equinoctial	297
5. The meridian of any place	297
6. The first meridian	297
7. The zenith of any place	297
8. A vertical circle	297
9. The horizon	297
10. The ecliptic	298
11. The equinoctial points	298
12. The declination of a celestial object	299
13. The tropics of Cancer and Capricorn	299
14. Polar circles	299
15. The right ascension of a heavenly body	299
16. The right ascension of the meridian	299
17. The culminating point of a star or planet	299
18. The geocentric latitudes and longitudes of the planets	299
19. The aphelion	299
20. The perihelion	300
21. The line of the apices	300
22. The radius vector of the earth	300
23. The occultation of a star or planet	300
24. The transit of the first point of Aries	300
25. The nodes	300
26. The aspect of the stars or planets	300

Explanatory Articles.

1. The division of time	301
2. The portion of time called a <i>year</i>	301
3. A solar or tropical year	301
4. A sidereal year	301
5. The recession of the equinoctial points	301
6. The number of revolutions that the earth takes to complete the solar and the sidereal years	304
7. An apparent solar day	305
8. A natural day	305
9. A mean solar day	305
10. A sidereal day	305
11. Arc which the earth must describe <i>beyond one diurnal revolution on its axis</i> to bring the sun upon the same meridian	305
12. Anticipation of the fixed stars	306
13. A lunar day, and arc of excess which the earth must describe to bring the moon upon the same meridian	306
14. A synodical lunation	306
15. A periodical lunation	306

Explanatory Articles.

	<i>Page.</i>
16. The synodical lunation longer than the periodical	307
17. A lunar year, and the epact	307
18. The lunar cycle, or golden number	307
19. The solar cycle	307
20. The dominical or sunday letter	307
21. The epact, or moon's age, on the first of January	308
22. The Roman indiction	308
23. The Julian period	308
24. The grand celestial period, or the Platonic day	308
25. To find the solar cycle	308
26. To find the golden number	308
27. To find the Julian period	308
28. To find the epact	308
29. To find the dominical letter	308
30. Apparent time, or the sun's meridian distance	309
31. Mean time	309
32. Sidereal time, or right ascension of the meridian	309
33. The equation of time	309
34. The length of the solar days always varying	310
35. Illustration of apparent time, mean time, and the equation of time, by a diagram	310
36. Signs of the equation of time to be marked contrary in relation to mean time	313
37. Mean sun, and sidereal time	313
38. Mean sun synonymous to the first point of Aries	314
39. Mean time converted into sidereal time, and conversely	314
40. The first "Table of Equivalents" in the Nautical Almanac	314
41. The second "Table of Equivalents" in ditto	314
42. The mean sun's right ascension, or the "sidereal time," in page II. of the month in the Ephemeris, and the "mean time of transit of the first point of Aries" may be deduced from each other	315
43. Equation of time expressed by the difference between the <i>mean</i> sun's right ascension and the true sun's right ascension	315
44. "Sidereal time," page II. of the month in the Ephemeris, called the <i>mean</i> sun's right ascension, and why	315
45, 46. The adjustment and use of nautical instruments	316
47. To set the index-glass perpendicular to the plane of the sextant, &c.	317
48. To set the horizon-glass perpendicular to the plane of the sextant, &c. ..	317
49. To set the horizon-glass by means of the sun's image	318
50. To set the horizon-glass parallel to the index-glass	318
51. To set the horizon-glass by means of the sun's image	319
52. To make the line of collimation parallel to the plane of the sextant	320
53. To make the line of collimation &c., by another method	320
54. Excellence of <i>Berge's</i> sextants	321
55. Relative to the index error of a sextant	321
56. The manner of finding the index error of a sextant	322
57. A more correct method of finding ditto, and how to read off to the <i>right-</i> <i>hand of zero</i> , or off the arch	322
58. On the elasticity or spring of the index bar	323
59. The true method of finding the index error of a sextant, so as to guard against the errors arising from the flexibility and friction of the index- bar	324
60. Relative to celestial observations	326
61. To take the sun's altitude at sea	327
62. To take the moon's altitude at sea	327

<i>Explanatory Articles.</i>	<i>Page.</i>
63. To take the altitude of a star at sea	327
64. To take the altitude of a planet at sea	328
65. To take the altitude of a celestial object on shore	328
66. Relative to artificial horizons	328
67. On the use of the artificial horizon	329
68. The necessity of using the inverting telescope	329
69. Rays of light proceeding from the sun &c., physically parallel	330
70. An artificial horizon obviates horizontal depression	330
71. The principles upon which the artificial horizon is established	330
72. Altitudes beyond 60° cannot be taken in an artificial horizon	331
73. <i>The same side of the artificial horizon to be kept next to the observer</i>	331
74. Altitudes may be taken by reflection from a basin of water, &c.	332
75. <i>Make-shift</i> horizons are always <i>erroneous</i>	332
76. To observe the distance between the sun and moon	332
77. To observe the distance between the moon and a star	333
78. To observe the distance between the moon and a planet	334
79. The method of taking a complete set of lunar observations	335
80. Relative to the new Tables and the equation of second differences for correcting the Greenwich time	335, 336
81. To compute the equation of second differences in Table A.	337
82. Table B., or to reduce the geographical to the geocentric latitude	337
The method of computing the equations in Table B.	338
83. Table C., or the logarithmical radius of the earth, for reducing the moon's horizontal parallax	339
The method of determining the logarithms in Table C.	340
84. Conclusion of the Explanatory Articles	341

PROMISCUOUS, OR UNCLASSED ASTRONOMICAL ELEMENTS.

Depression or dip of the horizon	3
Dip of the horizon at different distances from the observer	6
Augmentation of the moon's semidiameter	8
Contraction of the semidiameters of the sun and moon	11
Refraction of the heavenly bodies	13
Receding and advancing of the sun with respect to the poles	23
Parallax of the heavenly bodies	30
The moon's parallax readily determined	31
The sun's parallax determined by the transit of Venus	33
Equation of the second difference of the moon's place	35
Correction of the moon's apparent altitude	38
The line of collimation	47
Ditto, defined in the first paragraph, page 320.	
Retardation of the moon's passage over the meridian	100
Ditto, explained in Article 13, page 306.	
Ratio of the attractive forces of the moon and sun in raising the tides ..	102
The moon's equatorial horizontal parallax affected by the oblate figure of the earth (See Article 83, page 339)	104
The latitude deduced from spherical principles always greater than the truth (See Article 82, page 337)	105
Acceleration of the fixed stars (See second paragraph, page 306)	117

Note.—The Definitions of the remaining Astronomical Elements will be found in the Explanatory Articles between pages 301 and 340, or prefixed to the Solution of the Practical Problems in Nautical Astronomy, between pages 296 and 316.

INTRODUCTORY PROBLEMS IN NAUTICAL ASTRONOMY.

I. To convert longitude or parts of the equator into time	341
II. To convert time into longitude	342

<i>Problem.</i>	<i>Page.</i>
III. To reduce the mean time at any place to the meridian of Greenwich	342
IV. To reduce the mean time at Greenwich to the time under a known meridian	343
V. To reduce the MEAN sun's right ascension to any given meridian, and to any time under that meridian	344
VI. Given the mean time at ship and the longitude, to find the right ascension of the meridian &c.	345
VII. Given the right ascension of the meridian and the longitude, to find the mean time at ship, &c.	347
VIII. To find the mean time of a star's transit over the meridian of any known place	348
IX. To find what stars will be on, or nearest, to the meridian of a ship or place at any given time	349
X. To compute the mean time of the moon's transit over the meridian of Greenwich	350
XI. Given the mean time of the moon's transit over the meridian of Greenwich; to find the mean time of her transit over any other meridian. . . .	352
XII. To compute the mean time of a planet's transit over the meridian of Greenwich	353
XIII. Given the mean time of a planet's transit over the meridian of Greenwich; to find the time of its transit over any other meridian	355
XIV. To reduce the sun's right ascension and declination, the equation of time, and the sun's longitude, as given in the Nautical Almanac; to any given mean time under a known meridian	357
XV. To reduce the moon's semidiameter, horizontal parallax, longitude, and latitude, as given in the Nautical Almanac; to any given mean time under a known meridian	361
XVI. To reduce the moon's right ascension and declination, as given in the Nautical Almanac, to any given mean time under a known meridian . .	364
XVII. To reduce the geocentric right ascension and declination of a planet, as given in the Nautical Almanac, to any give mean time under a known meridian	366
XVIII. Given the observed mean time, <i>per watch</i> , of the sun's transit over the meridian of a ship; to find the correct mean time of transit.	368
XIX. Given the mean time at ship, <i>per watch</i> , and the sun's horary distance from the meridian; to find the correct mean time	369
XX. Given the mean time, <i>per watch</i> , and the horary distance of the moon, a fixed star, or a planet, from the meridian; to find the correct mean time	370
XXI. Given the mean time; to find the sun's horary distance from the meridian	372
XXII. Given the mean time; to find the horary distance of the moon, a fixed star, or a planet, from the meridian	373
XXIII. Given the observed altitude of the lower or upper limb of the sun; to find the true altitude of its centre	374
XXIV. Given the observed altitude of the upper or lower limb of the moon; to find her true central altitude	376
XXV. Given the observed altitude of a planet; to find its true altitude	377
XXVI. Given the observed altitude of a fixed star; to find its true altitude.	378
XXVII. To deduce the true altitude of a celestial object from its <i>double</i> altitude observed by means of an artificial horizon	379
XXVIII. To find the obliquity of the ecliptic	381
XXIX. Given the latitude of a place and the meridional altitude of the sun; to find his declination.	382
XXX. Given the mean time and the central distance between the moon and sun, a fixed star, or a planet; to find the mean time at Greenwich and the longitude	383

PROBLEMS RELATIVE TO THE LATITUDE.

I.	Given the sun's meridional altitude; to find the latitude of the place of observation	386
II.	Given the moon's meridional altitude; to find the latitude.....	387
III.	Given the meridional altitude of a planet; to find the latitude	390
IV.	Given the meridional altitude of a fixed star; to find the latitude.....	391
V.	Given the meridional altitude of a celestial object observed below the pole; to find the latitude of the place of observation.....	392
VI.	Given the altitude of the north polar star, taken at any hour of the night, to find the latitude of the place of observation	394
VII.	To find the latitude by <i>double altitudes of the sun</i>	398
VIII.	To find the latitude by the altitudes of two known fixed stars, observed at any time of the night.....	405
IX.	To find the latitude by an altitude of the sun's limb observed near the meridian	413
X.	To find the latitude by an altitude of the moon's lower or upper limb observed near the meridian	418
XI.	To deduce the latitude from an altitude of a planet observed near the meridian	419
XII.	To deduce the latitude from the altitude of a celestial object observed near the meridian below the pole	421
	Captain William Fitzwilliam Owen's general Problem for finding the latitude	423
XIII.	Given the interval of time between the rising or setting of the sun's upper and lower limbs; to find the latitude	424

PROBLEMS RELATIVE TO MEAN TIME, &c. 426

	A general Rule for finding the equation of time to an extreme degree of astronomical exactness	427
I.	Given the latitude and longitude of a ship, and the observed altitude of the sun, or of any other celestial object; to find its horary distance from the meridian, and the mean time of observation.....	428
	Method I. of computing the horary distance of a celestial object from the meridian	430
	Method II. of computing the horary distance, &c. &c.	431
	Method III. of computing the horary distance, &c. &c.	432
	Method IV. of computing the horary distance, &c. &c.	433
	Method V. of computing the horary distance, &c. &c.....	434
II.	Given the latitude and longitude of a ship, and the observed altitude of the sun's limb; to find the mean time of observation and the error of the watch	435
III.	Given the latitude and longitude and the observed altitude of the moon's limb; to find the mean time and the error of the watch	437
IV.	Given the latitude and longitude and the observed altitude of a planet; to find the mean time and the error of the watch	439
V.	Given the latitude and longitude and the observed altitude of a star; to find the mean time and the error of the watch.....	441

PROBLEMS RELATIVE TO FINDING THE ALTITUDES OF THE HEAVENLY BODIES.

	Theory and principles of the computation	443
I.	To find the true and the apparent altitude of the sun's centre	445
II.	To find the true and the apparent altitude of the moon's centre	446
III.	To find the true and the apparent altitude of a planet's centre	448
IV.	To find the true and the apparent altitude of a fixed star	449

CONTENTS.

xxx

Page.

PROBLEMS RELATIVE TO THE LONGITUDE.

	Definitions and general observations	451
	Of the error and the rate of a chronometer	453
I.	To find the error and rate of a chronometer by equal altitudes	455
II.	To find the error and rate of a chronometer by an altitude of the sun....	462
	<i>Remarks</i> , or Rule for finding the position most favourable for observation	467
III.	Given the altitude of the sun; to find the longitude by chronometer	468
IV.	Given the altitude of the moon; to find the longitude by chronometer ..	471
V.	Given the altitude of a planet; to find the longitude by chronometer	473
VI.	Given the altitude of a fixed star; to find the longitude by chronometer ..	475
VII.	To find the longitude by lunar observations	477
	Principles upon which the computation is founded	479
	Method I. of reducing the apparent to the true central distance	481
	Method II. of reducing ditto	483
	Method III. of reducing ditto	485
	Method IV. of reducing ditto	486
	Method V. of reducing ditto.....	487
	Method VI. of reducing ditto	488
	Method VII. of reducing ditto.....	489
	Method VIII. of reducing ditto	491
	Method IX. of reducing ditto	492
	Method X. of reducing ditto.....	493
	Method XI. of reducing ditto	494
	Method XII. of reducing ditto.....	495
	General remarks on the lunar observations, between pages 496 and 500.	
	Remark on <i>impossible</i> spherical triangles	500
IX.	Given the latitude and longitude by <i>account</i> ; the observed distance between the moon and sun, a fixed star, or a planet; to find the true longitude	501
	<i>Remarks</i> relative to the moon's distance from Venus	517
X.	To deduce the longitude from a lunar distance, when the altitudes are determined by calculation.....	519
XI.	To determine both latitude and longitude from the same set of lunar observations.....	525
XII.	To find the longitude by the eclipses of Jupiter's satellites	529
XIII.	To find the longitude by an eclipse of the moon	531
XIV.	To deduce the longitude from the transit of the moon's limb.....	532
XV.	To deduce the longitude from an occultation of a fixed star	536
XVI.	To deduce ditto from an eclipse of the sun.....	557
	<i>Remarks</i> relative to the external and internal contacts of the planets	561
	Example of an <i>occultation</i> from the Nautical Almanac.....	563

PROBLEMS RELATIVE TO THE VARIATION OF THE COMPASS.

	Definitions &c.	565
I.	To find the true amplitude and the variation.....	566
II.	To find the true azimuth and the variation	569
III.	To find the variation of the compass by a circumpolar star	573
IV.	To find the variation by the transit of a celestial object	574
V.	To reduce the true course to the magnet or compass course	576
VI.	To reduce the magnetic course to the true course.....	577
	Description of an improved azimuth compass-card	578

PROBLEMS RELATIVE TO FINDING THE LATITUDES AND LONGITUDES &c. OF THE HEAVENLY BODIES.

I.	Given the right ascension and declination of a celestial object; to find its latitude and longitude	580
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<i>Problem.</i>	<i>Page.</i>
II. Given the latitude and longitude of a celestial object; to find its right ascension and declination	584
III. Given the latitudes and longitudes of the moon and sun, or a fixed star; to find the true central distance between them	588
PROBLEMS RELATIVE TO THE RISING AND SETTING OF THE CELESTIAL BODIES, &c. &c.	
I. To find the mean times of the sun's rising and setting	593
II. To find the mean times of the moon's rising and setting	596
III. To find the beginning and the ending of twilight.....	601
IV. To find the time of the shortest twilight	603
V. To find when real night or darkness ceases, &c.	604
VI. To find the interval between the times of the sun's limbs touching the horizon of a given place	605
PROBLEMS IN GNOMONICS OR DIALLING.	
I. To find the angles which the hour-lines make with the sub-style or meridian line of a horizontal sun-dial	607
II. To find the angles on the plane of an erect dial	610
PROBLEMS IN MENSURATION.	
I. To find the height of an accessible object	613
II. To find the observer's horizontal distance from an object	614
III. To find the height of an inaccessible object	615
IV. To find the distance of an object which can neither be receded from nor approached in a perpendicular line of direction	616
V. To find the distance between the inaccessible objects	618
VI. To find the distance between any point and three given objects	620
VII. To find the distance between a certain point and three objects	623
VIII. To find the distance between a certain point and three objects in the same line	625
IX. To find the distance of the visible horizon.....	627
X. To find the allowance for the curvature of the earth	628
XI. To reduce an elevated base line to the level of the sea.....	630
XII. To find the height and distance of a mountain	633
XIII. To find the height of a mountain by means of two barometers, &c. &c. ..	634
XIV. To find the distance of an object by observing the flash of a gun, &c. &c...	635
XV. To find the course steered by a ship seen at sea	636
PROBLEMS IN GUNNERY.	
I. Given the diameter of an iron ball; to find its weight.....	640
II. To find the diameter of an iron ball	641
III. Given the diameter of a leaden ball; to find its weight	641
IV. To find the diameter of a leaden ball	642
V. Given the diameters of an iron shell; to find its weight	643
VI. To find how much powder will fill a shell	643
VII. To find the size of a shell to contain a given weight of powder.....	644
VIII. To find how much powder will fill a rectangular box	644
IX. To find the size of a cubical box to contain a given weight of powder....	645
X. To find how much powder will fill a cylinder	645
XI. To find what length of a cylinder will be filled with a given weight of powder	646
XII. To find the number of balls in a triangular pile	646
XIII. To find the number of balls in a square pile	647
XIV. To find the number of balls in a rectangular pile.....	648

CONTENTS.

xxxiii

Problem.	Page.
XV. To find the number of balls in an incomplete triangular pile.....	648
XVI. To find the number of balls in an incomplete square pile	649
XVII. To find the number of balls in an incomplete rectangular pile	650
XVIII. To find the velocity of any shot or shell.....	651
XIX. To find the terminal velocity of a shot or shell	652
XX. To find the height from which a body must fall in <i>vacuo</i> to acquire a given velocity	653
XXI. To find the greatest range and the elevation to produce that range	655
XXII. Given the range at one elevation, to find the range at another elevation..	657
XXIII. Given the elevation for one range; to find the elevation for another range	657
XXIV. Given the charge for one range; to find the charge for another range	658
XXV. Given the range for one charge; to find the range for another charge	659
XXVI. Given the range and the elevation; to find the impetus	660
XXVII. Given the elevation and the range; to find the time of flight.....	660
XXVIII. Given the range and the elevation; to find the greatest altitude	662
XXIX. Given the inclination of the plane, &c.; to find the range	662
XXX. Given the inclination of the plane, &c.; to find the impetus	663
XXXI. To find the velocity of a shell when projected with a given charge of powder	664
XXXII. To find the impetus, velocity, and charge of powder	666
XXXIII. Given the inclination of the plane, &c.; to find the charge of powder....	667
XXXIV. Given the inclination of the plane, &c.; to find the time of flight.....	668
XXXV. Given the impetus, &c.; to find the horizontal range	669
XXXVI. Given the impetus, &c.; to find the time of flight on a horizontal plane..	670
XXXVII. Given the inclination of the plane, &c.; to find the elevation.....	671
XXXVIII. Given the time of flight, &c.; to find the elevation	674
XXXIX. Given the elevation, &c.; to find the horizontal range	674
XL. Given the elevation, &c.; to find the time of flight	675
XLI. Given the time of flight; to find the length of the fuze	676
XLII. To find the time that a red-hot ball will take to cool	677

PROBLEMS IN GAUGING.

I. To reduce the old wine measure into imperial measure	680
II. To reduce imperial measure into the old wine measure	680
III. To reduce the old ale measure into imperial measure	681
IV. To reduce imperial measure into the old ale measure	681
V. To find the contents of a cask in ale, wine, and imperial measure	682
VI. Given the depth of <i>the ullage</i> ; to find the quantity in the cask	683
VII. Given the depth of <i>the ullage</i> in a standing cask; to find the quantity of liquor in the cask	686

PROBLEMS IN PRACTICAL NAVIGATION.

I. To reduce the sun's declination to the time of mean noon.....	689
II. Given the sun's meridian altitude; to find the latitude	691
III. Given the difference of longitude between two places; to find their distance	693
IV. Given the distance between two places; to find their difference of longitude	694
V. Given the latitudes and longitudes of two places; to find the course and distance.....	694
VI. Given the latitude and longitude sailed from; to find those come to.....	695
VII. Given the latitudes and course; to find the distance, and the longitude come to.....	698
VIII. To make out a day's work at sea	700
Of the log-book	706
Of the measure of a knot on the log-line	716

MISCELLANEOUS PROBLEMS.

I.	Given the base and height of a triangle; to find its area	720
II.	Given two sides and the contained angle; to find the area	721
III.	Given the three sides; to find the area	722
IV.	Given the diameter of a circle; to find its circumference and conversely ..	722
V.	Given the diameter or circumference of the earth; to find the area of its surface	723
VI.	To find the length of any arc of a circle	724
VII.	Given the length of an arc, and number of degrees; to find the radius	725
VIII.	Given the length of an arc, and the radius; to find the number of degrees in it	725
IX.	To find the length of a pendulum for vibrating seconds at the equator and poles	726
X.	To find the length of a pendulum for vibrating seconds between the equator and poles	728
XI.	To find the length of a pendulum for vibrating half-seconds	729
XII.	Given the time of descent of a heavy body; to find the height from which it fell	729
XIII.	Given the circumference and length of a cable; to find its weight	730
XIV.	Given the diameter of a circle; to find its area	731
XV.	Given the area of a circle; to find its diameter	731
XVI.	Given the diameter of a circle; to find the side of a square equal in area .	732
XVII.	Given the diameter of a circle; to find the side of an inscribed square	732
XVIII.	Given both diameters of an ellipsis; to find its area	733
XIX.	Given both diameters of an ellipsis; to find that of a circle equal in area	733
XX.	Given both diameters of an ellipsis; to find its circumference	734
XXI.	Given the diameter of a sphere; to find its solidity	734
XXII.	Given the diameter of the earth; to find the height to see one-third of its surface	735
XXIII.	To find the height of the earth's atmosphere	736
XXIV.	Given the earth's semidiameter, and the sun's parallax; to find their distance	738
XXV.	Given the sun's distance from the earth, and apparent semidiameter; to find his true diameter	738
XXVI.	Given the diameters of the earth and sun; to find the ratio of their magnitudes	739
XXVII.	Given the earth's circumference; to find the rate of motion of the equator	740
XXVIII.	To find the rate of motion at any parallel of latitude	740
XXIX.	To find the length of the solar year	741
XXX.	To find the rate of the earth's annual motion in the ecliptic	742
XXXI.	Given the moon's distance and apparent semidiameter; to find her true diameter	742
XXXII.	Given the diameters of the earth and moon; to find the ratio of their magnitudes	743
XXXIII.	To find how much larger the earth appears at the moon, than the moon at the earth	743
XXXIV.	To find the height of a mountain in the moon	744
XXXV.	To find the rate of the moon's revolution	745
XXXVI.	To find the mean distance of a planet from the sun	745
XXXVII.	To find the proportionate light and heat received by the planets from the sun	746
XXXVIII.	Given the apparent diameter of a planet; to find its true diameter	747
XXXIX.	To find the time in which the sun revolves on its axis	748
XL.	To find in what time the sun <i>could</i> revolve round the earth	749

THE
DESCRIPTION AND USE
OF THE
TABLES ;
WITH THE
PRINCIPLES UPON WHICH THEY HAVE BEEN COMPUTED.

TABLE I.

To convert Longitude, or Degrees, into Time, and conversely.

THIS Table consists of six compartments, each of which is divided into two columns: the left-hand column of each compartment contains the longitude, expressed either in degrees, minutes, or seconds; and the right-hand column the corresponding time, either in hours, minutes, seconds, or thirds. The proper signs, for degrees and time, are placed at the top and bottom of their respective columns in each compartment, with the view of simplifying the use of the Table:—hence it will appear evident that if the longitude be expressed in degrees, the corresponding time will be either in hours or minutes; if it be expressed in minutes, the corresponding time will be either in minutes or seconds; and if it be expressed in seconds, the corresponding time will be expressed either in seconds or thirds. The converse of this takes place in converting time into longitude.

The extreme simplicity of the Table dispenses with the formality of a rule in showing its use, as will obviously appear by attending to the following examples.

Example 1.

Required the time corresponding to $47^{\circ}47'47''$ of longitude?

47 degrees,	time answering to which in the Table is	3 ^h 8 ^m 0 ^s 0 ^t
. 47 minutes,	answering to which is 0. 3. 8. 0
. . 47 seconds,	answering to which is 0. 0. 3. 8

Lon. $47^{\circ}47'47''$, the time corresponding to which is . 3^h 11^m 11^s 8^t

Example 2.

Required the longitude corresponding to the given time $8^h 52^m 28^s$?

8 hours, longitude answering to which in the Table is $120^{\circ} 0' 0''$

. 52 minutes, answering to which is $13.0.0$

. . 28 seconds, answering to which is $0.0.7$

Time $8^h 52^m 28^s$, the longitude corresponding to which is . $133^{\circ} 0' 7''$

Besides the use of this Table in the reduction of longitude into time, and the contrary, it will also be found very convenient in problems relating to the Moon, where it becomes necessary to turn the right ascension of that object into time.

Example.

The right ascension of the Moon is $355^{\circ} 44' 48''$; required the corresponding time ?

355 degrees, time answering to which

in the Table is $23^h 40^m 0^s 0''$

. 44 minutes, answering to which is $0. 2. 56. 0$

. . 48 secs., answering to which is $0. 0. 3. 12$

Right ascension $355^{\circ} 44' 48''$, the time corresponding to

which is $23^h 42^m 59^s 12''$

Since the Earth makes one complete revolution on its axis in the space of 24 hours, it is evident that every part of the equator will describe a great circle of 360 degrees in that time, and, consequently, pass the plane of any given meridian once in every 24 hours; whence it is manifest that any given number of degrees of the equator will bear the same proportion to the great circle of 360 degrees that the corresponding time does to 24 hours; and that any given portion of time will be in the same ratio to 24 hours that its corresponding number of degrees is to 360.

Now since 24 hours are correspondent or equal to 360 degrees, 1 hour must, therefore, be equal to 15 degrees; 1 minute of time equal to 15 minutes of a degree; 1 second of time to 15 seconds of a degree, and so on. And as 1 minute of time is thus evidently equal to 15 minutes or one fourth of a degree, it is very clear that 4 minutes of time are exactly equal to 1 degree; wherefore since degrees and time are similarly divided, we have the following general rule for converting longitude into time, and *vice versa*.

Multiply the given degrees by 4, and the product will be the corresponding time:—observing that seconds multiplied by 4 produce thirds; minutes, so multiplied, produce seconds, and degrees minutes; which, divided by 60, will give hours. The converse of this is evident:—thus,

reduce the hours to minutes; then these minutes, divided by 4, will give degrees; the seconds, so divided, will give minutes, and the thirds, if any, seconds. Hence the principles upon which the Table has been computed. The following examples are given for the purpose of illustrating the above rule.

Example 1.

Required the time corresponding to $36^{\circ}44'32''$?

Given degrees = $36^{\circ}44'32''$

Multiplied by $\underline{4}$

Corresponding time $2^h26^m58^s8^{\frac{1}{2}}$

Example 2.

Required the degrees corresponding to $3^h45^m48^s20^{\frac{1}{2}}$?

Given time = $3^h45^m48^s20^{\frac{1}{2}}$

$\underline{60}$

Divide by $4 \) \ 225.48.20$

Corresponding degs. $56^{\circ}27'5''$

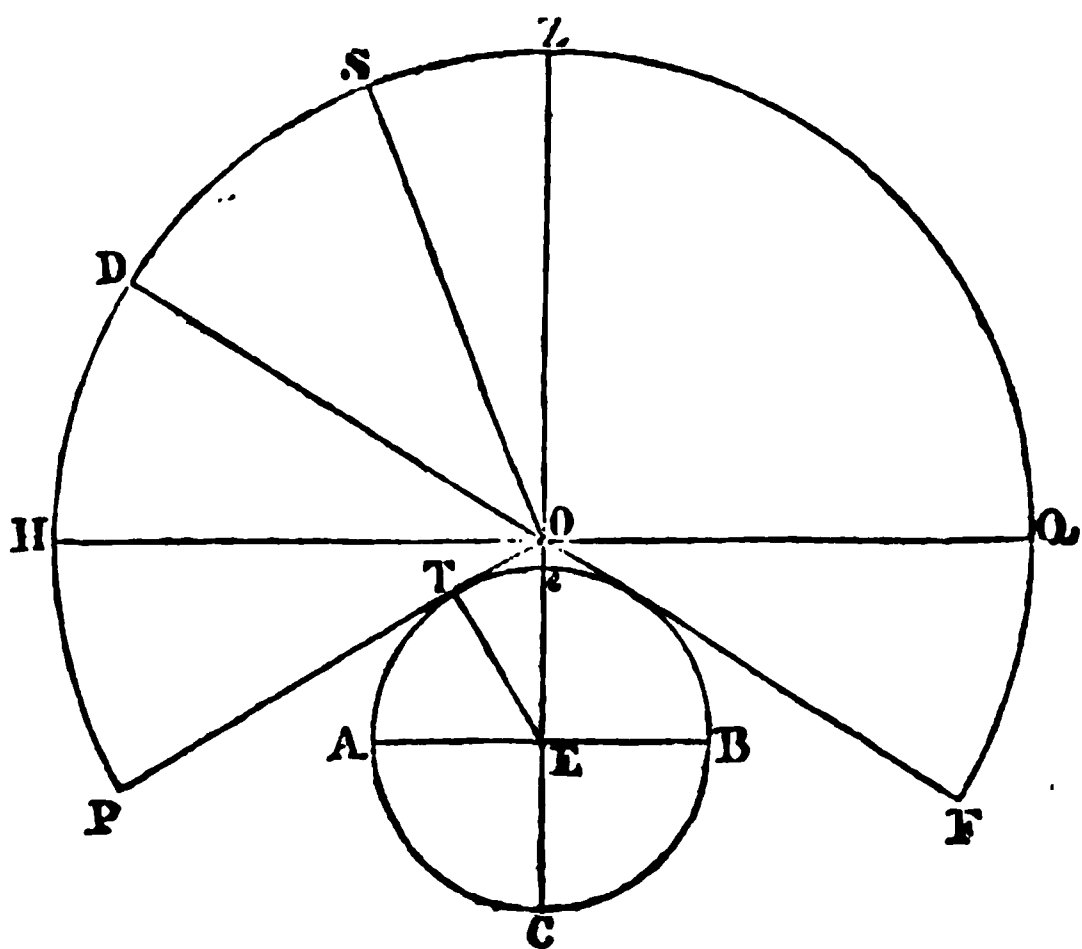
TABLE II.

Depression of the Horizon.

The depression or dip of the horizon is the angle contained between a horizontal line passing through the eye of an observer, and a line joining his eye and the visible horizon.

This Table contains the measure of that angle, which is a correction expressed in minutes and seconds answering to the height of the observer's eye above the horizon; and which being subtracted from the observed central altitude of a celestial object, when the fore observation is used, or added thereto in the back observation, will show its apparent central altitude. The corrections in this Table were deduced from the following considerations, and agreeably to the principles established in the annexed diagram.

Let the small circle $ABCe$ represent the terrestrial globe, and eO the height of the observer's eye above its surface; then HOQ , drawn parallel to a tangent line to the surface at e , will be the true or sensible horizon of the observer at O ; and OP , touching the surface at T , the apparent horizon.



Let S be an object whose altitude is to be taken by a fore observation, by bringing its image in contact with the apparent horizon at P ; then the angle SOP will be the apparent altitude, which is evidently greater than the true altitude SOH by the arc PH , expressed by the angle of horizontal depression POH . But if the altitude of the object S is to be taken by a back observation, then, the observer's back being necessarily turned to the object, his apparent horizon will be in the direction OF , and his whole horizontal plane represented by the line DOF ; in which case his back horizon OD , to which he brings the object S , will be as much elevated above the plane of the true horizon HOQ as the apparent horizon OF will be depressed below it; because, when two straight lines intersect each other, the opposite angles will be equal. (Euclid, Book I., Prop. 15.) In this case it is evident that the arc or apparent altitude SD is too little; and that it must be augmented by the arc $DH =$ the angle of horizontal depression FOQ , in order to obtain the true altitude SH . Hence it is manifest that altitudes taken by the fore observation must be diminished by the angle of horizontal depression, and that in back observations the altitudes must be increased by the value of that angle.

The absolute value of the horizontal depression may be established in the following manner:—From where the apparent horizon OP becomes a tangent to the earth's surface at T (the point of contact where the sky and water seem to meet) let a straight line be drawn to the centre E , and it will be perpendicular to OP (Euclid, Book III., Prop. 18): hence it is obvious that the triangle ETO is right-angled at T . Now, because OT is a straight line making angles from the point O upon the same side of the straight line OE , the two angles EOT and TOH are together equal to the angle EOH (Euclid, Book I., Prop. 13); but the angle EOH is a right angle; therefore the angle of depression TOH is the complement of the angle EOT , or what the latter wants of being a right angle: but the angle TEO is also the complement of the angle EOT (Euclid, Book I., Prop. 32); therefore the angle TEO is equal to the angle of horizontal depression; for magnitudes which coincide with one another, and which exactly fill up the same space, are equal to one another. Then, in the right-angled rectilineal triangle ETO , there are given the perpendicular TE , = the earth's semidiameter, and the hypotenuse EO , = the sum of the earth's semidiameter and the height of the observer's eye, to find the angle TEO = the angle of horizontal depression TOH :—hence the proportion will be, as the hypotenuse EO is to radius, so is the perpendicular TE to the cosine of the angle TEO , which angle has been demonstrated to be equal to the angle of horizontal depression $HO P$. But because very small arcs cannot be strictly determined by cosines, on account of the differences being so very trivial at the beginning of the quadrant as to run several seconds without producing any sensible alteration, and there being no rule for showing

why one second should be preferred to another in a choice of so many, the following method is therefore given as the most eligible for computing the true value of the horizontal depression, and which is deduced from the 36th Prop. of the third Book of Euclid.

Because the apparent horizon OP touches the earth's surface at T , the square of the line OT is equal to the rectangle contained under the two lines CO and eO . Now as the earth's diameter is known to be 41804400 English feet, and admitting the height of the observer's eye eO to be 290 feet above the plane of the horizon; then, by the proposition, the square root of CO , $41804690 \times eO, 290 =$ the line OT , 110105.75 feet; the distance of the visible horizon from the eye of the observer independent of terrestrial refraction.

Then, in the right-angled rectilineal triangle ETO , there are given the perpendicular $ET = 20902200$ feet, the earth's semidiameter, and the base $OT = 110105.75$, to find the angle TEO . Hence,

As the perpendicular $TE = 20902200$ feet,	log. arith. compt. =	2.679808
Is to the radius $90^{\circ}0'0''$	log. sine	10.000000
So is the base $OT = . . . 110105.75$ feet,	log.	5.041810
<hr/>		
To the angle $TEO = . . . 18'7'' =$	log. tang.	7.721618

But it has been shown that the angle TEO , thus found, is equal to the angle HOP ; therefore the true value of the angle of horizontal depression HOP , is $18'7''$. Now, according to Dr. Maskelyne, the horizontal depression is affected by terrestrial refraction, in the proportion of about one-tenth of the whole angle; wherefore, if from the angle of horizontal depression $18'7''$ we take away the one-tenth, viz. $1'49''$, the allowance for terrestrial refraction, there will remain $16'18''$ for the true horizontal depression, answering to 290 feet above the level of the sea. The principles being thus clearly established, it is easy to deduce many simple formulæ therefrom, for the more ready computation of the horizontal depression; of which the following will serve as an example.

To the proportional log. of the height of the eye in feet, (estimated as seconds,) add the constant log. .4236, and half the sum will be the proportional log. of an arc; which being diminished by one-tenth, for terrestrial refraction, will leave the true angle of horizontal depression.

Example.

Let the height of the eye above the level of the sea be 290 feet, required the depression of the horizon corresponding thereto?

Height of the eye 290 feet, esteemed as secs. = $4'.50''$, propor.log. = 1.5710
 Constant log.4236

Sum = 1.9946

Arc = $18'.7''$ Proportional log. .9973
 Deduct one-tenth = 1.49, for terrestrial refraction.

True horizontal depression $16'.18''$, the same as by the direct method.

In using the Table, it may not be unnecessary to remark that it is to be entered with the height of the eye above the level of the sea, in the column marked *Height, &c.*; opposite to which, in the following column, stands the corresponding correction; which is to be subtracted from the observed altitude of a celestial object when taken by the fore observation; but to be added thereto when the back observation is used, as before stated. Thus the dip, answering to 20 feet above the level of the sea, is $4'.17''$

TABLE III.

Dip of the Horizon at different Distances from the Observer.

If a ship be nearer to the land than to the visible horizon when unconfined, and that an observer on board brings the image of a celestial object in contact with the line of separation betwixt the sea and land, the dip of the horizon will then be considerably greater than that given in the preceding Table, and will increase as the distance of the ship from the land diminishes: in this case the ship's distance from the land is to be estimated, with which and the height of the eye above the level of the sea, the angle of depression is to be taken from the present Table. Thus, let the distance of a ship from the land be 1 mile, and the height of the eye above the sea 30 feet; with these elements enter the Table, and in the angle of meeting under the latter and opposite to the former will be found $17'$ which, therefore, is the correction to be applied by subtraction to the observed altitude of a celestial object when the fore observation is used, and *vice versa*.

The corrections in this Table were computed after the following manner; viz.,—

Let the estimated distance of the ship from the land represent the base of a right-angled triangle, and the height of the eye above the level of the sea its perpendicular; then the dip of the horizon will be expressed

by the measure of the angle opposite to the perpendicular : hence, since the base and perpendicular of that triangle are known, we have the following general

Rule.—As the base or ship's distance from the land, is to the radius, so is the perpendicular, or height of the eye above the level of the sea to the tangent of its opposite angle, which being diminished by one-tenth, on account of terrestrial refraction, will leave the correct horizontal dip, as in the subjoined example.

Let the distance of a ship from the land be 1 mile, and the height of the eye above the level of the sea 25 feet, required the corresponding horizontal dip

As distance 1 mile, or 5280 feet,	Logarithm Ar. Comp.=	6.277366
Is to radius	90°, Logarithmic Sine . . .	10.000000
So is height of the eye 25 feet,	Logarithm	1.397940

To Angle	16'. 17"=Log. Tang.=	7.675306
Deduct one-tenth for terrestrial		
refraction	1.37	

True horizontal dip =	14'. 40", or 15' nearly as in the Table.
-------------------------------	--

Remark.—Although a skilful mariner can always estimate the distance of a ship from the shore horizon to a sufficient degree of accuracy for taking out the horizontal dip from the Table, yet since some may be desirous of obtaining the value of that dip independently of the ship's distance from the land, and consequently of the Table, the following rule is given for their guidance in such cases :—

Let two observers, the one being as near the mast head as possible, and the other on deck immediately under, take the sun's altitude at the same instant. Then to the arithmetical complement of the logarithm of the difference of the heights, add the logarithm of their sum, and the logarithmic sine of the difference of the observed altitudes ; the sum, rejecting 10 from the index, will be the log. sine of an arch ; half the sum of which and the difference of the observed altitudes will be the horizontal dip corresponding to the greatest altitude, and half their difference will be that corresponding to the least altitude.

Example.

Admit the height of an observer's eye at the main-topmast head of a ship close in with the land, to be 96 feet, that of another (immediately under) on deck 24 feet ; the altitude of the sun's lower limb found by the former to be $39^{\circ}37'$, and by the latter, taken at the same instant, $39^{\circ}21'$; required the dip of the shore horizon corresponding to each altitude ?

Height of mast head observer 96 feet.

Height of deck observer . 24 do.

Difference of heights . . 72 do., Log. Ar. Comp.=8. 142667

Sum of ditto 120 do. Logarithm . 2. 079181

Difference of altitudes . 16' Log. sine . 7. 667845

Arch = 26'.40" Log. sine 7. 889693

Sum = 42'.40", $\frac{1}{2}$ = 21'.20" = dip to the greatest height

Diff. = 10.40, $\frac{1}{2}$ = 5.20 = dip to the least height.

Note.—When the dip answering to an obstructed horizon is thus carefully determined, the ship's distance from the land may be ascertained to the greatest degree of accuracy by the following rule: *viz.* As the Log. tangent of the horizontal dip of the shore horizon is to the logarithm of the height of the eye at which that dip was determined, so is radius to the true distance.

Thus, in the above example where the horizontal dip has been determined to the corresponding height of the eye and difference of altitudes,

As horizontal dip = 5'.20" Log. tang. ar. compt.=2. 809275

Is to the height of the eye 24 feet, Logarithm . . . 1. 380211

So is radius 90° Logarithmic sine . 10. 000000

To true distance . . 15469.8 feet . Logarithm=4. 189486

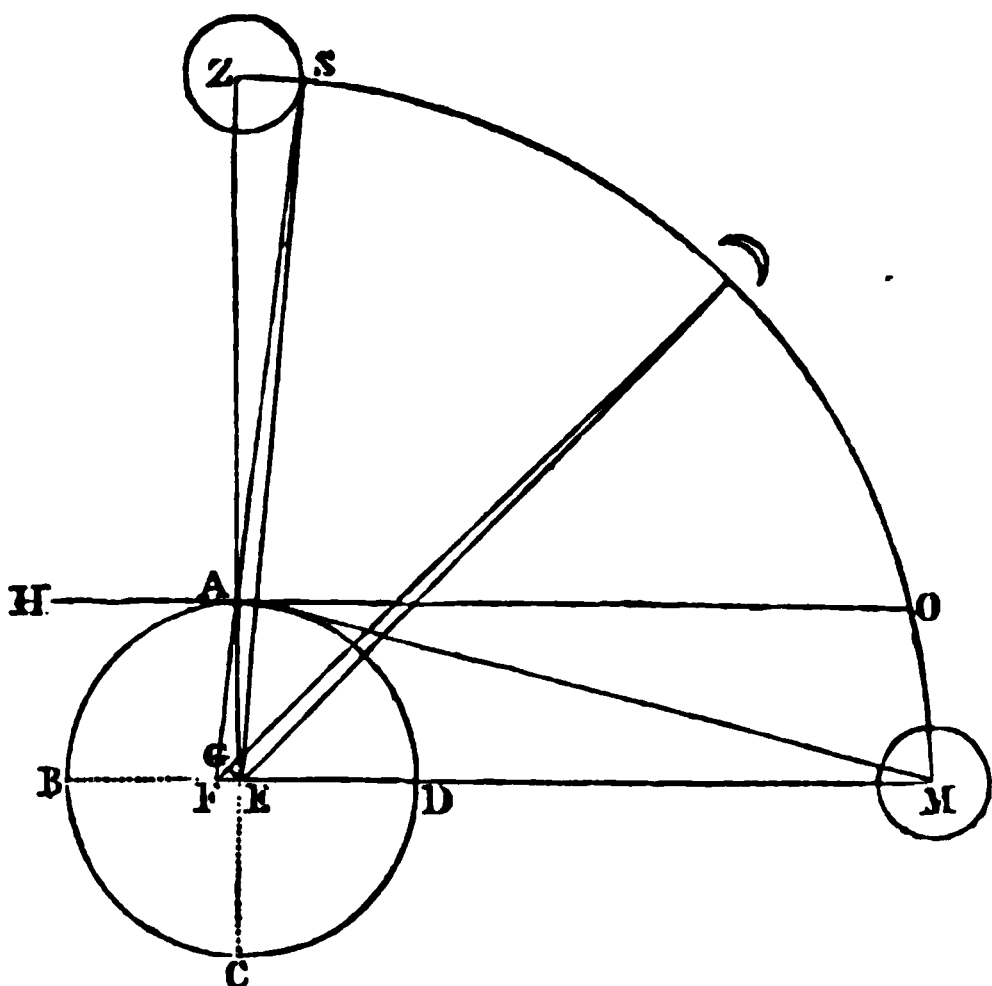
The same result will be obtained by using the greatest dip and its corresponding height; and since the operation is so very simple, it cannot fail of being extremely useful in determining a ship's true distance from the shore.

TABLE IV.

Augmentation of the Moon's Semidiameter.

Since it is the property of an object to increase its apparent diameter in proportion to the rate in which its distance from the eye of an observer is diminished; and, since the moon is nearer to an observer, on the earth, when she is in the zenith than when in the horizon, by the earth's semidiameter; she must, therefore, increase her semidiameter by a certain quantity as she increases her altitude from the horizon to the zenith. This increase is called the augmentation of the moon's semidiameter, and depends upon the following principles.

Let the circle $A B C D$ represent the earth; $A E$ its semidiameter, and M the moon in the horizon. Let A represent the place of an observer on the earth's surface; $B D M$ his rational horizon, and $H A O$, drawn parallel thereto, his sensible horizon extended to the moon's orbit; join $A M$, then $A M E$ is the angle under which the earth's semidiameter $A E$ is seen from the moon M , which is



equal to the angle $M A O$, the moon's horizontal parallax; because the straight line $A M$ which falls upon the two parallel straight lines $E M$ and $A O$ makes the alternate angles equal to one another. (Euclid, Book I. Prop. 29.) Let the moon's horizontal parallax be assumed at $57'.30''$, which is about the parallax she has at her mean distance from the earth; then in the right angled triangle $A E M$, there are given the angle $A M E = 57'.30''$, the moon's horizontal parallax, and the side $A E = 3958.75$ miles, the earth's semidiameter; to find the hypotenuse $A M$ = the moon's distance from the observer at A : hence by trigonometry,

As the angle at the moon, $A M E = 57'.30''$ Log. sine ar. comp. 1.776626
 Is to the earth's semidiameter $= A E = 3958.75$ miles, Log. . 3.597558
 So is radius 90° Log. sine 10.000000

To moon's horizontal distance $A M = 236692.35$ miles, Log. . 5.374184

Now, because the moon is nearer to the observer at A , by a complete semidiameter of the earth when in the zenith Z , than she is when in the horizon M , as appears very evident by the projection; and, because the earth's semidiameter $A E$ thus bears a sensible ratio to the moon's distance; it hence follows that the moon's semidiameter will be apparently increased when in the zenith, by a small quantity called its augmentation; and which may be very clearly illustrated as follows, viz.

Let the arc $Z O M$ represent a quarter of the moon's orbit; Z her place in the zenith, and $Z S$ her semidiameter: join $E Z$, $A S$, and $E S$; then the angles $Z E S$ and $Z A S$ will represent the angles under which the moon's semidiameter is seen from the centre and surface of the earth; their diffe-

rence, viz., the angle $A S E$ is, therefore, the augmentation of the moon's semidiameter, which may be easily computed; thus—

In the oblique angled triangle $A S E$, there are given the side $A E = 3958.75$ miles, the earth's semidiameter; the side $A S, = A M - A E = 232733.6$ miles, the moon's distance when in the zenith from the observer at A ; and the angle $A E S = 15' 30''$, the moon's mean semidiameter; to find the angle $A S E =$ the greatest augmentation corresponding to the given horizontal parallax and horizontal semidiameter: therefore,

As moon's zenith distance = $A Z = 232733.6$ miles,	Log. ar. co.	4.633141
Is to moon's semidiameter $A E S = 15' 30''$	Log. sine	7.654056
So is earth's semidiameter $E A = 3958.75$ miles,	Log. . .	3.597558
To augment. of semidiam. $A S E = 0' 16''$	Log. sine	5.884755

Now, having thus found the augmentation of the Moon's semidiameter, when in the zenith, answering to the assumed horizontal parallax and horizontal semidiameter; the increase of semidiameter at any given altitude, from the horizon to the zenith, may be computed in the following manner.

Let $S A$ be produced to F . and draw $E F$ parallel to $Z S$; then will $E F$ represent the greatest augmentation to the radius $E Z$. Let the moon be in any other part of her orbit, as at D with an altitude of 45 degrees; join $D E$, and $D F$, and make $D G = D E$; then will $E G$ (the measure of the angle $E D G$ to the radius $E D$,) be the augmentation corresponding to the given altitude. Then, in the right angled triangle $E G F$, right angled at G , there are given the angle $E F G = 45$ degrees, the moon's apparent altitude, and the side $E F = 16$ seconds, the augmentation of semidiameter when in the zenith; to find the side $E G$, which expresses the augmentation of semidiameter at the given altitude. And, since the angles expressing the augmentations are so very small, the measure of each may be substituted for its sine, which will simplify the calculation; thus,

As radius	$90^{\circ} 0' 0''$	Log. sine ar. comp.	0.000000
Is to moon's greatest augment. of semidiam. = $E F$	$16''$,	Log. =	1.204120
So is moon's given apparent alt. = $\angle E F G$,	45°	Log. sine =	9.849483
To the augmentation, or side	$E G = 11''.31$.	Log. =	1.053605

which, therefore, is the augmentation of the moon's semidiameter corresponding to the given apparent altitude of 45 degrees; horizontal semidiameter $15' 30''$ and horizontal parallax $57' 30''$.

Explanation of the Table.

This Table contains the augmentation of the moon's semidiameter (determined after the above manner,) to every third degree of altitude: the

augmentation is expressed in seconds, and is to be taken out by entering the Table with the moon's horizontal semidiameter at the top, as given in the Nautical Almanac, and the apparent altitude in the left-hand column; in the angle of meeting will be found a correction, which being applied by addition to the moon's horizontal semidiameter will give the true semidiameter, corresponding to the given altitude. Thus the augmentation answering to moon's apparent altitude 30 degrees, and horizontal semidiameter $16'.30''$ is 9 seconds; and that corresponding to altitude 60° and semidiameter $16'$ is 14 seconds.

TABLE V.

Contraction of the semidiameters of the Sun and Moon.

Since all parts of the horizontal semidiameter of the sun or moon are equally elevated above the horizon, all those parts must be equally affected by refraction, and thereby cause the horizontal semidiameter to remain invariable. But when the semidiameter is inclined to the plane of the horizon, the lower extremity will be so much more affected by refraction than the upper, as to suffer a sensible contraction, and thus cause the semidiameter, so inclined, to be something less than the horizontal semidiameter given in the Nautical Almanac. Hence it is manifest that the semidiameter of a celestial object, measured in any other manner than that parallel to the plane of the horizon will be always less than the true semidiameter by a certain quantity:—this quantity, called the contraction of semidiameter is contained in the present Table; the arguments of which are, the apparent altitude of the object in the left-hand column, and at the top the angle comprehended between the measured diameter and that parallel to the plane of the horizon; in the angle of meeting will be found a correction, which being subtracted from the horizontal semidiameter in the Nautical Almanac, will leave the true semidiameter. Thus, let the sun's or moon's apparent altitude be 5 degrees, and the inclination of its semidiameter 72 degrees; now, in the angle of meeting, of these arguments, stands 23 seconds; which, therefore, is the contraction of semidiameter, and which is to be applied by subtraction to the semidiameter given in the Nautical Almanac.

To compute the contraction of Semidiameter.

Rule.—Find by Table VIII. the refraction corresponding to the object's apparent central altitude, and also the refraction answering to that altitude augmented by the semidiameter; (which, for this purpose, may be estimated

at 16 minutes,) and their difference will be the contraction of the vertical semidiameter. Now, having thus found the contraction corresponding to the vertical semidiameter, that answering to a semidiameter which forms any given angle with the plane of the horizon, will be found by multiplying the vertical contraction by the square of the angle of inclination.

Example.

Let the sun's or moon's apparent central altitude be 3° and the inclination of its semidiameter to the plane of the horizon 72° ; required the contraction of the semidiameter?

Apparent central altitude . $3^{\circ} 0'$ Refraction = $14'.36''$

Do. augmented by semidiam. = $3^{\circ} 16'$ Ditto . = 13.46 .

Contraction of the vertical semidiameter . . . $0'.50''$ Log. = 1.698970

Inclination of semidiameter . = 72° twice the log. sine . = 19.956412

Required contraction of semidiameter . . . $45''.22$ Log. = 1.655382

And so on of the rest.—It is to be remarked, however, that the correction arising from the contraction of the semidiameter of a celestial object is very seldom attended to in practice at sea.

TABLE VI.

Parallax of the Planets in Altitude.

The arguments of this Table are the apparent altitude of a planet in the left or right-hand margin, and its horizontal parallax at the top; under the latter, and opposite the former, stands the corresponding parallax in altitude; which is always to be applied by addition to the planets apparent altitude. Hence, if the apparent altitude of a planet be 30 degrees, and its horizontal parallax 27 seconds, the corresponding parallax in altitude will be 23 seconds; additive to the apparent altitude.

The parallaxes of Altitude in this Table were computed by the following

Rule.—To the proportional logarithm of the planet's horizontal parallax add the log. secant of its apparent altitude, and the sum, abating 10 in the index, will be the proportional logarithm of the parallax in altitude.

Example.

If the horizontal parallax of a planet be 23 seconds, and its apparent altitude 30 degrees; required the parallax in altitude?

Horizontal parallax of the planet=23 Seconds, proportional log.=	2.6717
Apparent altitude of ditto . . . 30 Degrees, log. secant . . .	10.0625
Parallax in altitude 20 Seconds, proportional log.	2.7342

TABLE VII.

Parallax of the Sun in Altitude.

The difference between the places of the sun, as seen from the surface and centre of the earth at the same instant, is called his parallax in altitude, which may be computed in the following manner.

To the log. cosine of the sun's apparent altitude, add the constant log. 0.945124, (the log. of the sun's mean horizontal parallax estimated at 8".813,) and the sum, rejecting 10 from the index, will be the log. of the parallax in altitude; as thus,

Given the sun's apparent altitude 20 degrees; required the corresponding parallax in altitude?

Sun's apparent altitude 20 degrees, log. cosine . . .	9.972986
Constant log.	0.945124

Parall. corresponding to the given altitude 8".282 Log. 0.918110

This Table, which contains the correction for parallax, is to be entered with the sun's apparent altitude in the left-hand column; opposite to which, in the adjoining column, stands the corresponding parallax in altitude;—thus, the parallax answering to 10° apparent altitude is 9 seconds; that answering to 40° apparent altitude is 7 seconds, &c. &c.—And since the parallax of a celestial object causes it to appear something lower in the heavens, than it really is; this correction for parallax, therefore, becomes always additive to the sun's apparent altitude.

TABLE VIII.

Mean Astronomical Refraction.

Since the density of the atmosphere increases in proportion to its proximity to the earth's surface, it therefore causes the ray of light issuing from a celestial object to describe a curve, in its passage to the horizon; the convex side of which is directed to that part of the heavens to which a tangent to that curve at the extremity of it which meets the earth, would

be directed. Hence it is, that the celestial objects are apparently more elevated in the heavens than they are in reality ; and this apparent increase of elevation or altitude is called the refraction of the heavenly bodies ; the effects of which are greatest at the horizon, but gradually diminish as the altitude increases, so as to entirely vanish at the zenith.

In this Table the refraction is computed to every minute in the first 8 degrees of apparent altitude ; consequently this part of the Table is to be entered with the degrees of apparent altitude at the top or bottom, and the minutes in the left-hand column : in the angle of meeting, stands the refraction.

In the rest of the Table the apparent altitude is given in the vertical columns, opposite to which in the adjoining columns will be found the corresponding refraction. Thus, the refraction answering to $3^{\circ}27'$ apparent altitude, is $13'.14''$; that corresponding to $9^{\circ}46'$ is $5'.52''$; that corresponding to $17^{\circ}55'$ is $2'.54''$, and so on. The refraction is always to be applied by subtraction to the apparent altitude of a celestial object, on account of its causing such object to appear under too great an angle of altitude. The refractions in this Table are adapted to a medium state of the atmosphere ; that is, when the Barometer stands at 29.6 inches, and the Thermometer at 50 degrees ; and were computed by the following *general rule*, the horizontal refraction being assumed at 33 minutes of a degree.

To the constant $\log. 9.999279$ (the $\log.$ cosine of 6 times the horizontal refraction) add the $\log.$ cosine of the apparent altitude ; and the sum, abating 10 in the index, will be the $\log.$ cosine of an arch. Now, one-sixth the difference between this arch and the given apparent altitude will be the mean astronomical refraction answering to that altitude.

Example.

Let the apparent altitude of a celestial object be 45° , required the corresponding refraction ?

Constant $\log.$	9.999279
Given apparent altitude	$45^{\circ}0'.0''$	$\log.$ cosine 9.849485
<hr/>		
Arch $45^{\circ}5'.42''$	$\log.$ cosine 9.848764

$\text{Difference} 0^{\circ}5'.42'' + 6 = 0'.57''$; which, therefore, is the mean astronomical refraction answering to the given apparent altitude.

TABLE IX.

Correction of the Mean Astronomical Refraction.

Since the refraction of the heavenly bodies depends on the density and temperature of the atmosphere, which are ever subject to numberless variations; and since the corrections contained in the foregoing Table are adapted to a medium state of the atmosphere, or when the barometer stands at 29.6 inches, and the thermometer at 50 degrees: it hence follows, that when the density and temperature of the atmosphere differ from those quantities, the amount of refraction will also differ, in some measure, from that contained in the said foregoing Table. To reduce, therefore, the corrections in that Table to other states of the atmosphere, the present Table has been computed; the arguments of which are, the apparent altitude in the left or right hand margin, the height of the thermometer at the top, and that of the barometer at the bottom of the Table; the corresponding corrections will be found in the angle of meeting of those arguments respectively, and are to be applied, agreeably to their signs, to the mean refraction taken from Table VIII, in the following manner:—

Let the apparent altitude of a celestial object be 5 degrees; the height of the barometer 29.15 inches, and that of the thermometer 48 degrees; required the true atmospheric refraction?

Apparent altitude 5 degrees,—mean refraction in Table VIII = . .	9'54"
Opposite to 5 degrees, and over 29.15, in Table IX, stands . .	— 0. 9
Opposite to 5 degrees, and under 48 degrees, in ditto . . .	+ 0. 3
	<hr/>
True atmospheric refraction, as required	9'48"

The correction of the mean astronomical refraction, may be computed by the following rule, viz.

As the mean height of the barometer, 29.6 inches, is to its observed height, so is the mean refraction to the corrected refraction; now, the difference between this and the mean refraction will be the correction for barometer, which will be affirmative or negative, according as it is greater or less than the latter.—And,

As 350 degrees* increased by the observed height of Fahrenheit's thermometer, are to 400 degrees†, so is the mean refraction to the corrected refraction; the difference between which, and the mean refraction, will be the correction for thermometer; which will be affirmative or negative, according as it is greater or less than the latter.

* Seven times 50 degrees, the mean temperature of the atmosphere.

† Eight times 50 degrees, the mean temperature of the atmosphere.

Example 1.

Let the apparent altitude be 1 degree, the mean refraction $24'.29''$, the height of the barometer 28.56 inches, and that of the thermometer 32 degrees; required the respective corrections for barometer and thermometer?

As mean height of barometer . . .	29.60.	Log. ar. co. . .	8.528708
Is to observed height of ditto . . .	28.56.	Log.	1.455758
So is mean refraction $24'.29'' =$. . .	1469"	Log.	3.167022

To corrected refraction	1417"	Log.	3.151488
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Correction for barometer $- 52''$, which is negative, because the corrected refraction is the least.

And

As $350^\circ + 32^\circ =$	382°	Log. ar. co. . .	7.417937
Is to	400°	Log.	2.602060
So is mean refraction $24'.29'' =$. . .	1469"	Log.	3.167022

To corrected refraction	1538"	Log.	3.187019
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Correction for thermometer . . . $+ 69'' = 1'.9''$, which is affirmative, because the corrected refraction is the greatest.

Example 2.

Let the apparent altitude be 7 degrees, the mean refraction $7'.20''$, the height of the barometer 29.75 inches, and that of the thermometer 72 degrees; required the respective corrections for barometer and thermometer?

As mean height of barometer . . .	29.60.	Log. ar. co. . .	8.528708
Is to observed height of ditto . . .	29.75.	Log.	1.473487
So is mean refraction $7'.20'' =$. . .	440"	Log.	2.643453

To corrected refraction	442"	Log.	2.645648
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Correction for barometer $+ 2''$, which is affirmative.

And

As $350^\circ + 72^\circ =$	422°	Log. ar. co. . .	7.374688
Is to	400°	Log.	2.602060
So is mean refraction $7'.20'' =$. . .	440"	Log.	2.643453

To corrected refraction	417"	Log.	2.620201
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Correction for thermometer $- 23''$, which is negative.

TABLE X.

To find the Latitude by an Altitude of the North Polar Star.

The correction of altitude, contained in the third column of this Table, expresses the difference of altitude between the north polar star, and the north celestial pole, in its apparent revolution round its orbit, as seen from the equator: the correction of altitude is particularly adapted to the beginning of the year 1836; but by means of its annual variation, which is determined for the sake of accuracy to the hundredth part of a second, it may be readily reduced to any subsequent period, (with a sufficient degree of exactness for all nautical purposes,) for upwards of half a century, as will be seen presently.

The Table consists of five compartments; the left and right hand ones of which are each divided into two columns, containing the right ascension of the meridian: the second compartment, which forms the third column in the Table, contains the correction of the polar star's altitude: the third compartment consists of five small columns, in which are contained the proportional parts corresponding to the intermediate minutes of right ascension of the meridian; by means of which the correction of altitude, at any given time, may be accurately taken out at the first sight: the fourth compartment contains the annual variation of the polar star's correction, which enables the mariner to reduce the tabular correction of altitude to any future period: for, the product of the annual variation, by the number of years and parts of a year elapsed between the beginning of 1836, and any given subsequent time, being applied to the correction of the polar star's altitude by addition or subtraction, according to the prefixed sign, will give the true correction at such subsequent given time. 36

Note.—In taking out the proportional parts for the intermediate minutes of right ascension from the *upper part* of the Table, or *between the double horizontal line and the top*, whenever the odd minutes of R. A. exceed 5, let the double of any one, or the sum of any two proportional parts, be taken (answering to the two minutes that will make up the odd minutes of R. A.) in the line opposite to the nearest tabular R. A., and the result will be the required proportional part.

Example.

Required the correction of the polar star's altitude on the first day of January in 1848, the R. A. of the meridian being $6^{\text{h}} 22^{\text{m}}$?

Correction of altitude answer to $6^{\text{h}} 20^{\text{m}}$, is	16'.45"
Proportional part to 2 minutes of R. A.	— 0.48
		<hr/>
Correction of altitude at $6^{\text{h}} 22^{\text{m}}$, in January 1836=	15'.57"
Annual variation of correction=	+2'.95
Number of years after 1836=12, multiply by	12
		<hr/>
Product =	+35'.40 . . . +35
		<hr/>
Correction of polar star's altitude January 1st, 1848=	16'.32"

The corrections of altitude contained in the present Table were computed in conformity with the following principles:—

Since to an observer placed at the equator, the poles of the world will appear to be posited in the horizon, the polar star will, to such observer, apparently revolve round the north celestial pole in its diurnal motion round its orbit. In this apparent revolution round the celestial pole, the star's meridional or greatest altitude above the horizon will be always equal to its distance from that pole; which will ever take place, when the right ascension of the meridian is equal to the right ascension of the star. In six hours after this, the star will be seen in the horizon, west of the pole; in six hours more it will be depressed beneath the horizon (on the meridian below the pole), the angle of depression being equal to its polar distance; in six hours after, it will be seen in the horizon east of the pole; and in six hours more, it will be seen again on the meridian above the pole: allowance being made, in each case, for its daily acceleration.

Now, since the north celestial pole represents a fixed point in the heavens, and that the star apparently moves round it in an uniform manner, making determinable angles with the meridian; it is, therefore, easy to compute what altitude the star will have, as seen from the equator, in every part of its orbit; for, in this computation, we have a spherical triangle to work in, whose three sides are expressed by the complement of the latitude, the complement of the polar star's altitude, and the complement of its declination; in which there are given two sides and the included angle to find the third side; viz. the star's co-declination or polar distance and the complement of the latitude, with the comprehended angle, equal to the star's distance from the meridian, to find the star's co-altitude; the difference between which and 90 degrees will be the correction of altitude, or the difference of altitude between the polar star and the north celestial pole, as seen from the equator.

For the sake of conciseness, the polar star's altitude, as *seen from the equator*, may be determined at once by the following formula:—

To the log. rising of the star's horary distance from the meridian, add

the log. sine of its polar distance ; then, the natural number corresponding to the sum of these two logarithms being subtracted from the natural sine of the star's polar distance, the result will be the natural sine of the polar star's altitude, as *seen from the equator* ; or the correction of altitude.

Example.

In January, 1836, the mean right ascension of the polar star will be $1^{\text{h}}1^{\text{m}}5^{\text{s}}$, and its distance from the north celestial pole $1^{\circ}33'55''$; now, admitting the right ascension of the meridian to be $5^{\text{h}}15^{\text{m}}0^{\text{s}}$, the correction of the polar star's altitude, as seen from the equator, is required ?

Right ascension of meridian $5^{\text{h}}12^{\text{m}}0^{\text{s}}$

Right ascension of polar star $1^{\text{h}}1^{\text{m}}5^{\text{s}}$

Star's horary distance = $.4.13.55. =$ Logarithmic rising = 5.743100

Star's polar distance $. . 1^{\circ}33'55'' -$ Log. $. .$ Sine $. . 8.436415$

Natural number = $. . -15119$ Log. = 4.179515

Natural sine of polar distance $. 027316$

Natural sine of polar star's altitude = $. . 012197 = 0^{\circ}41'56''$, which is, the correction of the polar star's altitude, as seen from the equator, and in the same manner were all the corrections of altitude in Table X. obtained.

Note.—For further information on this subject, the reader is referred to the author's Treatise on the Sidereal and Planetary Parts of Nautical Astronomy, page 144 to 156.

TABLE XI.

Correction of the Latitude deduced from the preceding Table.

Although the latitude deduced from Table X. will be always sufficiently correct for most nautical purposes, yet, since observation has shown that it will be something less than the truth in places distant from the equator, the present Table has been computed ; which contains the number of minutes and seconds that the latitude, so deduced, will be less than what would result from actual observation at every tenth or fifth degree from the equator, to within five degrees of the north pole of the world.

The elements of this Table are, the approximate latitude, deduced from Table X., at top, and the right ascension of the meridian in the left or right hand column ; in the angle of meeting will be found the corresponding correction, which is always to be applied by *addition* to the approxi-

mate latitude. Hence, if the approximate latitude be 50 degrees, and the right ascension of the meridian $6^h 40^m$, the corresponding correction will be $1' 38''$ additive.

Remark.—Since the corrections of altitude in Table X. have been computed on the assumption that the motions of the polar star were witnessed from the equator, they ought, therefore, to show what altitude that star will have at any given time, in north latitude, when applied to such latitude with a contrary sign to that expressed in the Table; this, however, is not the case; because when the altitude of the polar star is computed by spherical trigonometry, or otherwise, it will always prove to be something less than that immediately deduced from Table X.; it is this difference, then, that becomes the correction of latitude in Table XI., and which is very easily determined, as may be seen in the following

Example.

Let the right ascension of the meridian in January 1824 be $6^h 40^m$, and the latitude 60 degrees north; required the true altitude of the polar star, and thence the correction of latitude?

Latitude or elevation of the pole $60^\circ 0' 0''$ north.
Correction in Table X., answ. to $6^h 40^m$, is + $0.7.41$

Altitude of polar star, per Table X.= . . . $60^\circ 7' 41''$

Now, to compute the true altitude of the polar star, on spherical principles, at the given time and place, we may proceed as follows:—

Right ascension of the merid. $6^h 40^m 0^s$
Star's right ascension . . . $0.58.1$

Star's dist. from the meridian $5^h 41^m 59^s$. . . Log. rising 5.964481
Star's polar distance . . . $1^\circ 37' 48''$. . . Log. sine 8.454006
Complement of the latitude 30. 0. 0 . . . Log. sine 9.698970

Difference $28^\circ 22' 12''$ Nat.cos. 879897
Natural number 013106 Log.=4.117457

Star's true altitude . . . $60^\circ 5' 16''$ Nat. sine 866791
Star's alt. per Tab. as above $60.7.41$

Difference $0^\circ 2' 25''$; which, therefore, is the correction of latitude.

Note.—Since the corrections of latitude in Table XI. will *not differ one minute from the truth* in half a century, they are, therefore, of the same value as those that were given in the first edition of this work.

TABLE XII.

The Mean Sun's approximate Right Ascension, or Sidereal Time to the nearest Minute.

This Table may be used for the purpose of finding the approximate time of transit of a fixed star, when a Nautical Almanac is not at hand; it may also be employed in finding the right ascension of the meridian, or mid-heaven, when the latitude is to be determined by an altitude of the north polar star: for, if to the mean sun's right ascension, as given in this Table, the mean time be added, the sum (rejecting 24 hours if necessary) will be the right ascension of the meridian, sufficiently near the truth for the purpose of determining the latitude.

TABLE XIII.

Equations to equal Altitudes.—FIRST PART.

The arguments of this Table are, the interval between the observations at top or bottom, and the latitude in either of the side columns; in the angle of meeting stands the corresponding equation, expressed in seconds and thirds: hence the equation to interval 6 hours 40 minutes and latitude 50 degrees, is 15 seconds and 33 thirds.

The equations in this Table were computed by the following rule, viz.:—

To the log. co-tangent of the latitude, add the log. sine of half the interval in degrees; the proportional log. of the whole interval in time (esteemed as minutes and seconds), and the constant log. 8.8239;* the sum of these four logs., rejecting 29 from the index, will be the proportional log. of the corresponding equation in minutes and seconds, which are to be considered as seconds and thirds.

Example.

Let the latitude be 50 degrees, and the interval between the observed equal altitudes of the sun 4 hours; required the corresponding equation?

Latitude	50°0'0"	Log. co-tang.	9.9238
Half int. = 2 hours, in degs.=30.0.0		Log. sine .	9.6990
Whole interval 4 hours, esteemed as 4 min., propor. log.			1.6532
Constant log.			8.8239
<hr/>			
Required equation	14"18"	Propor. log.	1.0999

* The arithmetical complement of 12 hours considered as minutes.

TABLE XIV.

Equations to equal Altitudes.—PART SECOND.

In this Table, the interval between the observations is marked at top or bottom, and the sun's declination in the left or right-hand margin; under or over the former, and opposite to the latter, stands the corresponding equation, expressed in seconds and thirds: thus, the equation answering to 6 hours 40 minutes, and declination $18^{\circ}30'$, is 2 seconds and 48 thirds.

The equations contained in this Table were computed as follows, viz.:—

To the log. co-tangent of the declination, add the log. tang. of half the interval in degrees; the proportional log. of the whole interval in time (esteemed as minutes and seconds), and the constant log. 8.8239;* the sum of these four logs., rejecting 29 from the index, will be the proportional log. of the corresponding equation in minutes and seconds, which are to be considered as seconds and thirds.

Example.

Let the sun's declination be $18^{\circ}30'$, and the interval between the observed equal altitudes of the sun 4 hours; required the corresponding equation?

Sun's declination	. . . $18^{\circ}30'$	Log. co-tang.	10.4755
Half interval=2 ho. in degs.=30. 0.		Log. tang.	9.7614
Whole interval 4 ho. esteemed as 4 min.		Prop. log.	1.6532
Constant log.		8.8239
<hr/>			
Required equation =	. . . $3^{\prime}29''$	Prop. log.	1.7140

To find the Equation of Equal Altitudes by Tables XIII. and XIV.

Rule.

Enter Table XIII., with the latitude in the side column and the interval between the observations at top; and find the corresponding equation, to which prefix the sign + if the sun be *receding* from the elevated pole, but the sign — if it be advancing towards that pole.

Enter Table XIV., with the declination in the side column, and the interval between the observations at top, and take out the corresponding equation, to which prefix the sign + when the sun's declination is *increasing*, but the sign — when it is *decreasing*.

* The arithmetical complement of 12 hours considered as minutes.

Now, if those two equations are of the same signs ; that is, both affirmative or both negative, let their sum be taken ; but if of contrary signs, namely, one affirmative and the other negative, their difference is to be taken : then,

To the proportional log. of this sum or difference, considered as minutes and seconds, add the proportional log. of the daily variation of the sun's declination ; and the sum, rejecting 1 from the index, will be the proportional log. of the true equation of equal altitudes in minutes and seconds, which are to be esteemed as *seconds and thirds*, and which will be always of the same name with the greater equation.

Example 1.

In latitude 49° south, the interval between equal altitudes of the sun was $7^{\text{h}}20^{\text{m}}$; the sun's declination 18° north, increasing, and the variation of declination $15'12''$; required the true equation of equal altitudes ?

Opposite lat. 49° under $7^{\text{h}}20^{\text{m}}$ Tab. XIII. stands $+ 15^{\text{m}}27^{\text{s}}$

Opposite dec. 18° under $7^{\text{h}}20^{\text{m}}$ Tab. XIV. stands $+ 2.30$

Sum	$17^{\text{m}}57^{\text{s}}$	Pro. log. 1.0012
Variation of declination . . .	$15'12''$	Pro. log. 1.0734
True equation, as required	$+ 15^{\text{m}}10^{\text{s}}$	Pro. log. 1.0746

Example 2.

In latitude 50° north, the interval between equal altitudes of the sun was $5^{\text{h}}20^{\text{m}}$; the sun's declination $18^{\circ}30'$ north, increasing, and the daily variation of declination $14'34''$; required the true equation of equal altitudes ?

Op. lat. 50° under $5^{\text{h}}20^{\text{m}}$ Tab. XIII. stands $- 14^{\text{m}}50^{\text{s}}$

Op. dec. $18^{\circ}30'$ under $5^{\text{h}}20^{\text{m}}$ Tab. XIV. stands $+ 3.11$

Difference	$- 11^{\text{m}}39^{\text{s}}$	Pro. log. = 1.1889
Variation of declination	$14'34''$	Pro. log. = 1.0919
True equation, as required	$- 9^{\text{m}}26^{\text{s}}$	Pro. log. = 1.2808

Remark.—In north latitude the sun *recedes* from the elevated pole from the summer to the winter solstice ; that is, from the 21st June to the 21st December ; but *advances* towards that pole from the winter to the summer solstice : viz., from the 21st December to the 21st June. The converse of this takes place in south latitude : thus, from the 21st June to the 21st December, the sun *advances* towards the south elevated pole ; but *recedes* from that pole the rest of the year, viz., from the 21st December to the 21st June.

Here it may be necessary to observe, that in taking out the equations from Tables XIII. and XIV. allowance is to be made for the excess of the given, above the next less tabular arguments, as in the following examples:—

Example 1.

Required the equation from Table XIII., answering to latitude $50^{\circ}48'$, and interval between the observations 5 hours 10 minutes?

$$\begin{array}{rcl}
 \text{Equation to latitude } 50^{\circ}, \text{ and interval } 4^{\text{h}}40^{\text{m}} & = & . \quad . \quad 14^{\circ}33'' \\
 \text{Tabular diff. to } 1^{\circ} \text{ of lat.} & = +31''; \text{ now, } \frac{31'' \times 48''}{60'} & = +0.24\frac{1}{2} \\
 \text{Tab. diff. to } 40' \text{ of inter.} & = +17''; \text{ now, } \frac{17'' \times 30'}{40'} & = +0.12\frac{1}{2} \\
 \text{Equation, as required} & . \quad . \quad . \quad . \quad . \quad . \quad . \quad . & \underline{15^{\circ}10''}
 \end{array}$$

Example 2.

Required the equation from Table XIV., answering to sun's declination $20^{\circ}47'$, and interval between the observations 5 hours 10 minutes?

$$\begin{array}{rcl}
 \text{Equation to declination } 20^{\circ}30' \text{ and interval } 4^{\text{h}}40^{\text{m}} & = & 3^{\circ}44'' \\
 \text{Tabular diff. to } 30' \text{ declination} & = +6''; \text{ now, } \frac{6'' \times 17'}{30'} & = +0.3\frac{1}{2} \\
 \text{Tabular diff. to } 40' \text{ interval} & = -10''; \text{ now, } \frac{10'' \times 30'}{40'} & = -0.7\frac{1}{2} \\
 \text{Equation, as required} & . \quad . \quad . \quad . \quad . \quad . \quad . \quad . & \underline{3^{\circ}40\frac{1}{4}''}
 \end{array}$$

Note.—Should the latitude exceed the limits of Table XIII., which is only extended so far as to comprehend the ordinary bounds of navigation, viz., to 60 degrees, the first part of the equation, in this case, must be determined by the rule under which that Table was computed, as in page 21.

TABLE XV.

To reduce the Sun's Longitude, Right Ascension, and Declination; and also the Equation of Time, as given in the Nautical Almanac, to any given Meridian, and to any given Time under that Meridian.

This Table is so arranged, that the proportional part corresponding to any given time, or longitude, and to any variation of the sun's right ascension, declination, &c. &c., may be taken out to the greatest degree of accuracy.

Precepts.

In the general use of this Table it will be advisable to abide by the solar day ; and hence, to estimate the time from noon to noon, or from 0 to 24 hours, after the manner of astronomers, without paying any attention to either the nautical or the civil division of time at midnight. And to guard against falling into error, in applying the tabular proportional part to the sun's right ascension, declination, &c. &c., it will be best to reduce the mean time at ship or place, to Greenwich time ; as thus :

Turn the longitude into time (by Table I.), and *add* it to the given time at ship or place, if it be *west* ; but *subtract* it if *east* ; and the sum, or difference, will be the corresponding time at Greenwich.

From page II. or III. of the month in the Nautical Almanac, take out the sun's right ascension, declination, longitude, &c., for the *noons immediately preceding and following the Greenwich time*, and find their difference, which will express the variation of those elements in 24 hours ; then,

Enter the Table with the variation, thus found, at top, and the Greenwich time in the left-hand column ; under the former and opposite the latter will be found the corresponding equation, or proportional part. And, since the Greenwich time may be estimated in hours, minutes, or seconds, and the variation of right ascension, &c. &c. &c., either in minutes or seconds : the sum of the several proportional parts making up the whole of such time and variation will, therefore, express the required proportional part. The proportional part, so obtained, is always to be applied by *addition* to the sun's longitude and right ascension at the *preceding noon* ; but it is to be applied by *addition*, or *subtraction*, to the sun's declination and the equation of time at that noon, according as they are *increasing* or *decreasing*.

Example.

Required the sun's right ascension and declination, and also the equation of time, May 6th, 1824, at 5^h 10^m, in longitude 64° 45' west of the meridian of Greenwich ?

Mean time at ship or place.	5 ^h 10 ^m
Longitude 64° 45' west, in time = . . . +	4. 19
Greenwich time	<hr/> 9 ^h 29 ^m

To find the Sun's Declination :—

Sun's declination at noon, May 6th, 1824, per Nautical

Almanac 16°36'5"

North, increasing, and var: in 24 ho.=16'38"

Pro. part to 9^h 0^m and 16' 0" = 6' 0" 0" 0"

Do. to 0.29 and 16. 0 = 0.19.20. 0

Do. to 9. 0 and 0.30 = 0.11.15. 0

Do. to 0.29 and 0.30 = 0. 0.36.15

Do. to 9. 0 and 0. 8 = 0. 3. 0. 0

Do. to 0.29 and 0. 8 = 0. 0. 9.40

Pro. part to 9^h29^m and 16'38" is 6.34.20.55 = + 6'34"

Sun's declination, as required 16°42'39"

To find the Equation of time :—

Equation of time at noon, May 6th, 1824, per Nautical

Almanac, 3^m36' 6"increasing, and variation in 24 hours = 4^m30"Pro. part to 9^h 0^m and 4^m 0" = 1^m30" 0"

Do. to 0.29 and 4. 0 = 0. 4.50

Do. to 9. 0 and 0.30 = 0.11.15

Do. to 0.29 and 0.30 = 0. 0.36

Pro. part to 9^h29^m is 4^m30" = 1.46.41 = + 1^m47"

Equation of time, as required 3^m37'53"

Remark.—Should the proportional part corresponding to the daily variation of the sun's longitude and any given time be required, it may be taken from the first page of the Table, by esteeming the seconds of variation, in that page, as minutes, and then raising the signs of the corresponding proportional parts one grade higher than what are marked at the top of the said page: the seconds of variation are of course to be taken out in the usual manner.

Note.—The present Table was computed agreeably to the established principles of the rule of proportion; viz. As one day, or 24 hours, is to the variation of the sun's right ascension, declination, &c. &c., in that time, so is any other portion of time to the corresponding proportional part of such variation.

TABLE XVI.

To reduce the Moon's Longitude, Latitude, Semidiameter, and Horizontal Parallax, as given in the Nautical Almanac, to any given Meridian, and to any given Time under that Meridian.

This Table is so arranged that the proportional part corresponding to any given time and change of longitude &c. in 12 hours may be taken out to the most rigid degree of astronomical exactness.

Precepts.

In the general use of this Table it will be advisable to abide by the solar day; and hence to estimate the time from noon to noon, or from 0 to 24 hours, after the manner of astronomers, without paying any attention to either the nautical or the civil division of time at midnight. And to guard against falling into an error, in applying the tabular proportional part to the moon's longitude &c. &c., it will be best to reduce the mean time at ship to the Greenwich mean time, as thus:—

Turn the longitude into time (by Table I.), and *add* it to the given mean time at ship or place, if it be *west*; but *subtract* it if *east*: and the sum or difference will be the corresponding mean time at Greenwich.

Take from pages III. and IV. of the month, in the Nautical Almanac, the moon's longitude, latitude, semidiameter, and horizontal parallax, (or any one of these elements, according to circumstances,) for the noon and midnight immediately preceding and following the Greenwich time, and find their difference; which difference will express the variation of those elements in 12 hours.

Enter the Table with the variation, thus found, at top, and the Greenwich time in the left-hand column: in the angle of meeting will be found the corresponding equation, or proportional part, which is always to be *added* to the moon's longitude at the *preceding* noon or midnight, but to be applied by *addition*, or *subtraction*, to the moon's latitude, semidiameter, and horizontal parallax, according as they are *increasing* or *decreasing*. And, since the Greenwich time and the variation in 12 hours will be very seldom found to correspond exactly; it is the sum, therefore, of the several equations making up those terms, that will, in general, express the required proportional part.

Example.

Required the moon's longitude, and latitude, semidiameter, and horizontal parallax, August 2nd, 1824, at 3^h 10^m, mean time, in longitude 60° 30' west of the meridian of Greenwich?

Mean time at ship or place	3 ^h 10 ^m
Longitude 60° 30' west, in time =	4. 2
Greenwich mean time =	<u>7^h 12^m</u>

To find the Moon's Longitude:—

Moon's longitude at noon, August 2nd, 1824, per Nautical

Almanac, 7:17:16:27"

Variation in 12^h = 6° 31' 59"Propor. part to 7^h 0^m and 6° 0' 0" = 3:30' 0" 0"

Do. to 0.12 and 6. 0. 0 = 0. 6. 0. 0

Do. to 7. 0 and 0.30. 0 = 0.17.30. 0

Do. to 0.12 and 0.30. 0 = 0. 0.30. 0

Do. to 7. 0 and 0. 1. 0 = 0. 0.35. 0

Do. to 0.12 and 0. 1. 0 = 0. 0. 1. 0

Do. to 7. 0 and 0. 0.50 = 0. 0.29.10

Do. to 0.12 and 0. 0.50 = 0. 0. 0.50

Do. to 7. 0 and 0. 0. 9 = 0. 0. 5.15

Do. to 0.12 and 0. 0. 9 = 0. 0. 0. 9

Propor. part to 7^h 12^m and 6° 31' 59" is 3.55.11.24 = + 3° 55' 11"

Moon's longitude, as required 7:21:11:38"

To find the Moon's Latitude:—

Moon's latitude at noon, August 2nd, 1824, per Nautical

Almanac, 4° 6' 59"

South, decreasing, and var. in 12 hours = 23' 35"

Proportional part to 7^h 0^m and 20' 0" = 11' 40" 0"

Do. to 0.12 and 20. 0 = 0.20. 0

Do. to 7. 0 and 3. 0 = 1.45. 0

Do. to 0.12 and 3. 0 = 0. 3. 0

Do. to 7. 0 and 0.30 = 0.17.30

Do. to 0.12 and 0.30 = 0. 0.30

Do. to 7. 0 and 0. 5 = 0. 2.55

Do. to 0.12 and 0. 5 = 0. 0. 5

Proportional part to 7^h 12^m and 23' 35" is 14. 9. 0 = - 14' 9"

Moon's latitude, as required (South) 3° 52' 50"

Note.—In consequence of the unequal motion of the moon in 12 hours, (when her place is to be determined with astronomical precision,) the proportional part of the variation of her longitude and latitude, found as above, must be corrected by the equation of second difference contained in Table XVII.

To find the Moon's Semidiameter :—

Moon's semidiameter at noon, August 2nd, 1824, per Nautical	
Almanac,	15'33"
decreasing, and var. in 12 hours=6"	
Proportional part to 7 ^h 0 ^m and 6" = 3 ^m 30"	
Do. to 0.12 and 6 = 0. 6	
<hr/>	
Proportional part to 7 ^h 12 ^m and 6" is 3.36	= — 4"
<hr/>	
Moon's semidiameter, as required	15'29"

To find the Moon's Horizontal Parallax :—

Moon's horizontal parallax at noon, August 2nd, 1824, per	
Nautical Almanac,	57'6"
decreasing and var. in 12 hours = 23"	
Proportional part to 7 ^h 0 ^m and 20" = 11 ^m 40"	
Do. to 0.12 and 20 = 0.20	
Do. to 7. 0 and 3 = 1.45	
Do. to 0.12 and 3 = 0. 3	
<hr/>	
Proportional part to 7 ^h 12 ^m and 23" is 13 ^m 48"	= — 14"
<hr/>	
Moon's horizontal parallax, as required	56'52"

Remarks.—1. It is evident that, in the above operations, the greater part of the figures might have been dispensed with, by taking out two or more of the proportional parts at once ; but since they were merely intended to simplify and render familiar the use of the Table, the whole of the proportional parts have been put down at length.

2. This Table was computed according to the rule of proportion, viz.—As 12 hours are to the variation of the moon's longitude, latitude, right ascension, &c. &c. &c.; in that interval, so is any other given portion of time to the corresponding proportional part of such variation.

Parallax of the Heavenly Bodies.

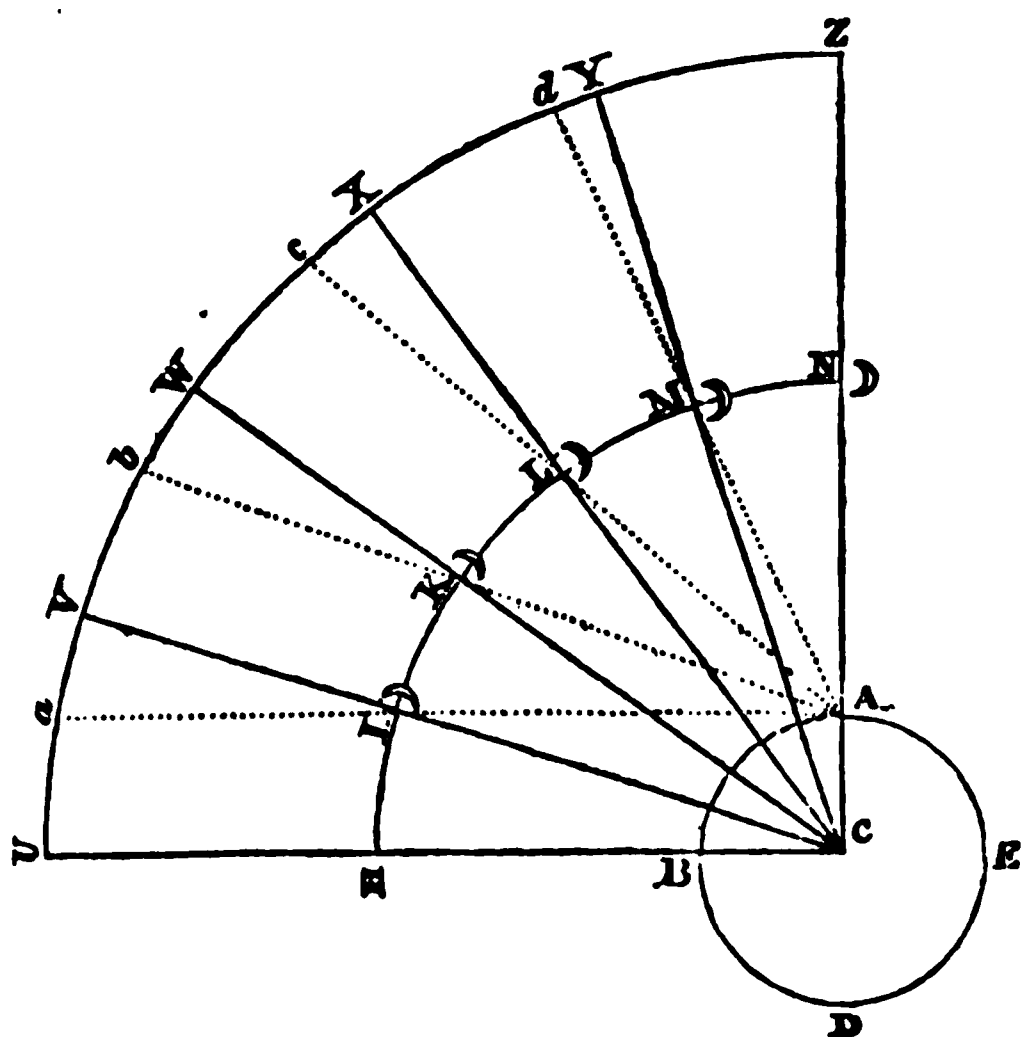
As the nature of celestial parallax is but little understood by those who have not had an opportunity of studying the elementary parts of astronomy, the following concise and practicable method of finding the lunar horizontal parallax is, therefore, submitted to their consideration ; and,

although remarkably familiar, yet, if duly attended to, it cannot fail of giving a correct idea of the cause and effects of celestial parallax in general.

Definition.—The parallax of the moon, sun, or planet, is the distance between its true and apparent places in the starry heavens. The true place of any celestial object, referred to the sphere of the fixed stars, is that in which it would appear if seen from the centre of the earth; the apparent place is that in which it appears to an observer on the earth's surface.

Illustration.

In the annexed diagram, let $A B D E$ be the earth, and C its centre; the quadrant $H I K L M N$ the concave crystalline arch, or the azure sky in which the moon is seen; and the quadrant $U V W X Y Z$ an arc of the sphere of the fixed stars. Let the dotted line $A I a$ be the sensible horizon of an observer on the earth's surface at A : to him the moon at $I D$ will ap-



pear in the horizon, extended to the starry heavens, at the horizontal point a ; but to an observer at the centre C (supposing the earth to be transparent) she will appear above the horizon at V , which is her true place. The angle $A I C$ is called the moon's horizontal parallax; and is equal to the opposite or parallactic angle $V I a$. To an observer at the centre C , the moon $K D$ will appear at W , her true place; but to an observer at A , she will appear below her true place at b : the difference, or parallactic angle $W K b$, equal to the angle $A K C$, is the moon's parallax at the altitude $K D$; and so on to the zenith Z , where the parallax entirely vanishes: for, it is evident that the ray of light flowing from the moon, when in the zenith at $N D$, must be in the same right line with the points A and C . From this it will appear manifest that the parallax decreases from the horizon to the zenith in proportion to the co-sine of the altitude; and, *vice versa*, that it increases from the zenith to the horizon in proportion to the sine of the zenith distance of the object.

The parallax causes the moon, sun, or planet, to appear *nearer* to the

horizon than it really is ; hence it increases the zenith distances of those objects. The fixed stars have no sensible parallax ; because their distance from the earth is so inconceivably great that, though seen from opposite points of the earth's orbit, they always appear under the same angle. Hence the diameter of the earth's orbit, which is upwards of 190 millions of miles in extent, is but as a *dimensionless point* compared with the immeasurable distance of those refulgent luminaries.

These being premised, we will now proceed to the proposed method of finding the moon's horizontal parallax.

Rule.

1. Reduce the mean time of the moon's transit over the meridian of Greenwich, as given in page IV. of the month in the Nautical Almanac, to the meridian of the place of observation ; as thus :—

Find the difference of transit between the given day and the day *following*, if the longitude be *west*, but the day *preceding* if *east* ; and it will be the daily retardation of transit. Then say, As the sum of 24 hours and the retardation of transit, is to the retardation ; so is the longitude, in *time*, to a correction : which being applied by *addition* to the time of transit over the meridian of Greenwich, on the given day, if the longitude be *west*, but by *subtraction* if *east* ; the sum, or difference, will be the mean time of transit over the meridian of the place of observation. To this time, let the moon's declination and semidiameter be carefully reduced. Then,

2. If the latitude of the place of observation and the moon's corrected declination be of the same name, take their difference ; but if of contrary names, their sum : in either case the moon's correct meridional zenith distance will be obtained.

3. Let the meridional altitude of the moon's lower or upper limb be very carefully taken at the moment of transit ; to which apply her corrected semidiameter, and the corresponding refraction ; the result will be the apparent meridional altitude of the moon's centre ; the difference between which and 90 degrees will be the moon's apparent meridional zenith distance ; this will be always *greater* than the true meridional zenith distance ; the excess will be the parallactic angle corresponding to the apparent meridional zenith distance. The parallactic angle being thus known, the horizontal parallax may be readily computed in the following manner, viz. :—

4. Suppose the moon's apparent place in the concave arch of the firmament to be at K in the preceding diagram, then her apparent zenith distance is Zb , and her true zenith distance ZW ; the difference between these, viz., the angle $W Kb$, is the parallactic angle in altitude. Then,

As the sine of the apparent zenith distance ZAb , is to the sine of the parallactic angle $W Kb$, so is the sine of the zenith distance or right angle

ZAa , to the sine of the horizontal parallactic angle $V Ia$; but, as this angle is equal to its opposite angle $A I C$, it is therefore equal to the true value of the moon's horizontal parallax, or the angle which the earth's semidiameter, AC , subtends at the moon.

Note.—In the above method of finding the moon's horizontal parallax, it is indispensably necessary that the latitude of the place of observation be very strictly established; and that the meridional altitude be determined with *the most rigid degree of exactness* by means of a sextant and an artificial horizon; both of which must be perfectly free from errors; or the value of the errors, if any, correctly known. If these precautions be attended to, the moon's horizontal parallax may be very accurately obtained; for, since the moon at the instant of her transit over the meridian has *no* parallax in right ascension, but *all* in declination; and since the circle of declination, at that moment, is in the plane of the meridian which cuts the horizon at right angles, the parallax can only affect the meridional zenith distance in a vertical manner, by making it *more than the truth*; and therefore the difference between the apparent and the true zenith distances must be the correct parallactic angle.

The most proper time for determining the lunar parallax, is when the moon's declination is at its maximum; because, then, that luminary, like the sun at the solstices, seems to make a momentary pause before returning towards the equinoctial. At that moment, and *no other*, her *parallel* of declination is *duly bisected by the meridian* of the place of observation; and therefore the agency of her parallax must *depress* her apparent place in the plane of the observer's meridian in a true vertical manner.

Example.

November 6th, 1834, in latitude $51^{\circ}30'49''$ North, and nearly under the meridian of Greenwich, the moon passed the meridian at $4^h49^m36^s$ mean time; at which moment her correct declination was $24^{\circ}28'37''$ South, and her true semidiameter $15'.36''$; required the moon's parallactic angle, and her horizontal parallax?

Latitude of the place of observation	. . .	$51^{\circ}30'49''$ North.
Moon's reduced declination	$24.28.37$ South.

Moon's true meridional zenith distance = . $75^{\circ}59'26''$

The altitude of the moon's lower limb, taken by means of a good sextant and an artificial horizon, at the moment of transit on the given day, was $25^{\circ}46'20''$; the half of which, or $12^{\circ}53'10''$ = the correct

observed altitude of the moon's lower limb. Let this be reduced to the apparent central altitude in the following manner:—

Observed altitude of moon's lower limb =	12° 53' 10"
Moon's semidiameter at time of transit	+ 15. 36
Augmentation of semidiameter, Table IV. =	+ 0. 3
<hr/>	
Approximate altitude	13° 8' 49"
Refraction corresponding to ditto in Table VIII. =	— 4. 1
<hr/>	
Apparent altitude of the moon's centre	13° 4' 48"
<hr/>	
Moon's apparent meridional zenith distance	76° 55' 12"
Moon's correct meridional zenith distance	75. 59. 26
<hr/>	
Difference of ditto =	0° 55' 46"

Which, therefore, is the value of the moon's parallax in altitude, or the true measure of her meridional parallactic angle. This being known, the horizontal parallax is to be deduced therefrom in conformity with the *fourth* section of the rule.—As thus:—

As moon's apparent zenith distance 76° 55' 12", Co-secant =	0. 011416
: Parallactic angle 55' 46" — Sine	8. 210082
:: The zenith distance of 90° 0' 0" — Sine	10. 000000
<hr/>	
: Horizontal parallax 57' 15" — Sine =	8. 221498

Hence, the moon's horizontal parallax is 57' 15"; which differs but *one second* from that given in the Nautical Almanac.

Remarks.—Were the sun not so very remote from the earth, its horizontal parallax could be determined with as much ease as that of the moon; but since the sun is, at a mean rate, about 400 times farther from us than the moon is, its horizontal parallax, or the difference between its true and apparent places in the heavens as seen by two observers at the same instant, the one on the surface and the other at the centre of the earth, becomes so exceedingly acute as not to be easily determined with any degree of accuracy.

The only correct method of finding the sun's horizontal parallax is by means of the transits of Venus over the face of the sun; these, however, are but very rarely seen; for they can only happen when Venus is between the earth and the sun, and when the earth is in the same right line with one of her nodes: and she cannot be in this favourable position with respect to the earth, but after the long intervals of 105, 235, or 243 years. The last transit was in the year 1769; and there cannot be another till the year

1874. However, from a careful comparison of a great number of observations of the transits which took place in the years 1761 and 1769, it has been found that the mean horizontal parallax of the sun is $8''.58$. There is every reason to believe that this is the correct value of the solar parallactic angle reduced to the horizon; because the observations were made in so many different parts of the world, and by men so highly renowned in the annals of science that no reasonable doubts can be entertained of its correctness. The transits were observed at London, at Hackney, and at Liskeard in Cornwall; at Paris, at Stockholm, and at Hernosand in Sweden; at Tobolsk in Siberia; at the Cape of Good Hope; at Madras, and at Calcutta in the East Indies; and yet, in all those places, so widely distant from each other, there was such a remarkable coincidence in the results of the observations as to justify astronomers in coming to the conclusion that the mean value of the sun's horizontal parallax is not more than $8''.58$; which is the measure of the angle that the earth's semidiameter of $3958\frac{1}{4}$ miles, subtends at the sun.

Now, the moon's horizontal parallax being duly established, as stated in page 33, her absolute distance from the earth may be readily determined; for, in the right angled triangle C A I (diagram page 30), the earth's semidiameter A C is given = 3958.75 miles, and the opposite angle A I C = the moon's horizontal parallax $57'.15''$, to find the side C I = the moon's distance from the centre of the earth. Hence, by plane trigonometry:—

As moon's horizontal parallax $57'.15''$, sine comp. arith.	. 1.7785185
Is to the earth's semidiameter—3958.75, logarithm . . .	3.5975581
So is radius 90 degrees—sine	10.0000000

To the moon's true distance = $237726\frac{1}{2}$ miles logarithm = 5.3760766

But since the moon's horizontal parallax varies from about $53'.55''$ to $61'.25''$, the moon's distance from the earth, found as above, will decrease or increase according as her horizontal parallax may be greater or less than $57'.15''$; and hence it is that the distance thus determined is 1034 miles greater than that which is established in page 9.

Note.—Because the distances of the sun and planets are in the *inverse ratio* of the sines of their horizontal parallaxes; therefore, since the solar and lunar parallactic angles are given, and that the moon's distance from the earth is known, the sun's distance from the earth may be readily computed in the following manner, viz.:—

As the sun's horizontal parallax = $8''.58$ sine comp. arith. . .	14.3816667
Is to the moon's horizontal parallax = $57'.15''$ sine . . .	8.2214815
So is the moon's distance from the earth 237726.5 miles log. =	5.3760766

To the sun's distance from the earth = 95328955 miles log. = 7.9792248

If the sun's horizontal parallax be assumed at $8''.65$, its distance from the earth will be 94546196 miles: and thus the small difference of $0''.07$ in the horizontal parallax will produce a difference of 782759 miles in the distance.

TABLE XVII.

Equation of Second Difference.

Since the moon's longitude and latitude require to be strictly determined on various astronomical occasions, and since the reduction of these elements, to a given instant, cannot be performed by even proportion, on account of the great inequalities to which the lunar motions are subject;—a *correction*, therefore, resulting from these inequalities, must be applied to the proportional part of the moon's longitude or latitude, answering to a given period after noon or midnight, as deduced from the preceding Table or otherwise, in order to have it truly accurate. This *correction* is contained in the present Table, the arguments of which are,—the mean second difference of the moon's place at top; and the apparent or Greenwich time past noon, or midnight, in the left or right hand column; in the angle of meeting stands the corresponding equation or correction.

The Table is divided into two parts: the upper part is adapted to the mean second difference of the moon's place in seconds of a degree, and in which the equations are expressed in seconds and decimal parts of a second; the lower part is adapted to minutes of mean second difference; the equations being expressed in minutes and seconds, and decimal parts of a second.

In using this Table, should the mean second difference of the moon's place exceed its limits, the sum of the equations corresponding to the several terms which make up the mean second difference, in both parts of the Table, is, in such case, to be taken. The manner of applying the equation of second difference to the proportional part of the moon's motion in latitude and longitude, as deduced from the preceding Table, or obtained by even proportion, will be seen in the solution of the following

PROBLEM.

To reduce the Moon's Latitude and Longitude, as given in the Nautical Almanac, to any given Time under a known Meridian.

Rule.

Turn the longitude into time (by Table I.), and apply it, to the mean time at ship or place by *addition* in *west*, or *subtraction* in *east* longitude; and the *sum*, or *difference*, will be the corresponding time at Greenwich.

Take from the Nautical Almanac the two longitudes and latitudes, immediately *preceding* and *following* the Greenwich time, and find the difference between each pair successively; find also the second difference, and let its mean be taken.

Find the proportional part of the middle *first* difference (the variation of the moon's motion in 12 hours) by Table XVI., answering to the Greenwich time.

With the mean second difference, found as above, and the Greenwich time, enter Table XVII., and take out the corresponding equation. Now, this equation being *added* to the proportional part of the moon's motion if the first *first difference* is *greater* than the third *first difference*, but *subtracted* if it be *less*, the sum or difference will be the correct proportional part of the moon's motion in 12 hours.

The correct proportional part, thus found, is always to be *added* to the moon's longitude at the noon or midnight preceding the Greenwich time; but to be applied by *addition* or *subtraction* to her latitude, according as it may be *increasing* or *decreasing*.

Example.

Required the moon's correct longitude and latitude, August 2nd, 1824, at 3^h 10^m mean time, in longitude 60° 30' west of the meridian of Greenwich?

Mean time at ship or place	3 ^h 10 ^m
Longitude 60° 30' west in time =	4. 2
Greenwich time	<u>7^h 12^m</u>

To find Moon's correct Longitude :—

			First Diff.	Second Diff.	Mean 2d Diff.
Moon's long. Aug. 1st, at midnt.	7 : 10° 38' 49"				
Do. 2 at noon	7. 17. 16. 27	} 6° 37' 38"		5' 39"	
Do. 2 at midnt.	7. 23. 48. 26	} 6. 31. 59			5' 27½"
Do. 3 at noon	8. 0. 15. 9	} 6. 26. 43		5. 16	
Propor. part from Table XVI., ans. to 7 ^h 12 ^m and 6° 31' 59"	is 3° 55' 11" 24"				
Eq. from Tab. XVII., corres. to 7 ^h 12 ^m and 5' 0"	= 36". 0				
	and 0. 20 = 2 . 4				
	and 0. 7½ = 0 . 9				
Eq. of mean second diff. ans. to 7 ^h 12 ^m and 5' 27½"	is 39". 3 = + 39". 18"				
Correct proportional part of the moon's motion in longitude	3° 55' 50" 42"				
Moon's longitude at noon, August 2d, 1824	7 : 17. 16. 27. 0				
Moon's correct longitude, at the given time	<u>7 : 21 : 12 : 17 : 42"</u>				

To find the Moon's correct Latitude :—

			First Diff.]	Second Diff.	Mean 2d Diff.
Moon's lat Aug. 1st, at midnt.	4°27'37" S.				
Do. 2 at noon	4. 6. 59		20'38"	2'57"	} 2'44"
Do. 2 at midnt.	3. 43. 24		23. 35		
Do. 3 at noon	3. 17. 18		26. 6	2. 31	

Pro. part. from Table XVI., ans. to 7^h12^m and 23'35" is 0°14'9".

Eq. from Tab. XVII., cor. to 7^h12^m and 2' 0 = 14".4

and 0.40 = 4 .8

and 0. 4 = 0 .5

Eq. of mean sec. diff., ans. to 7^h12^m and 2'44" is 19".7 = - 19".7

Correct proportional part of the moon's motion in lat. 0°13'49".3

Moon's latitude at noon, August 2d, 1824 . . . 4. 6. 59 .0 S.

Moon's correct latitude at the given time . . . 3°53' 9".7 south.

Note.—It frequently happens that the three *first differences* first increase and then decrease, or *vice versa*, first decrease and then increase ; in this case *half the difference* of the two second differences is to be esteemed as the mean second difference of the moon's place : as thus,

			First Diff.	Second Diff.	Mean 2d Diff.
Mu's dec. Aug. 18th, 1824, at midt.	24°23'26" N.				
Do. 19 at noon	24. 41. 47		18'21"	14'26"	} 4'26"
Do. 19 at midt.	24. 37. 52		3. 55	23. 18	
Do. 20 at noon	24. 10. 39		27. 13		

Here the two second differences are 14'26", and 23'18" respectively ; therefore half their difference, viz., 8'52" ÷ 2 = 4'26" is the mean second difference. Now, if the Greenwich time be 5^h40^m past noon of the 19th, the corresponding equation in Table XVII. will be 33" *subtractive*, because the first *first difference* is less than the third *first difference* ; had it been greater, the equation would be *additive*.

The equation of second difference, contained in the present Table, was computed by the following

Rule.

To the constant log. 7.540607 add the log. of the mean second difference reduced to seconds ; the log. of the time from noon, and the log. of the difference of that time to 12 hours (both expressed in hours and decimal parts of an hour) : the sum, rejecting 10 from the index, will be the log. of the equation of second difference in seconds of a degree.

Example.

Let the mean second difference of the moon's place be 8 minutes, and the mean time past noon or midnight $3^h 20^m$; required the corresponding equation?

Mean second difference, 8 minutes	= 480 seconds.	Log. = 2.681241
Mean time past noon or midnight	= $3^h.333$	Log. = 0.522835
Difference of do. to 12 hours	$8^h.666$	Log. = 0.937819
Constant log. (ar. co. of log. of 288 = 24×12)	. . .	= 7.540607
<hr/>		
Required equation	$48''.14$	Log. = 1.682502

TABLE XVIII.

Correction of the Moon's apparent Altitude.

By the correction of the moon's apparent altitude is meant the difference between the parallax of that object, at any given altitude, and the refraction corresponding to that altitude.

This correction was computed by the following rule; viz.

To the log. secant of the moon's apparent altitude, add the proportional log. of her horizontal parallax; and the sum, abating 10 in the index, will be the proportional log. of the parallax in altitude; which, being diminished by the refraction, will leave the correction of the moon's apparent altitude.

Example.

Let the moon's apparent altitude be $25^\circ 40'$, and her horizontal parallax 59 minutes; required the correction of the apparent altitude?

Moon's apparent altitude	$25^\circ 40'$	Log. secant = 10.0451
Moon's horizontal parallax	0.59	Propor. log. = 0.4844
<hr/>		
Moon's parallax in altitude	$53' 11''$	= Propor. log. = 0.5295
Refraction ans. to app. alt. in Tab. VIII.	1.58	

Correction of the moon's appar. altitude $51' 13''$

The correction, thus computed, is arranged in the present Table, where it is given to every tenth minute of apparent altitude, and to each minute of horizontal parallax. The proportional part for the excess of the given above the next *less* tabular altitude, is contained in the right-hand column of each page; and that answering to the seconds of parallax is given in the intermediate part of the Table.

This correction is to be taken out of the Table in the following manner ; viz.

Enter the Table with the moon's apparent altitude in the left-hand column, or the altitude next *less* if there be any odd minutes ; opposite to which, and under the minutes of the moon's horizontal parallax, will be found the approximate correction. Enter the compartment of the "Proportional parts to seconds of parallax," abreast of the approximate correction, with the tenths of seconds of the moon's horizontal parallax in the vertical column, and the units at the top ; in the angle of meeting will be found the proportional part for seconds, which *add* to the approximate correction. Then,

Enter the last or right-hand column of the page, abreast of the approximate correction or nearly so, and find the proportional part corresponding to the odd minutes of altitude. Now, this being added to or subtracted from the approximate correction, according to its sign, will leave the true correction of the moon's apparent altitude. And since the apparent altitude of a celestial object is depressed by parallax and raised by refraction, and the lunar parallax being always greater than the refraction to the same altitude, it hence follows that the correction, thus deduced, is always to be applied by *addition* to the moon's apparent altitude.

Example 1.

Let the moon's apparent altitude be $8^{\circ}38'$, and her horizontal parallax $57'.46''$; required the corresponding correction ?

Correction to alt. $8^{\circ}30'$, and horiz. parallax $57'.0''$ is	$50'.14''$
Propor. part to 46 seconds of horiz. parallax . . . +	0.46
Do. to 8 min. of alt. $(8' \times 0''.5 = 4''.0)$ = +	0.4
	<hr/>

Correction of the moon's apparent altitude, as required $51'.4''$

Example 2.

Let the moon's apparent altitude be $33^{\circ}16'$, and her horizontal parallax $59'.34''$; required the corresponding correction ?

Correction to alt. $33^{\circ}10'$ and horiz. parallax $59'.0''$ is	$47'.56''$
Propor. part to 34 seconds of horiz. parallax . . . +	28
Do. to 6 minutes of altitude -	3
	<hr/>

Correction of the moon's apparent altitude, as required $48'.21''$

TABLE XIX.

To reduce the True Altitudes of the Sun, Moon, Stars, and Planets, to their Apparent Altitudes.

This Table is particularly useful in that method of finding the longitude by lunar observations, where the distance only is given, and where, of course, the altitudes of the objects must be obtained by computation.

The Table consists of two pages, each page being divided into two parts: the left-hand part contains four columns; the first of which comprehends the true altitude of the sun or star; the second the reduction of the sun's true altitude; the third the reduction of a star's true altitude; and the fourth the common difference of those reductions to 1 minute of altitude for sun or star.

The other part of the Table is appropriated to the moon; in which the true altitude of that object is given in the column marked "Moon's true altitude," and her horizontal parallax at top or bottom; the two last or right-hand columns of each page contain the difference to 1 minute of altitude, and 1 second of parallax respectively; by means of which the reduction may be easily taken out to minutes of altitude and seconds of horizontal parallax.

The first part of the Table is to be entered with the sun's or star's true altitude (or the altitude next *less* when there are any odd minutes, as there generally will be), in the left-hand column; abreast of which, in the proper column, will be found the approximate reduction; from which let the product of the difference to 1 minute by the excess of the odd minutes above the tabular altitude, be subtracted, and the remainder will be the true reduction of altitude for sun or star.

Example 1.

Let the true altitude of the sun's centre be $8^{\circ}15'$; required the reduction to apparent altitude?

Correction corresponding to altitude 8 degrees	6' 15"
Cor. for min. of alt.; viz. diff. to 1 min. of alt.	$= 0''70 \times 15' = 10''.5 = -10$	
Required reduction =	<u>6' 5"</u>

Example 2.

Given the true altitude of a star $19^{\circ}45'$; the reduction to apparent altitude is required?

Correction corresponding to altitude 19 degrees	2'44"
Cor. for min. of alt.; viz. diff. to 1 min. of alt. = $0''.15 \times 45' = 6''.75 = -7$	
Required reduction =	<u>2'37"</u>

In the case of a planet, proceed the same as if it were a fixed star; then, the tabular reduction being diminished by the value of the planet's parallax in altitude, Table VI., the result will be its true correction of altitude.

The reduction of the moon's true altitude is to be taken from the second part of the Table, by entering that part with the true altitude in the proper column (or the altitude next *less* when there are any odd minutes) and the horizontal parallax at top or bottom; in the angle of meeting will be found a correction; to which apply the product of the difference to 1 minute by the excess of the odd minutes above the tabular altitude by *subtraction*,* and the product of the difference to 1 second by the odd seconds of parallax by *addition*: and the true reduction will be obtained, as may be seen in the following

Example.

Let the true altitude of the moon's centre be $29^{\circ}13'$, and her horizontal parallax $58'37''$; required the corresponding reduction to apparent altitude?

Correc. corres. to alt. 29 degs., and horiz. parallax $58' =$. .	49'22"
Cor. for min. of alt.; viz., diff. to 1 min. of alt. = $0''.41 \times 13' = 5''.3 = -5$	
Cor. for secs. of par.; viz. diff. to 1 sec. of par. = $0''.90 \times 37'' = 33''.3 = +33$	
Required reduction	<u>49'50"</u>

Remark.—The reduction of the sun's true altitude is obtained by increasing that altitude by the difference between the refraction and parallax corresponding thereto: then, the difference between the refraction and parallax answering to that augmented altitude, will be the reduction of the true altitude.

Example.

Let the true altitude of the sun's centre be 5 degrees; required the reduction to apparent altitude?

* When the moon's altitude is less than 15 degrees, the product is to be applied by *addition*.

Sun's true altitude	5° 0' 0"	
Refract. Tab. VIII.	= 9' 54"	} diff. + 9' 45"	
Paral. Table VII.	0. 9		
<hr/>			
Augmented altitude	5° 9' 45"	refrac. ans. to which is 9' 38"
			and parallax . 0. 9
<hr/>			
Required reduction =		9' 29"

The correction for reducing a star's true altitude to its apparent, is obtained in the same manner, omitting what relates to parallax. Thus, if the true altitude of a star be 8 degrees, and the corresponding refraction 6' 29", their sum, viz., 8° 6' 29" will be the augmented altitude; the refraction answering to this is 6' 24", which, therefore, is the reduction of the true to the apparent altitude of the star.

The correction for reducing the true altitude of the moon to the apparent, is found by diminishing the true altitude by the difference between the parallax and refraction answering thereto; then the difference between the parallax and refraction corresponding to the altitude so diminished, will be the reduction of the true to the apparent altitude. As thus:—

Let the true altitude of the moon's centre be 10 degrees, and her horizontal parallax 57 minutes; required the reduction to apparent altitude?

Moon's true altitude	10° 0' 0"	Log. secant	10.0066
Do. horizontal parallax	57' 0"	Propor. log.	0.4994
<hr/>				
Parallax in altitude	56' 8"	Propor. log.	0.5060
Refrac. to altitude 10°, Table VIII.	=	5.15		
<hr/>				
Difference between parallax and refrac.	=	50' 53"		
<hr/>				
Diminished altitude	9° 9' 7"	Log. secant	10.0056
Horizontal parallax	57. 0	Propor. log.	0.4994
<hr/>				
Parallax in altitude	56' 16"	Propor. log.	0.5050
Refrac. to diminished alt. Table VIII.		5.42		
<hr/>				
Difference	50' 34"	; which, therefore, is the required reduction.	

TABLE XX.

Auxiliary Angles.

Since the solution of the Problem for finding the longitude at sea, by celestial observation, is very considerably abridged by the introduction of

an *auxiliary angle* into the operation, the true central distance being hence readily determined to the nearest second of a degree by the simple addition of five natural versed sines ; this Table has, therefore, been computed ; and to render it as convenient as possible, it is extended to every tenth minute of the moon's apparent altitude, and to each minute of her horizontal parallax ; with proportional parts adapted to the intermediate minutes of altitude, and to the seconds of horizontal parallax.

This Table was calculated in the following manner :—

To the moon's apparent altitude apply the correction from Table XVIII., and the sum will be her true altitude ; from the log. cosine of which (the index being augmented by 10) subtract the log. cosine of her apparent altitude, and the remainder will be a log., which, being diminished by the constant log. .300910,* will give the logarithmic cosine of the auxiliary angle.

Example.

Let the moon's apparent altitude be 4 degrees, and her horizontal parallax 55 minutes ; required the corresponding auxiliary angle ?

Moon's apparent altitude .	4° 0' 0"	Log. cosine . .	9.998941
Correction from Table XVIII. +	43. 2		

Moon's true altitude . .	4° 43' 2"	Log. cosine .	9.998527
		Log. . . .	9.999586
		Constant log.	0.300910

Auxiliary angle, as required $60^{\circ} 1' 21'' = \text{Log. cosine} . 9.698676$

The correction of the auxiliary angle for the sun's or star's apparent altitude, given at the bottom of each page of the Table, was computed by the following rule—viz.

From the log. cosine of the sun's or star's true altitude subtract the log. cosine of the apparent altitude, and find the difference between the remainder and the constant log. .000120.† Now, this difference being subtracted from the log. cosine of 60 degrees, will leave the log. cosine of an arch ; the difference between which and 60 degrees will be the correction of the auxiliary angle depending on the apparent altitude of the sun or star.

* This is the log. secant, less radius, of 60 degrees diminished by .000120, the difference between the log. cosines of a star's true and apparent altitude betwixt 30 and 90 degrees.

† This is the difference between the log. cosines of a star's true and apparent altitude between 30 and 90 degrees.

Example.

Let the sun's or star's apparent altitude be 3 degrees ; required the correction of the auxiliary angle ?

Sun's apparent altitude 3° 0' 0" Log. cosine 9.999404

Refract. Table VIII. 14'36" }
Parallax. Table VII. 9 } difference—14'27"

Sun's true altitude 2°45'33" Log. cosine 9.999497

Remainder 0.000093

Const. log. 0.000120

Difference 0.000027

60° 0' 0" Log. cosine 9.698970

Arch 60. 0. 8 Log. cosine 9.698943

Difference 0° 0' 8"; which, therefore, is the required correction of the auxiliary angle.

In this Table the auxiliary angle is given to every tenth minute of the moon's apparent altitude (as has been before observed) from the horizon to the zenith, and to each minute of horizontal parallax. The proportional part for the excess of the given, above the next *less* tabular altitude is contained in the right-hand column of each page; and that answering to the seconds of parallax is given in the intermediate part of the Table. The correction depending on the sun's or star's apparent altitude is placed at the bottom of the Table in each page.

As the size of the paper would not admit of the complete insertion of the auxiliary angle, except in the first vertical column of each page under or over 53' ; therefore, in the eight following columns, it is only the excess of the auxiliary angle above 60 degrees that is given : hence, in taking out the auxiliary angle from those columns, it is always to be prefixed with 60 degrees.

The auxiliary angle is to be taken out of the Table, as thus :—

Enter the Table with the moon's apparent altitude in the left-hand column of the page, or the altitude next *less* if there be any odd minutes, opposite to which and under the minutes of the moon's horizontal parallax at top, will be found the approximate auxiliary angle.

Enter the compartment of the "Proportional parts to seconds of parallax," abreast of the approximate auxiliary angle, with the tenths of seconds of the moon's horizontal parallax in the vertical column, and the units at the top ; in the angle of meeting will be found a correction, which place

under the approximate auxiliary angle ; then enter the last or right-hand column of the page abreast of where the approximate auxiliary angle was found, or nearly so, and find the proportional part corresponding to the odd minutes of altitude, which place under the former. To these *three* let the correction, at the bottom of the Table, answering to the sun's or star's apparent altitude, be applied, and the *sum* will be the correct auxiliary angle.

Example.

Let the moon's apparent altitude be $25^{\circ}37'$, the sun's apparent altitude $58^{\circ}20'$, and the moon's horizontal parallax $59'.47''$; required the corresponding auxiliary angle ?

Aux. angle ans. to moon's app. alt. $25^{\circ}30'$, and hor. par. $59'$ is $60^{\circ}13'.47''$	
Proportional parts to 47 seconds of horizontal parallax is	12
Proportional part to 7 minutes of altitude is	4
Correction corresponding to sun's app. alt. ($58^{\circ}20'$) is	4
<hr/>	
Auxiliary angle, as required	$60^{\circ}14'.7''$

TABLE XXI.

Correction of the Auxiliary Angle when the Moon's Distance from a Planet is observed.

The arguments of this Table are, a planet's apparent altitude in the left or right hand column, and its horizontal parallax at top ; in the angle of meeting stands the correction, which is always to be applied by *addition* to the auxiliary angle deduced from the preceding Table : hence, if the apparent altitude of a planet be 26 degrees, and its horizontal parallax 23 seconds, the correction of the auxiliary angle will be 6 seconds, additive.

This Table was calculated by a modification of the rule (page 43) for computing the correction of the auxiliary angle, answering to the sun's or star's apparent altitude ; as thus :—

To the logarithmic secant of the planet's apparent altitude, add the logarithmic cosine of its true altitude, and the constant logarithm 9.698850 ;* and the sum (abating 20 in the index) will be the logarithmic cosine of an arch ; the difference between which and 60 degrees will be the required correction.

* This is the log. cosine of 60 degrees diminished by .000120, the difference between the log. cosines of the true and apparent altitude of a fixed star between 30 and 90 degrees:

Example

Let the apparent altitude of a planet be 30 degrees, and its horizontal parallax 23 seconds : required the correction of the auxiliary angle ?

Planet's apparent altitude . . . 30° 0' 0" Log. secant 10.062469

Refrac. Table VIII. 1'38" }
Parallax, Table VI. 0.20 } difference — 1'18"

True altitude of the planet . . . 29°58'42" Const. log. 9.698850
Log. cosine 9.937626

Arch = 60° 0' 7" = Log. cosine 9.698945
60. 0. 0

Difference 0° 0' 7" ; which is the required correction.

TABLE XXII.

Error arising from a Deviation of one Minute in the Parallelism of the Surfaces of the Central Mirror of the Circular Instrument of Reflection.

This Table contains the error of observation arising from a deviation of *one minute* in the parallelism of the surfaces of the central mirror of the reflecting circle, the axis of the telescope being supposed to make an angle of 80 degrees with the horizon mirror; it is very useful in finding the verification of the parallelism of the surfaces of the central mirror in the reflecting circle, or of the index-glass in the sextant ; as thus :—

Let the instrument be carefully adjusted, and then take four or five observations of the angular distance between two *well-defined* objects, whose distance is not less than 100 degrees ; the sum of these, divided by their number, will be the mean observation. Then,

Take out the central mirror, and turn it so that the edge which was before uppermost may now be downwards, or next the plane of the instrument ; rectify its position, and take an equal number of observations of the angular distance between the same two objects, and find their mean, as before : now, half the difference between the mean of these and that of the former, will be the error of the mirror answering to the observed angle. If the first mean exceeds the second, the error is subtractive ; otherwise additive : the mirror being in its first or natural position. Hence, if the

mean of the first set of observations be $115^{\circ}0'40''$, and that of the second $114^{\circ}59'20''$, half their difference, viz., $1'20'' \div 2 = 40''$, will be the error of the observed angle, and is subtractive; because the first mean angular distance, or that taken with the mirror in its natural position, is greater than the second, or that taken with the mirror inverted.

Having thus determined the error of the observed angle, that answering to any given angle may be readily computed by means of the present Table, as follows:—

Enter the left-hand column of the Table with the angular distance, by which the error of the central mirror was determined, and take out the corresponding number from the adjoining column, or that marked “Observation to the right;” in the same manner take out the number answering to the given angle; then,

To the arithmetical complement of the proportional log. of the *first* number, add the proportional log. of the second, and the proportional log. of the observed error; the sum of these three logs., rejecting 10 from the index, will be the proportional log. of the error answering to such given angle.

Example.

Having found the error arising from a defect of parallelism in the central mirror, at an angle of 115 degrees, to be 40 seconds subtractive; required the error corresponding to an angle of 85 degrees?

Obs. ang. 115 deg. opp. to which is $3'23''$	Arith. comp. prop. log.	=	8.2741
Given ang. 85 deg. opp. to which is $1'15''$	Propor. log. . .	=	2.1584
Observed error of central mirror 0.40	Propor. log. . .	=	2.4313
<hr/>			
Required error =	— $0'15''$	=	Propor. log. . = 2.8638

TABLE XXIII.

Error of Observation arising from an Inclination of the Line of Collimation to the Plane of the Sextant, or to that of the circular Instrument of Reflection.

If the line of sight is not parallel to the plane of the instrument, the angle measured by such instrument will always be greater than the true angle. This Table contains the error arising from that cause, adapted to the most probable limits of the inclination of the line of collimation, and to any angle under 120 degrees: hence the arguments of the Table are, the observed angle in the left-hand column, and the inclination of the line

of collimation at top ; opposite the former, and under the latter, will be found the corresponding correction.

Thus, if the observed angle be 80 degrees, and the inclination of the line of collimation 30 minutes, the corresponding error will be 13 seconds. The error or correction taken from this Table is always to be applied by *subtraction* to the observed angle.

The corrections in this Table were computed by the following

Rule.

To the log. sine of half the observed angle, add the log. cosine of the inclination of the line of collimation ; and the sum, rejecting 10 in the index, will be the log. sine of an arch. Now, the difference between twice this arch and the observed angle, will be the error of the line of collimation.

Example.

Let the observed angle be 80 degrees, and the inclination of the line of collimation 1°30' ; required the corresponding correction ?

Observed angle	80° 0' 0"	+ 2 =	40°	Log. sine	9.808068
Inclinat. of line of collim.		1°30'	Log. cosine	9.999851

Arch =	39°59' 1"	=	Log. sine	9.807919
--------	-----------	---	-----------	----------

Twice the arch = 79°58' 2"

Difference 0° 1'58", which, therefore, is the required error.

TABLE XXIV.

Logarithmic Difference.

This Table contains the logarithmic difference, adapted to every tenth minute of the moon's apparent altitude from the horizon to the zenith, and to each minute of horizontal parallax. The proportional part for the excess of the given above the next *less* tabular altitude, is contained in the right-hand compartment of each page, and that answering to the seconds of parallax is given in the intermediate part of the Table.

As the size of the paper would not admit of the complete insertion of the logarithmic difference, except in the first vertical column of each page, under or over 53', therefore in the eight following columns it is only the

four last figures of the logarithmic difference that are given: hence, in taking out the numbers from these columns, they are always to be prefixed by the characteristic, and the two leading figures in the first column. The logarithmic difference is to be taken out in the following manner.

Enter the Table with the moon's apparent altitude in the left-hand column of the page, or the altitude next *less* if there be any odd minutes, opposite to which, and under the minutes of the moon's horizontal parallax, at top, will be found a number, which call *the approximate logarithmic difference*.

Enter the compartment of the "Proportional parts to seconds of parallax," abreast of the approximate logarithmic difference, with the tenths of seconds of the moon's horizontal parallax in the vertical column, and the units at the top, and take out the corresponding correction. Enter the right-hand compartment of the page,* abreast of where the approximate logarithmic difference was found, or nearly so, with the odd minutes of altitude, and take out the corresponding correction, which place under the former. Enter Table XXV. or XXVI., with the sun's, star's, or planet's apparent altitude, and take out the corresponding correction, which also place under the former. Now, the sum of these three corrections being taken from the approximate logarithmic difference, will leave the correct logarithmic difference.

Example 1.

Let the moon's apparent altitude be $19^{\circ}25'$, her horizontal parallax $60'.38''$, and the sun's apparent altitude 33 degrees; required the logarithmic difference?

Log. difference to app. alt. $19^{\circ}20'$, and hor. par. $60'$ is	9.997669	
Propor. part to 38 seconds of parallax is	. 28	} sum = - 49
Propor. part to 5 minutes of altitude is	. . 11	
Cor. from Tab. XXV. ans. to sun's apparent alt. is	10	

Logarithmic difference, as required 9.997620

* In taking out the correction corresponding to the odd minutes of altitude in this compartment, attention is to be paid to the moon's horizontal parallax: thus, if the parallax be between $53'$ and $56'$, the correction is to be taken out of the first column, or that adjoining the minutes of altitude; if it be between $56'$ and $59'$, the correction is to be taken out of the second, or middle column; and if it be between $59'$ and $62'$, the correction is to be taken out of the third, or last column.

Example 2.

Let the moon's apparent altitude be $63^{\circ}37'$, her horizontal parallax $58'.43''$, the apparent altitude of a planet $35^{\circ}10'$, and its horizontal parallax $23''$; required the logarithmic difference?

Log. difference to appar. alt. $63^{\circ}30'$, and hor. par. $58'$, is	9.993622
Propor. part to $43''$ of parallax is 83
Propor. part to $7'$ of altitude is 7
Cor. from Tab. XXVI. ans. to planet's appar. alt.	28
} sum = -118	

Logarithmic difference, as required 9.993504

Remark.—The logarithmic difference was computed by the following

Rule.

To the logarithmic secant of the moon's apparent altitude, add the logarithmic cosine of her true altitude, and the constant log. .000120;* the sum of these three logs., abating 10 in the index, will be the logarithmic difference.

Example.

Let the moon's apparent altitude be $19^{\circ}20'$, and her horizontal parallax 60 minutes; required the logarithmic difference?

Moon's apparent altitude $19^{\circ}20' 0''$	Log. secant	10.025208
Correction from Table XVIII.	53.56	Constant log.	0.000120
<hr/>			
Moon's true altitude $20. 13. 56$	Log. cosine	9.972341
<hr/>			
Logarithmic difference, as required		9.997669

* The difference between the log. cosines of the true and apparent altitude of a star betwixt 30 and 90 degrees.

TABLE XXV.

Correction of the Logarithmic Difference.

This Table is divided into two parts: the first, or left-hand part, contains the correction of the logarithmic difference when the moon's distance from the sun is observed; and the second, or right-hand part, the correction of that log. when the moon's distance from a star is observed. Thus, if the sun's apparent altitude be 35 degrees, the corresponding correction will be 11; if a star's apparent altitude be 20 degrees, the corresponding correction will be 1; and so on. These corrections are always to be applied by *subtraction* to the logarithmic difference deduced from the preceding Table.

The corrections contained in this Table were obtained in the following manner, viz.

To the log. secant of the apparent altitude, add the log. cosine of the true altitude; and the sum, rejecting 10 from the index, will be a log.; which being subtracted from the constant log. .000120,* will leave the tabular correction.

Example 1.

Let the sun's apparent altitude be 35 degrees; required the tabular correction?

Given apparent altitude	=	35° 0' 0"	Log. secant	=	10.086635
Refrac. Table VIII.	1'21"	} diff. = -1.14			
Parallax Table VII.	7				
<hr/>					
Sun's true altitude	34°58'46"	Log. cosine	9.913474
				Sum 0.000109
				Constant log.	0.000120
<hr/>					
Tabular correction, as required				0.000011

Example 2.

Let the apparent altitude of a star be 10 degrees; required the tabular correction?

* See Note, page 50.

Star's apparent altitude	10° 0' 0"	Log. secant =	10.006649
Refraction Table VIII.	— 5.15		
<hr/>			
Star's true altitude	. . 9.54.45	Log. cosine =	9.993467
<hr/>			
		Sum = . .	0.000116
		Constant log.	0.000120
<hr/>			
Tabular correction, as required		0.000004

TABLE XXVI.

Correction of the Logarithmic Difference when the Moon's Distance from a Planet is observed.

The arguments of this Table are, the apparent altitude of a planet in the left or right-hand marginal column, and its horizontal parallax at top; in the angle of meeting stands the corresponding correction, which is to be applied by *subtraction* to the logarithmic difference deduced from Table XXIV., when the moon's distance from a planet is observed. Hence, if the apparent altitude of a planet be 20 degrees, and its horizontal parallax 21 seconds, the corresponding correction will be 16 subtractive, and so on.

This Table was computed by the rule in page 51, under which the correction corresponding to the sun's apparent altitude in Table XXV. was obtained, as thus :—

Let the apparent altitude of a planet be 23 degrees, and its horizontal parallax 21 seconds; required the correction of the logarithmic difference?

Planet's apparent altitude	23° 0' 0"	Log. secant	10.035974
Refrac. Table VIII. = 2'.14"	} diff. = - 1.54		
Parallax Table VI. = 20"			
<hr/>			
Planet's true altitude	22°58' 6"	Log. cosine	9.964128
<hr/>			
		Sum . .	0.000102
		Constant log.	0.000120
<hr/>			
Correction of the logarithmic difference, as required			0.000018

TABLE XXVII.

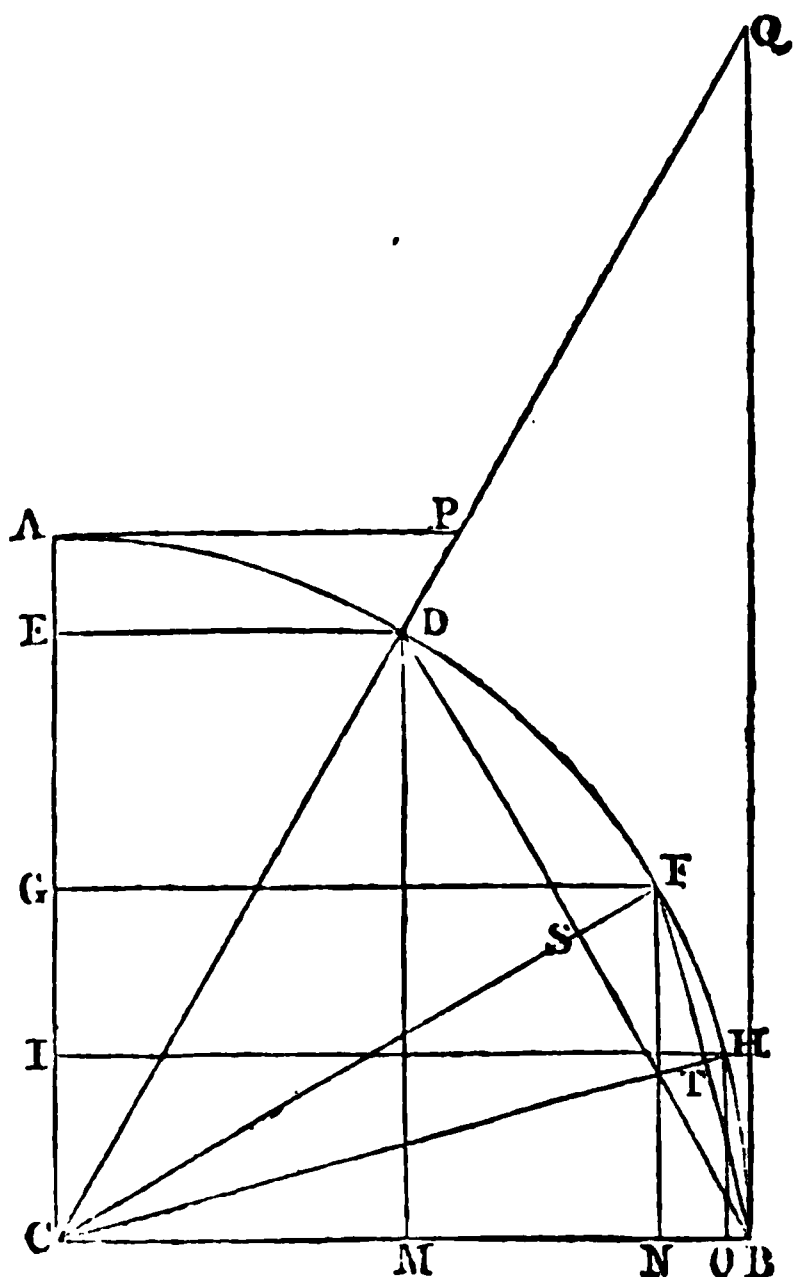
Natural Versed Sines, and Natural Sines.

Since the methods of computing the true altitudes of the heavenly bodies, the apparent time at ship or place, and the true central distance between the moon and sun, or a fixed star, are considerably facilitated by the application of natural versed sines, or *natural sines*, this Table is given; which, with the view of rendering it generally useful and convenient, is extended to every tenth second of the semicircle, with proportional parts corresponding to the intermediate seconds; so that either the natural versed sine, natural versed sine supplement, natural co-versed sine, *natural sine* or *natural cosine* of any arch, may be readily taken out at sight.

The numbers expressed in this Table may be obtained in the following manner:—

Let $A B C$ represent a quadrant, or the fourth part of a circle; and let the radius $C B =$ unity or 1, be divided into an indefinite number of decimal parts: as thus, 1.0000000000, &c. Make $B D =$ the radius $C B$; and since the radius of a circle is equal to the chord of 60 degrees, the arc $B D$ is equal to 60 degrees: draw $D M$, the sine of the arc $B D$, and, at right-angles thereto, the cosine $D E$: bisect the arc $B D$ in F , and draw $F N$ and $F G$ at right-angles to each other; then will the former represent the sine, and the latter the cosine of the arc $B F = 30$ degrees: bisect $B F$ in H ; then $H O$ will express the sine, and $H I$ the cosine of the arc $B H = 15$ degrees.

Proceeding in this manner, after 12 bisections, we come to an arc of $0^{\circ}0'52''44'''3''''45'''''$, the cosine of which approximates so very closely to the radius C B, that they may be considered as being of equal value. Now, the absolute measure of this arc may be obtained by numerical calculation, as follows, viz.



Because the chord line BD is the side of a hexagon, inscribed in a circle, it is the subtense of 60 degrees, and, consequently, equal to the radius CB (corollary to Prop. 15, Book IV., of Euclid); wherefore half the radius $BS = BM$, will be the sine of 30 degrees $= FN$, which, therefore, is .5000000000. Now, having found the sine of 30 degrees, its cosine may be obtained by Euclid, Book I., Prop. 47: for in the right-angled triangle FCN , the hypotenuse FC is given $=$ the radius, or 1.0000000000, and the perpendicular $FN =$ half the radius, or .5000000000, to find the base $CN =$ the cosine GF ; therefore

$\sqrt{FC \times FC - FN \times FN} = CN .8660254037$, or its equal GF ; hence the sine of 30 degrees is .5000000000, and its cosine .8660254037. Again,

In the triangle $FN B$, the perpendicular FN is given $= .5000000000$, and the base $CB - CN = NB = .1339745963$, to find the hypotenuse BF : but half the side of a polygon, inscribed in a circle, is equal to the sine of half the circumscribing arc; therefore its half, $BT = HO$, will be the sine of the arc of 15 degrees: hence $\sqrt{FN \times FN + NB \times NB} = BF .5176380902$; the half of which, viz., .2588190451, is therefore equal to BT , or to its equal HO , the sine of 15 degrees, and from which its cosine HI may be easily obtained; for, in the triangle COH , the hypotenuse CH is given $= 1.0000000000$, and the perpendicular $HO = .2588190451$, to find the base $CO =$ the cosine HI . Now,

$\sqrt{CH \times CH - HO \times HO} = CO .9659258263 =$ the cosine HI ; hence the sine of 15 degrees is .2588190451, and its cosine .9659258263.

Thus proceeding, the sine of the 12th bisection, viz., $52^{\circ}44'3''45'''$, will be found $= .0002556634$. And because small arcs are very nearly as their corresponding sines, the measure of 1 minute may be easily deduced from the sine of the small arc, or 12th bisection determined as above; for,

As the arc of $52^{\circ}44'3''45'''$ is to an arc of 1 minute, so is the sine of the former to the sine of the latter: that is, as $52^{\circ}44'3''45''' : 1' :: .0002556634 : .0002908882$; which, therefore, is the sine of 1 minute, the cosine of which is .9999999577; but this approximates so very closely to the radius, that it may be esteemed as being actually equal to it in all calculations; and hence, that the cosine of $1'$ is 1.0000000000.

Now, having thus found the sine and cosine of *one minute*, the sines of every minute in the quadrant may be obtained by the following rule; viz.

As radius is to twice the cosine of 1 minute, so is the sine of a mean arc to the sum of the sines of the two equidistant extremes; from which let either extreme be subtracted, and the remainder will be the sine of the other extreme: as thus,

To find the Sine of the Arc of 2 Minutes.

As radius = 1 : 2 :: .0002908882 to .0005817764, and .0005817764 — .0000000000 = .0005817764; which, therefore, is the sine of the arc of 2 minutes.

To find the Sine of 3 Minutes.

As radius = 1 : 2 :: .0005817764 to .0011635528, and .0011635528 — .0002908882 = .0008726646; which, therefore, is the sine of an arc of 3 minutes.

To find the Sine of 4 Minutes.

As radius = 1 : 2 :: .0008726646 to .0017453292, and .0017453292 — .0005817764 = .0011635528; which, therefore, is the sine of the arc of 4 minutes.

To find the Sine of 5 Minutes.

As radius = 1 : 2 :: .0011635528 to .0023271056, and .0023271056 — .0008726646 = .0014544407; which, therefore, is the sine of the arc of 5 minutes.

In this manner, the sines may be found to 60 degrees; from which, to the end of the quadrant, they may be obtained by addition only; for the sine of an arc greater than 60 degrees, is equal to the sine of an arc as much less than 60, augmented by the sine of the excess of the given arc above 60 degrees: thus,

All the sines being found to 60 degrees; required the sine of 61 degrees?

Solution.—Sine of 59° = .8571673, and sine of 1° = .0174524; their sum = .8746197; which, therefore, is the sine of 61 degrees, as required. Again,

All the sines being found to 60 degrees; required the sine of 62 degrees?

Solution.—Sine of 58° = .8480481, and sine 2° = .0348995; their sum = .8829476; which, therefore, is the sine of 62 degrees, as required.

Now, the natural sines being thus found, the natural versed sines, natural tangents, and natural secants, may be readily deduced therefrom, agreeably to the principles of similar triangles, as demonstrated in Euclid, Book VI., Prop. 4. Thus,

To find the Natural Versed Sine of 30 Degrees = NB, in the Diagram.

Since the versed sine of an arc is represented by that part of the diameter which is contained between the sine and the arc ; therefore NB is the versed sine of the arc BF, which is the arc of 30 degrees ; and since the versed sines are measured upon the diameter, from the extremity B to C continued to the other extremity, the natural versed sines under 90 degrees are expressed by the difference between the radius and the cosine, and those above 90 degrees by the sum of the radius and the sine : hence, the radius CB 1.0000000 — the cosine FG, or its equal NC .8660254 = NB .1339746 ; which, therefore, is the natural versed sine of 30 degrees.

To find the Natural Tangent of 60 degrees = BQ, in the Diagram.

As the cosine CM is to the sine DM, so is the radius CB to the tangent BQ : that is,

As CM .5000000 : DM .8660254 :: CB 1.0000000 : BQ = 1.7320508 ; which is the natural tangent of 60 degrees.

To find the Natural Secant of 60 Degrees = CQ, in the Diagram.

As the cosine CM is to the radius CD, so is the radius CB to the secant CQ : that is,

As CM .5000000 : CD 1.0000000 :: CB 1.0000000 : CQ = 2.0000000 ; which is the natural secant of 60 degrees. Hence, the manner of computing the natural co-tangent AP, the natural co-secant CP, and the natural co-versed sine EA, will be obvious. The versed sine supplement of an arc is represented by the difference between the versed sine of that arc and the diameter or twice the radius : thus, the versed sine supplement of the arc BF is expressed by the difference between twice the radius CB, and the versed sine NB ; viz., twice CB = 2.0000000 — NB .1339746 = 1.8660254 ; which, therefore, is the natural versed sine supplement of the arc BF or the arc of 30 degrees, and so of any other.

Now, the natural sines, versed sines, tangents, and secants, found as above, being principally decimal numbers, on account of the radius being assumed at unity or 1 ; therefore, in order to render these numbers all affirmative, they are to be multiplied by ten thousand millions respectively ; and then the common logs. corresponding thereto will be the logarithmic sines, versed sines, tangents, and secants, which are generally given in the different mathematical Tables under these denominations.

Of the Table.

In this Table, the natural versed sines are given to every tenth second of the semicircle; the corresponding arcs being arranged at the top, in numerical order, from 0 to 180 degrees. The natural versed sines supplement are given to the same extent; but their corresponding arcs are placed at the bottom of the Table, and numbered from the right hand towards the left, or contrary to the order of the versed sines. The natural co-versed sines begin at the end of the first quadrant, or of the 90th degree of the versed sines; the arcs corresponding to which are given at the bottom of the page and numbered, like the versed sines supplement, towards the left hand from 0 to 90 degrees, and then continued at top of the page from 90 to 180 degrees, towards the right hand, until they terminate at the 90th degree of the versed sines, where they first began. The *natural sines* begin where the co-versed sines end; viz., at the end of the first quadrant, or 90th degree of the versed sines, with which they increase by equal increments; the arcs corresponding to those are placed at the top of the page to every tenth second of the quadrant, the 90th degree of which terminates with the 180th of the versed sines. The *natural cosines* begin with the versed sines supplement; the arcs corresponding to which are given at the bottom of the page, being numbered, like the latter, contrary to the order of the versed sines and *natural sines*, to every tenth second from 0 to 90 degrees, or to the end of the first quadrant of the versed sines, thus ending where the co-versed sines begin.

Note.—In the general use of this Table, it is to be remarked, that the natural versed sine supplement, natural co-versed sine under 90 degrees, or *natural cosine*, of a given degree, is found in the same page with the *next less* degree in the column marked 0" at top, it being the first number in that column; that answering to a given degree and minute is found on the same line with the *next less* minute in the column marked 60" at the bottom of the page; and that corresponding to an arch expressed in degrees, minutes, and seconds, is obtained by deducting the proportional part, at bottom of the page, from the natural versed sine supplement, natural co-versed sine under 90 degrees, or *natural cosine* of the given degree, minute, and *less* tenth second.

PROBLEMS TO ILLUSTRATE THE USE OF THE TABLE.

PROBLEM.

To find the Natural Versed Sine, Natural Versed Sine Supplement, Natural Co-versed Sine, Natural Sine, and Natural Cosine, of any given Arch, expressed in Degrees, Minutes, and Seconds.

RULE.

Enter the Table, and find the natural versed sine, versed sine supplement, co-versed sine, *natural sine*, or *natural cosine*, answering to the given degree, minute, and *next less tenth second* ; to which add the proportional part answering to the odd seconds, found at the bottom of the page, if a natural versed sine, co-versed sine above 90°, or *natural sine* be wanted ; but subtract the proportional part, if a versed sine supplement, co-versed sine under 90°, or *natural cosine*, be required : and the sum, or remainder, will be the natural versed sine, *natural sine*, natural versed sine supplement, co-versed sine, or *natural cosine*, of the given arch.

Example 1.

Required the natural versed sine, versed sine supplement, co-versed sine, *natural sine*, and *natural cosine*, answering to 42°12'36"?

To find the Natural Versed Sine :—

Natural versed sine to	42°12'30"	=	259293
Proportional part to	.	6"	=	Add	20
<hr/>									
Given Arch	42°12'36"	Natural versed sine =	259313						

To find the Versed Sine Supplement :—

Versed sine supplement to	42°12'30"	1.740707
Proportional part to	6"	.	.	.	Subtract				20
<hr/>									
Given arch	42°12'36"	Versed sine sup. =					1.740687		

To find the Co-versed Sine :—

Co-versed sine to	42°12'30"		328172
Proportional part to	6"	Subtract	21
<hr/>			
Given arch	42°12'36"	Co-vers. sine =	328151

To find the Natural Sine :—

Natural sine to	42°12'30"		671828
Proportional part to	6"	Add	21
<hr/>			
Given arch	42°12'36"	Nat. sine =	671849

To find the Natural Cosine :—

Natural cosine to	42°12'30"		740707
Proportional part to	6"	Subtract	20
<hr/>			
Given arch	42°12'36"	Nat. cosine =	740687

Example 2.

Required the natural versed sine, versed sine supplement, co-versed sine, *natural sine*, and *natural cosine*, answering to 109°53'45"?

To find the Natural Versed Sine :—

Natural versed sine to	109°53'40"	=	1.340288
Proportional part to	5"	is Add	23
<hr/>			
Given arch	109°53'45"	Nat. versed sine =	1.340311

To find the Versed Sine Supplement :—

Versed sine sup. to	109°53'40"		659712
Proportional part to	5"	Subtract	23
<hr/>			
Given arch	109°53'45"	Vers. sine sup. =	659689

To find the Co-versed Sine :—

Co-versed sine to	109°53'40"		059679
Proportional part to	5"	Add	8
<hr/>			
Given arch	109°53'45"	Co-versed sine =	059687

To find the Natural Sine :—

Natural sine to	70°6'10"	Sup. to 109°53'50"	940305
Proportional part to	5"	Add 8

Supplement 70°6'15" to given arch, nat. sine = 940313

To find the Natural Cosine :—

Natural cosine to	70°6'10"	Sup. to 109°53'50"	. 340334
Proportional part to	5"	Subtract 23

Supplement 70°6'15" to given arch, nat. cosine = 340311

Remark.—Since the *natural sines* and *natural cosines* are not extended beyond 90 degrees, therefore, when the given arch exceeds that quantity, its supplement, or what it wants of 180 degrees, is to be taken, as in the above example. And when the given arch is expressed in degrees and minutes, the corresponding versed sine supplement, co-versed sine under 90 degrees, and *natural cosine*, are to be taken out agreeably to the note in page 57, which see.

PROBLEM II.

To find the Arch corresponding to a given Natural Versed Sine, Versed Sine Supplement, Co-versed Sine, Natural Sine, and Natural Cosine.

RULE.

Enter the Table, and find the arch answering to the *next less* natural versed sine, or *natural sine*, but to the next greater versed sine supplement, co-versed sine, or *natural cosine*; the difference between which and that given, being found in the bottom of the page, will give a number of seconds, which, being added to the arch found as above, will give the required arch.

Example 1.

Required the arch answering to the natural versed sine 363985 ?

Solution.—The next *less* natural versed sine is 363959, corresponding to which is $50^{\circ}30'10''$; the difference between 363959 and the given natural versed sine, is 26; corresponding to which, at the bottom of the Table, is $7''$, which, being added to the above-found arch, gives $50^{\circ}30'17''$, the required arch.

Note.—The arch corresponding to a given *natural sine* is obtained precisely in the same manner.

Example 2.

Required the arch corresponding to the natural versed sine supplement 1.464138?

Solution.—The next *greater* natural versed sine supplement is 1.464155; corresponding to which is $62^{\circ}20'40''$; the difference between 1.464155 and the given natural versed sine supplement, is 17; answering to which, at the bottom of the Table, is $4''$, which, being added to the above-found arch, gives $62^{\circ}20'44''$, the required arch.

Note.—The arch corresponding to a given co-versed sine, or *natural cosine*, is obtained in a similar manner.

Remark 1.

The logarithmic versed sine of an arch may be found by taking out the common logarithm of the product of the natural versed sine of such arch by 10000000000; as thus:

Required the logarithmic versed sine of $78^{\circ}30'45''$?

The natural versed sine of $78^{\circ}30'45''$ is .800846, which, being multiplied by 10000000000, gives 8008460000; the common log. of this is 9.903549; which, therefore, is the logarithmic versed sine of the given arch, as required.

Remark 2.

The Table of Logarithmic Rising may be readily deduced from the natural versed sines; as thus:

Reduce the meridian distance to degrees, by Table I., and find the natural versed sine corresponding thereto; now, let this be esteemed as an integral number, and its corresponding common log. will be the logarithmic rising.

Example.

Required the logarithmic rising answering to $4^{\circ}50'45''$?

$4^{\circ}50'45'' = 72^{\circ}41'15''$, the natural versed sine of which is 702417 ; the common log. of this is 5.846595, which, therefore, is the logarithmic rising required.

TABLE XXVIII.

Logarithms of Numbers.

Logarithms are a series of numbers invented, and first published in 1614, by Lord Napier, Baron of Merchiston in Scotland, for the purpose of facilitating troublesome calculations in plane and spherical trigonometry. These numbers are so contrived, and adapted to other numbers, that the sums and differences of the former shall correspond to, and show, the products and quotients of the latter.

Logarithms may be defined to be the numerical exponents of ratios, or a series of numbers in arithmetical progression, answering to another series of numbers in geometrical progression ; as,

Thus :

0.	1.	2.	3.	4.	5.	6.	7.	8. ind. or log.
1.	2.	4.	8.	16.	32.	64.	128.	256. geo. prog.

Or,

0.	1.	2.	3.	4.	5.	6.	7.	8. ind. or log.
1.	3.	9.	27.	81.	243.	729.	2187.	6561. geo. pro.

Or,

0.	1.	2.	3.	4.	5.	6.	7.	8. ind. or log.
1.	10.	100.	1000.	10000.	100000.	1000000.	10000000.	100000000 ge. pro.

Whence it is evident, that the same indices serve equally for any geometrical series ; and, consequently, there may be an endless variety of systems of logarithms to the same common number, by only changing the second term 2. 3. or 10. &c. of the geometrical series of whole numbers.

In these series it is obvious, that if any two indices be added together,

their sum will be the index of that number which is equal to the product of the two terms, in the geometrical progression to which those indices belong: thus, the indices 2. and 6. being added together, make 8; and the corresponding terms 4. and 64. to those indices (in the first series), being multiplied together, produce 256, which is the number corresponding to the index 8.

It is also obvious, that if any one index be subtracted from another, the difference will be the index of that number which is equal to the quotient of the two corresponding terms: thus, the index 8. minus the index 3 = 5; and the terms corresponding to these indices are 256 and 8, the quotient of which, viz., 32, is the number corresponding to the index 5, in the first series.

And, if the logarithm of any number be multiplied by the index of its power, the product will be equal to the logarithm of that power; thus, the index, or logarithm of 16, in the first series, is 4; now, if this be multiplied by 2, the product will be 8, which is the logarithm of 256, or the square of 16.

Again,—if the logarithm of any number be divided by the index of its root, the quotient will be equal to the logarithm of that root: thus, the index or logarithm of 256 is 8; now, 8 divided by 2 gives 4; which is the logarithm of 16, or the square root of 256, according to the first series.

The logarithms most convenient for practice are such as are adapted to a geometrical series increasing in a tenfold ratio, as in the last of the foregoing series; being those which are generally found in most mathematical works, and which are usually called *common logarithms*, in order to distinguish them from other species of logarithms.

In this system of logarithms, the index or logarithm of 1, is 0; that of 10, is 1; that of 100, is 2; that of 1000, is 3; that of 10000, is 4, &c. &c.; whence it is manifest, that the logarithms of the intermediate numbers between 1 and 10, must be 0, and some fractional parts; that of a number between 10 and 100, must be 1, and some fractional parts; and so on for any other number: those fractional parts may be computed by the following

Rule.—To the geometrical series 1. 10. 100. 1000. 10000. &c., apply the arithmetical series 0. 1. 2. 3. 4. &c., as logarithms. Find a geometrical mean between 1 and 10, or between 10 and 100, or any other two adjacent terms of the series between which the proposed number lies. Between the mean thus found and the nearest extreme, find another geometrical mean in the same manner, and so on till you arrive at the number whose logarithm is sought. Find as many arithmetical means, according to the order in which the geometrical ones were found, and they will be the logarithms

of the said geometrical means ; the last of which will be the logarithm of the proposed number.

Example.

To compute the Log. of 2 to eight Places of Decimals :—

Here the proposed number lies between 1 and 10.

- First, The log. of 1 is 0, and the log. of 10 is 1 ;
therefore $0 + 1 + 2 = .5$ is the arithmetical mean,
and $\sqrt{1 \times 10} = 3.1622777$ is the geometrical mean :
hence the log. of 3.1622777 is .5.
- Second, The log. of 1 is 0, and the log. of 3.1622777 is .5 ;
therefore $0 + 5 + 2 = .25$ is the arithmetical mean,
and $\sqrt{1 \times 3.1622777} = 1.7782794$ the geometrical mean :
hence the log. of 1.7782794 is .25.
- Third, The log. of 1.7782794 is .25, and the log. of 3.1622777 is .5 ;
therefore $.25 + .5 + 2 = .375$ is the arithmetical mean,
and $\sqrt{1.7782794 \times 3.1622777} = 2.3713741$ the geo. mean :
hence the log. of 2.3713741 is .375.
- Fourth, The log. of 1.7782794 is .25, and the log. of 2.3713741 is .375 ;
therefore $.25 + .375 + 2 = .3125$ is the arithmetical mean,
and $\sqrt{1.7782794 \times 2.3713741} = 2.0535252$ the geo. mean :
hence the log. of 2.0535252 is .3125.
- Fifth, The log. of 1.7782794 is .25, and the log. of 2.0535252 is .3125 ;
therefore $.25 + .3125 + 2 = .28125$ is the arith. mean,
and $\sqrt{1.7782794 \times 2.0535252} = 1.9109530$ the geo. mean :
hence the log. of 1.9109530 is .28125.
- Sixth, The log. of 1.9109530 is .28125, & the log. of 2.0535252 is .3125 ;
therefore $.28125 + .3125 + 2 = .296875$ is the arith. mean,
and $\sqrt{1.9109530 \times 2.0535252} = 1.9809568$ the geo. mean :
hence the log. of 1.9809568 is .296875.
- Seventh, The log. of 1.9809568 is .296875, & the log. of 2.0535252 is .3125 ;
therefore $.296875 + .3125 + 2 = .3046875$ is the arith. mean,
and $\sqrt{1.9809568 \times 2.0535252} = 2.0169146$ the geo. mean :
hence the log. of 2.0169146 is .3046875.
- Eighth, The log. of 2.0169146 is .3046875, & log. of 1.9809568 is .296875 ;
therefore $.3046875 + .296875 + 2 = .30078125$ is the ar. mean,
and $\sqrt{2.0169146 \times 1.9809568} = 1.9988548$ the geo. mean :
hence the log. of 1.9988548 is .30078125.

Proceeding in this manner, it will be found, after 25 extractions, that the log. of 1.9999999 is .30103000; and since 1.9999999 may be considered as being essentially equal to 2 in all the practical purposes to which it can be applied, therefore the log. of 2 is .30103000.

If the log. of 3 be determined, in the same manner, it will be found that the twenty-fifth arithmetical mean will be .47712125, and the geometrical mean 2.9999999; and since this may be considered as being in every respect equal to 3, therefore the log. of 3 is .47712125.

Now, from the logs. of 2 and 3, thus found, and the log. of 10, which is given=1, a great many other logarithms may be readily raised; because the sum of the logs. of any two numbers gives the log. of their product; and the difference of their logs. the log. of the quotient; the log. of any number, being multiplied by 2, will give the log. of the square of that number; or, multiplied by 3, will give the log. of its cube; as in the following examples:—

Example 1.

To find the Log. of 4:—

To the log. of 2 = .30103000
 Add the log. of 2 = .30103000

 Sum is the log. of 4 = .60206000

Example 2.

To find the Log. of 5:—

From the log. of 10=1.00000000
 Take the log. of 2 = .30103000

 Rem. is the log. of 5 = .69897000

Example 3.

To find the Log. of 6:—

To the log. of 3 = .47712125
 Add the log. of 2 = .30103000

 Sum is the log. of 6 = .77815125

Example 4.

To find the Log. of 8:—

To the log. of 4 = .60206000
 Add the log. of 2 = .30103000

 Sum is the log. of 8 = .90309000

Example 5.

To find the Log. of 9:—

To the log. of 3 = .47712125
 Add the log. of 3 = .47712125

 Sum is the log. of 9 = .95424250

Example 6.

To find the Log. of 15:—

To the log. of 5 = .69897000
 Add the log. of 3 = .47712125

 Sum is the log. of 15 = 1.17609125

Example 7.

To find the Log. of 81 = the square of 9:—

Log. of 9 =95424250
 Multiply by2

 Pro. is the log. of 81 = 1.90848500

Example 8.

To find the Log. of 729 = the cube of 9.

Log. of 9 =95424250
 Multiply by3

 Pro. is the log. of 729 = 2.86272750

Since the odd numbers 7. 11. 13. 17. 19. 23. 29. &c. cannot be exactly deduced from the multiplication or division of any two numbers, the logs. of those must be computed agreeably to the rule by which the logs. of 2 and 3 were obtained; after which, the labour attending the construction of a table of logarithms will be greatly diminished, because the principal part of the numbers may then be very readily found by addition, subtraction, and composition.

Of the Table.

This Table, which is particularly adapted to the reduction of the apparent to the true central distance, by certain concise methods of computation, to be treated of in the Lunar Observations, is divided into two parts: the *first* of which contains the decimal parts of the logs., to six places of figures, of all the natural numbers from unity, or 1, to 999999; and the *second*, the logs. to the same extent, of all the natural numbers from 1000000 to 1839999;—and although the logs. apparently commence at the natural number 100, yet the logs. of all the natural numbers under that are also given: thus, the log. of 1, or 10, is the same as that of 100; the log. of 2, or 20, is the same as that of 200; the log. of 3, or 30, is equal to that of 300; that of 11, to 110; that of 17, to 170; that of 99, to 990; and so on: using, however, a different index. And as the indices are not affixed to the logs., they must therefore be supplied by the computer: these indices are always to be considered as being *one less* than the number of *integer* figures in the corresponding natural number. Hence the index to the log. of any natural number, from 1 to 9 inclusive, is 0; the index to the log. of any number from 10 to 99 inclusive, is 1; that to the log. of any number from 100 to 999, is 2; that to the log. of any number from 1000 to 9999, is 3; &c. &c. &c. The second part of the Table will be found very useful in computing the lunar observations, by certain methods to be given hereafter, when the apparent distance exceeds 90 degrees, or when it becomes necessary to take out the log. of a natural number consisting of seven places of figures, and conversely.

In the left-hand column of the Table, and in the upper or lower horizontal row, are given the natural numbers, proceeding in regular succession; and, in the ten adjacent vertical columns, their corresponding logarithms.

As the size of the paper would not admit of the ample insertion of the logs., except in the first column, therefore only the four last figures of each log. are given in the nine following columns; the two preceding figures belonging to which will be found in the first column under 0 at top, or over 0 at bottom; and where these two preceding figures change, in the body of the Table, large dots are introduced instead of 0's, to catch the

eye and to indicate that from thence, through the rest of the line, the said two preceding figures are to be taken from the next lower line in the column under or over 0 : those dots are to be accounted as ciphers in taking out the logarithms.

The log. of any natural number consisting of four figures, or under, and conversely, is found directly by the Table ; but because the log. of a natural number consisting of five or more places of figures, and the converse, is frequently required in the reduction of the apparent to the true central distance, and also in many other astronomical calculations ; proportional parts are, therefore, adapted to the Table, and arranged in the nine small columns on the right-hand side of each page ; by means of which the logarithms of all the natural numbers, not consisting of more than seven places of figures, and *vice versa*, may be found to a sufficient degree of accuracy for all nautical purposes, as may be seen in the following problems.

PROBLEM I.

Given a Natural Number consisting of five, six, or seven Places of Figures, to find the corresponding Logarithm.

RULE.

Look for the three first figures of the given natural number in the left-hand column ; opposite to which, and under the fourth figure, in the horizontal column at top, will be found the log. to the four first figures of the given natural number : on the same line with this, and under the fifth figure of the natural number at top, in the proportional parts, will be found a number, which, being added to the above, will give the log. to five places of figures of the given natural number ; on the same line of proportional parts, and under the sixth figure of the natural number at top, will be found a number, which, being divided by 10, and the quotient added to the last found log., will give the log. to six places of figures of the given natural number. In the same manner, the log. may be taken out to seven places of figures ; observing, that the number in the proportional parts, corresponding to the seventh figure of the natural number, is to be divided by 100.

Note.—In dividing by 10 or 100, we have only to strike off the right-hand, or two right-hand figures.

Example.

Required the log. corresponding to the given natural number 1878978 ?

Log. corresponding to 1378	(four first figures) is	139249
5th fig. of the nat. num.	. 9	ans. to which in the pro. parts is	284
6th fig. of do.	. . 7	ans. to which in the pro. parts is	
		221, which, divided by 10,	
		gives 22. 1	22
7th fig. of do.	. . . 8	ans. to which in the pro. parts is	
		252, which, divided by 100,	
		gives 2. 52	2
<hr/>			
Given natural number 1378978	Corresponding log. =	. . 6. 139557*	

PROBLEM II.

To find the Natural Number to five, six, or seven Places of Figures, corresponding to a given Logarithm.

RULE.

Find the next *less* log. answering to the given one in the column under 0; continue the sight along the horizontal line, and a log., either the same as that given, or somewhat near it, will be found; then, the three first figures of the corresponding natural number will be found in the left-hand column, and the fourth figure, above the log., at the top of the Table. Should the given log. be found exactly, let one, two, or three ciphers be annexed to the natural number found as above, according to the number of figures wanted, and it will be the natural number required. But, if the log. cannot be exactly found (which in general will be the case), find the difference between the given log. and the next *less* log. in the Table: with this difference, enter the proportional parts, on the same horizontal line in which the next *less* log. was found, and find the next *less* proportional part; answering to which, at the top or bottom, will be found the fifth figure of the required natural number: find the difference between the above-found difference and the aforesaid next *less* proportional part; which being multiplied by 10, and the *product* found in the same line of proportional parts, the number corresponding thereto, at top or bottom, will be the sixth figure of the required natural number. Now, the difference between the above *product* and its next *less* proportional part, being multiplied by 10, also, and the product found in the same line of proportional parts, the number answering thereto at top or bottom will be the seventh figure of the required natural number.

* The index 6 is prefixed, because the given natural number consists of seven places of figures.

Example.

Required the natural number corresponding to the given log. 6.119558?

Given log.	6.119558
1316 = four first figs. of the required nat. num.		
answering to next less log.119256
Difference	302
.9 = fifth fig. of the required nat. num. ans.		
to the pro. part next less	297
Difference	5 × 10 = 50 product.
. . 1 = sixth fig. of the required nat. num.		
ans. to pro. part next less	33
Difference	17 × 10 = 170
. . . 5 = seventh fig. of the required nat. num. ans. to the		
nearest pro. part	165
<hr/>		
1316915 which is the natural number corresponding to the given log.		
6.119558, as required.		

Note.—From the above Problems, the manner of using the second part of the Table will appear obvious.

Remarks.

1. The whole of the operation is inserted at length, for the purpose of illustrating, more clearly, the use of the Table; but in practice, the logs. may, in most cases, be taken out at sight, and conversely; particularly from the *second part*, where the natural numbers are given to five places of figures, from 1000000 to 1839999.

2. In taking out the log. of a decimal fraction, or any number less than unity, if the first decimal place be a significant figure, the index of its log. is to be accounted as 9; but if the first significant figure of the decimal stands in the second, third, or fourth place, &c., the index of the corresponding log. is to be taken as 8, 7, or 6, &c. The converse of this,—that is, finding the significant decimals corresponding to a given log., will appear obvious.

3. The arithmetical complement of a log. is what that log. wants of the

radius of the Table; viz., of 10.000000: this is most easily found, by beginning at the left hand, and subtracting each figure from 9, except the last significant one, which is to be taken from 10; as thus:

The arithmetical complement of the log. 4.372853 is 5.627147; and so on.

PROBLEM III.

To perform Multiplication by Logarithms.

RULE.

To the log. of the multiplicand, add the log. of the multiplier, or add the logs. of the factors together, and the sum will be the log. of the product; the natural number corresponding to which will be the product required.

Example 1.

Multiply 436 by 19.7.

436	Log. =	. .	2.639486
19.7	Log. =	. .	1.294466
<hr/>			
Prod.=8589.18	Log.=		3.933952

Example 2.

Multiply 437.8 by 14.07, and also by 0.239.

437.8	Log. =	. .	2.641276
14.07	Log. =	. .	1.148294
0.239	Log. =	. .	9.378398
<hr/>			
Pro.=1472.204	Log.=		3.167968

Example 3.

What is the product of 0.049, 9.875, and 0.753?

0.049	Log. =	. .	8.690196
9.875	Log. =	. .	0.994537
0.753	Log. =	. .	9.876795
<hr/>			
Prod.=0.3642	Log.=		9.561528

Example 4.

What is the product of 0.0567 and 0.00339?

0.0567	Log. =	. .	8.753583
0.00339	Log. =	. .	7.530200
<hr/>			
Pro.=0.0001922	Log.=		6.283783

Note.—Respecting the index of a decimal fraction, and conversely, see Remark 2, page 69.

PROBLEM IV.

To perform Division by Logarithms.

RULE.

From the log. of the dividend, subtract the log. of the divisor, and the remainder will be the log. of the quotient; the natural number corresponding to which will be the quotient required.

Example 1.

Divide 1497 by 93.

1497	Log. =	. . .	3.175222
93	Log. =	. . .	1.968483

Quo.=16.0968	Log.=1.206739
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Example 2.

Divide 469.76 by 0.937.

469.76	Log. =	. . .	2.671876
0.937	Log. =	. . .	9.971740

Quo.=501.343	Log.=2.700136
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Example 3.

Divide 49.73 by 0.0632.

49.73	Log. =	1.696618
0.0632	Log. =	8.800717

Quo.=786.869	=Log.=2.895901
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Example 4.

Divide 0.00815 by 0.000275.

0.00815	Log. =	. . .	7.911158
0.000275	Log. =	. . .	6.439333

Quo.=29.6363	Log.=1.471825
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PROBLEM V.

To perform Proportion, or the Rule of Three, or Golden Rule, by Logarithms.

RULE.

To the arithmetical complement of the log. of the first term, add the logs. of the second and third terms; and the sum will be the log. of the fourth term, or answer.

Example 1.

If a ship sails $19\frac{1}{2}$ miles in $2\frac{1}{4}$ hours, how many miles will she run, at the same rate, in 24 hours?

As 2.25 hours, arith. comp. log. = . . . 9.647817

Is to 19.5 miles, log. 1.290035

So is 24 hours, log. 1.380211

To 208 miles, log. =	2.318063
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Example 2.

If the interest of 100*l.* for 365 days be 4*l.* 10*s.*, what will be the interest of 178*l.* 15*s.* for 213 days ?

As	{ 100	Arith. comp. of log.	=	. .	8.000000
	{ 365	Do. do.		. .	7.437707
Is to	{ 178.75	Log.	2.252246
	{ 213.	Log.	2.328380
So is	4.5	Log.	0.653213
To	4.69403	Log.	0.671546

Example 3.

A man of war, sailing at the rate of 9 knots an hour, descried a ship, distant 26 miles, sailing at the rate of $6\frac{1}{2}$ knots, to which she gave chase : after two hours' chase, the breeze freshened, and increased the man of war's rate of sailing to $11\frac{1}{2}$ knots, and that of the chase to $8\frac{1}{2}$. In what time did the man of war come up with the chase ?

Solution.—Since the man of war gained, at the commencement, $2\frac{1}{2}$ miles an hour on the chase, therefore, at the end of the first two hours, the distance between them was reduced to 21 miles ; during the rest of the chase, the hourly gain of the man of war was $2\frac{1}{2}$ miles.

Hence,	As the hourly gain	2.75	Ar. comp. log.	9.560667
	Is to	1 hour,	Log. . . 0.000000
	So is distance	. 21 miles,	Log.	. . 1.322219
				<hr/>
	To	7.6363	Log. = 0.882886,

or 7 hours and 38 minutes from the time the breeze freshened.

PROBLEM VI.

To perform Involution by Logarithms.

RULE.

Multiply the log. of the given number by the index of the power to which it is to be raised, and the product will be the log. of the required power.

Example 1.

Required the square of 346 ?

346 Log. . . . 2.539076
 Ind. of the power = 2

Answer 119716 Log. = 5.078152

Example 2.

Required the cube of 754 ?

754 Log. . . . 2.877371
 Ind. of the power = 3

Ans. 428661064 Log. = 8.632113

PROBLEM VII.

To perform Evolution by Logarithms.

RULE.

Divide the log. of the given number by the index of the power, and the quotient will be the log. of the root.

Example 1.

Required the square root of 76176 ?

76176 Log. . . 4.881818

Ans. 276 Log. = 2.440909

Example 2.

Required the cube root of 21952000 ?

21952000 Log. . 7.341475

Ans. 280 Log. = 2.447158½

PROBLEM VIII.

To find the Tonnage of a Ship by Logarithms.

RULE.

To the log. of the length of the keel, *reduced to tonnage*, add the log. of the breadth of the beam, the log. of half the breadth of the beam, and the constant log. 8.026872* ; the sum, rejecting 10 from the index, will be the log. of the required tonnage.

Example.

Let the length of a ship's keel, *reduced to tonnage*, be 120.5 feet, and the breadth of the beam 35.75 feet ; required the ship's tonnage ?

* This is the arithmetical complement of the log. of 94 ; the common divisor for finding the tonnage of ships.

Length of the keel for tonnage .	120.5	feet	Log. 2.080987
Breadth of the beam	35.75	feet	Log. 1.553276
Half ditto	17.875	feet	Log. 1.252246
Constant log.			8.026872

Required tonnage 819.18 . . . Log. 2.913381

PROBLEM IX.

Given the Measured Length of a Knot, the Number of Seconds run by the Glass, and the Distance sailed per Log, to find the true Distance by Logarithms.

RULE.

To the arithmetical complement of the log. of the number of seconds run by the glass, add the log. of the measured length of a knot, the log. of the distance sailed, and the constant log. 9.795880*; the sum of these four logs., rejecting 20 from the index, will be the log. of the true distance.

Example 1.

The distance sailed by the log is 180 miles; the measured length of a knot is 43 feet, and the time by the glass 32 seconds; required the true distance?

32 seconds, arith. comp. log.	. . .	8.494850
43 feet, log. =	1.633469
180 miles, log. =	2.255273
Constant log. =		9.795880

True distance = 151.2 miles. Log. = 2.179472

Example 2.

The distance sailed by the log is 210 miles; the measured length of a knot is 51 feet, and the time by the glass 27 seconds; required the true distance?

27 seconds, arith. comp. log. =	. . .	8.568636
51 feet, log.	1.707570
210 miles, log.	2.322219
Constant log.		9.795880

True distance = 247.9 miles. Log. = 2.394305

* This is the sum of the arithmetical complement of the log. of 48 (the general length of a knot) and the log. of 30 seconds, the true measure of the half-minute glass.

TABLE XXIX.

Proportional Logarithms.

This Table contains the proportional log. corresponding to all portions of time under three hours, and to every second under three degrees. It was originally computed by Dr. Maskelyne, and particularly adapted to the operation for finding the apparent time at Greenwich answering to a given distance between the moon and sun, or a fixed star; but it is now applied to many other important purposes, as will be seen hereafter.

Proportional Logarithms may be computed by the following

RULE.

From the common log. of 3 hours, reduced to seconds, subtract the common log. of the given time in seconds; and the remainder will be the proportional log. corresponding thereto.

Example.

Required the proportional log. corresponding to $0^h 40^m 26^s$?

3 hours reduced to seconds	=	10800"	Log.	=	4.033424
$40^m 26^s$ given time, in secs.	=	2426"	Log.	=	3.384891

Proportional log. corresponding to the given time = 0.6485.33

As hours and degrees are similarly divided, therefore, in the general use of this Table, the hours and parts of an hour, may be considered as degrees and parts of a degree, and conversely. And to render the use of it more extensive, one minute may be esteemed as being either one degree, or one second, and *vice versa*.

Since proportion is performed by adding together the arithmetical complement of the proportional logarithm of the first term, and the proportional logarithms of the second and third terms, rejecting 10 from the index, the present Table is of great use in reducing the altitudes of the moon and sun, or a fixed star, to the mean time and distance, when the observations are made by one person, as will appear evident by the following

Example.

Let the first altitude of the moon's lower limb be $27^{\circ} 25' 20''$, and the corresponding time per watch $21^h 42^m 8^s$, and the last altitude $25^{\circ} 24' 20''$,

and its corresponding time $21^{\text{h}}55^{\text{m}}57^{\text{s}}$; it is required to reduce the first altitude to what it should be at $21^{\text{h}}49^{\text{m}}33^{\text{s}}$, the time at which the mean lunar distance was taken?

1st time $21^{\text{h}}42^{\text{m}}8^{\text{s}}$ 1st time $21^{\text{h}}42^{\text{m}}8^{\text{s}}$ 1st alt. $27^{\circ}25'20''$ $27^{\circ}25'20''$
 Last do. $21.55.57$ Mean do. $21.49.33$ Last do. $25.24.20$

Diff. . $0.13.49$ Diff. . $0.7.25$ Diff. . $2.1.0$

As $13^{\text{m}}49^{\text{s}}$, arithmetical comp. prop. log. $= 8.8851$

Is to $7^{\text{m}}25^{\text{s}}$ proportional log. . . . $= 1.3851$

So is $2^{\circ}1'$ proportional log. . . . $= 0.1725$

To prop. log. of reduction of Moon's alt. . $= 0.4427 = - 1^{\circ}4'57''$

Moon's alt. reduced to mean time of observation . . $= 26^{\circ}20'23''$

And in the same manner may the altitude of the sun, or a fixed star, be reduced to the time of taking the mean lunar distance.

Remark.—Although this Table is only extended to 3 hours or 3 degrees, yet by taking such terms as exceed those quantities one grade lower, that is, the hours, or degrees, to be esteemed as minutes, and the minutes as seconds, the proportion may be worked as above: hence it is evident that the Table may be very conveniently applied to the reduction of the sun's, moon's, or a planet's right ascension and declination to any given time after noon or midnight; and, also, to the equation of time;—for the illustration of which the following Problems are given.

PROBLEM I.

To reduce the Sun's Longitude, Right Ascension and Declination; and, also, the Equation of Time, as given in the Nautical Almanac, to any given Meridian, and to any given time under that Meridian.

RULE.

To the apparent time at ship, or place, (to be always reckoned from the preceding noon *,) add the longitude, in time, if it be west, but subtract it if east; and the sum, or difference, will be the Greenwich time.

From page II. of the month in the Nautical Almanac, take out the sun's

* See precepts to Table XV.—page 25.

longitude, right ascension, declination, or equation of time for the noons immediately preceding and following the Greenwich time, and find their difference; then,

To the proportional log. of this difference, add the proportional log. of the Greenwich time (reckoning the hours as *minutes*, and the minutes as *seconds*), and the constant log. 9.1249*; the sum of these three logs., rejecting 10 from the index, will be the proportional log. of a correction which is always to be *added* to the sun's longitude and right ascension at the *noon preceding* the Greenwich time; but to be applied by addition or subtraction to the sun's declination and the equation of time, at that noon, according as they may be increasing or decreasing.

Example 1.

Required the sun's longitude, right ascension and declination, and also the equation of time, May 6th, 1824, at 5^h 10^m, in longitude 64° 45' west of the meridian of Greenwich?

$$\begin{array}{rcl}
 \text{Apparent time at ship or place,} & = & \dots\dots\dots 5^h 10^m \\
 \text{Longitude } 64^\circ 45' \text{ west, in time,} & = & \dots\dots\dots +4.19 \\
 \hline
 \text{Greenwich time,} & = & \dots\dots\dots 9^h 29^m
 \end{array}$$

To find the Sun's Longitude.

$$\begin{array}{rcl}
 \text{Diff. in 24 hours} & = & 57'.59'' \text{ prop. log. } \dots\dots\dots = .4920 \\
 \text{Greenwich time} & = & 9^h 29^m \text{ prop. log. } \dots\dots\dots = 1.2783 \\
 & & \text{Constant log. } \dots\dots\dots = 9.1249 \\
 & & \hline
 \text{Correction of sun's long.} & = & \dots\dots\dots + 22'.55'' \text{ p. log.} = 0.8952 \\
 \text{Sun's long. at noon, May 6, 1824} & = & 1^h 15^m 51'.13'' \\
 & & \hline
 \text{Sun's long. as required} & = & 1^h 16^m 14'.8''
 \end{array}$$

To find the Sun's Right Ascension.

$$\begin{array}{rcl}
 \text{Diff. in 24 hours} & = & 3'.52'' \text{ prop. log. } \dots\dots\dots = 1.6679 \\
 \text{Greenwich time} & = & 9^h 29^m \text{ prop. log. } \dots\dots\dots = 1.2783 \\
 & & \text{Constant log. } \dots\dots\dots = 9.1249 \\
 & & \hline
 \text{Correction of sun's right asc.} & = & \dots\dots\dots + 1'.32'' \text{ p. log.} = 2.0711 \\
 \text{Sun's right asc. at noon, May 6, 1824,} & = & 2^h 53^m 31'.7 \\
 & & \hline
 \text{Sun's right asc. as required} & = & 2^h 55^m 3'.7
 \end{array}$$

* The arithmetical complement of the proportional log. of 24 hours esteemed as minutes.

To find the Sun's Declination.

Diff. in 24 hours = 16'38" prop. log.	= 1.0343
Greenwich time = 9 ^h 29 ^m prop. log.	= 1.2783
Constant log.	= 9.1249
<hr/>	
Correction of sun's dec.	= + 6'34" p. log. = 1.4375
Sun's dec. at noon, May 6, 1824, . . .	= 16°36' 5" north.
<hr/>	
Sun's dec. as required	= 16°42'39" north.

To find the Equation of Time.

Diff. in 24 hours = 4 ^s .5 prop. log.	= 3.3829
Greenwich time = 9 ^h 29 ^m prop. log.	= 1.2783
Constant log.	= 9.1249
<hr/>	
Correction of the equation of time . . .	= + 1 ^m .8 p. log. = 3.7861
Equation of time, May 6, 1824 . . .	= 3 ^m 36 ^s .1
<hr/>	
Equation of time as required	= 8 ^m 37 ^s .9

Remark.—Since the daily difference of the equation of time is expressed, in the Nautical Almanac, in seconds and tenths of a second; if, therefore, these tenths be multiplied by 6, the daily difference will be reduced to seconds and thirds:—Now, if those seconds and thirds be esteemed as minutes and seconds, the operation of reducing the equation of time will become infinitely more simple; because the necessity of making proportion for the tenths, as above, will then be done away with:—remembering, however, that the minutes and seconds corresponding to the sum of the three logs. are to be considered as seconds and thirds.

Example 2.

Required the sun's longitude, right ascension, and declination, and also the equation of time, August 2d, 1824, at 19^h22^m, in longitude 98°45' east of the meridian of Greenwich?

Apparent time at ship, or place,	19 ^h 22 ^m
Longitude 98°45' east, in time, =	— 6.35 ₄
<hr/>	
Greenwich time,	12 ^h 47 ^m

To find the Sun's Longitude :—

Diff. in 24 hours=	57'.28"	prop. log.	=	.4959
Greenwich time =	12 ^h .47 ^m	prop. log.	=	1.1486
		Constant log.	=	9.1249
<hr/>					
Correction of sun's longitude,	. .	=	+ 30'.37"	p. log.	= 0.7694
Sun's longitude at noon, Aug. 2, 1824	=4'.10° 3'. 8"				
<hr/>					
Sun's long. as required	=	4'.10°33'.45"		

To find the Sun's Right Ascension :—

Diff. in 24 hours =	3'52"	prop. log.	=	1.6679
Greenwich time =	12 ^h 47 ^m	prop. log.	=	1.1486
	Constant log.	=	9.1249	
<hr/>					
Correction of sun's right asc.	. . .	= + 2' 4"	p. log.	. . .	= 1.9414
Sun's right asc. at noon, Aug. 2, 1824	= 8 ^h 50 ^m 0. 8				
<hr/>					
Sun's right asc. as required	. . .	= 8 ^h 52 ^m 4'. 8			

To find the Sun's Declination :—

Diff. in 24 hours =	15'.36"	prop. log.	=	1.0621
Greenwich time =	12 ^h .47 ^m	prop. log.	=	1.1486
		Constant log.	=	9.1249
<hr/>					
Correction of sun's dec.	=	— 8'.19"	p. log.	= 1.3356
Sun's dec. at noon, Aug. 2, 1824	. = 17° 44'.41" north.				
<hr/>					
Sun's dec. as required	=	17° 36'.22"	north.	

To find the Equation of Time :—

Diff. in 24 hours	=	4".30"	prop. log.
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PROBLEM II.

To reduce the Moon's Longitude, Latitude, Semidiameter and Horizontal Parallax, as given in the Nautical Almanac, to any given Meridian, and to any given time under that Meridian.

RULE.

To the mean time at ship, or place, (reckoned from the preceding noon or midnight,) add the longitude in time, if it be west, but subtract it if east, and the sum, or difference, will be the Greenwich time past that noon or midnight, according as it may be.

Take from pages III. and IV. of the month in the Nautical Almanac, the moon's longitude, latitude, semidiameter, or horizontal parallax, for the noon and midnight immediately *preceding* and *following* the Greenwich time, and find their difference; then,

To the proportional log. of this difference, add the proportional log. of the Greenwich time past the preceding noon or midnight, (reckoning the hours as *minutes*, and the minutes as *seconds*,) and the constant logarithm 8.8239 *; the sum of these three logs., abating 10 in the index, will be the proportional log. of a correction, which is always to be *added* to the moon's longitude at the noon or midnight preceding the Greenwich time; but to be applied by addition or subtraction to her latitude, semidiameter, or horizontal parallax, at that noon or midnight, according as it may be increasing or decreasing.

Note.—Since the difference of the moon's longitude in 12 hours, will always exceed the limits of the Table, if, therefore, the one-half or one-third of such difference be taken, and the correction, resulting therefrom, multiplied by 2 or 3, the required correction will be obtained.

Example.

Required the moon's longitude, latitude, semidiameter and horizontal parallax, Aug. 2d, 1824, at 3^h 10^m mean time past noon, in longitude 60° 30' west of the meridian of Greenwich?

Mean time at ship, or place	=	3 ^h 10 ^m
Longitude 60° 30' west, in time . . .	= +	4. 2.
Greenwich time, past noon, Aug. 2, 1824	=	7 ^h 12 ^m

* The arithmetical complement of the proportional log. of 12 hours esteemed as *minutes*.

To find the Moon's Longitude:—

Diff. in 12 hours = $6^{\circ}31'59'' \div 3 = 2^{\circ}10'39\frac{1}{3}''$, prop. log.	=	.1391
Greenwich time = 7^h12^m = prop. log.	=	1.3979
Constant log.	=	8.8239

One third the corr. of the moon's long. = $1^{\circ}18'25''$ p. log.	=	0.3609
Multiply by 3.		

Corr. of moon's long. + $3^{\circ}55'15''$		
Moon's long. at noon, Aug. 2, 1824 .	=	$7^{\circ}17'16'27''$

Moon's long. as required		$7^{\circ}21'11'42''$
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To find the Moon's Latitude:—

Diff. in 12 hours = $23'35''$ prop. log.	=	.8827
Greenwich time = 7^h12^m prop. log.	=	1.3979
Constant log.	=	8.8239

Correction of moon's lat. $-14'9''$ p. log.	=	1.1045
Moon's lat. at noon, Aug. 2, 1824 .	=	$4^{\circ}6'59''$ south.

Moon's lat. as required		$3^{\circ}52'50''$ south.
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Note.—The correction, or proportional part of the moon's motion, found as above, must be corrected by the Equation of Second Differences contained in Table XVII., as explained in pages 33, 34, &c.

To find the Moon's Semidiameter:—

Diff in 12 hours = $6''$ prop. log.	=	3.2553
Greenwich time = 7^h12^m prop. log.	=	1.3979
Constant log.	=	8.8239

Corr. of the moon's semidiameter $-4''$ p. log.	=	3.4771
Moon's semidiameter at noon, Aug. 2, 1824	=	$15'33''$

Moon's semidiameter as required		$15'29''$
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To find the Moon's Horizontal Parallax:—

Diff. in 12 hours = $23''$ prop. log.	=	2.6717
Greenwich time = 7^h12^m prop. log.	=	1.3979
Constant log.	=	8.8239

Corr. of moon's horiz. paral. $-14''$ p. log.	=	2.8935
Moon's horiz. paral. at noon, Aug. 2, 1824	=	$57'6''$

Moon's horiz. paral. as required		$56'52''$
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Note.—The moon's semidiameter, found as above, must be augmented by the correction contained in Table IV., as explained in page 10.

Moon's declination at 7 hours = — $23^{\circ}54'21''$ south.

Ditto at 8 hours = . — 24. 2.35 ditto.

Difference $0^{\circ} 8'14''$ prop. log. = . 1.3397

Minutes, &c. in Greenwich time = $14^{\text{h}}36'$ ditto . . 1.0909

Constant log. 9.5229

Correction of declination = . $2' 0''$ = prop. logarithm = 1.9535

Moon's declination at 7 hours = $23^{\circ}54'21''$ south.

Moon's correct declination = . $23^{\circ}56'21''$ south, as required.

PROBLEM IV.

To reduce the Right Ascension and Declination of a Planet, as given in the Nautical Almanac, to any given time under a known Meridian.

RULE.

Turn the longitude into time, and add it to the mean time at ship, if it be west, but subtract it if east; the sum or difference will be the corresponding mean time at Greenwich.

As it is the four bright planets, *viz.* Venus, Mars, Jupiter, and Saturn, that are of any importance to the mariner; the places of which are given, in the order thus named, for every noon between pages 278 and 302, and between 323 and 346 of the Nautical Almanac: therefore, enter those pages and take out the planet's right ascension and declination for the noons immediately preceding and following the Greenwich mean time, and find their difference; then,—

To the proportional logarithm of the difference of right ascension, or of declination, add the proportional logarithm of the Greenwich time, (esteeming the hours as *minutes*, and the minutes as *seconds*,) and the constant logarithm 9.1249*; the sum, abating 10 in the index, will be the proportional logarithm of a correction, which being applied by addition or subtraction to the right ascension or declination at *the noon preceding* the Greenwich time, according as those elements may be increasing or decreasing, the sum, or difference, will be the planet's correct right ascension, or declination, at the given time and place.

Example.

Required the right ascension, and the declination of the planet Venus at $10^{\text{h}}20^{\text{m}}$ mean time, on the 3rd of July, 1836; the longitude being $75^{\circ}30'$ west of the meridian of Greenwich?

* This is the prop. log. ar. comp. of 24 hours, esteemed as *minutes*.

Mean time at ship or place	10 ^h 20 ^m
Longitude 75° 30' west, in time = . . .	+ 5. 2
Mean time at Greenwich	15 ^h 22 ^m
Venus's right ascension, July 3rd . =	8 ^h 52 ^m 44 ^s
Ditto, ditto, the 4th . —	8. 52. 34
Difference	0. 0. 10 prop. log. . 3. 0334
Greenwich mean time =	15. 22. 0 ditto . 1. 0687
Constant log.	9. 1249
Correction of right ascension . . =	— 0 ^m 6 ^s prop. log. = . 3. 2270
Right ascension at noon, July 3rd =	8. 52. 44
Venus's correct right ascension . =	8 ^h 52 ^m 38 ^s as required.

To find the Declination :—

Venus's declination July 3rd . .	16° 3' 19" north.
Ditto, ditto, the 4th . .	15. 50. 49 ditto.
Difference	0° 12' 30" prop. log. . . = 1. 1584
Greenwich mean time	15 ^h 22 ^m ditto . . = 1. 0687
Constant log.	9. 1249
Correction of declination . . . =	8' 0" = prop. log. . . 1. 3520
Declination at noon, July 3rd =	16° 3' 19" north.
Venus's reduced declination =	15° 55' 19" north, as required.

TABLE XXX.

Logarithmic Half-elapsed Time.

This Table is useful in finding the latitude by two altitudes of the sun ; and also in other astronomical calculations, as will be shown hereafter. The Table is extended to every fifth second of time under 6 hours, with proportional parts, adapted to the intermediate seconds, in the right-hand margin of each page ; by means of which, the logarithmic half-elapsed time answering to any given period, and conversely, may be readily obtained at sight.

As the size of the page would not admit of the indices being prefixed to the logs. except in the first column, under 0', therefore where the indices change in the other columns, a bar is placed over the 9, or left-hand figure of the log., as thus, $\overline{9}$, to catch the eye, and to indicate that from thence, through the rest of the line, the index is to be taken from the next lower line in the first column, or that marked 0' at top and bottom. It is to be observed, however, that the indices are only susceptible of change when the half-elapsed time is under 23 minutes.

The logarithmic half-elapsed time corresponding to any given period, is to be taken out by entering the Table with the hours and fifths of seconds at the top, or next less fifth if there be any odd seconds, and the minutes in the left-hand column; in the angle of meeting will be found a number, which being *diminished* by the proportional part answering to the odd seconds, in the right hand margin, will give the required logarithm.

Example.

Required the logarithmic half-elapsed time answering to $2^{\text{h}}47^{\text{m}}28^{\text{s}}?$
 $2^{\text{h}}47^{\text{m}}25^{\text{s}}$ answering to which is 0.17572
 Odd seconds . . 3. pro. part answering to which is . . — 11

 Given time = $2^{\text{h}}47^{\text{m}}28^{\text{s}}$ corresponding log. hf.-elapsed time . 0.17561

In the converse of this, that is, in finding the time corresponding to a given log.;—if the given log. can be exactly found, the corresponding hours, minutes, and seconds, will be the time required:—but if it cannot be exactly found (which in general will be the case), take out the hours, minutes, and seconds answering to the *next greater* log.; the difference between which, and that given, being found in the column of proportional parts, abreast of where the *next greater* log. was found, or nearly so, will give a certain number of seconds, which being added to the hours, minutes and seconds, found as above, will give the required time.

Example.

Required the time corresponding to the logarithmic half-elapsed time 0.14964?

Solution.—The next *greater* log. is 0.14973, corresponding to which is $3^{\text{h}}0^{\text{m}}25^{\text{s}}$; the difference between this log. and that given is 9; answering to which in the column of proportional parts is 3 seconds, which being added to the above found time gives $3^{\text{h}}0^{\text{m}}28^{\text{s}}$ for that required.

Remark.—The numbers in this Table are expressed by the logarithmic co-secants adapted to given intervals of time, the index being diminished by radius, as thus:

Let the half-elapsed time be $3^{\text{h}}20^{\text{m}}45^{\text{s}}$; to compute the corresponding logarithm.

Given time = $3^{\text{h}}20^{\text{m}}45^{\text{s}}$ in degrees = $50^{\circ}11'15''$; log. co-secant less radius = 0.114557; which, therefore, is the required log.; and since it is not necessary that this number should be extended beyond five decimal places, the sixth, or right hand figure, may be struck off; observing, however, to increase the fifth figure by unity or 1, when the right hand figure, so struck off, amounts to 5 or upwards:—hence, the tabular number corresponding to $3^{\text{h}}20^{\text{m}}45^{\text{s}}$ is 0.11456; and so of others.

TABLE XXXI.

Logarithmic Middle Time.

This Table is, also, useful in finding the latitude by two altitudes of the sun; for which purpose it is extended to every fifth second under 6 hours, with proportional parts for the intermediate seconds, in the right-hand margin of each page; by means of which the logarithmic middle time answering to any given period, and conversely, may be readily taken out at sight.

As the indices are only prefixed to the logs. in the first column, therefore where those change in the other columns a bar is placed over the cypher, as thus, $\bar{0}$, to catch the eye, and to indicate that from thence through the rest of the line, the index is to be taken from the next lower line, in the first column.

The logarithmic middle time answering to any given period is to be taken out by entering the Table with the hours and fifths of seconds at the top, or the *next less fifth* second (when there are any odd seconds, as there generally will be), and the minutes in the left-hand column; in the angle of meeting will be found a number, which being *augmented* by the proportional part answering to the odd seconds, in the compartment abreast of the angle of meeting, will give the log. required.

Example.

Required the logarithmic middle time answering to		$3^h 17^m 23^s$?
		$3^h 17^m 20^s$ answering to which is 6.18099
Odd seconds	. . 3. pro. part answering to which is	. . + 8
Given time = $3^h 17^m 23^s$ corresponding log. middle time		. . 6.18107

The time corresponding to a given logarithmical number, is found by taking out the hours, minutes, and seconds, answering to the *next less* tabular number; the difference between which and that given, being found in the compartment of proportional parts, abreast of the said *next less* tabular number, will give a certain number of seconds, which being added to the hours, minutes, and seconds found as above, will be the time required.

Example.

Required the time corresponding to the log. middle time, 6.01767 ?

Solution.—The *next less* tabular log. is 6.01757, answering to which is $2^h 5^m 30^s$; the difference between this log. and that given is 10, answering to which in the column of proportional parts is 2 seconds, which being added to the time found, as above, gives $2^h 5^m 32^s$, for that required.

Remark.—The logarithmic middle time may be readily computed by the following rule ; viz :—

To the logarithmic sine of the given time expressed in degrees, add the constant log. 6. 301030, and the sum, abating 10 in the index, will be the required logarithm.

Example.

Let the middle time be $4^{\text{h}}10^{\text{m}}25^{\text{s}}$, required the corresponding log. ?

Given time = $4^{\text{h}}10^{\text{m}}25^{\text{s}}$, in degs. = $62^{\circ}36'15''$ log. sine = 9.948339

Constant log. = 6.301030

Logarithmic middle time, as required = 6.249369 ; and since it is not necessary that this log. should be extended beyond five places of decimals, the sixth, or right-hand figure may, therefore, be struck off ; observing, however, to increase the fifth figure by unity or 1, when the right-hand figure, so struck off, amounts to 5 or upwards ; hence the tabular number corresponding to $4^{\text{h}}10^{\text{m}}25^{\text{s}}$, is 6.24937, and so on.

TABLE XXXII.

Logarithmic Rising.

This Table, with the two preceding, is particularly useful in finding the latitude by two altitudes of the sun ; it is also of considerable use in many other astronomical calculations, such as in computing the apparent time from the altitude of a celestial object ; determining the altitude of a celestial object from the apparent time, &c. &c.—The arrangement of the present Table is so very uniform with the preceding, that it is not deemed necessary to enter into its description any farther than by observing that the indices are only prefixed to the logs. in the first column :—that where those change in the other columns, large dots are introduced instead of 0's to catch the eye, for the purpose of indicating that from thence through the rest of the line, the index is to be taken from the next lower line in the first column ; and that, in the general use of the Table, these dots are to be accounted as cyphers.

Example.

Required the logarithmic rising answering to $1^{\text{h}}43^{\text{m}}27^{\text{s}}$?

$1^{\text{h}}43^{\text{m}}25^{\text{s}}$, answering to which is 5.00040

Odd seconds . . . 2, pro. part answering to which is 28

Given time = $1^{\text{h}}43^{\text{m}}27^{\text{s}}$, corresponding logarithmic rising 5.00068

The converse of this, that is, finding the time corresponding to a given log. will appear obvious ; thus,

Let the given logarithmic rising be 5.69088, to find the corresponding time.

The *next less* tabular log. is 5.69071, answering to which is $3^{\text{h}}57^{\text{m}}30^{\text{s}}$; the difference between this log. and that given is 17, answering to which in the column of proportional parts, abreast of the tabular log., is 3 seconds ; now, this being added to the time found, as above, gives $3^{\text{h}}57^{\text{m}}33^{\text{s}}$; which, therefore, is the time corresponding to the given logarithmic rising.

Note.—The numbers in this Table were computed by the following rule, viz :—

To twice the logarithmic sine of half the meridian distance, in degrees, add the constant log. 6.301030, and the sum, rejecting 20 from the index, will be the logarithmic rising.

Example.

Required the logarithmic rising answering to $4^{\text{h}}10^{\text{m}}45^{\text{s}}$?

Given meridian distance = $4^{\text{h}}10^{\text{m}}45^{\text{s}}$, in degrees = $62^{\circ}41'15''$

Half the meridian distance, in degrees = $31.20.37\frac{1}{2}$, twice the

logarithmic sine 19.432293

Constant log. 6.301030

Logarithmic rising answering to the given meridian distance = 5.733323

The numbers in the present Table may be also computed by means of the natural versed sines contained in Table XXVII., as thus ;

Reduce the meridian distance to degrees, and find the natural versed sine corresponding thereto ; the common log. of which will be the logarithmic rising.

Example.

Required the logarithmic rising answering to $4^{\circ}22'30''$, or $65^{\circ}37'30''$?
 Meridian distance in degrees = $65^{\circ}37'30''$, natural versed sine = 587293,
 log. = 5.768855; which, therefore, is the logarithmic rising answering
 to the given meridian distance.

In this method of computing the logarithmic rising, the natural versed sine is to be conceived as being multiplied by 1000000, the radius of the Table, and thus reduced to a whole number.

TABLE XXXIII.

To reduce Points of the Compass to Degrees, and conversely.

This Table is divided into six columns; the two first and two last of which contain the names of the several points and quarter points of the compass; the third column contains the corresponding number of points and quarter points reckoned from the meridian; and the fourth column the degrees and parts of a degree answering thereto.—The manner of using this Table is obvious; and so is the method by which it was computed:—for since the whole compass card is divided into 32 points, and the whole circle into 360 degrees; it is evident that any given number of points will be to their corresponding degrees in the ratio of 32 to 360; and *vice versa*, that any given number of degrees will be to their corresponding points as 360 is to 32:—Hence, to find the degrees corresponding to one point.—As $32^{\circ} : 360^{\circ} :: 1^{\circ} : 11^{\circ}15'$; so that one point contains 11 degrees and 15 minutes;—two points, 22 degrees 30 minutes, &c. &c.

TABLE XXXIV.

Logarithmic Sines, Tangents, and Secants, to every Point and Quarter Point of the Compass.

In this Table the points and quarter points are contained in the left and right hand marginal columns, and the log. sines, tangents, and secants, corresponding thereto, in the intermediate columns.

If the course be given in points, it will be found more convenient to take the log. sine, tangent, or secant of it from this Table, than to reduce those points to degrees, and then find the corresponding log. sine, &c. &c. in either of the following Tables.—The manner of using this Table must appear obvious at first sight.

TABLE XXXV.

Logarithmic Secants.

In the first 10 degrees of this Table, the logarithmic secants are given to every *tenth* second, with proportional parts, answering to the intermediate seconds, in the right hand marginal column.—Thence to 88 degrees, the log. secants are given to every *fifth* second, with proportional parts, adapted to the intermediate seconds, in the right hand column of each page :—and because the numbers increase rapidly between 80 and 88 degrees, producing very considerable differences between any two adjacent logs. ; therefore betwixt those limits, there are two pages allotted to a degree ; every page being divided into two parts of 15 minutes each, so that no portion whatever of the proportional parts might be lost, and that the whole might have room to be fully inserted.—In the two last degrees, viz. from 88 to 90, the log. secants are given to every second.—The Table is so arranged as to be extended to every second in the semicircle, or from 0 to 180 degrees ; as thus : the arcs corresponding to the log. secants are given in regular succession at top from 0 to 90 degrees, and then continued at bottom, reckoning towards the left hand, from 90 to 180 degrees :—the arcs corresponding to the co-secants are placed at the bottom of the Table, in numerical order, from the right hand towards the left (like the secants in the second quadrant), from 0 to 90 degrees, and then continued at top, agreeably to the order of the secants in the first quadrant, from 90 degrees to the end of the semicircle.—This mode of arrangement, besides doing away with the necessity of finding the supplement of an arch when it exceeds 90 degrees, possesses the peculiar advantage of enabling the reader to take out the log. secant, or co-secant of any arch whatever, and conversely, at sight, as will appear evident by the following problems.

Note.—The log. co-secant of a given degree, or secant of a degree above 90, will be found in the same page with the *next less* degree in the first column under 0" at top, it being the first number in that column ; and the log. co-secant of a given degree and minute, or secant of a degree and minute above 90, will be found on the same line with the *next less* minute in the column marked 60" at bottom of the Table.

PROBLEM I.

To find the Logarithmic Secant, and Co-secant of any given Arch, expressed in Degrees, Minutes, and Seconds.

Rule.

If the given arch be comprised within the limits of the two last degrees of the first quadrant, that is, between 88 and 90 degrees, the Table will directly exhibit its corresponding log. secant or co-secant;—but when it falls without those limits, then find the log. secant, or co-secant, in the angle of meeting made by the given degree and next less fifth or tenth second at top, and the minutes in the left hand column; to which, add the proportional part corresponding to the odd seconds from the right hand column abreast of the angle of meeting, if a secant be wanted, or a co-secant above 90 degrees; but subtract that part when a co-secant is required, or a secant above 90 degrees; and the sum, or difference, will be the log. secant or co-secant answering to the given arch.

Example 1.

Required the logarithmic secant, and co-secant, corresponding to $23^{\circ}14'23''$?

To find the Log. Secant:—

	$23^{\circ}14'20''$, ans. to which is . . .	10.036747
Odd seconds	3 propor. part to which is +	3
<hr/>		
Given arch =	$23^{\circ}14'23''$ Corres. log. secant =	10.036750

To find the Log. Co-secant:—

	$23^{\circ}14'20''$, ans. to which is . . .	10.403881
Odd seconds	3 propor. part to which is —	15
<hr/>		
Given arch =	$23^{\circ}14'23''$ Corres. log. co-secant =	10.403866

Example 2.

Required the log. secant, and co-secant, corresponding to $113^{\circ}23'47''$?

To find the Log. Secant :—

$$\begin{array}{rcl}
 & 113^{\circ}23'45'', \text{ ans. to which is} & . . 10.401121 \\
 \text{Odd seconds} & \underline{2} \text{ propor. part to which is} & \underline{-} \quad 10 \\
 \text{Given arch} = & 113^{\circ}23'47'' \text{ Corres. log. secant} = & 10.401111
 \end{array}$$

To find the Log. Co-secant :—

$$\begin{array}{rcl}
 & 113^{\circ}23'45'', \text{ ans. to which is} & . . 10.037260 \\
 \text{Odd seconds} & \underline{2} \text{ propor. part to which is} & \underline{+} \quad 2 \\
 \text{Given arch} = & 113^{\circ}23'47'' \text{ Corres. log. co-secant} = & 10.037262
 \end{array}$$

Note.—In that part of the Table which lies between 10 and 80 degrees, the size of the page would not admit of the indices being prefixed to any other logs. than those contained in the first column of each page; nor, indeed, is it necessary that they should be, since they are uniformly the same as those contained in the said first column; viz., 10 for each log. secant or co-secant.

PROBLEM II.

To find the Arch corresponding to a given Logarithmic Secant or Co-secant.

RULE.

If the given log. secant, or co-secant, exceeds the secant of 88 degrees, viz., 11.457181, its corresponding arch will be found at first sight in the Table; but if it be under that number, find the arch answering to the next less secant, or next greater co-secant; the difference between which and that given, being found in the column of proportional parts, abreast of the tabular log., will give a certain number of seconds, which, being added to the above-found arch, will give that required.

Example 1.

Required the arch corresponding to the given log. secant 10.235421?

Solution.—The *next less* secant, in the Table, is 10.235412, corresponding to which is $54^{\circ}26'25''$; the difference between this log. secant, and that given, is 9; answering to which, in the column of proportional parts abreast of the tabular log., is $3''$; which, being *added* to the above-found arch, gives $54^{\circ}26'48''$ for that required.

Example 2.

Required the arch corresponding to the given log. co-secant 10.562114 ?

Solution.—The *next greater* co-secant, in the Table, is 10.562129, corresponding to which is $15^{\circ}45'25''$; the difference between this log. co-secant and that given, is 15; answering to which, in the column of proportional parts abreast of the tabular log., is $2''$; which, being added to the above-found arch, gives $15^{\circ}45'27''$ for that required.

Remark.—The log. secant of any arch is expressed by the difference between twice the radius and the log. co-sine of that arch; and the co-secant of an arch, by the difference between twice the radius and the log. sine of such arch. Hence, to find the log. secant of $50^{\circ}40'$.—The log. co-sine of $50^{\circ}40'$ is 9.801974, which, being taken from twice the radius, viz., 20.000000, leaves 10.198026 for the log. secant; from this, the manner of computing the co-secant will be obvious.

TABLE XXXVI.

Logarithmic Sines.

Of all the Logarithmic Tables in this work, this is, by far, the most generally useful, particularly in the sciences of Navigation and Nautical Astronomy; and, therefore, much pains have been taken in reducing it to that state of simplicity which appears to be best adapted to its direct application to the many other purposes for which it is intended, besides those above-mentioned.

In this Table, the log. sines of the two first degrees of the quadrant are given to every second. The next eight degrees, viz., from 2 to 10, have their corresponding log. sines to every fifth second, with proportional parts answering to the intermediate seconds in the *adjacent* right-hand column; and because the log. sines increase rapidly in those degrees, two pages are allotted to a degree; every page being divided into two parts, and each part containing 15 minutes of a degree: so that no portion whatever of the proportional parts might be lost, and that the whole might have room to be fully inserted. In the following seventy degrees, that is, from 10 to 80, the log. sines are also given to every fifth second, with proportional parts corresponding to the intermediate seconds in the right-hand column of each page. In this part of the Table, each page contains a degree; and, for want of sufficient room, the indices are only prefixed to the logs. expressed in the first column.

From 80 to 90 degrees, the log. sines are only given to every tenth second, because of the small increments by which the sines increase towards the end of the first quadrant; the proportional parts for the intermediate seconds are given in the right-hand column of each page, as in the preceding part of the Table.

The Table is so arranged, as to be extended to every second in the semicircle, or from 0 to 180 degrees; as thus: the arcs corresponding to the log. sines are given in regular succession at top, from 0 to 90 degrees, and then continued, at bottom, reckoning towards the left hand, from 90 to 180 degrees. The arcs corresponding to the co-sines are given at bottom of the Table, and ranged in numerical order towards the left hand, from 0 to 90 degrees, (according to the order of the sines between 90 and 180 degrees,) and then continued at top, from 90 degrees to the end of the semicircle, agreeably to the order of the sines in the first quadrant. This mode of arrangement does away with the necessity of finding the supplement of an arch when it exceeds 90 degrees, and possesses the peculiar advantage of enabling the navigator to take out the log. sine or co-sine of any arch, and conversely, at sight, as will appear obvious by the following Problems.

Note.—The log. co-sine of a given degree is found in the same page with the *next less* degree in the column marked 0" at top, it being the first number in that column; and the co-sine of a given degree and minute is found on the same line with the *next less* minute in the column marked 60" at bottom of the page.

PROBLEM I.

To find the Logarithmic Sine, and Co-sine of any given Arch, expressed in Degrees, Minutes, and Seconds.

RULE.

If the given arch be comprised within the limits of the two first degrees of the quadrant, the Table will directly exhibit its corresponding log. sine or co-sine; but when it exceeds those limits, then find the log. sine, or co-sine, in the angle of meeting made by the given degree and next less fifth or tenth second at top, and the minutes in the left-hand column; to which add the proportional part corresponding to the odd seconds in the right-hand column abreast of the angle of meeting, if a sine be wanted, *or a co-sine above 90 degrees*; but subtract that part when a co-sine is required, *or a sine above 90 degrees*: and the sum, or difference, will be the log. sine, or co-sine, answering to the given arch.

Example 1.

Required the log. sine, and co-sine, corresponding to $23^{\circ}14'23''$?

To find the Log. Sine :—

	$23^{\circ}14'20''$, ans. to which is . . .	9.596119
Odd seconds	3 propor. part to which is +	15
<hr/>		
Given arch =	$23^{\circ}14'23''$ Corresponding log. sine	9.596134

To find the Log. Co-sine :—

	$23^{\circ}14'20''$, ans. to which is . . .	9.968253
Odd seconds	3 propor. part to which is —	8
<hr/>		
Given arch =	$23^{\circ}14'23''$ Corresponding log. co-sine	9.968250

Example 2.

Required the log. sine, and co-sine, corresponding to $113^{\circ}23'47''$?

To find the Log. Sine :—

	$113^{\circ}23'45''$, ans. to which is . . .	9.962740
Odd seconds	2 propor. part to which is —	2
<hr/>		
Given arch =	$113^{\circ}23'47''$ Corresponding log. sine	9.962738

To find the Log. Co-sine :—

	$113^{\circ}23'45''$, ans. to which is . . .	9.598879
Odd seconds	2 propor. part to which is +	10
<hr/>		
Given arch =	$113^{\circ}23'47''$ Corresponding log. co-s.	9.598889

PROBLEM II.

To find the Arch corresponding to a given Logarithmic Sine, or Co-sine.

RULE.

If the given log. sine, or co-sine, be less than the sine of 2 degrees, viz., 8.542819, its corresponding arch will be found at first sight in the Table; but if it exceeds that number, find the arch answering to the next less sine, or next greater co-sine; the difference between which and that given, being found in the column of proportional parts abreast of the tabular log., will give a certain number of seconds, which, being *added* to the above-found arch, will give that required.

Note.—Since the arcs corresponding to the sines between 90 and 180 degrees are found at the bottom of the Table, and those corresponding to the co-sines between the same limits at its top; if, therefore, it be required to find the arch above 90 degrees answering to a given log. sine, or co-sine, the first term is to be taken out as if it were a co-sine under 90 degrees, and the other term as if it were a sine under 90 degrees.

Example 1.

Required the arch corresponding to the given log. sine 9.437886?

Solution.—The next *less* log. sine in the Table is 9.437871, corresponding to which is $15^{\circ}54'25''$; the difference between this and that given, is 15; answering to which, in the column of proportional parts, abreast of the tabular log., is $2''$; which, being added to the above-found arch, gives $15^{\circ}54'27''$ for that required.

Example 2.

Required the arch corresponding to the given log. co-sine 9.764579?

Solution.—The next *greater* co-sine in the Table is 9.764588, corresponding to which is $54^{\circ}26'25''$; the difference between this and that given, is 9; answering to which, in the column of proportional parts, abreast of the tabular log., is $3''$; which, being added to the above-found arch, gives $54^{\circ}26'28''$ for that required.

Remark.—The log. sines are deduced directly from the natural sines; as thus:—

Multiply the natural sine by 10000000000; find the common log. of the product, and it will be the log. sine.

Example 1.

Required the log. sine of $39^{\circ}30'$?

Solution.—The natural sine of $39^{\circ}30'$ is .636078, which, being multiplied by 10000000000, gives 6360780000.000000, the common log. of which is 9.803511; which, therefore, is the log. sine of 39 degrees and 30 minutes, as required.

Example 2.

Required the log. co-sine of 68 degrees?

Solution.—The natural co-sine of 68 degrees is .374607, which, being multiplied by 10000000000, gives 3746070000.000000, the common log. of which is 9.573575; which, therefore, is the log. co-sine of 68 degrees, as required.

TABLE XXXVII.

Logarithmic Tangents.

This Table is arranged in a manner so very nearly similar to that of the log. sines, that it is not deemed necessary to enter into its description any farther than by observing, that it is computed to every second in the two first and two last degrees of the quadrant, or semicircle, and to every fifth second in the intermediate degrees. The log. tangent, or co-tangent, of a given arch, and conversely, is to be found by the rules for the log. sines in pages 94 and 95.

Example 1.

Required the log. tangent, and co-tangent, corresponding to $31^{\circ}10'47''$?

To find the Log. Tangent:—

	$31^{\circ}10'45''$, ans. to which is	. . .	9.781845
Odd seconds	2	propor. part to which is +	10
<hr/>			
Given arch =	$31^{\circ}10'47''$	Corresponding log. tang. =	9.781855

To find the Log. Co-tangent:—

	$31^{\circ}10'45''$, ans. to which is	. . .	10.218155
Odd seconds	2	propor. part to which is —	10
<hr/>			
Given arch =	$31^{\circ}10'47''$	Corres. log. co-tang. =	10.218145

Example 2.

Required the log. tangent, and co-tangent, corresponding to $139^{\circ}11'53''$?

To find the Log. Tangent:—

	$139^{\circ}11'50''$, ans. to which is	. . .	9.936142
Odd seconds	3	propor. part to which is —	13
<hr/>			
Given arch =	$139^{\circ}11'53''$	Corres. log. tang. =	9.936129

To find the Log. Co-tangent:—

	$139^{\circ}11'50''$, ans. to which is	. . .	10.063858
Odd seconds	3	propor. part to which is +	13
<hr/>			
Given arch =	$139^{\circ}11'53''$	Corres. log. co-tang. =	10.063871

Example 3.

Required the arch corresponding to the given log. tang. 10. 155436?

Solution.—The next *less* log. tangent in the Table is 10. 155423, corresponding to which is $55^{\circ}2'25''$; the difference between this log. tangent and that given, is 13; answering to which, in the column of proportional parts abreast of the tabular log., is $3''$; which, being added to the above-found arch, gives $55^{\circ}2'28''$ for that required.

Example 4.

Required the arch corresponding to the given log. co-tang. 9. 792048?

Solution.—The next *greater* log. co-tangent in the Table is 9. 792057, corresponding to which is $58^{\circ}13'15''$; the difference between this log. co-tangent and that given, is 9; answering to which, in the column of proportional parts abreast of the tabular log., is $2''$; which, being added to the above-found arch, gives $58^{\circ}13'17''$ for that required.

Remark.

The arch corresponding to a given log. tangent may be found by means of a Table of log. sines, in the following manner; viz.,

Find the natural number corresponding to twice the given log. tangent, rejecting the index, to which add the radius, and find the common log. of the sum; now, half this log. will be the log. secant, less radius, of the required arch; and which, being subtracted from the given log. tangent, will leave the log. sine corresponding to that arch.

Example.

Let the given log. tangent be 10. 084153; required the arch corresponding thereto by a table of log. sines?

Given log. tang. . 084153 $\times 2 =$. 168306, Nat. num. $=$ 1. 473349
to which add the radius $=$ 1. 000000

Sum $=$ 2. 473349, the
common log. of which is 0. 393286; the half of this is 0. 196648, the
secant, less radius of the required arch.

Given log. tangent $=$ 10. 084153

Corresponding log. sine $=$ 9. 887510,
answering to which is $50^{\circ}31'$; and which, therefore, is the
required arch corresponding to the given log. tangent.

The arch corresponding to a given log. tangent may also be found in the following manner, which, it is presumed, will prove both interesting and instructive to the student in this department of science.

Find the natural tangent, that is, the natural number corresponding to the given log. tangent, to the square of which add the square of the radius; extract the square root of the sum, and it will be the natural secant corresponding to the required arch; then, say, as the natural secant, thus found, is to the natural tangent, so is the radius to the natural sine: now, the degrees, &c. answering to this in the *Table of Natural Sines*, will be the arch required, or that corresponding to the given log. tangent.

Example.

Let the given log. tangent be 10.084153; it is required to find the arch corresponding thereto by a Table of Natural Sines?

Solution.—Given log. tangent = .084153; the natural number corresponding to this is 1.213816; which, therefore, is the natural tangent answering to the given log. tangent.

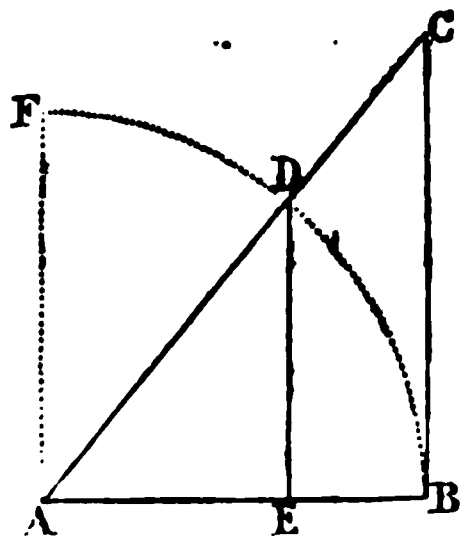
In the annexed diagram, let BC represent the natural tangent = 1.213816, and AB the radius = 1.000000. Now, since the base and perpendicular of the right-angled triangle ABC are known, the hypotenuse or secant AC may be determined by Euclid, Book I., Prop. 47. Hence

$$\sqrt{BC^2 + AB^2} = \sqrt{1.213816^2 + 1.000000^2} = AC = 1.572689,$$

the natural secant corresponding to the given log. tangent. Having thus found the natural secant AC , the natural sine DE may be found agreeably to the principles of similar triangles, as demonstrated in Euclid, Book VI., Prop. 4; for, as the natural secant AC is to the natural tangent BC , so is the radius $AD = AB$ to the natural sine DE : hence,

As AC 1.572689 : BC 1.213816 :: AD 1.000000 : DE = 771810, the corresponding natural sine; now, the arch answering to this, in the Table of Natural Sines, is $50^\circ 31'$; which, therefore, is the arch corresponding to the given log. tangent, as required.

Note.—The Table of log. tangents may be very readily deduced from Tables XXXV. and XXXVI., as thus:—To the log. secant of any given arch, add its log. sine; and the sum, abating 10 in the index, will be the log. tangent of that arch; the difference between which and twice the radius, will be its co-tangent.



Example.

Required the log. tangent, and co-tangent, of $25^{\circ}27'35''$?

Log. secant of the given arch $25^{\circ}27'35''$	=	10.044366
Log. sine of ditto		9.633344
		<hr/>
Log. tangent corres. to the given arch	=	9.677710
		<hr/>
Log. co-tangent corres. to ditto	:	10.322290

The Table of log. tangents may also be computed in the following manner ; viz.,

From the log. sine of the given arch, the index being increased by 10, subtract its log. co-sine, and the remainder will be the log. tangent of that arch ; the difference between which and twice the radius, will be its log. co-tangent.

Example.

Required the log. tangent, and co-tangent, of $32^{\circ}39'40''$?

Log. sine of the given arch $32^{\circ}39'40''$	=	. . 9.732128
Log. co-sine of ditto		9.925249
		<hr/>
Log. tangent corres. to the given arch	=	. . 9.806879
		<hr/>
Log. co-tangent corresponding to ditto . .		10.193121

TABLE XXXVIII.

To reduce the Time of the Moon's Passage over the Meridian of Greenwich, to the Time of her Passage over any other Meridian.

The daily retardation of the moon's passage over the meridian, given at the top of the Table, signifies the difference between two successive transits of that object over the same meridian, diminished by 24 hours ; as thus: the moon's passage over the meridian of Greenwich, July 22d, 1824, is $21^{\text{h}}7^{\text{m}}$, and that on the following day $22^{\text{h}}9^{\text{m}}$; the interval of time between these two transits is $25^{\text{h}}2^{\text{m}}$, in which interval it is evident that the moon is $1^{\text{h}}2^{\text{m}}$ later in coming to the meridian ; and which, therefore, is the daily retardation of her passage over the meridian.

This Table contains the proportional part corresponding to that retardation and any given interval of time or longitude ; in computing which, it is easy to perceive that the proportion was,

As 24 hours, augmented by the daily retardation of the moon's transit over the meridian, are to the said daily retardation of transit, so is any given interval of time, or longitude, to the corresponding proportional part of such retardation. The operation was performed by proportional logs., as in the following

Example.

Let the daily retardation of the moon's transit over the meridian be $1^{\circ}2'$; required the proportional part corresponding thereto, and $9^{\text{h}}40'$ of time, or 145 degrees of longitude ?

As 24 hours + $1^{\circ}2'$ (daily retard.) =	$25^{\circ}2'$	Ar. comp. pro. log.	9.1432
Is to daily retardation of transit =	. 1. 2	Propor. log.	. . 0.4629
So is given interval of time =	. . . 9.40	Propor. log.	. . 1.2700

To corresponding proportional part = $23^{\circ}57'$ = Pro. log. = 0.8761;
and in this manner were all the numbers in the Table obtained.

The corrections or proportional parts contained in this Table are expressed in minutes and seconds, and are extended to every twentieth minute of time, or fifth degree of longitude : these are to be taken out and applied to the time of the moon's transit, as given in the Nautical Almanac, in the following manner :—

Find, in page VI. of the month in the Nautical Almanac, the difference between the moon's transit on the given day (reckoned astronomically) and that on the day *following*, if the longitude be *west*; but on the day *preceding*, if it be *east*. With this difference enter the Table at the top, and the given time in the left-hand, or the longitude in the right-hand column ; in the angle of meeting will be found a correction, which, being applied by *addition* to the time of transit on the given day, if the longitude be *west*, but by *subtraction*, if *east*, the sum, or difference, will be the reduced time of transit.

Example 1.

Required the apparent time of the moon's passage over a meridian 80 degrees west of Greenwich, July 22d, 1824 ?

Mn's pas. over mer. of Greenw. on giv. day is $21^{\circ} 7'$. . . $21^{\circ} 7'$
 Ditto on the day following = 22.9

Retardation of moon's transit = . . . $1^{\circ} 2'$; ans. to which
 and 80 degs. is + 13

Apparent time of the moon's transit over the given meridian = $21^{\circ} 20'$

Example 2.

Required the apparent time of the moon's passage over a meridian
 degrees east of Greenwich, August 20th, 1824?

Mn's pas. over mer. of Greenw. on giv. day is $20^{\circ} 54'$. . . $20^{\circ} 54'$
 Ditto on the day preceding = 19.54

Retardation of the moon's transit = . . . $1^{\circ} 0'$; ans. to which
 and 120 degs. is - 19.

Apparent time of the moon's transit over the given meridian = $20^{\circ} 34'$

TABLE XXXIX.

*Correction to be applied to the Time of the Moon's Transit in finding
 the Time of High Water.*

Since the moon is the principal agent in raising the tides, it might be expected that the time of high water would take place at the moment of her passage over the meridian; but observation has shown that this is not the case, and that the tide does not cease flowing for some time after her passage, for, since the attractive influence of the moon is only diminished, and not entirely destroyed, in passing the meridian of any place, the ascending impulse previously communicated to the waters at that place must, therefore, continue to act for some time after the moon's meridional passage. The ascending impulse, thus imparted to the waters, ought to cause the time of the highest tide to be about 80 minutes after the moon's passage over the meridian; but owing to the disturbing force of the sun, the actual time of high water differs, at times, very considerably from that period.

The effect of the moon in raising the tides exceeds that of the sun in the ratio of about $2\frac{1}{2}$ to 1; but this effect is far from being uniform: since the moon's distance from the earth bears a very sensible proportion to the diameter of this planet, and since she is constantly changing her distance, (being sometimes nearer, and at other times more remote in

lunation,) it is evident that she must attract the waters of the ocean with very unequal forces: but the sun's distance from the earth being so very immense, that, compared with it, the diameter of this planet is rendered nearly insensible, his attraction is consequently more uniform, and therefore it affects the different parts of the ocean with nearly an equal force.

By the combined effect of these two forces, the tides come on *sooner* when the moon is in her *first* and *third* quarters, and *later* when in the *second* and *fourth* quarters, than they would do if raised by the sole lunar agency: it is, therefore, the mean quantity of this acceleration and retardation that is contained in the present Table, the arguments of which are, the apparent times of the moon's reduced transit; answering to which, in the adjoining column, stands a correction, which, being applied to the apparent time of the moon's passage over the meridian of any given place by addition or subtraction, according to its title, the sum, or difference, will be the corrected time of transit. Now, to the corrected time of transit, thus found, let the time of high water on full and change days, at any given place in Table LVI., be applied by *addition*, and the *sum* will be the time of high water at that place, reckoning from the noon of the given day: should the *sum* exceed $12^h 24^m$, or $24^h 48^m$, subtract one of those quantities from it, and the remainder will be the time of high water very near the truth.

Example 1.

Required the time of high water at Cape Florida, America, March 7th, 1824; the longitude being $80^{\circ} 5'$ west, and the time of high water on full and change days $7^h 30^m$?

Moon's transit over the meridian of Greenwich, per Nautical

Almanac, March 7th, 1824, is $5^h 2^m 0^s$

Correction from Table XXXVIII., answering to retardation of

transit 58^m , and longitude $80^{\circ} 5'$ west = + 12.23

Moon's transit reduced to the meridian of Cape Florida . . . $5^h 14^m 23^s$

Correction answering to reduced transit ($5^h 14^m 23^s$) in Table

XXXIX., is - $1.9.0$

Corrected time of transit $4^h 5^m 23^s$

Time of high water at Cape Florida on full and change days $7.30.0$

Time of high water at Cape Florida on the given day = . . $11^h 35^m 23^s$

Example 2.

Required the time of high water in Queen Charlotte's Sound, New Zealand, April 13th, 1824; the longitude being $174^{\circ} 56'$ east, and the time of high water on full and change days $9^h 0^m$?

Moon's transit over the meridian of Greenwich, per Nautical Almanac, April 13th, 1824, is	10 ^h 27 ^m 0 ^s
Correction from Table XXXVIII., answering to retardation of transit 50 ^m , and longitude 174° 56' west =	— 23. 29
<hr/>	
Moon's transit reduced to the meridian of Queen Charlotte's Sound	10 ^h 3 ^m 31 ^s
Correction answering to reduced time of transit (10 ^h 3 ^m 31 ^s) in Table XXXIX., is	+ 23. 0
<hr/>	
Corrected time of transit	10 ^h 26 ^m 31 ^s
Time of high water at given place on full and change days	9. 0. 0
<hr/>	
Time of high water at Queen Charlotte's Sound, past noon of the given day	19 ^h 26 ^m 31 ^s
	Subtract 12. 24. 0
<hr/>	
Time of high water at given place, as required	7 ^h 2 ^m 31 ^s
<hr/>	

TABLE XL.

Reduction of the Moon's Horizontal Parallax on account of the Spheroidal Figure of the Earth.

Since the moon's equatorial horizontal parallax, given in the Nautical Almanac, is determined on spherical principles, a *correction* becomes necessary to be applied thereto, in places distant from the equator, in order to reduce it to the spheroidal principles, on the assumption that the polar axis of the earth is to its equatorial in the ratio of 299 to 300; and, when *very great accuracy* is required, this *correction* ought to be attended to, since it may produce an error of seven or eight seconds in the computed lunar distance. The correction, thus depending on the spheroidal figure of the earth, is contained in this Table; the arguments of which are, the moon's horizontal parallax at the top, and the latitude in the left-hand column; in the angle of meeting will be found a correction, expressed in seconds, which being *subtracted* from the horizontal parallax given in the Nautical Almanac, will leave the horizontal parallax agreeably to the spheroidal hypothesis.

Thus, if the moon's horizontal parallax, in the Nautical Almanac, be 57' 58", and the latitude 51° 48'; the corresponding correction will be 7 seconds *subtractive*. Hence the moon's horizontal parallax on the spheroidal hypothesis, in the given latitude, is 57' 51".

Remark.—The corrections contained in this Table may be computed by the following

Rule.

To the logarithm of the moon's equatorial horizontal parallax, reduced to seconds, add twice the log. sine of the latitude, and the constant log. 7.522879 ;* the sum, rejecting the tens from the index, will be the logarithm of the corresponding reduction of parallax.

Example.

Let the moon's horizontal parallax be $57'58''$, and the latitude $51^{\circ}48'$; required the reduction of parallax agreeably to the spheroidal hypothesis?

Moon's equatorial horiz. par.	$57'58'' = 3478''$	Log. =	3.541330
Latitude	$51^{\circ}48'$	Twice the log. sine =	19.790688
		Constant log.	7.522879
<hr/>			
Reduction of horizontal parallax =	$7''.159$	Log. =	0.854897

TABLE XLI.

Reduction of Latitude on account of the Spheroidal Figure of the Earth.

Since the figure of the earth is that of an oblate spheroid, the latitude of a place, as deduced directly from celestial observation, agreeably to the spherical hypothesis, must be greater than the true latitude expressed by the angle, at the earth's centre, contained between the equatorial radius and a line joining the centre of the earth and the place of observation. This excess, which is extended to every second degree of latitude from the equator to the poles, is contained in the present Table; and which, being *subtracted* from the latitude of any given place, will reduce that latitude to what it would be on the spheroidal hypothesis: thus, if the latitude be 50 degrees, the corresponding reduction will be $11'42''$, subtractive; which, therefore, gives $49^{\circ}48'18''$ for the reduced or spheroidal latitude.

Remark.—The corrections contained in this Table may be computed by the following rule; viz.,

To the constant log. .003003,† add the log. co-tangent of the latitude,

* The arithmetical complement of the log. of the earth's ellipticity assumed at $\frac{1}{288}$.

† The excess of the spherical above the elliptic arch in the parallel of 45 degrees from the equator, is $11'.887$, or $11'53''$ (Robertson's Navigation, Book VIII., Article 134): hence $45^{\circ} - 11'53'' = 44^{\circ}48'7''$, the log. co-tangent of which, rejecting the index, is .003003.

and the sum will be the log. co-tangent of an arch ; the difference between which and the given latitude will be the required reduction.

Example.

Let it be required to reduce the spherical latitude $50^{\circ}48'$ to what it would be if determined on the spheroidal principles ; and, hence, to find the reduction of that latitude,

Latitude	$50^{\circ}48' 0''$	Log. co-tang.=	9.911467
		Constant log.=	<u>.003003</u>
Reduced or spheroidal latitude =	$50^{\circ}36'.21''$	Log. co-tang.=	<u>9.914470</u>
Reduction of latitude, as required	$0^{\circ}11'.39''$		

TABLE XLII.

A General Traverse Table ; or Difference of Latitude and Departure.

This Table, so exceedingly useful in the art of navigation, is drawn up in a manner quite different from those that are given, under the same denomination, in the generality of nautical books ; and, although it occupies but 38 pages, yet it is more extensive than the two combined Tables of 61 pages, which are contained in those books. In this Traverse Table, every page exhibits all the angles that a ship's course can make with the meridian, expressed both in points and degrees ; which does away with the necessity of consulting two Tables in finding the difference of latitude and the departure corresponding to any given course and distance. If the course be *under* 4 points, or 45 degrees, it will be found in the left-hand compartment of each page ; but that *above* 4 points, or 45 degrees, in the right-hand compartment of the page. The distance is given, in numerical order, at the top and bottom of the page, from unity, or 1, to 304 miles ; which comprehends all the probable limits of a ship's run in 24 hours ; and, by this arrangement, the mariner is spared the trouble of turning over and consulting twenty-three additional pages. Although the manner of using this Table must appear obvious at first sight, yet since its mode of arrangement differs so very considerably from the Tables with which the reader may have been hitherto acquainted, the following Problems are given for its illustration.

PROBLEM I.

Given the Course and Distance sailed, or between two Places, to find the Difference of Latitude and the Departure.

RULE.

Enter the Table with the course in the left or right-hand column, and the distance at the top or bottom; opposite to the former, and under or over the latter, will be found the corresponding difference of latitude and departure: these are to be taken out as marked at the top of the respective columns if the course be under 4 points or 45 degrees, but as marked at the bottom if the course be more than either of those quantities.

Note.—If the distance exceed the limits of the Table, an aliquot part thereof may be taken, as a half, third, fourth, &c.; then the difference of latitude and departure corresponding to this and the given course, being multiplied by 2, 3, 4, &c., (that is, the figure by which such aliquot part was found,) the product will be the difference of latitude and departure answering to the given course and distance.

Example 1.

A ship sails S.S.W. $\frac{1}{4}$ W. 176 miles; required the difference of latitude and the departure?

Opposite $2\frac{1}{4}$ points and under 176 miles, stand 155.2 and 83.0: hence the difference of latitude is 155.2, and the departure 83.0 miles.

Example 2.

A ship sails N. 57° E. 236 miles; required the difference of latitude and the departure?

Opposite to 57° , and under 236 miles, stand 128.5 and 197.9: hence the difference of latitude is 128.5, and the departure 197.9 miles.

Example 3.

The course between two places is E. b. S. $\frac{3}{4}$ S., and the distance 540 miles; required the difference of latitude and the departure?

Distance divided by 2, gives 270 miles; under or over which, and opposite to $6\frac{1}{4}$ points, stand . . . 91.0 and 254.2

Multiply by 2 2

Products = 182.0 and 508.4: hence the difference of latitude is 182.0, and the departure 508.4 miles.

Example 4.

The course between two places is N. 61 W. and the distance 1176 miles; required the difference of latitude and the departure ?

Distance 1176 divided by 4, gives 294 miles; under or over which, and opposite to 61°, stand . . . 142.5 and 257.1

Multiply by 4 4

Product = 570.0 and 1028.4 : hence the difference of latitude is 570.0, and the departure 1028.4 miles.

PROBLEM II.

Given the Difference of Latitude and the Departure, to find the Course and Distance.

RULE.

With the given difference of latitude and departure, enter the Table and find, in the proper columns abreast of each other, the tabular difference of latitude and departure either corresponding or nearest to those given; then the course will be found on the same horizontal line therewith in the left or right-hand column, and the distance at the top or bottom of the compartment where the tabular numbers were so found.

Note.—If the difference of latitude be *greater* than the departure, the course will be *less* than 4 points, or 45 degrees; and, therefore, it is to be taken from the left-hand column: but when the difference of latitude is *less* than the departure, the course will be *more* than 4 points or 45 degrees, and, consequently, it must be taken from the right-hand column.

Note, also, that when the difference of latitude and the departure, or either of them, exceed the limits of the Table, aliquot parts are to be taken, as a half, third, fourth, &c., with which find the course and distance as before; then the *distance*, thus found, being multiplied by 2, 3, 4, &c., the product will be the *whole distance* corresponding to the given difference of latitude and departure. The course is *never* to be multiplied, because the angle will be the same whether determined agreeably to the whole difference of latitude and the departure, or according to their corresponding aliquot parts.

Example 1.

If the difference of latitude made by a ship in 24 hours be 177.4 miles north, and the departure 102.6 miles east, required the course and distance made good?

Solution.—The tabular difference of latitude and departure, nearest corresponding to those given, are 177.5 and 102.5 respectively: these are found in the compartment under or over 205, and opposite to 30 degrees; hence the course made good is N. 30 E., and the distance 205 miles.

Example 2.

The difference of latitude made by a ship in 24 hours, is 98.5 miles south, and the departure 140.6 miles west; required the course and distance made good?

Solution.—The tabular difference of latitude and departure, nearest to those given, are 98.7 and 140.9 respectively: these are found in the compartment under or over 172, and opposite to 55 degrees; hence the course made good is S. 55° W., and the distance 172 miles.

Example 3.

The difference of latitude is 700 miles south, and the departure 928 miles west; required the course and distance?

Solution.—Since the difference of latitude and the departure exceed the limits of the Table, take therefore any aliquot part of them, as one fourth, and they will be 175 and 232 respectively: now, the tabular numbers, answering nearest to those, are 175.1 and 232.4; these are found in the compartment under or over 291, and opposite to 53 degrees: hence the course is S. 53° W., and the distance $291 \times 4 = 1164$ miles, as required.

Remark.—Whenever it becomes necessary to take aliquot parts of the difference of latitude, the same must be taken of the departure, whether it falls without the limits of the Table or not; and, *vice versa*, whenever it becomes necessary to take aliquot parts of the departure, the same must be taken of the difference of latitude.

And, in all cases where the tabular numbers differ considerably from those given, proportion must be made for that difference.

PROBLEM III.

Given the proper Difference of Latitude between two Places, the Meridional Difference of Latitude, and the Departure, to find the Course, Distance, and Difference of Longitude.

RULE.

With the proper difference of latitude and the departure, find the course and distance by Problem II.; then, with the course thus found and the meridional difference of latitude, (in a latitude column,) take out the corresponding departure, and it will be the difference of longitude required; as thus: run the eye along the horizontal line answering to the course, from where the proper difference of latitude was found, (*always to the right hand,*) and find, in a latitude column, the tabular difference of latitude answering nearest to the given meridional difference of latitude; abreast of which, in the departure column, will be found the difference of longitude.

Example.

The proper difference of latitude between two places, is 142 miles north, the departure 107 miles west, and the meridional difference of latitude 169 miles; required the course, distance, and difference of longitude?

Solution.—The tabular difference of latitude and departure answering nearest to those given, are 142.2 and 107.3 respectively; these are found in the compartment under or over 178, and opposite to 37 degrees: hence the course is N. 37° W., and the distance 178 miles. Now, with the course 37 degrees, and the meridional difference of latitude 169 miles, the difference of longitude is found, as thus: from where the proper difference of latitude was found, run the eye along the horizontal line answering to 37 degrees, (*always towards the right hand,*) and the tabular difference of latitude answering nearest to the given meridional difference of latitude will be found in the compartment under or over 212, viz. 169.3; corresponding to which, in the departure column, is 127.6; and which, therefore, is the difference of longitude, as required.

PROBLEM IV.

Given the proper Difference of Latitude, the Meridional Difference of Latitude, and the Difference of Longitude, to find the Course and Distance.

RULE.

With the meridional difference of latitude and the difference of longitude, esteemed as difference of latitude and departure, find the course by Problem II.; then with the course, thus found, and the proper difference of latitude, the distance is to be obtained, as thus : run the eye (*always to the left hand*) along the horizontal line answering to the course, from where the meridional difference of latitude was found, and seek, in the proper column, the difference of latitude answering nearest to that given ; over or under which, at the top or bottom of the column, will be found the required distance.

Note.—When the meridional difference of latitude exceeds the difference of longitude, the course is to be taken from the left-hand column ; but otherwise from the right.

Example.

The proper difference of latitude between two places is 78 miles south, the meridional difference of latitude 107 miles south, and the difference of longitude 119 miles east ; required the course and distance ?

Solution.—The tabular difference of latitude and departure, answering nearest to the meridional difference of latitude and the difference of longitude, are 107.1 and 118.9 respectively ; these are found in the compartment under or over 160, and opposite to 48 degrees : hence the course is S. 48° E. Now, the eye being run along the horizontal line answering to 48, (*towards the left hand,*) the nearest tabular difference of latitude, answering to the proper difference of latitude, will be found in the compartment under or over 117 : hence the distance is 117 miles.

PROBLEM V.

Given the middle Latitude, and the Meridian Distance or Departure, to find the Difference of Longitude.

RULE.

Enter the Table with the middle latitude, *taken as a course*, and the departure in a latitude column ; run the eye along the horizontal line

answering to that course (towards the right hand or the left, according as the first tabular difference of latitude which meets the eye therein is greater or less than the given departure), and find a difference of latitude that either agrees with, or comes nearest to, the given departure; then the distance corresponding to this, at the top or bottom of the column, will be the difference of longitude.

Example.

The middle latitude between two places is 20° north, and the meridian distance or departure 140 miles; required the difference of longitude?

Solution.—The middle latitude, 20 degrees, taken as a course, and the departure 140, as difference of latitude, will be found to correspond in the compartment under or over 149: hence the difference of longitude is 149 miles, as required.

PROBLEM VI.

Given the middle Latitude, the Difference of Latitude, and the Difference of Longitude between two Places, to find the Course and Distance.

RULE.

Enter the Table with the difference of longitude, esteemed as distance, at the top or bottom of the page, and the middle latitude, *taken as a course*, in the left or right-hand column; answering to which, in the *difference of latitude column*, will be found the departure. Now, with this departure and the given difference of latitude, the course and distance are to be found by Problem II.

Example.

The middle latitude is 26 degrees north, the difference of latitude 200 miles north, and the difference of longitude 208 miles east; required the course and distance?

Solution.—In the compartment under or over 208 miles (the given longitude), and opposite to 26 degrees (the middle latitude taken as a course), stands 186.9 in the difference of latitude column, which, therefore, is the departure. Now, the tabular numbers answering nearest to the given difference of latitude and the departure, thus found, are 200.4 and 186.9 respectively; these are found in the compartment under or over 274, and opposite to 43° : hence the course is N. 43° E., and the distance 274 miles.

Remark.—The numbers in the general Traverse Table were computed agreeably to the following rule; viz.,

As radius is to the distance, so is the co-sine of the course to the difference of latitude; and so is the sine of the course to the departure.

Example.

Given the course 35 degrees, and the distance 147 miles; to compute the difference of latitude and the departure.

To find the Difference of Latitude.

As radius	=	90° log. sine . . .	=	10.000000
Is to distance		147 miles . . . log.	=	2.167317
So is the course	=	35° log. co-sine . . .	=	9.913365
<hr/>				
To difference of lat.	=	120.4, miles . . . log.	=	2.080682

To find the Departure.

As radius	=	90° log. sine . . .	=	10.000000
Is to distance		147 miles . . . log.	=	2.167317
So is the course	=	35°, log. sine . . .	=	9.758591
<hr/>				
To departure	=	84.3 miles . . . log.	=	1.925908

TABLE XLIII.

Meridional Parts.

This Table contains the meridional parts answering to each degree and minute of latitude from the equator to the poles; the arguments of which are, the degrees at the top, and the minutes in the left or right hand marginal columns; under the former, and opposite to the latter, in any given latitude, will be found the meridional parts corresponding thereto, and conversely. Thus, if the latitude be 50°48', the corresponding meridional parts will be 3549.8 miles.

Remark.—The Table of meridional parts may be computed by the following rule; viz.,

Find the logarithmic co-tangent *less radius* of half the complement of any latitude, and let it be esteemed as an *integral number*; now, from the

The places of the stars, as given in this Table, may be reduced to any future period by multiplying the annual variation by the number of years and parts of a year elapsed between the beginning of 1824, and such future period : the product of right ascension is to be *added* to the right ascensions of all the stars, except β and δ , in Ursa Minor, from whose right ascensions it is to be subtracted : but the product of declination is to be applied, according to *the sign* prefixed to the annual variation in the Table, to the declinations of all the stars without any exception ;—thus,

To find the right ascension and the declination of α Arietis, Jan. 1st, 1834.

R. A. of α Arietis, per Tab.	$1^h 57^m 16^s$, and its dec.	$22^\circ 37' 33''$ N.
Annual var.	$+3''.35$	Ann. var. $+17''.40$.
Number of years		Num. of yrs.
after 1824 =	10	after 1824 = 10

$$\text{Product} \quad +33''.5 \quad +0'.33''.5 \quad \text{Prod.} +174''.0 = +2'.54''$$

Rt. asc. of α Arietis, as req. $1^h 57^m 49^s.5$, and its declination $22^\circ 40' 27''$ N.

Should the places of the stars be required for any period antecedent to 1824, it is evident that the products of right ascension and declination must be applied in a contrary manner.

The eighth column of this Table contains the *true spherical distance* and the approximate bearing between the stars therein contained and those preceding, or abreast of them on the same horizontal line ; and the ninth, or last column of the page, the annual variation of that distance expressed in seconds and decimal parts of a second.—By means of the last column, the tabular distance may be reduced very readily to any future period, by multiplying the years and parts of a year between any such period and the epoch of the Table, by the annual variation of distance ; the product being applied by addition or subtraction to the tabular distance, according as the sign may be affirmative or negative, the sum or difference will be the distance reduced to that period.

Example.

Required the distance between α Arietis and Aldebaran, Jan. 1st, 1844 ?

Tabular dist. between the two given stars = $35^\circ 32' 7''$

Annual var. of distance $-0''.02$

Number of years after 1824 = 20

$$\text{Product} \quad \cdot \quad -0''.40 = \cdot \quad -0''.40$$

True spherical distance between the two given stars, as required. $35^\circ 32' 6''.60$.

Remark.—The true spherical distance between any two stars, whose right ascensions and declinations are known, may be computed by the following rule; viz.,

To twice the log. sine of half the difference of right ascension, in degrees add the log. sines of the polar distances of the objects; from half the sum of these three logs. subtract the log. sine of half the difference of the polar distances, and the remainder will be the log. tangent of an arch; the log. sine of which being subtracted from the half sum of the three logs., will leave the log. sine of half the true distance between the two given stars.

Example.

Let it be required to compute the true spherical distance between α Arietis and Aldebaran, January 1, 1844.

R. A. of α Arietis red. to 1844 = $1^h 58^m 23^s$, and its dec. = $22^\circ 43' 21''$ N.

R. A. of Aldebaran red. to 1844 = 4. 26. 58. 6, and its dec. = $16. 11. 28$ N.

Difference of right ascension = $2^h 28^m 35^s. 6 =$

$37^\circ 8' 54'' + 2 = 18^\circ 34' 27''$

Half difference of R. A. in degrees = $18^\circ 34' 27''$ Twice the Log. sine } 19. 0063060

N. polar dist. of α Arietis = { 67. 16. 39 { Log. sine } 9. 9649129

N. polar dist. of Aldebaran = { 73. 48. 32 { Log. sine } 9. 9824236

Sum . . 38. 9536425

Diff. of Polar dists. $6^\circ 31' 53''$ Half = $19. 4768212\frac{1}{2}$. . . 19. 4768212. 5

Half diff. of ditto $3^\circ 15' 56\frac{1}{2}''$ Log S. 8. 7556177 $\frac{1}{2}$

Arch $79^\circ 14' 27''. 5826$ log. tang. . 10. 7212035 Log. S. 9. 9922976. 3

Half the req. dist. . . . 17 $^\circ$ 46' 3''. 4424 . Log. S. 9. 4845236. 2

True spher. dist. between

the two given stars . . 35 $^\circ$ 32' 6''. 8848 on Jan. 1, 1844.

Now, by comparing this computed distance with that directly deduced from the Table, as in the preceding example, it will be seen that the difference amounts to very little more than the fifth part of a second in twenty years; which evidently demonstrates that the tabular distances may be reduced to any subsequent period, for a considerable series of years, with all the accuracy that may be necessary for the common purposes of navigation.

Note.—The tabular distances will be found particularly useful in determining the latitude, at sea, by the altitudes of two stars, as will be shown hereafter.

TABLE XLV.

Acceleration of the Fixed Stars; or to reduce Sidereal to Mean Solar Time.

Observation has shown that the interval between any two consecutive transits of a fixed star over the same meridian is only $23^{\text{h}}56^{\text{m}}4^{\text{s}}.09$, whilst that of the sun is 24 hours:—the former is called a sidereal day, and the latter a solar day; the difference between those intervals is $3^{\text{m}}55^{\text{s}}.91$, and which difference is called the acceleration of the fixed stars.

This acceleration is occasioned by the earth's annual motion round its orbit: and since that motion is from west to east at the mean rate of $59'.8''.3$ of a degree each day; if, therefore, the sun and a fixed star be observed on any day to pass the meridian of a given place at the same instant, it will be found the next day when the star returns to the same meridian, that the sun will be nearly a degree short of it; that is, the star will have gained $3^{\text{m}}56^{\text{s}}.55$ sidereal time, on the sun, or $3^{\text{m}}55^{\text{s}}.91$ in mean solar time; and which amounts to one sidereal day in the course of a year:—for $3^{\text{m}}55^{\text{s}}.91 \times 365^{\text{d}}5^{\text{h}}48^{\text{m}}48^{\text{s}} = 23^{\text{h}}56^{\text{m}}4^{\text{s}}:—$ hence in 365 days as measured by the transits of the sun over the same meridian, there are 366 days as measured by those of a fixed star.

Now, because of the earth's equable or uniform motion on its axis, any given meridian will revolve from any particular star to the same star again in every diurnal revolution of the earth, without the least perceptible difference of time shewn by a watch, or clock, that goes well:—and this presents us with an easy and infallible method of ascertaining the error and the rate of a watch or clock:—to do which we have only to observe the instant of the disappearance of any bright star, during several successive nights, behind some fixed object, as a chimney or corner of a house at a

As one sidereal day, is to 3^m55'.91, so is any given portion of sidereal time to its corresponding portion of mean solar time :—and hence, the method by which the Table was computed.

TABLE XLVI.

To reduce Mean Solar Time into Sidereal Time.

Since this Table is merely the converse of the preceding, it is presumed that it does not require any explanation farther than by observing, that the correction is to be applied by addition to the corresponding mean solar time, in order to reduce it into sidereal time ; as thus.

Required the sidereal time corresponding to 20^h15^m33^s: mean solar time?

Given mean solar time =	20 ^h 15 ^m 33 ^s :
Corresponding to 20 hours is	3 ^m 17'.13	} Sum = . + 3 ^m 19'.68
Do. 15 minutes	0. 2 .46	
Do. 33 seconds	0. 0 .09	
Sidereal time as required	20 ^h 18 ^m 52'.68

TABLE XLVII.

Time from Noon when the Sun's Centre is in the Prime Vertical ; being the instant at which the Altitude of that Object should be observed in order to ascertain the apparent Time with the greatest Accuracy.

Since the change of altitude of a celestial object is quickest when that object is in the prime vertical, the most proper time for observing an altitude from which the apparent time is to be inferred, is therefore when the object is due east or west ; because then the apparent time is not likely to be affected by the unavoidable errors of observation, nor by the inaccuracy of the assumed latitude.—This Table contains the apparent time when a celestial object is in the above position.—The declination is marked at top and bottom, and the latitude in the left and right hand marginal columns : hence, if the latitude be 50 degrees, and the declination 10 degrees, both being of the same name, the object will be due east or west at 5^h.26^m from its time of transit or meridional passage.

Remark.—This Table was computed by the following rule ; viz.,

To the log. co-tangent of the latitude, add the log. tangent of the declination; and the sum, abating 10 in the index, will be the log. co-sine of the hour angle, or the object's distance from the meridian when its true bearing is either east or west.

Example.

Let the latitude be 50 degrees, north or south, and the sun's declination 10 degrees, north or south; required the apparent time when that object will bear due east or west?

Given latitude = 50° log. co-tangent = 9.923814

Declination of the sun = 10° log. tangent = 9.246319

Hour angle = 81°29'30" = log. co-sine = 9.170133

In time = 5^h25^m58^s; which, therefore, is the apparent time when the sun bears due east or west.

Note.—During one half of the year, or while the sun is on the other side of the equator, with respect to the observer, that object is not due east or west while above the horizon; in this case, therefore, the observations for determining the apparent time must be made while the sun is near to the horizon; the altitude, however, should not be under 3 or 4 degrees, on account of the uncertainty of the effects of the atmospheric refraction on low altitudes.

TABLE XLVIII.

Altitude of a Celestial Object (when its centre is in the Prime Vertical,) most proper for determining the apparent Time with the greatest Accuracy.

This Table is nearly similar to the preceding; the only difference being that that Table shows the apparent time when a celestial object bears due east or west, and this Table the true altitude of the object when in that position; being the altitude most proper to be observed in order to ascertain the apparent time with the greatest accuracy:—thus, if the latitude be 50 degrees, and the declination 10 degrees, both being of the same name, the altitude of the object will be 13°6', when it bears due east or west from the observer; which, therefore, is the altitude most proper to be observed, for the reasons assigned in the explanation to Table XLVII.

Note.—This Table was computed by the following rule; viz.,

If the declination be less than the latitude ; from the log. sine of the former (the index being increased by 10), subtract the log. sine of the latter, and the remainder will be the log. sine of the altitude of the object when its centre is in the prime vertical :—But, if the latitude be less than the declination, a contrary operation is to be used ; viz., from the log. sine of the latitude, the index being increased by 10, subtract the log. sine of the declination, and the remainder will be the log. sine of the altitude of the object when its centre is in the prime vertical, or when it bears due east or west.

Example 1.

Let the latitude be 50° , and the declination of a celestial object 10° , both being of the same name ; required the altitude of that object when its centre is in the prime vertical.

Declination of the object = 10°	log. sine = 9.239670
Latitude 50.	log. sine = 9.884254
Altitude required . . . $13^{\circ}6'6''$	log. sine = <u>9.355416</u>

Example 2.

Let the latitude be 3° , and the declination of a celestial object 14° , both being of the same name ; required the altitude of that object when its in the prime vertical.

Latitude 3°	log. sine = 8.718800
Declination of the object = 14	log. sine = 9.383675
Altitude required . $12^{\circ}29'38''$	log. sine = <u>9.335125</u>

Note.—Altitudes under 3 or 4 degrees should not be made use of in computing the apparent time, on account of the uncertainty of the atmospheric refraction near the horizon.

And since the Table only shows the altitude of a celestial object most favourable for observation when the latitude and declination are of the same name ; therefore during that half of the year in which the sun is on the other side of the equator, with respect to the observer, and in which he does not come to the prime vertical while above the horizon, the altitude is to be taken whenever it appears to have exceeded the limits ascribed to the uncertainty of the atmospheric refraction in page 120.

TABLE XLIX.

Amplitudes of a Celestial Object, reckoned from the true East, or West Point of the Horizon.

The arguments of this Table are, the declination of a celestial object at top or bottom, and the latitude in the left, or right hand column; in the angle of meeting will be found the amplitude: proportion, however, is to be made for the excess of the minutes above the next less tabular arguments.

Example 1.

Let the latitude be $50^{\circ}48'$ north, and the sun's declination $10^{\circ}25'$ north; required the sun's true amplitude at its setting?

True amplitude corresp. to lat. 50° , and dec. 10° , = W. $15^{\circ}40'$ N.

Tab. diff. to 1° of lat. = $21'$; now $\frac{21' \times 48'}{60'} = + 17$, nearly;

T.diff. to 1° of dec. = $1^{\circ}36'$, or $96'$; now $\frac{96' \times 25'}{60'} = + 40$

Sun's true amplitude as required = W. 16.37 . N.

Example 2.

Let the latitude be $34^{\circ}24'$ north, and the sun's declination $16^{\circ}48'$ south; required the sun's true amplitude at the time of its rising?

True amplitude corresponding to latitude 34° N. and

declination $16^{\circ}30'$ S. = E. $20^{\circ}2'$ S.

Tab. diff. to 1° of lat. = $15'$; now $\frac{15' \times 24'}{60'} = . . + 6$

Tab. diff. to $30'$ of dec. = $37'$; now $\frac{37' \times 18'}{30'} = . . + 22$, nearly.

Sun's true amplitude as required = E. $20^{\circ}30'$ S.

Remark.

This Table was computed agreeably to the following rule; viz.,

To the log. secant of the latitude, add the log. sine of the declination, and the sum, abating 10 in the index, will be the log. sine of the true amplitude.

Example.

Let the latitude be $50^{\circ}48'$, and the declination of a celestial object $10^{\circ}25'$; required the true amplitude of that object?

Latitude $50^{\circ}48'$ log. secant 10.199263

Declination 10.25 log. sine 9.257211

True amplitude as required $16^{\circ}37'22''$ log. sine . . 9.456474

TABLE L.

To find the Times of the Rising and Setting of a Celestial Object.

This Table contains the semidiurnal arch, or the time of half the continuance of a celestial object above the horizon when its declination is of the same name with the latitude of the place of observation; or the time of half its continuance below the horizon when its declination and the latitude are of different denominations.—*The semi-diurnal arch expresses the time that a celestial object takes in ascending from the eastern horizon to the meridian; or of its descending from the meridian to the western horizon.*

As the Table is only extended to $23\frac{1}{2}$ degrees of declination, being the greatest declination of the sun, and to no more than 60 degrees of latitude; therefore, when the declination of any other celestial object and the latitude of the place of observation exceed those limits, the semi-diurnal arch is to be computed by the following rule; viz.,

To the log. tangent of the latitude, add the log. tangent of the declination, and the sum, rejecting 10 in the index, will be the log. sine of an arch; which being converted into time, and added to 6 hours when the latitude and declination are of the same name; or subtracted from 6 hours when these elements are of contrary names; the sum, or difference, will be the semi-diurnal arch.

Example 1.

Let the latitude be 61 degrees, north, and the declination of a celestial object $25^{\circ}10'$, north; required the corresponding semi-diurnal arch?

Latitude $61^{\circ} 0'$ north, log. tangent 10.256248

Declination 25.10 north, log. tangent 9.671963

Arch = $57^{\circ}57'21''$ = log. sine . . . 9.928211

Arch conv. into time $3^h51^m49^s + 6^h = 9^h51^m49^s$, the semidiurnal arch, as required,

Example 2.

Let the latitude be $20^{\circ}40'$, south, and the declination of a celestial object $30^{\circ}29'$, north; required the corresponding semi-diurnal arch?

Latitude. $20^{\circ}40'$ south, log. tangent . . 9.576576

Declination 30.29 north, log. tangent . . 9.769860

Arch = $12^{\circ}49'45''$ = log. sine . . . 9.346436

Arch conv. into time $0^h 51^m 19^s$; and $6^h - 0^h 51^m 19^s = 5^h 8^m 41^s$, the semi-diurnal arch.

The present Table has been computed agreeably to the first example; but as in most nautical computations, it is not absolutely necessary that the semi-diurnal arch should be determined to a greater degree of accuracy than the nearest minute; the seconds have, therefore, been rejected, and the nearest minute retained accordingly.

Since the Table for finding the time of the rising or setting of a celestial object (commonly called a Table of semi-diurnal and semi-nocturnal arcs,) is scarcely applied to any other purpose, by the generality of nautical persons, than that of merely finding the approximate time of the rising or setting of the sun; the following problems are, therefore, given for the purpose of illustrating and simplifying the use of this Table; and of showing how it may be employed in determining the apparent times of the rising and setting of all the celestial objects whose declinations come within its limits.

PROBLEM I.

Given the Latitude and the Sun's Declination, to find the Time of its Rising or Setting.

RULE.

Let the sun's declination, as given in the Nautical Almanac, be reduced to the meridian of the given place by Table XV., or by Problem I., page 76; then,

Enter the Table with this reduced declination at top, or bottom, and the latitude in either of the side columns; under or over the former, and opposite to the latter, will be found the approximate time of the sun's setting when the latitude and declination are of the same name; or that of its rising when they are of contrary names.—The time of setting being taken from 12 hours will leave the time of rising, and *vice versa*, the time of rising being taken from 12 hours will leave that of setting.

Note.—Proportion must be made, as usual, for the excess of the minutes of latitude and declination above the next less tabular arguments.

Example 1.

Required the approximate times of the sun's rising and setting July 13, 1824, in latitude $50^{\circ}48'$, north, and longitude 120 degrees west?

Sun's declination July 13th. per Nautical

Almanac, is $21^{\circ}49'51''$ north.

Correction from Table XV., answering to

var. of dec. $8'58''$, and long. 120° W. — $2'59''$

Sun's dec. reduced to given meridian . . $21^{\circ}46'52''$; or $21^{\circ}47'$, N. *

Time, in Table L., ans. to lat. 50° , north, and

dec. $21^{\circ}30'$, north = 7^h52^m

Tabular difference to 1° of lat. = $4'$; now $\frac{4' \times 48'}{60'} = + 3$

Tab. difference to $30'$ of dec. = $3'$; now $\frac{3' \times 17'}{30'} = + 2$, nearly.

Approximate time of the sun's setting 7^h57^m

Approximate time of the sun's rising $4^h 3^m$

Note.—Twice the time of the sun's setting will give the length of the day; and twice the time of its rising will give the length of the night.

Example 2.

Required the approximate times of the sun's rising and setting October 1st, 1824, in latitude $40^{\circ}30'$ north, and longitude 105 degrees east?

Sun's declination October 1st. per Nautical

Almanac, is $3^{\circ}16' 6''$ south.

Correction from Table XV., answering to

var. of dec. $23'20''$, and long. 105° E. — $6'48''$

Sun's dec. reduced to the given meridian $3^{\circ} 9'18''$, or $3^{\circ}9'$ south.

Time in Table L., ans. to lat. 40° north, and

dec. 3° south, is 6^h10^m

Tab. diff. to 1° of lat. = $0'$; now $\frac{0' \times 30'}{60'} = 0$

Tab. diff. to 1° of dec. = $3'$; now $\frac{3' \times 9'}{60'} = 0$

Approximate time of the sun's rising 6^h10^m

Approximate time of the sun's setting 5^h50^m

* The nearest minute of declination is sufficiently exact for the purpose of finding the approximate times of the rising and setting of a celestial object.

Remark.

Since the times of the sun's rising and setting, found as above, will differ a few minutes from the observed, or apparent times in consequence of no notice having been taken of the combined effects of the horizontal refraction and the height of the observer's eye above the level of the sea, by which the time of rising of a celestial object is accelerated, and that of its setting retarded; nor of the horizontal parallax which affects these times in a contrary manner; a correction, therefore, must be applied to the approximate times of rising and setting, in order to reduce them to the apparent times.—This correction may be computed by the following rule; by which the apparent times of the sun's rising and setting will be always found to within a few seconds of the truth.

Rule.—To the approximate times of rising and setting, let the longitude, in time, be applied by addition or subtraction, according as it is west or east, and the corresponding times at Greenwich will be obtained; to these times, respectively, let the sun's declination be reduced by Table XV., or by Problem I., page 76; then,

Find the sum and the difference of the natural sine of the latitude, and the natural co-sine of the declination (rejecting the two right hand figures from each term), and take out the common log. answering thereto, rejecting also the two right hand figures from each:—now, to half the sum of these two logs. add the proportional log. of the sum of the horizontal refraction and the dip of the horizon diminished by the sun's horizontal parallax, and the constant log. 1. 1761*; the sum of these three logs., abating 4 in the index, will be the proportional log. of a correction; which being subtracted from the approximate time of rising, and added to that of setting, the apparent times of the sun's rising and setting will be obtained.

Thus,—Let it be required to reduce the approximate times of the sun's rising and setting, as found in the last Example, to the respective apparent times; the horizontal refraction being 33'; the dip of the horizon 5' 15", and the sun's horizontal parallax 9 seconds.

The sun's declination reduced to the approximate time of rising, is 3° 3' 37", and to that of setting 3° 14' 58" south.

* This is the proportional log. of 12 hours esteemed as minutes.

Latitude . . 40°30' 0" nat. sine . . = 6494

Declination . 3° 3'37" nat. co-sine . = 9986

Sum 16480 log. = 4.2170

Difference 3492 log. = 3.5431

Sum 7.7601

Half-sum = . . . 3.8800½

33' + 5'15" - 9" = 38'6", prop. log. . . . 0.6743

Constant log. 1.1761

Correction - 3'21" prop. log. = 1.7304½

Approximate time of rising = . . . 6^h 10^m 0^s

Apparent time of sun's rising = . . 6^h 6^m 39^s

Latitude . . 40°30' 0" nat. sine . . = 6494

Declination . 3°14'58" nat. co-sine . = 9984

Sum 16478 log. = 4.2169

Difference 3490 log. = 3.5428

Sum 7.7597

Half sum = . . . 3.8798½

33' + 5'15" - 9" = 38'6", prop. log. = . . 0.6743

Constant log. 1.1761

Correction + 3'21" prop. log. = 1.7302½

Approximate time of setting = . . 5^h 50^m 0^s

Apparent time of sun's setting = . 5^h 53^m 21^s

Note.—In this method of reducing the approximate to the apparent time of rising or setting, it is immaterial whether the latitude and declination be of the same, or of contrary names:—nor is it of any consequence whether the declination be reduced to the approximate times of rising and setting or not, since the declination at noon will be always sufficiently exact to determine the correction within two or three seconds of the truth, on account of its natural co-sine being only required to four places of figures:—this will appear evident by referring to the above example, where,

although there is a difference of $11^{\circ}21'$ between the reduced declinations at the approximate times of rising and setting; yet this difference has no sensible effect on the correction corresponding to those times.

PROBLEM II.

Given the Latitude of a Place and the Declination of a fixed Star, to find the Times of its Rising and Setting.

RULE.

Let the right ascension and declination of the star, as given in Table XLIV, be reduced to the given day; then, from the right ascension of the star, increased by 24 hours if necessary, subtract that of the sun, at noon of the given day; and the remainder will be the approximate time of the star's transit, or passage over the meridian; from which, let the correction answering thereto and the daily variation of the sun's declination (Table XV.,) be subtracted, and the apparent time of the star's transit will be obtained.

If much accuracy be required, and the place of observation be under a meridian different from that of Greenwich, a correction depending on the longitude and variation of the sun's right ascension (Table XV.,) must be applied to the time of transit:—this correction is subtractive in west, and additive in east longitude; the time being always reckoned from the *preceding* noon: now,

Enter Table L., with the declination at top or bottom, and the latitude in the side column; and in the angle of meeting will be found the semi-diurnal arch, or the time of half the star's continuance above the horizon, when the latitude and declination are of the same name; but if these elements are of different names, the time, so found, is to be subtracted from 12 hours, in order to obtain the half continuance above the horizon: then this half continuance* being applied by subtraction and addition to the apparent time of transit, will give the approximate times of the star's rising and setting.

Example 1.

At what times will the star α Arietis rise and set January 1st. 1824, in latitude $50^{\circ}48'$ north?

* In strictness the semi-diurnal arch, or half continuance above the horizon ought to be corrected by subtracting therefrom the proportional part (Table XV.,) corresponding to it and the variation of the sun's right ascension for the given day.

Star's dec. on given day is $22^{\circ}37'33''$, or $22^{\circ}38'$ north, and its right ascension	$1^h57^m16^s$
Sun's right ascension at noon of the given day is	$18.43.58$
Approximate time of star's transit	$7.13.18$
Correction from Tab. XV., ans. to $7^h13^m18^s$, and $4'24''$, the var. of the sun's right ascension	$- 1.20$
Apparent time of star's transit, or passage over the meridian Time, in Tab. L. ans. to lat. 50° N., and dec.	$7^h11^m58^s$
$22^{\circ}30'$ N. =	7^h58^m
Tabular diff. to 1° of lat. = $5'$; now $\frac{5' \times 48'}{60'} = + 4$	
Tab. diff. to $30'$ of dec. = $4'$; now $\frac{4' \times 8'}{30'} = + 1$	
Semi-diurnal arch, or time of half the star's continuance above the horizon	$8^h 3^m . 8^h 3^m 0^s$
Approx. time of star's rising, past noon of Dec. 31st, 1823	$23^h 8^m58^s$
Approx. time of star's setting, past noon of the given day	$15^h14^m58^s$

Example 2.

At what times will the star Sirius rise and set January 1st, 1824, in lat. $40^{\circ}30'$ north, and long. 120 degrees, west of the meridian of Greenwich?

Star's dec. on given day is $16^{\circ}28'53''$ or $16^{\circ}29'$ south, and its right ascension	$6^h37^m23^s$
Sun's right ascension at noon of the given day is	$18.43.58$
Approximate time of the star's transit	$11.53.25$
Corr. from Table XV., ans. to $11^h53^m25^s$, and $4'24''$, the var. of the sun's right ascension	$- 2.11$
Corr. from ditto, ans. to long. 120° west, and $4'24''$ the var. of the sun's right ascension	$- 1.28$
Appar. time of star's transit over the given meridian	$11^h49^m46^s$
Time, in Table L., ans. to lat. 40° north, and declination 16° S. =	6^h56^m
Tab. diff. to 1° of lat. = $2'$; now $\frac{2' \times 30'}{60'} = + 1$	
Tab. diff. to $30'$ of dec. = $2'$; now $\frac{2' \times 29'}{30'} = + 2$, nearly.	
Semi-nocturnal arch	6^h59^m ,
which being subtracted from 12^h leaves	$5^h 1^m 0^s$
Approximate time of the star's rising	$6^h48^m46^s$
Approximate time of the star's setting	$16^h50^m46^s$

Remark.—The approximate times of the rising and setting of a fixed star may be readily reduced to the respective apparent times by the rule given for those of the sun, in page 126 ; omitting, however, the first part, or that which relates to the reduction of declination : and, since the fixed stars have no sensible parallax, the words “horizontal parallax” are, also, to be omitted ; thus :—

To reduce the approximate times of rising and setting, as found in the last example, to the respective apparent times, the dip of the horizon being assumed at 6'.30"

Lat. of place of observ. 40°30' Nat. sine = 6494

Declin. of the star = 16. 29 Nat.co-sine=9589

Sum = . . . 16083 Log. = . 4. 2064

Difference = . 3095 Log. = . 3. 4907

Sum = . 7. 6971

Half sum = 3. 8485½

Horiz. refrac.=33' + dip of horiz.=6'.30"=39'.30" Prop. log.=0. 6587

Constant log. = 1. 1761

Correction =. 3'.44" Prop. log.= 1. 6833½

Now, this correction being subtracted from the approximate time of rising, and added to that of setting, shows the former to be 6^h.45^m.2^s., and the latter 16^h.54^m.30^s.

PROBLEM III.

Given the Latitude of a Place, and the Declination of a Planet, to find the Times of its Rising and Setting.

RULE.

Take, from page IV. of the month in the Nautical Almanac, the times of the planet's transits for the days nearest preceding and following the given day, and find their difference ; then say, as 6 days are to this difference, so is the interval between the given day and the nearest preceding

day, to a correction; which, being applied by addition or subtraction to the time of transit on the nearest preceding day, according as it is increasing or decreasing, the sum or difference will be the approximate time of transit. Find the interval between the times of transit on the days nearest preceding and following the given day; and then say, as the interval between the times of transit is to the difference of transit in that interval, so is the longitude, in time, to a correction; which, being added to the approximate time of transit if the longitude be west and the transit increasing, or subtracted if decreasing, the sum or difference will be the apparent time of the planet's transit over the meridian of the given place; but if the longitude be east, a contrary process is to be observed: that is, the correction is to be subtracted from the approximate time of transit if the transit be increasing, but to be added thereto if it be decreasing.

To the apparent time of transit, thus found, apply the longitude, in time, by addition or subtraction, according as it is west or east; and the sum or difference will be the corresponding time at Greenwich. To this time, let the planet's declination be reduced by Problem III., page 83; or as thus:—

Take, from the Nautical Almanac, the planet's declination for the days nearest preceding and following the Greenwich time, and find the difference; find, also, the difference between the Greenwich time and the nearest preceding day: then say, as 6 days are to the difference of declination, so is the difference between the Greenwich time and the nearest preceding day, to a correction; which, being applied to the declination on the nearest preceding day, by addition or subtraction, according as it may be increasing or decreasing, the sum or difference will be the planet's correct declination at the time of its transit over the given meridian. Now,

With the planet's declination and the latitude of the given place, enter Table L., and find the corresponding semidiurnal arch* by Problem II., page 128; and, thence, the approximate times of rising and setting, in the same manner as if it were a fixed star that was under consideration.

Example 1.

At what times will the planet Jupiter rise and set, January 4th, 1824, in latitude 36° north, and longitude 135° west of the meridian of Greenwich?

* In strictness the semidiurnal arch ought to be corrected by adding thereto, or subtracting therefrom, the proportional part corresponding to it and the daily variation of transit, according as the transit may be increasing or decreasing.

Time of preced. trans. Jan. 1, is $11^h 38^m$ nearest prec. day 1st, trans. $11^h 38^m 0^s$

Time of follow. trans. Jan. 7, is 11. 8. given day 4th

As 6^d is to $0^h 30^m$, so is 3^d to $15^m 0^s$

Approximate time of transit on the given day = $11^h 23^m 0^s$

Time of preceding transit = $1^h 11^m 38^s$

Time of following transit = 7. 11. 8

Interval between the times of trans. = $5^h 23^m 30^s$

As interval between times of trans. = $5^h 23^m 30^s$: diff. of

transit = 30^m :: longitude in time = 9^h to 1.53

Apparent time of transit over given merid. Jan. 4th, 1824 = $11^h 21^m 7^s$

Longitude 135 degrees west, in time = 9. 0. 0

Corresponding time at Greenwich = $20^h 21^m 7^s$

Planet's dec. Jan. 1 is = $23^\circ 17' N.$; near. prec. $1^d 0^h 0^m 0^s$ dec. $23^\circ 17' 0'' N.$

Ditto 7 is = $23. 20 N.$; Gr. tim. = 4. 20. 21. 7

As 6^d is to $0^\circ 3'$ so is 3^d to $20^h 21^m 7^s$ to $+ 1.55$

Jupiter's dec. reduced to his app. time of transit over the

given meridian = $23^\circ 18' 55'' N.$

Time, in Table L., ans. to lat. 36° north, and dec. $23^\circ N.$ = $7^h 12^m 0^s$

Tabular difference to $30'$ of dec. = $2'$; now, $\frac{2' \times 19'}{30'} = + 1.16$

Semidiur. arch, or time of half planet's contin. above the hor. = $7^h 13^m 16^s$

Apparent time of Jupiter's transit over the given meridian = 11. 21. 7

Approximate time of Jupiter's rising at the given meridian = $4^h 7^m 51^s$

Approximate time of Jupiter's setting at ditto = $18^h 34^m 23^s$

Example 2.

At what times will the planet Mars rise and set, January 16th, 1824, in latitude 40° north, and longitude 140° east of the meridian of Greenwich?

Time of preced. trans. 13th, is $16^h 54^m$; near. prec. day 13th, trans. = $16^h 54^m 0^s$

Time of follow. trans. 19th, is 16.34; given day 16th

As $6'$ is to $0^h 20^m$, so is $3'$ to $10^m 0^s$

Approximate time of transit on the given day = $16^h 44^m 0^s$

Interval between the times of transit = $5^h 23^m 40^s$

As interval between times of transit = $5^h 23^m 40^s$: diff. of
trans. = 20^m :: long. in time = $9^h 20^m$, to $+ 1.18$

Apparent time of trans. over given merid. Jan. 16th, 1824 = $16^h 45^m 18^s$

Longitude of the given merid. = 140° east, in time = $9.20. 0$

Corresponding time at Greenwich = $7^h 25^m 18^s$

Dec. of Mars, Jan. 13, is $0^\circ 37' S.$; near. prec. 13^d $0^h 0^m 0^s$: dec. $0^\circ 37' 0^s S.$

Ditto 19, is 1.11 S.; Gr. time = 16. 7.25.18

As $6'$ is to $0^\circ 34'$, so is $3'$ $7^h 25^m 18^s$ to $+ 18.45$

Dec. of Mars reduced to his apparent time of transit over
the given meridian = $0^\circ 55' 45^s S.$

Semidiurnal arch in Table L., answering to lat. $40^\circ N.$ and

dec. $0^\circ 55' 45^s S.$, is $6^h 2^m 48^s$; sub. from 12^h leaves $5^h 57^m 12^s$

Apparent time of the planet's transit over the given meridian = 16.45.18

Approximate time of rising of the planet Mars = $10^h 48^m 6^s$

Approximate time of setting of ditto = $22^h 42^m 30^s$

Remark.—The approximate times of a planet's rising and setting may be reduced to the respective apparent times, by the rule in page 126, for reducing those of the sun; omitting, however, the first part, or that which relates to the reduction of declination, and reading planet's instead of sun's horizontal parallax: this, it is presumed, does not require to be illustrated by an example.

PROBLEM IV.

Given the Latitude of a Place, and the Moon's Declination, to find the Times of her Rising and Setting.

RULE.

Take, from page VI. of the month in the Nautical Almanac, the moon's transit, or passage over the meridian of Greenwich, on the given day, and also her declination. Let the time of transit be reduced to the meridian of the given place by Table XXXVIII.; to which apply the longitude, in time, by addition or subtraction, according as it is west or east; and the sum, or difference, will be the corresponding time at Greenwich: to this time, let the declination be reduced by Table XVI., or by Problem II., page 80;—then,

With this reduced declination, and the latitude of the given place, find the moon's semidiurnal arch, or the time of half her continuance above the horizon, by Problem II., page 128, and, thence, the approximate times of rising and setting, in the same manner precisely as if it were a fixed star that was under consideration: call these the *estimated* times of rising and setting.

To the *estimated* times of rising and setting, thus found, let the longitude, in time, be applied by addition or subtraction, according as it is west or east; and the sum, or difference, will be the corresponding times at Greenwich.

To these times respectively, let the moon's declination be reduced by Table XVI., or by Problem II., page 80; with which, and the latitude, find the moon's semidiurnal arch at each of the *estimated* times.

To the respective semidiurnal arches, thus found, apply the corrections corresponding thereto, and the retardation of the moon's transit (Table XXXVIII.) by addition, and the correct semidiurnal arches will be obtained.

Now, the semidiurnal arch answering to the *estimated* time of rising, being subtracted from the moon's reduced transit, will leave the approximate time of her rising at the given place; and that corresponding to the *estimated* time of setting, being added to the moon's reduced transit, will give the approximate time of her setting at the said place.

Example 1.

Required the times of the moon's rising and setting, Jan. 17th, 1824, in latitude $51^{\circ}29'$ north, and longitude $78^{\circ}45'$ west of the meridian of Greenwich?

Moon's transit over merid. of Greenwich on the given day is $13^h 34^m 0^s$
 Corr. fr. Tab. XXXVIII., ans. to retard. $53'$, and long. 75° west $+ 10.39$

App. time of moon's transit reduced to the given meridian . $13^h 44^m 39^s$
 Longitude $78^\circ 45'$ west, in time = $5.15.0$

Corresponding time at Greenwich $18^h 59^m 39^s$
 Moon's dec. red. to Gr. time, by Table XVI., is $10^\circ 25' 30''$ N.

Semidiurnal arch, in Table L., answering to lat. $51^\circ 29'$ N.,
 and declination $10^\circ 25'$ N., is $6^h 54^m 0^s$
 Moon's reduced transit $13.44.39$

Estimated time of the moon's rising $6^h 50^m 39^s$

Estimated time of the moon's setting $20^h 38^m 39^s$

To find the approximate Time of Rising :—

Estimated time of rising $6^h 50^m 39^s$
 Longitude $78^\circ 45'$ west, in time = $5.15.0$

Greenwich time past noon of the given day $12^h 5^m 39^s$
 Moon's dec. reduced to Greenwich time, is $12^\circ 10' 53''$ N.

Time, in Table L., ans. to lat. $51^\circ 29'$ N. and dec. $12^\circ 11'$ N., is $7^h 3^m 0^s$
 Correction, Table XXXVIII., ans. to $53'$ and $7^h 3^m =$ $+ 15.0$

Moon's correct semidiurnal arch at rising $7^h 18^m 0^s$
 Moon's reduced transit $13.44.39$

Approximate time of moon's rising $6^h 26^m 39^s$

To find the approximate Time of Setting :—

Estimated time of setting $20^h 38^m 39^s$
 Longitude $78^\circ 45'$ west, in time = $5.15.0$

Greenwich time past noon of the 18th $1^h 53^m 39^s$
 Moon's dec. reduced to Greenwich time, is $8^\circ 41' 11''$ N.

Time, in Table L., answ. to lat. $51^\circ 29'$ N. and dec. $8^\circ 41'$ N., is $6^h 44^m 0^s$
 Correction, Table XXXVIII., ans. to $53'$ and $6^h 44^m =$ $+ 14.0$

Moon's correct semidiurnal arch at setting $6^h 58^m 0^s$
 Moon's reduced transit $13.44.39$

Approximate time of moon's setting $20^h 42^m 39^s$

Example 2.

Required the approximate times of the moon's rising and setting, January 20th, 1824, in latitude $40^{\circ}30'$ north, and longitude 80 degrees east of the meridian of Greenwich?

Moon's transit over the merid. of Greenwich on the given day is $16^h 6^m 0^s$
Cor. fr. Tab. XXXVIII., ans. to retard. $49'$ and long. 80° east — 10.32

Moon's transit reduced to the given meridian	$15^h 55^m 28^s$
Longitude 80 degrees east, in time =	$5.20.0$

Greenwich time	$10^h 35^m 28^s$
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Moon's dec. red. to Green. time, by Table XVI., is $5^{\circ}55'40''S$.

Seminoturnal arch, in Table L., answering to lat. $40^{\circ}30'N$.

and dec. $5^{\circ}56'S = 6^h 20^m$, subtracted from 12^h , leaves $5^h 40^m 0^s$
Moon's reduced transit $15.55.28$

<i>Estimated</i> time of the moon's rising	$10^h 15^m 28^s$
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<i>Estimated</i> time of the moon's setting	$21^h 35^m 28^s$
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To find the approximate Time of Rising:—

<i>Estimated</i> time of rising	$10^h 15^m 28^s$
Longitude 80 degrees. east, in time =	$5.20.0$

Greenwich time =	$4^h 55^m 28^s$
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Moon's dec. reduced to this time, is $4^{\circ}30'49''S$.

Time, in Table L., answering to lat. $40^{\circ}30'N$, and dec.

$4^{\circ}31'S$ is $6^h 15^m$, which, subtracted from 12^h , leaves . . $5^h 45^m 0^s$
Corr. Table XXXVIII., answering to $49'$ and $5^h 45^m$. . $+ 11.0$

Moon's correct semidiurnal arch at rising	$5^h 56^m 0^s$
Moon's reduced transit	$15.55.28$

Approximate time of moon's rising	$9^h 59^m 28^s$
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To find the approximate Time of Setting:—

<i>Estimated</i> time of setting	$21^h 35^m 28^s$
Longitude 80 degs. east, in time =	$5.20.0$

Greenwich time =	$16^h 15^m 28^s$
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Moon's dec. reduced to this time, is $7^{\circ}17'52''S$.

Time, in Table L., answering to lat. $40^{\circ}30'N.$ and dec.

$7^{\circ}18'S.$, is 6^h25^m , which, subtracted from 12^h , leaves	$5^h35^m 0^s$
Corr. Table XXXVIII., ans. to $49'$ and 5^h35^m	$+ 11. 0$
Moon's correct semidiurnal arch at setting	$5^h46^m 0^s$
Moon's reduced transit	$15.55.28$
Approximate time of moon's setting	$21^h41^m28^s$

Remark.—The approximate times of the moon's rising and setting may be reduced to the respective apparent times by the following rule ; viz.,

Find the sum and the difference of the natural sine of the latitude and the natural co-sine of the declination at the *estimated* times of rising and setting (rejecting the two right-hand figures from each term), and find the common log. answering thereto, rejecting also the two right-hand figures from each. Now, to half the sum of these two logs. add the constant log. 1. 1761,* and the proportional log. of the difference between the horizontal parallax and the sum of the horizontal refraction and dip of the horizon : the sum of these three logs., abating 4 in the index, will be the proportional log. of a correction, which, being *added* to the approximate time of rising and *subtracted* from that of setting, the respective apparent times of rising and setting will be obtained : thus,

Let it be required to reduce the approximate times of rising and setting, as found in the last example, to the respective apparent times, the dip of the horizon being $4'.50''$

Note.—The moon's horizontal parallax computed to the *reduced estimated* time of rising, is $59'.6''$, and that at the reduced time of setting $58'.40''$

Latitude $40^{\circ}30'$ Nat. sine 6494
Declination 4.31 Nat.co-sine 9969

Sum	16463	Log.	4.2165
Difference	3475	Log.	3.5410
		Sum	7.7575
		Half sum	$3.8787\frac{1}{2}$
$59'.6'' - 37'.50'' (33' + 4'.50'') = 21'.16''$		Prop. log.	0.9276
Constant log.			1.1761
Correction	$+ 1'.52''$	Prop. log.	$1.9824\frac{1}{2}$
Approximate time of rising =	$9^h59^m28^s$		
Apparent time of moon's rising =	$10^h 1^m20^s$		

* This is the proportional log. of 12 hours esteemed as minutes.

Latitude $40^{\circ}30'$ Nat. sine 6494
 Declination 7.18 Nat. co-sine 9919

Sum 16413

Difference 3425

Log. 4.2152

Log. 3.5347

Sum 7.7499

Half sum 3.8749 $\frac{1}{2}$

$58^{\circ}40' - 37^{\circ}50' (33' + 4^{\circ}50') = 20^{\circ}50'$ Prop. log. 0.9365

Constant log. 1.1761

Correction $-1^{\circ}51'$ Prop. log. 1.9875 $\frac{1}{2}$

Approximate time of setting = $21^{\circ}41'28'$

Apparent time of moon's setting = $21^{\circ}39'37'$

Note.—The direct method of solving this and the three preceding Problems, by spherical trigonometry, is given in some of the subsequent pages of this work.

TABLES LI. AND LII.

For computing the Meridional Altitude of a Celestial Object.

Since it frequently happens, at sea, that the meridional altitude of the sun, or other celestial object, cannot be taken, in consequence of the interposition of clouds at the time of its coming to the meridian; and since it is of the utmost importance to the mariner to be provided at all times with the means of determining the meridional altitude of the heavenly bodies, for the purpose of ascertaining the exact position of his ship with respect to latitude, these Tables have therefore been carefully computed; by means of which the meridional altitude of the sun, or any other celestial object whose declination does not exceed 28 degrees, may be very readily obtained to a sufficient degree of accuracy for all nautical purposes, provided the altitude be observed within certain intervals of noon, or time of transit, to be governed by the meridional zenith distance of the object: thus, *for the sun*, the number of minutes and parts of a minute contained in the interval between the time of observation and noon, must not exceed the number of degrees and parts of a degree contained in the object's meridional zenith distance at the place of observation. And since the meridional zenith distance of a celestial object is expressed by the difference between the latitude and the declination when they are of the same name, or by their sum

when of contrary names; therefore the extent of the interval from noon (within which the altitude should be observed) may be determined by means of the difference between the latitude and the declination when they are both north or both south, or by their sum when one is north and the other south. Thus, if the latitude be 40 degrees, and the declination 8 degrees, both of the same name, the interval between the time of taking the altitude and noon must not exceed 32 minutes; but if they be of different names, the altitude may be taken at any time within 48 minutes before or after noon; if the latitude be 60 degrees, and the declination 10 degrees, both of the same name, the interval between the time of observation and noon ought not to exceed 50 minutes; but if one be north and the other south, the interval may be extended, if necessary, to 70 minutes before or after noon, and so on.

The limits within which the altitudes of the other celestial objects should be observed, may be determined in the same manner; taking care, however, to estimate the interval from the time of transit or passage over the meridian, instead of from noon.

Now, if the altitude of the sun or other celestial object be observed at any time within the limits thus prescribed, and the time of observation be carefully noted by a well-regulated watch, the meridional altitude of such object may then be readily determined, to every desirable degree of accuracy, by the following rule; viz.,

Enter Table LI. or LII., according as the latitude and the declination are of the same or of contrary names, and with the latitude in the side column, and the declination (reduced to the meridian of the place of observation) at the top or bottom; take out the corresponding correction in seconds and thirds, which are to be esteemed as *minutes and seconds*;—then,

To the proportional log. of this correction,* add twice the proportional log. of the interval between the time of observation and noon, or time of transit, and the constant log. 7. 2730; and the sum will be the proportional log. of a correction, which, being added to the true altitude deduced from observation, will give the correct meridional altitude of the object.

Note 1.—In taking out the numbers from Tables LI. and LII., proportion must be made for the excess of the given latitude and declination above the next less tabular arguments.

* When the object either comes to, or within, one degree of the zenith, the angle of meeting made by the latitude and declination will fall within the zigzag double lines which run through the body of Table LI., and through the upper left-hand corner of Table LII.; in this case, since the interval between the time of observation and noon, or meridional passage, must not exceed one minute, the corresponding number will be the correction of altitude direct, independently of any calculation whatever.

2.—The interval between the time of observation and noon may be always known by means of a chronometer, or any well-regulated watch; making proper allowance, however, for the time comprehended under the change of longitude since the last observation for determining the error of such watch or chronometer.

Example 1.

In latitude 45° north, at $34^{\text{m}}40'$ before noon, the sun's true altitude was found to be $54^{\circ}12'49''$, when his declination was 10° north; required the meridional altitude?

Corr. in Table LI., ans. to lat. 45° and dec. 10° , is $2^{\text{m}}23^{\text{s}}.1$;

the propor. log. of which is 1.8778

Interval between time of obs. and noon, $34^{\text{m}}40'$, twice prop. log. = 1.4308

Constant log. 7.2730

Correction of altitude . . . $0^{\circ}47'10''$ Prop. log. = . . . 0.5816

True alt. at time of observ. $54.12.49$

Sun's meridional altitude $54^{\circ}59'59''$; which is but one second less than the truth.

Example 2.

In latitude 48° north, at $1^{\text{h}}5^{\text{m}}48'$ past noon, the sun's true altitude was found to be $20^{\circ}25'5''$, when his declination was 20 degs. south; required the meridional altitude?

Corr. in Table LII., answering to lat. 48° N. and dec. 20° S., is

$1^{\text{m}}19^{\text{s}}.9$, the propor log. of which is 2.1308

Interval between time of obs. and noon $1^{\text{h}}5^{\text{m}}48'$, twice prop. log. = 0.8740

Constant log. 7.2730

Correction of altitude . . . $1^{\circ}34'57''$ Prop. log. = . . . 0.2778

True alt. at time of observ. $20.25.5$

Sun's meridional altitude . $22^{\circ}0'2''$; which is but two seconds more than the truth.

Example 3.

At sea, March 22d, 1824, in latitude $51^{\circ}16'$ north, at $50^{\text{m}}32'$ past noon, the sun's true altitude was found to be $38^{\circ}20'56''$; required the meridional altitude, the declination being $0^{\circ}43'51''$ north?

Corr. in Table LI., answering to lat. $51^{\circ}16'$ and dec. $0^{\circ}43'51''$

is $1''35''.6$,* the propor. log. of which is 2.0530
Interval between time of obs. and noon $50^m32'$, twice prop. log. = 1.1034
Constant log. 7.2730

Correction of altitude . . . $1^{\circ}6'58''$ Prop. log. = . . . 0.4294
True alt. at time of observ. $38.20.56$

Sun's meridional altitude . $39^{\circ}27'54''$; which differs but three seconds from the truth.

Example 4.

At sea, December 21st, 1824, in latitude $60^{\circ}22'$ north, at $10^h36^m10^s$ A.M., or $1^h23^m50^s$ before noon, the sun's true altitude was found to be $4^{\circ}26'38''$; required his meridional altitude, the declination being $23^{\circ}27'45''$ south?

Corr. in Table LII., ans. to lat. $60^{\circ}22'$ and dec. $23^{\circ}27'45''$,

is $0''53''.8$,† the propor. log. of which is 2.3026
Interval between time of obs. and noon $1^h23^m50^s$, twice prop. log. = 0.6638
Constant log. 7.2730

Correction of altitude . . . $1^{\circ}43'43''$ Prop. log. = . . . 0.2394
True alt. at time of observ. $4.26.38$

Sun's meridional altitude . $6^{\circ}10'21''$; which differs but six seconds from the truth.

After this manner may the meridional altitude of the moon, a planet, or a fixed star be obtained, when the declination does not exceed the limits of the Table.

Remarks, &c.

From the above examples it is manifest, that by means of the present Tables the meridional altitude of a celestial object may be readily inferred

* Corr. to lat. 50° and dec. 0° = $1''38'''.8$
Diff. to 2° lat. = $6'''.8$; now, $6'''.8 \times 76' + 120' = - 4 . 3$
Diff. to 1° dec. = $1'''.5$; now, $1'''.5 \times 44' + 60' = + 1 . 1$

Corr. to lat. $50^{\circ}32'$ and dec. $0^{\circ}43'51''$ = $1''35'''.6$

† Corr. to lat. 60° and dec. 23° = $0''54'''.6$
Diff. to 2° lat. = $3'''.5$; now $3'''.5 \times 22' + 120' = . . - 0 . 6$
Diff. to 1° dec. = $0'''.5$; now $0'''.5 \times 28' + 60' = . . - 0 . 2$

Corr. to lat. $60^{\circ}22'$ and dec. $23^{\circ}27'45''$ = $0''53'''.8$

from its true altitude observed at a known interval from noon (within the limits before prescribed), with all the accuracy to be desired in nautical operations; and that it is immaterial whether the observation is made before or after noon, or time of transit, provided the time be but correctly known; and, since most sea-going ships are furnished with chronometers, there can be but very little difficulty in ascertaining the apparent time to within a few seconds of the truth.

It is to be observed, however, that the nearer to noon or time of transit the observation is made, the less susceptible will it be of being affected by any error in the time indicated by the watch: thus, in *example 4*, where the interval or time from noon is $1^{\text{h}}23^{\text{m}}50^{\text{s}}$, an error of one minute in that interval would produce an error of $2\frac{1}{2}$ minutes in the sun's meridional altitude; but if the observation had been made within a quarter of an hour of noon, an error of *five minutes* in the time would scarcely affect the meridional altitude to the value of 2 minutes: hence it is evident, that although the observation may be safely made at any time from noon to the full extent of the interval, when dependance can be placed on the time shown by the watch, yet when there is any reason to doubt the truth of that time, it will be advisable to take the altitude as near to noon, or the time of transit, as circumstances may render convenient.

In all narrow seas trending in an easterly or westerly direction, where the meridional altitude of a celestial object is of the greatest consideration, such as in the British Channel, the mariner will do well to avail himself of this certain method for its actual determination; particularly during the winter months, when the sun is so very frequently obscured by clouds at the time of its coming to the meridian.

These Tables were computed by the following rule; viz.,

To the constant log. 0. 978604,* add the log. co-sines of the latitude and the declination; the sum, rejecting 20 from the index, will be the log. of a natural number, which, being subtracted from the natural co-sine of the difference between the latitude and the declination, when they are of the same name, or from that of their sum if of contrary names, will leave the natural co-sine of an arch; now, the difference between this arch, and the difference or sum of the latitude and the declination, according as they are of the same or of contrary names, will be the change of altitude in one minute from noon.

Example 1.

Let the latitude be 13 degrees, and the declination of a celestial object 2 degrees, both of the same name; required the variation or change of altitude in one minute from noon?

* This is the log. versed sine, or log. rising, of one minute of time.

Constant log. =	0.978604
Latitude =	13 degrees.	Log. co-sine . 9.988724
Declination =	2 degrees.	Log. co-sine . 9.999735

Difference = 11 degrees. Nat. co-sine=981627
 Nat. number= 9.269=Log. 0.967063

Arch = . . 11° 0' 10" = Nat. co-s.=981617.731

Difference = . 0° 0' 10" ; which, therefore, is the change of altitude in one minute from noon.

Example 2.

Let the latitude be 40 degrees, and the declination of a celestial object 8 degrees, of a contrary name to that of the latitude ; required the variation or change of altitude in one minute from noon ?

Constant log. =	0.978604
Latitude =	40 degrees.	Log. co-sine . 9.884254
Declination =	8 degrees.	Log. co-sine . 9.995753

Sum = . . 48 degrees. Nat. co-sine=669131
 Nat. num. = 7.221 Log.=0.858611

Arch = . . 48° 0' 2" = Nat.co-sine=669123.779

Difference = 0° 0' 2" ; which, therefore, is the change of altitude in one minute from noon.

It is to be observed, however, that, with the view of introducing every possible degree of accuracy into the present Tables, the natural and log. co-sines, &c., employed in their construction, have had their respective numbers extended to seven places of decimals.

Note.—The difference between the meridional altitude of a celestial object and its altitude at a given interval from noon, is found, by actual observation, to be very nearly proportional to the square of that interval, under certain limitations, as pointed out in page 138 ; and hence the rule, in page 139, for computing the meridional altitude of a celestial object.

TABLE LIII.

The Miles and Parts of a Mile in a Degree of Longitude at every Degree of Latitude.

This Table consists of seven compartments: the first column in each compartment contains the degrees of latitude, and the second column the miles and parts of a mile in a degree of longitude corresponding thereto. In taking out the numbers from this Table, proportion is to be made, as usual, for the minutes of latitude; this proportion is subtractive from the miles, &c., answering to the given degree of latitude.

Example.

Required the number of miles contained in a degree of longitude in latitude $37^{\circ}48'$?

Miles in a degree of longitude, in latitude 37 degrees = . . . 47.92

Difference to 1 degree of latitude = .64; now $\frac{64 \times 48'}{60'} = \text{---} .51$

Miles in a degree of long. in latitude 37 degs. 48 min., as required = 47.41

Remarks.—Since the difference of longitude between two places on the earth is measured by an arch of the equator intercepted between the meridians of those places; and since the meridians gradually approach each other from the equator to the poles, where they meet, it hence follows that the number of miles contained in a degree of longitude will decrease in proportion to the increase of the latitude; the ratio of decrease being as radius to the co-sine of the latitude. Now, since a degree of longitude at the equator contains 60 miles, we have the following rule for computing the present Table; viz.,

As radius is to the co-sine of the latitude of any given parallel, so is the measure of a degree of longitude at the equator to the measure of a degree in the given parallel of latitude.

Example.

Required the number of miles contained in a degree of longitude in the parallel of latitude 37 degrees?

As radius . . . 90 degrees	Log. sine = . . . 10.000000
Is to latitude = 37 degrees	Log. co-sine . . . 9.902349
So is . . . 60 miles	Log. = . . . 1.778151

To 47.92 miles	Log. = . . . 1.680500;
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Hence the measure of a degree of longitude in the given parallel of latitude, is 47.92 miles.

TABLE LIV.

Proportional Miles for constructing Marine or Sea Charts.

In this Table the parallels of latitude are ranged in the upper horizontal column, beginning at 0° , and numbered 10° , 20° , 30° , &c., to 89° ; the horizontal column immediately under the parallels of latitude contains the number of miles of longitude corresponding to each parallel's distance from the equator; under which, in the horizontal column marked "Difference of the Parallels, &c.," stands the number of miles of longitude contained between the parallel under which it is placed and that immediately preceding it.

The left-hand vertical column contains the intermediate or odd degrees of latitude, from 0° to 10° ; opposite to which, and under the respective parallels of latitude, will be found the number of miles of longitude corresponding to each degree of latitude in those parallels: these are intended to facilitate, and render more accurate, the subdivision of the different parallels of latitude into degrees and minutes.

To make a Chart of the World, in which the Parallels of Latitude and Longitude are to consist of 10 Degrees each.

Draw a straight, or meridian, line along the right hand, or east margin of the paper intended to receive the projection; bisect that line, and from the point of bisection draw a straight line perpendicular to the former, which continue to the left-hand or west margin of the paper, and it will represent the equator.

From any diagonal scale of convenient size take 600 miles in the compasses (the number of miles of the equator contained in 10 degrees of longitude), and lay it off from the point of bisection along the equator, and it will graduate it into 36 equal parts of 10 degrees each; through which let straight lines be drawn at right angles to the equator, and parallel to that drawn along the right-hand margin, and they will represent the meridians or parallels of longitude. Take, from the same scale, 60 miles in the compasses, and it will subdivide each of those 36 divisions, or parallels of longitude, into ten equal parts consisting of one degree each; and then will the equator be divided into 360 degrees of 60 miles each.

On the meridian lines drawn along the right and left-hand margins of the paper, let the parallels of latitude be laid down, as thus:—For the first parallel, or 10 degrees from the equator, take 603.1 miles in the compasses (found in the horizontal column immediately under the parallels of latitude, and marked "Ditto in miles of the Equator, &c."); place one foot on the

equator, and where the other falls upon the right and left-hand marginal lines, when turned northward and southward, there make points; through which let straight lines be drawn parallel to the equator, and they will represent the parallels of latitude at 10 degrees north and south of the equator: in the same manner, for 20 degrees, lay off 1225.1 miles; for 30 degrees, 1888.4 miles; for 40 degrees, 2622.6 miles, and so on.

But since the common compasses are generally too small for taking off such high numbers, it will be found more convenient to lay down the parallels of latitude by the numbers contained in the third horizontal column, or that marked "Difference of the Parallels, &c." Thus, for 10 degrees, take 603.1 miles in the compasses; place one foot on the equator, and with the other make points north and south thereof on the east and west marginal lines, through which let straight lines be drawn, and they will represent the parallels of latitude at 10 degrees north and south of the equator. From these parallels respectively, lay off 622.0 miles, by placing one foot of the compasses on the respective parallels and the other on the east and west marginal lines; through the points thus made by the compasses draw straight lines, and they will represent the parallels of latitude at 20 degrees north and south of the equator. From the parallels, thus obtained, lay off 663.3 miles, and the parallel of 30 degrees will be determined: thence lay off 734.2 miles, and it will show the parallel of 40 degrees; and so on for the succeeding parallels.

The numbers for subdividing those parallels will be found in the vertical columns under each respectively, and are to be applied as follows; thus, to graduate the parallel between 50 and 60 degrees: take 94.3 miles in the compasses, and lay it off from 50 degrees towards 60 degrees, and it will give the parallel of 51 degrees; from which lay off 96.4 miles, and it will show the parallel of 52 degrees; from this lay off 98.6 miles, and the parallel of 53 degrees will be obtained; and so on of the rest. In the same manner let the other parallels of latitude be subdivided; then let the parallels of latitude be numbered along the east and west marginal columns, from the equator towards the poles, according to the number of degrees contained in that arc of the meridian which is intercepted between them and the equator, as 10°, 20°, 30°, 40°, &c. &c.; and let the parallels of longitude be numbered at the top and bottom, and also along the equator; these are to be reckoned east and west of the first meridian, as 10°, 20°, 30°, 40°, &c., to 180°, both ways; and since the first meridian is entirely arbitrary, it may be assumed as passing through any particular place on the earth, such as Greenwich Observatory: then will the chart be ready for receiving the latitudes and longitudes of all the principal places on the earth, and which are to be placed thereon by the following rule; viz.,

Lay a ruler over the given longitude found at the top and bottom of the

chart, and with a pair of compasses take the latitude from the east or west marginal columns ; which being applied to the edge of the ruler, placing one foot on the equator or on the parallel that the latitude was counted from, the other foot turned north or south according to the name of the latitude, will point out or fall upon the true position of the given latitude and longitude.

From what has been thus laid down, the manner of constructing a chart for any particular place or coast must appear obvious.

Note.—Since this Table is merely an extract from the Table of Meridional parts, the reader is referred to page 113 for the method of computing the different numbers contained therein.

TABLE LV.

To find the Distance of Terrestrial Objects at Sea.

If an observer be elevated to any height above the level of the earth or sea, he can not only discern the distant surrounding objects much plainer than he could when standing on its surface, but also discover objects which are still more remote by increasing his elevation. Now, although the great irregularity of the surface of the land cannot be subjected to any definite rule for determining the distance at which objects may be seen from different elevations ; yet, at sea, where there is generally an uniform curvature of the water, on account of the spherical figure of the earth, the distance at which objects may be seen on its surface may be readily obtained by means of the present Table ; in which the distance answering to the height of the eye, or to that of a given remote object, is expressed in nautical miles and hundredth parts of a mile ; allowance having been made for terrestrial refraction, in the ratio of the one-twelfth of the intercepted arch.

Note.—The distance between two objects whose heights are given, is found by adding together the tabular distances corresponding to those heights. And, when the given height exceeds the limits of the Table, an aliquot part thereof is to be taken ; as one fourth, one ninth, or one sixteenth, &c. ; then, the distance corresponding thereto in the Table, being multiplied by the *square root* of such aliquot part, viz., by 2, 3, or 4, &c., according as it may be, will give the required distance.

Example 1.

The look-out man at the mast-head of a man-of-war, at an elevation of 160 feet above the level of the sea, saw the top of a light-house in the horizon whose height was known to be 290 feet; required the ship's distance therefrom?

The distance answering to 160 feet is	. .	14.57 miles.
Ditto to 290 feet is	. .	19.62 do.

Required distance = 34.19 miles;
which, therefore, is the ship's distance from the light-house.

Example 2.

The Peak of Teneriffe is about 15300 feet above the level of the sea; at what distance can it be seen by an observer at the mast-head of a ship, supposing his eye to be 170 feet above the level of the water?

One ninth of 15300 is 1700, answering to which is 47.50 miles; this being multiplied by 3 (the square root of one ninth) gives 142.50 miles.

Distance ans. to 170 feet (height of the eye) is	. .	15.03 do.
--	-----	-----------

Required distance = 157.53 miles.

Remark 1.—Since the distances given in this Table are expressed in nautical miles, whereof 60 are contained in one degree, and there being 69.1 English miles in the same portion of the sphere; if, therefore, the distance be required in English miles, it is to be found as follows; viz.,

As 60, is to 69.1; so is the tabular distance to the corresponding distance in English miles; which may be reduced to a logarithmic expression, as thus:—

To the log. of the given tabular distance, add the constant logarithm 0.061327,* and the sum will be the log. of the given distance in English miles.

Example.

Let it be required to reduce 157.53 nautical miles into English miles?

Given distance in nautical miles = 157.53, log. =	2.197364
Constant log.	0.061327

Distance reduced to English miles 181.42 = Log. = 2.258691

* The log. of 69.1 = 1.839478, less the log. of 60 = 1.778151 is 0.061327; which, therefore, is the constant logarithm.

The converse of this (that is, to reduce English miles into nautical miles,) must appear obvious.

Remark 2.—This Table was computed by the following rule; viz.,

To the earth's diameter in feet, add the height of the eye above the level of the sea, and multiply the sum by that height; then, the square root of the product being divided by 6080 (the number of feet in a nautical mile), will give the distance at which an object may be seen in the visible horizon, independent of terrestrial refraction. This rule may be adapted to logarithms, as thus:—

Let the earth's diameter in feet be augmented by the height of the eye; then, to the log. thereof add the log. of the height of the eye; from half the sum of these two logs. subtract the constant log. 3.783904,* and the remainder will be the log. of the distance in nautical miles, which is to be increased by a twelfth part, of itself, on account of the terrestrial refraction.

Example.

At what distance can an object be seen, in the visible horizon, by an observer whose eye is elevated 290 feet above the level of the sea?

Diameter of the earth in feet =	41804400	
Height of the eye	290	Log. = 2.462898
<hr/>		
Sum =	41804690	Log. = 7.621225
<hr/>		
		Sum . 10.083623
<hr/>		
		Half sum 5.041811½
Constant log. =		3.783904
<hr/>		
Distance uncorrected by refraction	18.11	= Log. = 1.257907½
Add one-12th part on acc. of refraction	1.51	
<hr/>		
Distance, as required = . . .	19.62	nautical miles.

Note.—For the principles of this rule, see how the distance of the visible horizon, expressed by the line O T, is determined in page 5.

* This is the log. of 6080, the number of feet in a nautical mile.

TABLE LVI.

To reduce the French Centesimal Division of the Circle into the English Sexagesimal Division; or, to reduce French Degrees, &c., into English Degrees, &c., and conversely.

This Table is intended to facilitate the reduction of French degrees of the circle into English degrees, and conversely. The Table is divided into two parts: the first or upper part exhibits the number of English degrees and parts of a degree contained in any given number of French degrees and parts of a degree; and the second or lower part exhibits the number of French degrees, &c., contained in any given number of English degrees, &c.

Note.—In the general use of this Table, when any given number of French degrees exceeds the limits of the first part, take out for 100 degrees first, and then for as many more as will make up the given number; and, when any given number of English degrees exceeds the limits of the second part, take out for 90 degrees first, and then for as many more as will make up the given number.

Example 1.

If the distance between the moon and a fixed star, according to the French division of the circle, be $128^{\circ}93'96''$, required the distance agreeably to the English division of the circle?

100 French degrees	are equal to	. .	$90^{\circ} 0' 0''$	English.
28 Ditto	are equal to	. .	25. 12. 0	do.
93 French minutes	are equal to	. .	0. 50. 13 . 20	do.
96 French seconds	are equal to	. .	0. 0. 31 . 10	do.

Distance reduced to English degs., as required $116^{\circ} 2' 44'' . 30$

Example 2.

If the distance between the moon and sun, according to the English division of the circle, be $116^{\circ}53'47''$, required the distance agreeably to the French division of the circle?

90 English degrees	are equal to	. .	$100^{\circ} 0' 0''$	French.
26 Ditto	are equal to	. .	28. 88. 88 . 89	do.
53 English minutes	are equal to	. .	0. 98. 14 . 81	do.
47 English seconds	are equal to	. .	0. 1. 45 . 06	do.

Distance reduced to French degs. as required $= 129^{\circ} 88' 48'' . 76$

Remark 1.—This Table was computed in conformity with the following considerations and principles ; viz.,

The French writers on trigonometry have recently adopted the centesimal division of the circle, as originally proposed by our excellent countryman Mr. Henry Briggs, about the year 1600. In this division, the circle is divided into 400 equal parts or degrees, and the quadrant into 100 equal parts or degrees ; each degree being divided into 100 equal parts or minutes, and each minute into 100 equal parts or seconds : these degrees, &c. &c., are written in the usual manner and with the customary signs, as thus ; $128^{\circ}93'96''$.

Hence, the French degree is evidently less than the English, in the ratio of 100 to 90 ; a French minute is less than an English minute, in the ratio of $100^{\circ} \times 100'$ to $90^{\circ} \times 60'$; and a French second is less than an English second, in the ratio of $100^{\circ} \times 100' \times 100''$ to $90^{\circ} \times 60' \times 60''$: now, the converse of this being obvious, we have the following general rule for converting French degrees into English, and the contrary.

As 100, the number of degrees in the French quadrant, is to 90, the number of degrees in the English quadrant ; so is any given number of French degrees to the corresponding number of English degrees.

As 10000, the number of minutes in the French quadrant, is to 5400, the number of minutes in the English quadrant ; so is any given number of French minutes to the corresponding number of English minutes. And,

As 1000000, the number of seconds in the French quadrant, is to 324000, the number of seconds in the English quadrant ; so is any given number of French seconds to the corresponding number of English seconds.

English degrees, minutes, and seconds, are reduced into French by a converse proportion ; viz.,

As 90, is to 100 ; so is any given number of English degrees to the corresponding number of French degrees.

As 5400, is to 10000 ; so is any given number of English minutes to the corresponding number of French minutes. And,

As 324000, is to 1000000 ; so is any given number of English seconds to the corresponding number of French seconds.

Remark 2.—French degrees and parts of a degree may be turned into English, independently of the Table, by the following rule ; viz.,

Let the French degrees be esteemed as a whole number, to which annex the minutes and seconds as decimals ; then one-tenth of this mixed number, deducted from itself, will give the corresponding English degrees, &c.

Example.

The latitude of Paris, according to the French division of the quadrant, is $54^{\circ}26'36''$ north; required the latitude agreeably to the English division of the quadrant?

$$\begin{array}{r}
 \text{Given latitude} = 54^{\circ}26'36'' = 54^{\circ}.2636 \\
 \text{Deduct one-tenth} \quad . \quad . \quad . \quad 5 \quad .42636 \\
 \hline
 \text{English degrees, \&c.} \quad . \quad . \quad . \quad 48^{\circ}.83724 \\
 \quad \quad \quad \quad \quad \quad \quad \quad 60 \\
 \hline
 \quad \quad \quad \quad \quad \quad \quad 50'.23440 \\
 \quad \quad \quad \quad \quad \quad \quad \quad 60 \\
 \hline
 \quad \quad \quad \quad \quad \quad \quad 14''.06400
 \end{array}$$

Hence, the latitude of Paris, reduced to the English division of the quadrant, is $48^{\circ}50'14''$ north.

Remark 3.—English degrees and parts of a degree may be turned into French, independently of the Table, as thus:—

Reduce the English minutes and seconds to the decimal of a degree, and annex it to the given degrees; then one-ninth of this mixed number, being added to itself, will give the corresponding French degrees, &c.

Example.

The latitude of the Royal Observatory at Greenwich is $51^{\circ}28'40''$ north, agreeably to the English division of the quadrant; required the latitude according to the French division of the quadrant?

$$\begin{array}{r}
 \text{Given latitude} = 51^{\circ}28'40'' = 51^{\circ}.4777777, \&c. \\
 \text{Add one-ninth} \quad . \quad . \quad . \quad . \quad 5 \quad .7197530, \&c. \\
 \hline
 \text{French degrees, \&c.} \quad . \quad . \quad . \quad . \quad 57^{\circ}.1975307 = 57^{\circ}19'75''.307
 \end{array}$$

Hence, the latitude of Greenwich Observatory, according to the French division of the quadrant, is $57^{\circ}19'75''.307$ N.

TABLE LVII.

A general Table for Gauging, or finding the Content of all Circular-headed Casks.

Although this Table may not directly affect the interest of the mariner; yet, since it cannot fail of being exceedingly useful to officers in charge of

His Majesty's victualling stores (such as Purser's of the Royal Navy, Lieutenants commanding gun-brigs, &c. &c.), it has therefore been deemed advisable to give it a place in this work, particularly since it may be found interesting to those whom it immediately concerns.

This Table is divided into two parts: the first part consists of five compartments, and each compartment of three columns; the first of which contains the quotient of the head diameter of a cask divided by the bung diameter; the second the corresponding log. adapted to ale gallons; and the third the log. for wine gallons. The second part of the Table contains the bung diameter and its corresponding logarithm.

The use of this Table will be exemplified in the following

PROBLEM.

Given the Dimensions of a Cask, to find its Contents in Ale and Wine Gallons.

RULE.

Divide the head diameter by the bung diameter to two places of decimals in the quotient; then add together the log. for ale or wine gallons, corresponding to this quotient, in the first part of the Table; the log. corresponding to the bung diameter, in the second part of the Table, and the common log. of the length of the cask; the sum of these three logs., rejecting 10 in the index, will be the log. of the true content of the cask, in ale or wine gallons, according as the content may be required.

Example.

Let the bung diameter of a cask be 25 inches, the head diameter 19.5 inches, and its length 31 inches; required the contents in ale and wine gallons?

25)19.50(.78, quotient of the head diameter divided by the bung diameter.

175

200

200

...

.78 = quotient, log. for ale gallons = . . . 7.362671

25 inches, bung diameter, corresponding log. = 2.795880

31 inches, length of the cask, common log. = 1.491362

Content in ale gallons = 44.66 common log. = 1.649913

.78 = quotient, log. for wine gallons =	. . . 7.449340
25 inches, bung diameter, corresponding log. =	2.795880
31 inches, length of the cask, common log. =	1.491362
Content in wine gallons = 54.52 common log. =	<u>1.736582</u>

Remark.—Should the bung diameter not come within the limits of the second part of the Table; that is, should it be under 10 or above 50 inches, then twice the common log. corresponding thereto will express the log. of the said bung diameter, with which proceed as before: hence, the rule becomes universal for all circular-headed casks, be the size ever so great or ever so trivial.

This subject will be revived in a subsequent page of the present work.

TABLE LVIII.

Latitudes and Longitudes of the principal Sea-Ports, Islands, Capes, &c. &c., with the Time of High Water at the Full and Change of the Moon at all Places where it is known.

In drawing up this Table, the greatest pains have been taken to render it not only the most accurate, but also the most extensive of any now extant. Perfect accuracy, however, is not to be expected in a Table which principally depends on the observations made, at different periods, by the navigators of most civilized nations; because, in those periods, or at the time when a very considerable portion of the latitudes and longitudes were established, the nautical instruments and tables employed in their determination were far from being in that highly-improved state in which they are found at present: besides, it is a fact well known to the generality of nautical persons, that if two or more navigators be directed to ascertain the position of any particular place, they will, in most cases, differ four or five miles in the latitude, and perhaps thrice as many in the longitude.

In constructing all the other Tables in this work, there were fixed data to work upon, with certain means of detecting and exterminating errors; but, in this, there were no determinate means of ensuring the desired degree of accuracy, except in those positions where chance or professional duties happened, from time to time, to conduct the author. Hence, although every possible degree of attention has been paid in consulting the most approved works of the present day, and in collating *this* with the best modern Tables; yet the mariner must not expect to find it perfectly free from blemishes; though, doubtless, he will find it considerably less so than any with which he may have been hitherto acquainted.

Since this Table is not intended for general geographical purposes, the

positions of places *inland*, which do not concern the navigator, have, with one or two exceptions, been purposely omitted: hence, the latitudes and longitudes are limited to maritime places. These are so arranged as to exhibit to the mariner the whole line of coast along which he may chance to sail, or on which he may be employed, agreeably to the manner in which it unfolds to his view on a Mercator's chart. This mode of arrangement is evidently much better adapted to nautical purposes than the alphabetical mode.

With the view of keeping up the identity of the Table with the line of coast laid down on particular charts, a *few* positions have been inserted a second time. This, it is presumed, if not conducive to good, will not, at least, be productive of any evil, since the repetition is so very trivial as not to embrace, in the whole, more than ten or twelve positions.

The time of high water, at the full and change of the moon, is given at all places where it is known. This, it is hoped, will be found not a little convenient, since it does away with the necessity of consulting a separate Table for that particular purpose.

In order to render this Table still more complete, an alphabetical reference has been annexed, which will very essentially contribute towards assisting the mariner in readily finding out most of the principal coasts and islands contained in that Table.

The page which immediately follows the alphabetical reference to Table LVIII. contains the form of a Transit Table, and the next page a variety of numbers with their corresponding logarithms, &c., which may, perhaps, be found useful on many occasions. At the foot of these numbers there is a small Table, showing the absolute time at which the hour and minute hands of a well-regulated watch or clock should exactly be in conjunction, and also in opposition, in every revolution.

Having thus completed the Description and Use of the Tables contained in this work, it now remains to show their application to the different elements connected with the sciences of navigation and nautical astronomy. In doing this, since the author's design carries him no farther than that of giving an ample illustration of the various purposes to which they may be applied; the reader must not, therefore, expect to find the elementary part of the sciences treated of. Hence, in this part of the work, the author will endeavour to confine himself to such Problems and subject matters as may appear to be most interesting and useful to nautical persons, without entering into particulars or the minutiae of the sciences, and thus swelling the work to an unnecessary size;—a thing which he most anxiously wishes to avoid.

A CONCISE SYSTEM
OF
DECIMAL ARITHMETIC.

ALTHOUGH, from what has been said in the last paragraph, it may appear somewhat irregular, and even contrary to the general tenor of this work, to introduce any subject therein that does not come immediately under the cognizance of logarithms ; yet, since the reader may be desirous of having some little acquaintance with the nature of decimal fractions previously to his entering on the logarithmical computations, the following concise system is given for that purpose.—It has been deemed advisable to touch upon this subject for two cogent reasons ;—first, because a short account of decimals may be acceptable to the mariner whose early entrance on a sea life prevents him from going through a regular course of scholastic education on shore ; and, second, that he may have directly under his view all that is essentially necessary to be known in the practically useful branches of science, without being under the necessity of consulting any other author for the purpose of assisting him in the comprehension of the different subjects contained in this work.

DECIMAL FRACTIONS.

A decimal fraction signifies the artificial manner of setting down and expressing natural vulgar fractions as if they were whole numbers.—A decimal fraction has always for its denominator an unit (1,) with as many ciphers annexed to it as there are places in the numerator ; and it is generally expressed by setting down the numerator only, with a point before it, on the left hand ;—thus, $\frac{5}{10}$ is .5 ; $\frac{75}{100}$ is .75 ; $\frac{25}{1000}$ is .025 ; $\frac{114}{10000}$ is .00114, &c. &c. :—hence the numerator must always consist of as many figures as there are ciphers in the denominator.

A mixed number is made up of a whole number and a decimal fraction, the one being separated from the other by a point ; thus 5.75 is the same as $5 \frac{75}{100}$, or $\frac{575}{100}$.

Ciphers on the right hand of decimals do not increase their value ; for .5 .50 .500 .5000, &c., are decimal fractions of the same value, each being equal to $\frac{5}{10}$, or $\frac{1}{2}$.—But when ciphers are placed on the left hand of a decimal they decrease its value in a tenfold proportion ;—thus, .5 is $\frac{5}{10}$ or 5 tenths ; but .05 is only $\frac{5}{100}$ or 5 hundredths ; .005 is only $\frac{5}{1000}$ or 5 thousandths, and so on :—hence it is evident that in decimals as well as in whole numbers, the value of the place of the figure increases towards the left hand, and decreases towards the right, each being in the same tenfold proportion.

ADDITION OF DECIMALS.

Addition of decimals is performed in the same way as addition of whole numbers, observing to place the numbers right ; that is, all the decimal points under each other, units under units, tenths under tenths, hundredths under hundredths, &c. ; taking care to point off from the total or sum as many places for decimals as there are in the line containing the greatest number of decimal places.

Example 1.

Add together 41.37 ; 3.762 ; 137.03 ; 409, and .3976.

$$\begin{array}{r} 41.37 \\ 3.762 \\ 137.03 \\ 409. \\ .3976 \\ \hline \end{array}$$

591.5596, the sum.

Example 2.

Add together 3.268 ; 208.1 ; 276 ; 4.7845, and 1.07.

$$\begin{array}{r} 3.268 \\ 208.1 \\ 276. \\ 4.7845 \\ 1.07 \\ \hline \end{array}$$

493.2225, the sum.

SUBTRACTION OF DECIMALS.

Subtraction of decimals is likewise performed the same way as in whole numbers ; observing to place the numbers right ; that is, the decimal points under each other, units under units, tenths under tenths, hundredths under hundredths, &c. &c.

Example 1.

From	439.7265
Take	98.283
	<hr/>
Remains . . .	341.4435

Example 2.

From	179.037
Take	54.932468
	<hr/>
Remains . . .	124.104532

MULTIPLICATION OF DECIMALS.

Multiplication of decimals is also performed the same way as in whole numbers ; observing to cut off as many decimal places in the product as there are decimal places in both factors ; that is, in the multiplicand and multiplier.

Example 1.

Multiply	2.4362
By275
	<hr/>
	121810
	170534
	48724
	<hr/>
Product = .	0.6699550

Example 2.

Multiply	376.09
By	13.43
	<hr/>
	112827
	150436
	112827
	37609
	<hr/>
Product = .	5050.8887

Note.—If a decimal fraction be multiplied by a decimal fraction the product will be less than either the multiplicand or the multiplier.—And if any number either whole or mixed, be multiplied by a decimal fraction, the product will be always less than the multiplicand, as in example 1 ;—hence if a decimal fraction be multiplied by itself, its value will *decrease* in the proportion of its multiple:—thus,

Multiply25
By25
	<hr/>
	125
	50
	<hr/>
Product = .	.0625

Multiply75
By75
	<hr/>
	375
	525
	<hr/>
Product = .	.5625

DIVISION OF DECIMALS.

Division of decimals is performed in the same manner as in whole numbers ; observing to point off as many decimal places in the quo-

tient as the decimal places in the dividend exceed those in the divisor :— But if there be not as many figures in the quotient as there are in that excess, the deficiency must be supplied by prefixing ciphers, with a point before them ;—for the decimal places in the divisor and quotient taken together, must be always equal to those in the dividend.—When there happens to be a remainder after the division ; or when the decimal places in the divisor are more than those in the dividend, then ciphers may be annexed to the latter, and the quotient carried on as far as may be necessary.

Example 1.

Divide .6699550 by .275

	<i>Dividend.</i>	<i>Quotient.</i>
<i>Divisor</i> .275	.6699550	(2.4362
	550	
	1199	
	1100	
	..995	
	825	
	1705	
	1650	
	..550	
	550	
	...	

Example 2.

Divide 5050.8887 by 13.43

	<i>Dividend.</i>	<i>Quotient.</i>
<i>Div.</i> 13.43	5050.8887	(376.09
	4029	
	10218	
	9401	
	.8178	
	8058	
	.12087	
	12087	
	

Note.—If a decimal fraction be divided by a decimal fraction, the quotient will be greater than either the divisor or dividend, as in Example 1. And, if any whole, or mixed number be divided by a decimal fraction, the quotient will be greater than the dividend ; but if a decimal fraction be divided by a whole, or mixed number, the quotient will be less than the dividend.—If a decimal fraction be divided by itself, its value will increase in the proportion of its division, or of the *decrease of the parts* into which the decimal is divided ; because, in this case, the quotient will be a natural number :—thus, .25 divided by .25, quotes 1.—And, .5625, divided by .5625, quotes 1 also. Hence it is manifest that the dividing of a decimal fraction by itself increases its value.

REDUCTION OF DECIMALS.

CASE I.

To reduce a Vulgar Fraction to a Decimal Fraction of equal value.

RULE.

Annex a cipher or ciphers to the numerator; then divide by the denominator, as in whole numbers, and the quotient will be the required decimal.

Examples.

Reduce $\frac{1}{4}$ to a decimal fraction.

$$\begin{array}{r} 4 \overline{)100} \end{array}$$

Required dec. = .25

Reduce $\frac{3}{4}$ to a decimal fraction.

$$\begin{array}{r} 4 \overline{)300} \end{array}$$

Required dec. = .75

Examples.

Reduce $\frac{1}{2}$ to a decimal fraction.

$$\begin{array}{r} 2 \overline{)10} \end{array}$$

Required dec. = .5

Reduce $\frac{5}{8}$ to a decimal fraction.

$$\begin{array}{r} 8 \overline{)5000} \end{array}$$

Req. dec. = .625

CASE II.

To reduce Numbers of different Denominations, such as Degrees, Time, Coin, Measure, &c. into Decimals.

RULE.

Reduce the given degrees, time, coin, measure, &c. into the lowest denomination mentioned, for a dividend, annex ciphers thereto, and then divide by the integer, reduced also into the lowest denomination mentioned; the quotient will be the required decimal fraction.

Examples.

Reduce 30 minutes to the decimal of a degree.

The given number being in the lowest denomination required, annex a cipher and divide by 60, the number of minutes in a degree; the quotient will be the required decimal;—thus,

$$\begin{array}{r} 60 \overline{)300} \left(.5, \text{ the Answer.} \right. \\ \underline{300} \\ \dots \end{array}$$

Examples.

Reduce 49' 30" to the decimal of a degree.

The given number being reduced to the lowest denomination mentioned, gives 2970"; to this annex ciphers, and divide by 3600, the seconds in a degree; the quotient will be the required decimal:—thus,

$$\begin{array}{r} 3600 \overline{)2970.000} \left(.825, \text{ Answer.} \right. \\ \underline{28800} \\ \dots 9000 \\ \underline{7200} \\ 18000 \\ \underline{18000} \\ \dots \end{array}$$

Reduce 15^h50^m: to the decimal of an hour.

The given terms being reduced to the lowest denomination give 950 seconds; annex ciphers and divide by 3600, the seconds in an hour; as thus,

$$\begin{array}{r} 3600 \overline{) 950.0000} \left(.2639 \text{ nearly Ans.} \right. \\ \underline{7200} \\ 23000 \\ \underline{21600} \\ 14000 \\ \underline{10800} \\ 32000 \\ \underline{32400} \\ \dots \end{array}$$

Reduce 4^h10^m50^s: to the decimal of a day.

The given time being reduced to the lowest denomination mentioned is 15050 seconds; annex ciphers and divide by 86400, the seconds in a day, or 24 hours;—thus,

$$\begin{array}{r} 86400 \overline{) 15050.00000} \left(.17419 \text{ nearly Ans.} \right. \\ \underline{86400} \\ 641000 \\ \underline{604800} \\ 362000 \\ \underline{345600} \\ 164000 \\ \underline{86400} \\ 776000 \end{array}$$

Reduce 3^l4^s to the decimal of a pound sterling.

The given sum being reduced to the lowest denomination mentioned gives 40 pence, annex ciphers and divide by 240, the pence in a pound sterling; as thus,

$$\begin{array}{r} 240 \overline{) 40.0000} \left(.1666 \text{ Answer.} \right. \\ \underline{240} \\ 1600 \\ \underline{1440} \\ 1600 \\ \underline{1440} \\ 1600 \\ \underline{1440} \\ 160 \end{array}$$

Reduce 45 minutes to the decimal of an hour.

The given number being in the lowest denomination mentioned, annex ciphers and divide by 60, the minutes in an hour; as thus,

$$\begin{array}{r} 60 \overline{) 45.00} \left(.75 \text{ which is the Ans.} \right. \\ \underline{420} \\ 300 \\ \underline{300} \\ \dots \end{array}$$

Reduce 100 fathoms and 2 feet to the decimal of a nautical mile.

The given measure being reduced to the lowest denomination mentioned is 602 feet; annex ciphers and divide by 6080, the number of feet in a sea mile; as thus,

$$\begin{array}{r} 6080 \overline{) 602.00000} \left(.09901 \text{ Ans.} \right. \\ \underline{54720} \\ 54800 \\ \underline{54720} \\ 8000 \\ \underline{6080} \\ 1920 \end{array}$$

Reduce 3 qrs. 21 lb. to the decimal of a hundred weight.

The given weight being reduced to the lowest denomination mentioned is 105 lbs. annex ciphers, and divide by 112, the number of pounds in a hundred weight; as thus,

$$\begin{array}{r} 112 \overline{) 105.0000} \left(.9375 \text{ Ans.} \right. \\ \underline{1008} \\ 420 \\ \underline{336} \\ 840 \\ \underline{784} \\ 560 \\ \underline{560} \\ \dots \end{array}$$

CASE III.

To find the value of any Decimal Fraction in the known parts of an Integer ; such as Degrees, Time, Coin, Weight, Measure, &c.

RULE.

Multiply the given decimal by the number of parts contained in the next inferior denomination ; and, from the right hand of the product, point off so many figures as the given decimal consists of.—Multiply those figures so pointed off by the number of parts contained in the next inferior denomination, and from the result cut off the decimal places as before :—proceed in this manner till the least known, or required parts of the integer are brought out ;—then, the several denominations on the left hand of the decimal points, will express the value of the given decimal fraction.

Example 1.

Required the value of .825 of a degree.

Given decimal .825
 Multiply by 60 minutes.

 49'.500
 Multiply by 60 seconds.

 30".000

Hence, the required value is 49'.30"

Example 3.

Required the value of .166666 of a pound sterling.

Given decimal = .166666
 Multiply by 20 shill.

 3'.333320
 Multiply by 12 pence

 3'.999840

Hence, the required value is 3'.4' very nearly.

Example 2.

Required the value of .2639 of an hour.

Given decimal = .2639
 Multiply by 60 min.

 15".8340
 Multiply by 60 seconds.

 50'.040

Hence, the required value is 15".50'.040.

Example 4.

Required the value of .09901 of a nautical or sea mile.

Given decimal = .09901
 Multiply by 6080, the ft. in a sea mile.

 792080
 594060

 601.98080

Hence, the required value is 602 feet very nearly.

THE RULE OF PROPORTION IN DECIMALS.

Prepare the terms by reducing the fractional parts to the highest denomination mentioned; then state the question and proceed as in the common Rule of Three Direct;—thus, place the numbers in such order that the first and third may be of the same kind, and the second the same as the number required :—bring the first and third terms into the same name, and the second into the highest denomination mentioned.—Then,

Multiply the second and third terms together; divide the product by the first term, and the quotient will be the answer in the same denomination as the second number;—observing, however, to point off the decimal places; the value of which is to be found by the Rule to Case III., page 162.

Note.—In the rule of proportion there are always three numbers given to find a fourth proportional; two of these are of supposition and one of demand; the latter must ever be the third term in the statement of the question; and, as this is interrogatory, it may, therefore, be known by the words—What will? What cost? How many? How far? How much?, &c.—The first term must always be of the same name as the third; the fourth, or term sought, will be of the same kind and denomination as the second term in the proportion.

Example 1.

If a degree of longitude, measured on the surface of the earth under the equator, be 69.092 English miles; how many miles are contained in the earth's circumference under the same parallel, it being divided into 360 degrees?

$$\text{As } . . . 1^{\circ} : 69^{\text{m}}.092 :: 360^{\circ}$$

360

4145520

207276

Answer . . . 24873.120 English miles.

Example 2.

The earth turns round upon its axis in $23^{\text{h}}.56^{\text{m}}$; at what rate per hour are the inhabitants carried from west to east by this rotation under the

equator where the earth's circumference measures 24873.12 English miles ; and at what rate per hour are the inhabitants of London carried in the same direction, where a degree of longitude measures 42.99 miles.

FIRST.—For the Inhabitants at the Equator.

23 hours 56 minutes are equal to 23.9333 hours.—Now,
As $23^{\text{h}}.9333$: 24873 $^{\text{m}}.12$:: 1^{h} : 1039 miles.

$$\begin{array}{r}
 24873.1200 \\
 239333 \\
 \hline
 ..939820 \\
 717999 \\
 \hline
 2218210 \\
 2153997 \\
 \hline
 ..64213
 \end{array}$$

SECOND.—For the Inhabitants of London.

360 degrees multiplied by 42.99 miles, give 15476.4 miles ;—And,
As $23^{\text{h}}.9333$: 15476 $^{\text{m}}.4$:: 1^{h} : 646 miles.

$$\begin{array}{r}
 15476.4000 \\
 1435998 \\
 \hline
 .1116420 \\
 957332 \\
 \hline
 .1590880 \\
 1435998 \\
 \hline
 .154882
 \end{array}$$

Hence, the inhabitants under the equator are carried at the rate of 1039 miles every hour, and those of London 646 miles per hour, by the earth's motion round its axis.

Example 3.

If a ship sails at the rate of $11\frac{1}{2}$ knots per hour ; in what time would she circumnavigate the globe, the circumference of which is 24873.12 miles ?

11½ knots are equal to 11.25 miles.—Now,
 As 11.25 : 1 :: 24873.12 : 2210.9 hours.
 2250

$$\begin{array}{r}
 .2373 \\
 2250 \\
 \hline
 .1231 \\
 1125 \\
 \hline
 .10620 \\
 10125 \\
 \hline
 ..495
 \end{array}$$

Hence, the required time is 2210.9 hours ; or 92 days, 2 hours, and 54 minutes.

PROPORTION, AND PROPERTIES OF NUMBERS.

If three quantities be proportional, the product or rectangle of the two extremes will be equal to the square of the mean.

If four quantities be proportional, the product of the two extremes will be equal to the rectangle or product of the two means.—Thus,

Let 2. 4. 8. 16 be the four quantities ; then, the rectangle of the extremes, viz. 16×2 , is equal to the rectangle of the means, viz. 4×8 , or 32.

If the product of any two quantities be equal to the product of two others, the four quantities may be turned into a proportion by making the terms of one product the *means*, and the terms of the other product the *extremes*.—Thus,

Let the terms of two products be 10 and 6, and 15 and 4, each of which is equal to 60 ; then, As $10 : 4 :: 15 : 6$. As $4 : 6 :: 10 : 15$. As $6 : 15 :: 4 : 10$, &c. &c.

If four quantities be proportional, they shall also be proportional when taken inversely and alternately.

If four quantities be proportional, the sum, or difference, of the first and second will be to the second, as the sum, or difference of the third and fourth is to the fourth.—Thus, let 2. 4. 8. 16 be the four proportional quantities ; then

As $2 + 4 : 4 :: 8 + 16 : 16$; or, as $4 - 2 : 4 :: 16 - 8 : 16$,

If from the sum of any two quantities either quantity be taken, the remainder will be the other quantity.

If the difference of any two quantities be added to the less, the sum will be the greater quantity; or if subtracted from the greater, the remainder will be the less quantity.

If half the difference of any two quantities be added to half their sum, the total will give the greater quantity; or if subtracted, the remainder will be the less quantity.

If the product of any two quantities be divided by either quantity, the quotient will be the other quantity.

If the quotient of any two quantities be multiplied by the less, the product will be the greater quantity.

The rectangle or product of the sum and difference of any two quantities, is equal to the difference of their squares.—Thus,

Let 4 and 10 be the two quantities; then $4 + 10 = 14$; $10 - 4 = 6$, and $14 \times 6 = 84$.—Now, $10 \times 10 = 100$; $4 \times 4 = 16$, and $100 - 16 = 84$.

The difference of the squares of the sum and difference of any two quantities, is equal to four times the rectangle of those quantities.—Thus,

Let 10 and 6 be the two quantities; then $10 + 6 = 16$; $16 \times 16 = 256$;— $10 - 6 = 4$; $4 \times 4 = 16$.—Now, $256 - 16 = 240$; and $10 \times 6 \times 4 = 240$.

The sum of the squares of the sum and difference of any two quantities, is equal to twice the sum of their squares.—Thus,

$10 + 6 = 16$; $16 \times 16 = 256$; and $10 - 6 = 4$; $4 \times 4 = 16$; then $256 + 16 = 272$. Again, $10 \times 10 = 100$; $6 \times 6 = 36$, and $100 + 36 = 136 \times 2 = 272$.

If the sum and difference of any two numbers be added together, the total will be twice the greater number.—Thus,

$10 + 6 = 16$; and $10 - 6 = 4$; then $16 + 4 = 20$; and $10 \times 2 = 20$.

If the difference of any two numbers be subtracted from their sum, the remainder will be twice the less number.—Thus,

$10 - 6 = 4$; and $10 + 6 = 16$; then $16 - 4 = 12$;—and $6 \times 2 = 12$.

The square of the sum of any two numbers is equal to the sum of their squares, together with twice their rectangle.—Thus,

$10 + 6 = 16$; and $16 \times 16 = 256$. Again, $10 \times 10 = 100$; $6 \times 6 = 36$, and $100 + 36 = 136$; then, $10 \times 6 \times 2 = 120$; and $120 + 136 = 256$.

The sum, or difference, of any two numbers will measure the sum, or difference, of the cubes of the same numbers; that is, the sum will measure the sum, and the difference the difference.

The difference of any two numbers will measure the difference of the squares of those numbers.

The sum of any two numbers differing by an unit (1,) is equal to the difference of the squares of those numbers.—Thus,

$9 + 8 = 17$; and $9 \times 9 = 81$; $8 \times 8 = 64$; now, $81 - 64 = 17$.

If the sum of any two numbers be multiplied by each number respect-

ively, the sum of the two rectangles will be equal to the square of the sum of those numbers.

Thus, $10+6=16$; now, $16 \times 10 = 160$; $16 \times 6 = 96$; and $160+96=256$.

Again, $10+6=16$; and $16 \times 16 = 256$.

The square of the sum of any two numbers is equal to four times the square of half their sum.—Thus,

$10+6=16$; and $16 \times 16 = 256$; then $10+6=16 \div 2 = 8$, and $8 \times 8 \times 4 = 256$.

The sum of the squares of any two numbers is equal to the square of their difference, together with twice the rectangle of those numbers.—Thus,

$10 \times 10 = 100$; $6 \times 6 = 36$; and $100+36 = 136$.—Again,

$10-6=4$; and $4 \times 4 = 16$; $10 \times 6 \times 2 = 120$; and $120+16 = 136$.

The numbers 3, 4 and 5, or their multiples 6, 8 and 10, &c. &c., will express the three sides of a right angled plane triangle.

The sum of any two square numbers whatever, their difference, and twice the product of their roots, will also express the three sides of a right angled plane triangle.—Thus,

Let 9 and 49 be the two square numbers:—then $9+49=58$; $49-9=40$.—Now, the root of 9 is 3, and that of 49 is 7;—then $7 \times 3 \times 2 = 42$: hence the three sides of the right angled plane triangle will be 58, 40, and 42.

The sum of the squares of the base and perpendicular of a right angled plane triangle, is equal to the square of the hypotenuse.

The difference of the squares of the hypotenuse and one leg of a right angled plane triangle, is equal to the square of the other leg.

The rectangle or product of the sum and difference of the hypotenuse and one leg of a right angled plane triangle, is equal to the square of the other leg.

The cube of any number divided by 6 will leave the same remainder as the number itself when divided by 6.—The difference between any number and its cube will divide by 6, and leave no remainder.

Any even square number will divide by 4, and leave no remainder; but an uneven square number divided by 4 will leave 1 for a remainder.

PLANE TRIGONOMETRY.

The Resolution of the different Problems, or Cases, in Plane Trigonometry, by Logarithms.

ALTHOUGH it is not the author's intention (as has been already-observed,) to enter into the elementary parts of the sciences on which he may have occasion to touch in elucidating a few of the many important purposes to which these Tables may be applied; yet, since this work may, probably, fall into the hands of persons not very conversant with trigonometrical subjects, he therefore thinks it right briefly to set forth such definitions, &c. as appear to be indispensably necessary towards giving such persons some little insight into this particular department of science.

PLANE TRIGONOMETRY is that branch of the mathematics which teaches how to find the measures of the unknown sides and angles of plane triangles from some that are already known.—It is divided into two parts; right angled and oblique angled:—in the former case one of the angles is a right angle, or 90° ; in the latter they are all oblique.

Every plane triangle consists of six parts; viz., three sides and three angles; any three of which being given (except the three angles), the other three may be readily found by logarithmical calculation.

In every triangle the greatest side is opposite to the greatest angle; and, *vice versa*, the greatest angle opposite to the greatest side.—But, equal sides are subtended by equal angles, and conversely.

The three angles of every plane triangle are, together, equal to two right angles, or 180 degrees.

If one angle of a plane triangle be obtuse, or more than 90° , the other two are acute, or each less than that quantity: and if one angle be right, or 90° , the other two taken together, make 90° :—hence, if one of the angles of a right angled triangle be known, the other is found by subtracting the known one from 90° .—If one angle of any plane triangle be known, the sum of the other two is found by subtracting that which is given from 180° ; and if two of the angles be known, the third is found by subtracting their sum from 180° .

The complement of an angle is what it wants of 90° ; and the supplement of an angle is what it wants of 180° .

In every right angled triangle, the side subtending the right angle is called the *hypotenuse*; the lower or horizontal side is called the *base*, and that which stands upright, the *perpendicular*.

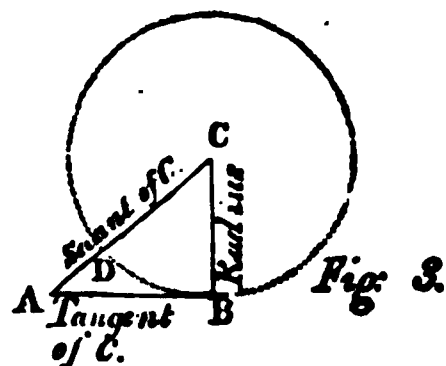
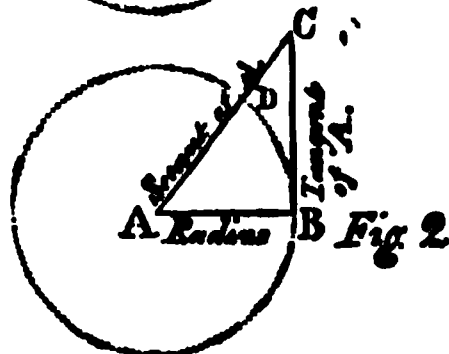
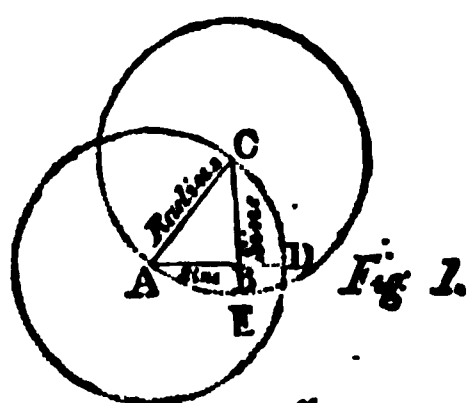
If the hypotenuse be assumed equal to the radius, the sides, that is, the base and the perpendicular, will be the sines of their opposite angles. And, if either of the sides be considered as the radius, the other side will be the tangent of its opposite angle, and the hypotenuse the secant of the same angle.

Thus.—Let ABC be a right angled plane triangle; if the hypotenuse AC be made radius, the side BC will be the sine of the angle A , and AB the sine of the angle C .—If the side AB be made radius, BC will be the tangent, and AC the secant, of the angle A :—And, if BC be the radius, AB will be the tangent, and AC the secant of the angle C .

For, if we make the hypotenuse AC radius (Fig. 1.), and upon A , as a centre, describe the arch CD to meet AB produced to D ; then it is evident that BC is the sine of the arch DC , which is the measure of the angle BAC ; and that AB is the co-sine of the same arch:—and if the arch AE be described about the centre C , to meet CB produced to E , then will AB be the sine of the arch AE , or the sine of the angle ACB , and BC its co-sine.

Again, with the extent AB as a radius (Fig. 2.), describe the circle BD ; then BC is the tangent of the arch BD , which is evidently the measure of the angle BAC ; and AC is the secant of the same arch, or angle.

Lastly, with CB as a radius (Fig. 3.), describe the arch BD ; then AB is the tangent of the arch BD , the measure of the angle ACB , and AC the secant of the same arch or angle.



In the computation of right angled triangles, any side, whether given or required, may be made radius to find a *side*; but a given side must be made radius to find an angle: thus,

To find a Side:—

Call any one of the sides of the triangle radius, and write upon it the word *radius*:—observe whether the other sides become sines, tangents, or secants, and write these words on them accordingly, as in the three preceding figures: then say, as the name of the given side, is to the given side; so is the name of the side required, to the side required.

And, to find an Angle :—

Call one of the *given sides* the radius, and write upon it the word radius: observe whether the other sides become sines, tangents, or secants, and write these words on them accordingly, as in the three foregoing figures; then say, as the side made radius, is to radius; so is the other *given side* to its name: that is, to the sine, tangent, or secant by it represented.

Now, since in plane trigonometry the sides of a triangle may be considered, without much impropriety, as being in a direct ratio to the sines of their opposite angles, and conversely; the proportion may, therefore, be stated agreeably to the established principles of the *Rule of Three Direct*, by saying

As the name of a given angle, is to its opposite given side; so is the name of any other given angle to its opposite side.—And, as a given side, is to the name of its opposite given angle; so is any other given side, to the name of its opposite angle.

The proportion, thus stated, is to be worked by logarithms, in the following manner; viz.,

To the arithmetical complement of the first term, add the logs. of the second and third terms, and the sum (rejecting 20, or 10 from the index, according as the required term may be a side or an angle,) will be the logarithm of the required, or fourth term.

Remarks.—1. The arithmetical complement of a logarithm is what that logarithm wants of the radius of the Table; viz., what it is short of 10.000000; and the arithmetical complement of a log. sine, tangent, or secant, is what such logarithmic sine, &c. &c. wants of twice the radius of the Tables, viz., 20.000000.

2. The arithmetical complement of a log. is most readily found by beginning at the left hand and subtracting each figure from 9 except the last significant one, which is to be taken from 10, as thus;—if the given log. be 2.376843, its arithmetical complement will be 7.623157:—if a given log. sine be 9.476284, its arithmetical complement will be 10.523716, and so on.

3. The arithmetical complement of the log. sine of an arch, is the log. co-secant of that arch;—the arithmetical complement of the log. tangent of an arch, is the log. co-tangent of that arch; and conversely, in both cases.

Solution of Right-angled Plane Triangles, by Logarithms.

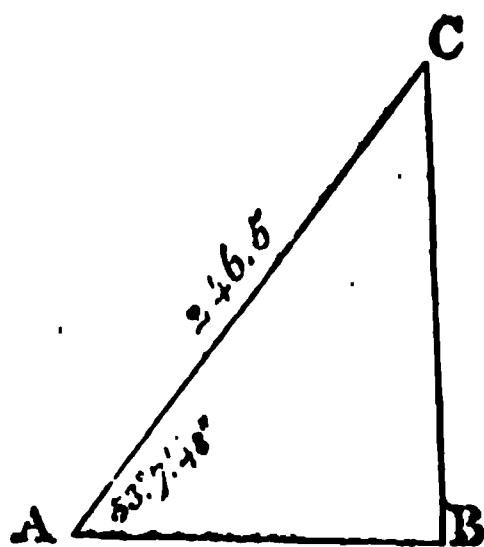
PROBLEM I.

Given the Angles and the Hypotenuse, to find the Base and the Perpendicular.

Example.

Let the hypotenuse A C, of the annexed triangle A B C, be 246.5, and the angle A $53^{\circ}7'48''$; required the base A B, and the perpendicular B C?

Note.—Since there is no more intended, in this place, than merely to show the use of the Tables; the geometrical construction of the diagrams is, therefore, purposely omitted.



By making the hypotenuse A C radius; B C becomes the sine of the angle A, and A B the co-sine of the same angle.—Hence,

To find the Perpendicular B C :—

As radius = 90° =	Log. sine =	10.000000
Is to hypotenuse A C = 246.5	Log. =	2.391817
So is the angle A = $53^{\circ}7'48''$	Log. sine =	9.903090

To the perpendicular B C = 197.2 = Log. = 2.294907

To find the Base A B :—

As radius = 90° =	Log. sine =	10.000000
Is to hypotenuse A C = 246.5	Log. =	2.391817
So is the angle A = $53^{\circ}7'48''$	Log. co-sine =	9.778153

To the base A B = 147.9 = Log. = 2.169970

Making the base A B radius; B C becomes the tangent of the angle A, and A C the secant of the same angle.—Hence,

To find the Perpendicular B C :—

As the angle A = $53^{\circ}7'48''$	Log. secant Ar. comp. =	9.778153
Is to hypotenuse A C = 246.5	Log. =	2.391817
So is the angle A = $53^{\circ}7'48''$	Log. tangent =	10.124937

To the perpendicular B C = 179.2 = Log. = 2.294907

To find the Base A B :—

As the angle A = 53° 7' 48"	Log. secant Ar. compt. = 9.778153
Is to hypotenuse A C = 246.5	Log. = 2.391817
So is radius = 90°	Log. sine = 10.000000
<hr/>	
To the base A B = 147.9 =	Log. = 2.169970

The perpendicular B C being made radius ; the base A B becomes the tangent of the angle C, or co-tangent of the angle A, and the hypotenuse A C the secant of the angle C, or co-secant of the angle A.—Hence,

To find the Perpendicular B C :

As the angle A = 53° 7' 48"	Log. co-secant Ar. compt. = 9.903090
Is to hypotenuse A C = 246.5	Log. 2.391817
So is radius = 90°	Log. sine 10.000000
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To the perpendicular B C = 197.2 =	Log. = 2.294907

To find the Base A B :—

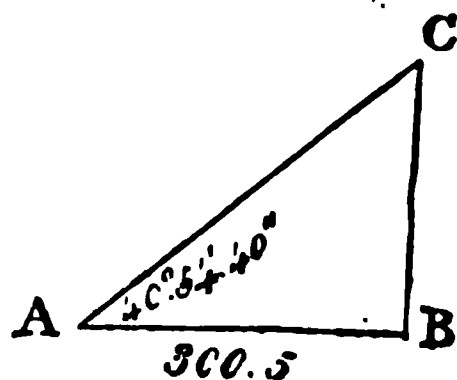
As the angle A = 53° 7' 48"	Log. co-secant Ar. compt. = 9.903090
Is to hypotenuse A C = 246.5	Log. = 2.391817
So is the angle A = 53° 7' 48"	Log. co-tangent . . . 9.875063
<hr/>	
To the base A B = 147.9 =	Log. = 2.169970

PROBLEM II.

Given the Angles and One Side, to find the Hypotenuse and the other Side.

Example.

Let the base A B of the annexed triangle A B C, be 300.5, and the angle A 40° 54' 40" ; required the hypotenuse A C, and the perpendicular B C ?



The hypotenuse A C being made radius ; the perpendicular B C will be the sine of the angle A, and the base A B the co-sine of the same angle.

To find the Hypothenuse A C :—

As the angle A = 40°54'40"	Log. co-sine Ar. compt. =	10.121635
Is to the base A B = 300.5	Log. =	2.477845
So is radius = 90°	Log. sine =	10.000000

To the hypotenuse A C = 397.6 = Log. =	2.599480
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To find the Perpendicular B C :—

As the angle A = 40°54'40"	Log. co-sine Ar. compt. =	10.121635
Is to the base A B = 300.5	Log. =	2.477845
So is the angle A = 40°54'40"	Log. sine =	9.816167

To the perpendicular B C = 260.4 = Log. =	2.415647
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The base A B being made radius ; the perpendicular B C will be the tangent of the angle A, and the hypotenuse A C the secant thereof.—Hence,

To find the Hypothenuse A C :—

As radius = 90°	Log. sine =	10.000000
Is to the base A B = 300.5	Log. =	2.477845
So is the angle A = 40°54'40"	Log. secant	10.121635

To the hypotenuse A C = 397.6 = Log. =	2.599480
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To find the Perpendicular B C :—

As radius = 90°	Log. sine =	10.000000
Is to the base A B = 300.5	Log.	2.477845
So is the angle A = 40°54'40"	Log. tangent	9.937802

To the perpendicular B C = 260.4 = Log. =	2.415647
---	----------

The perpendicular B C being made radius ; the base A B will be the tangent of the angle C, or co-tangent of the angle A, and the hypotenuse the secant of the angle C, or co-secant of A.—Hence,

To find the Hypothenuse A C :—

As the angle A = 40°54'40"	Log. co-tang. Ar. compt. =	9.937802
Is to the base A B = 300.5	Log. =	2.477845
So is the angle A = 40°54'40"	Log. co-secant =	10.183833

To the hypotenuse A C = 397.6 = Log. =	2.599480
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To find the Perpendicular B C :—

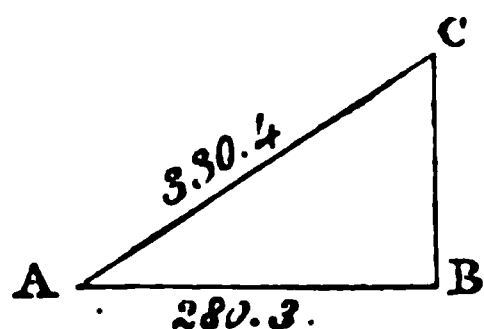
As the angle A = 40°54'40"	Log. co-tang. Ar. compt. =	9.937802
Is to the base A B = 300.5	Log. =	2.477845
So is radius = 90°	Log. sine =	10.000000
		2.415647
To the perpendicular B C = 260.4	Log. =	2.415647

PROBLEM III.

Given the Hypothenuse and One Side, to find the Angles and the Other Side.

Example.

Let the hypothenuse A C, of the annexed triangle A B C, be 330.4, and the base A B 280.3; required the angles A and C, and the perpendicular B C?



By making the hypothenuse A C radius; the perpendicular B C becomes the sine of the angle A, and the base A B the co-sine of the same angle.—Hence,

To find the Angle A :—

As the hypothenuse A C = 330.4	Log. Ar. compt. =	7.480960
Is to radius = 90°	Log. sine =	10.000000
So is the base A B = 280.3	Log. =	2.447623
		9.928583
To the angle A = 31°57'56"	Log. co-sine =	9.928583

To find the Perpendicular B C.

As radius = 90°	Log. sine =	10.000000
Is to hypothenuse A C = 330.4	Log. =	2.519040
So is the angle A = 31°57'56"	Log. sine =	9.723791
		2.242831
To the perpendicular B C = 174.9	Log. =	2.242831

The base A B being made radius; the perpendicular B C becomes the tangent of the angle A, and the hypothenuse A C the secant of that angle.—Hence,

To find the Angle A :—

As the base A B = 280.3	Log. Ar. compt. =	. . .	7.552377
Is to the radius = 90°	Log. sine =	10.000000
So is the hypotenuse A C = 330.4	= Log. =	. . .	2.519040
<hr/>			
To the angle A = 31°57'56"	Log. secant =	. . .	10.071417

To find the Perpendicular B C :—

As radius = 90°	Log. sine =	10.000000
Is to the base A B = 280.3	Log. =	2.447623
So is the angle A = 31°57'56"	Log. tangent =	. .	9.795208
<hr/>			
To the perpendicular B C = 174.9	= Log. =	. . .	2.242831

Remark.—The perpendicular B C may be found independently of the angles by the following rule (deduced from Euclid, Book I. Prop. 47, and Book II. Prop. 5), viz.,

To the log. of the sum of the hypotenuse and given side, add the log. of their difference; then, half the sum of these two logs. will be the log. of the required side :—as thus ;

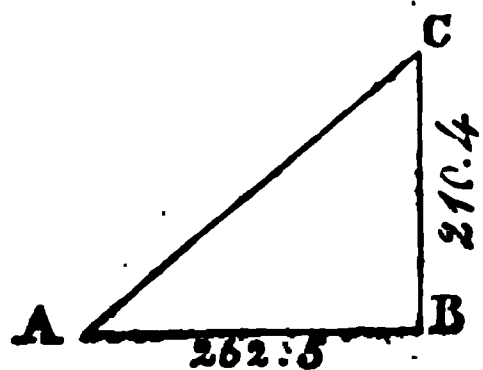
Hypotenuse A C =	330.4	
Base . . . A B =	280.3	
<hr/>		
Sum . . . =	610.7	Log. . . . = 2.785828
Difference . . . =	50.1	Log. . . . = 1.699838
<hr/>		
	Sum . . . =	4.485666
<hr/>		
Perpendicular B C =	174.9	= Log. . . = 2.242833

PROBLEM IV.

Given the Base and the Perpendicular, to find the Angles and the Hypotenuse.

Example.

Let the base A B, of the annexed triangle A B C, be 262.5, and the perpendicular B C 210.4; required the angles, and the hypotenuse A C?



By making the base AB radius; the perpendicular BC becomes the tangent of the angle A , and the hypotenuse AC the secant thereof.—Hence,

To find the Angle A :—

As the base $AB = 262.5$	Log. Ar. compt. =	7.580871
Is to radius = 90°	Log. sine =	10.000000
So is the perpendicular $BC = 210.4$	Log. =	,	2.323046
<hr/>			
To the angle $A = 38^\circ 42' 47''$	Log. tangent =	9.903917

To find the Hypotenuse AC :—

As radius = 90°	Log. sine =	10.000000
Is to the base $AB = 262.5$	Log. =	2.419129
So is the angle $A = 38^\circ 42' 47''$	Log. secant =	10.107745
<hr/>			
To the hypotenuse $AC = 336.4$	Log. =	2.526874

The perpendicular BC being made radius; the base AB will be the tangent of the angle C , or co-tangent of the angle A , and the hypotenuse AC will be the secant of C , or the co-secant of the angle A .—Hence,

To find the Angle A :—

As the perpendicular $BC = 210.4$	Log. Ar. compt. =	7.676954
Is to radius = 90°	Log. sine =	10.000000
So is the base $AB = 262.5$	Log. =	2.419129
<hr/>			
To the angle $A = 38^\circ 42' 47''$	Log. co-tangent =	10.096083

To find the Hypotenuse AC :—

As radius = 90°	Log. sine =	10.000000
Is to the perpendicular $BC = 210.4$	Log. =	2.323046
So is the angle $A = 38^\circ 42' 47''$	Log. co-secant =	10.203828
<hr/>			
To the hypotenuse $AC = 336.4$	Log. =	2.526874

The angle A subtracted from 90° leaves the angle C ; thus $90^\circ - 38^\circ 42' 47'' = 51^\circ 17' 13''$ the measure of the angle C .

Remark.—The hypotenuse AC may be found independently of the angles by the following rule, deduced principally from Euclid; Book I. Prop. 47; Book II. Prop. 5; and Book VI. Prop. 8, viz.,

From twice the log. of the base subtract the log. of the perpendicular, and add the corresponding natural number to the perpendicular; then, to the log. of this sum add the log. of the perpendicular, and half the sum of these two logs. will be the log. of the hypotenuse. As thus:—

$$\text{Base } A B = . . . 262.5 \text{ twice the log.} = 4.838258$$

$$\text{Perpendicular } B C = 210.4 \text{ Log.} . . = 2.323046 \quad . . 2.323046$$

$$\text{Natural number} = 327.5 \text{ Log.} . . = 2.515212$$

$$\text{Sum} . . . 537.9 \text{ Log.} = 2.780702$$

$$\text{Sum} = 5.053748$$

$$\text{Hypotenuse } A C = 336.4 \text{ Log.} = 2.526874$$

Solution of Oblique-angled Plane Triangles by Logarithms.

PROBLEM I.

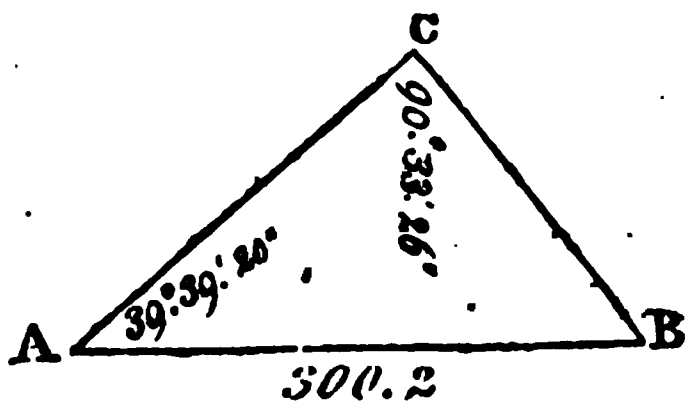
Given the Angles and One Side of an Oblique-angled Plane Triangle, to find the other Sides.

RULE.

As the Log. sine of any given angle, is to its opposite given side; so is the log. sine of any other given angle to its opposite side.

Example.

Let the side $A B$, of the triangle $A B C$, be 300.2, the angle A $39^{\circ}39'20''$ the angle C $90^{\circ}33'26''$ and, hence, the angle B $49^{\circ}47'14''$; to find the sides $A C$ and $B C$.



To find the Side $A C$:—

$$\text{As the angle } C = 90^{\circ}33'26'' \text{ Log. sine ar. compt.} = 10.000021$$

$$\text{Is to the side } B C = 300.2 \text{ Log.} 2.477411$$

$$\text{So is the angle } B = 49^{\circ}47'14'' \text{ Log. sine} 9.882895$$

$$\text{To the side } A C = 229.3 = \text{Log.} = 2.360327$$

To find the Side B C:—

As the angle C = 90°33'26"	Log. sine ar. compt. =	10.000021
Is to the side B C = 300.2	Log. =	2.477411
So is the angle A = 39°39'20"	Log. sine =	9.804937
To the side B C = 191.6	Log. =	2.282369

Note.—When a log. sine, or log. co-sine, is the first term in the proportion, the arithmetical complement thereof may be taken directly from the Table of secants by using a log. co-secant in the former case, and a log. secant in the latter.

PROBLEM II.

Given two Sides and an Angle opposite to one of them, to find the other Angles and the third Side.

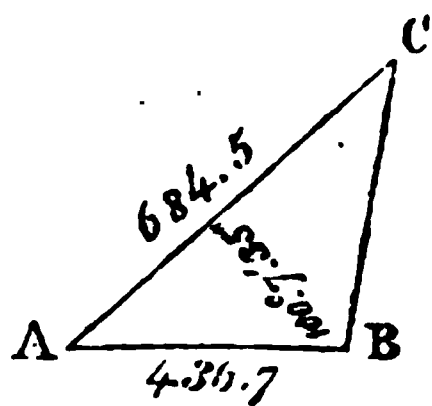
RULE.

As any given side of a triangle is to the log. sine of its opposite given angle, so is any other given side to the log. sine of the angle opposite thereto.

The angles being thus found, the third side is to be computed by the preceding Problem.

Example.

Let the side A B, of the triangle A B C, be 436.7, the side A C 684.5, and the angle B 100°7'35"; required the angles A and C, and the side B C?



To find the angle C:—

As the side A C =	684.5	Log. ar. comp. =	7.164626
Is to the angle A = 100°7'35"		Log. sine =	9.993181
So is the side A B = 436.7		Log. =	2.640183
To the angle C = 38°54'22"		Log. =	9.797990

To find the side B C :—

As the angle B =	100° 7' 35"	Log. sine ar. comp. =	10.006819.
Is to the side A C =	684.5	Log. = 2.835374
So is the angle A =	40° 58' 3"	Log. sine = 9.816659
To the side B C =	455.9 =	Log. = 2.658852

Note.—The angle A = 100° 7' 35" + the angle C = 38° 54' 22" = 139° 1' 57"; and 180° − 139° 1' 57" = the angle A = 40° 58' 3"

Remark.—An angle found by this rule is ambiguous when the given side opposite to the given angle is *less* than the other given side; that is, the angle opposite to the greater side may be either acute or obtuse: for trigonometry only gives the sine of an angle, which sine may either represent the measure of the angle itself, or of its supplement to 180 degrees. But when the given side opposite to the given angle is greater than the other given side, then the angle opposite to that (other given) side is always acute, as in the above example.

PROBLEM III.

Given two Sides and the included Angle, to find the other Angles and the third Side.

RULE.

Find the sum and difference of the two given sides; subtract the given angle from 180°; take half the remainder, and it will be half the sum of the unknown angles; then say,

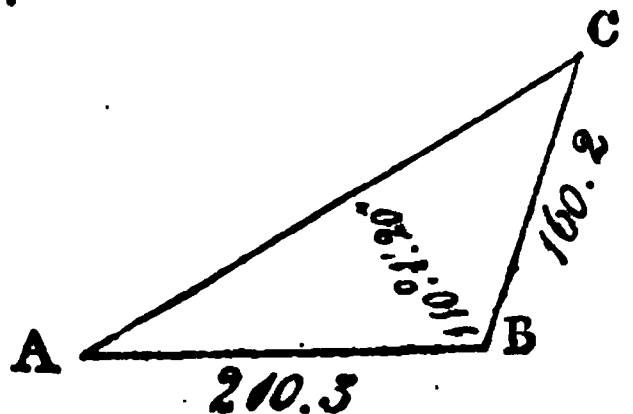
As the sum of the sides is to their difference; so is the log. tangent of half the sum of the unknown angles, to the log. tangent of half their difference.

Now, half the difference of the angles, thus found, added to half their sum, gives the greater angle, or that which is opposite to the greater side; and being subtracted, leaves the angle opposite to the less side.

The angles being thus determined, the third side is to be computed by Problem I., page 177.

Example.

Let the side A B, of the triangle A B C, be 210.3, the side B C 160.2, and the angle B 110° 1' 20"; required the angles A and C, and the side A C?



180° — the angle B $110^\circ 1' 20'' = 69^\circ 58' 40'' + 2 = 34^\circ 59' 20'' =$
half the sum of the angles A and C.

$$\text{Side AB} = 210.3$$

$$\text{Side BC} = 160.2$$

$$\text{As sum} = 370.5 \quad \text{Log ar. comp.} = 7.431212$$

$$\text{Is to difference} = 50.1 \quad \text{Log.} = 1.699838$$

$$\text{So is } \frac{1}{2} \text{ sum of angles} = 34^\circ 59' 20'' \quad \text{Log. tang.} = 9.845048$$

$$\text{To } \frac{1}{2} \text{ differ. of angles} = 5^\circ 24' 24'' \quad \text{Log. tang.} = 8.976098$$

$$\text{Angle C} = 40^\circ 23' 44''$$

$$\text{Angle A} = 29^\circ 34' 56''$$

To find the side AC :

$$\text{As the angle A} = . 29^\circ 34' 56'' \quad \text{Log. sine ar. comp.} = 10.306561$$

$$\text{Is to the side BC} = . 160.2 \quad \text{Log.} = 2.204663$$

$$\text{So is the angle B} = . 110^\circ 1' 20'' \quad \text{Log. sine} = 9.972925$$

$$\text{To the side AC} = . 304.9 = \text{Log.} = 2.484149$$

PROBLEM IV.

Given the three Sides of a Plane Triangle, to find the Angles.

RULE.

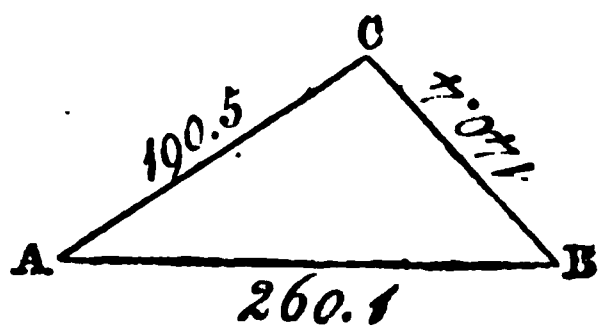
Add the three sides together, and take half their sum ; the difference between which and the side opposite to the required angle call the *remainder* ; then,

To the arithmetical complements of the logs. of the other two sides, add the logs. of the half sum and of the *remainder* : half the sum of these four logs. will be the log. co-sine of an arch ; which, being doubled, will give the required angle.

Now, one angle being thus found, either of the other two angles may be computed by Problem II., page 178.

Example.

Let the side AB, of the triangle ABC, be 260.1, the side AC 190.5, and the side BC 140.4 ; required the angles A, B, and C?



The side $AB = 260.1$
 $BC = 140.4$ Log. ar. comp. . . 7.852633
 $AC = 190.5$ Log. ar. comp. . . 7.720105

Sum = . . . 591.0

Half sum = . . 295.5 Log. = . . . 2.470558

Remainder = . . 35.4 Log. = . . . 1.549003

Sum = 19.592299

Arch = . . $51^{\circ}17'22''$ Log. co-sine = . . 9.796149 $\frac{1}{2}$

Angle C = . $102^{\circ}34'44''$

To find the angle B:—

As the side $AB = 260.1$ Log. ar. comp. = 7.584860

Is to the angle $C = 102^{\circ}34'44''$ Log. sine = . . 9.989448

So is the side $AC = 190.5$ Log. = . . . 2.279895

To the angle $B = 45^{\circ}37'45''$ Log. = . . . 9.854203

Now, angle $C 102^{\circ}34'44'' + \text{angle } B 45^{\circ}37'45'' = 148^{\circ}12'29''$;
 and $180^{\circ} - 148^{\circ}12'29'' = 31^{\circ}47'31'' = \text{the angle } A$.

THE RESOLUTION OF THE DIFFERENT PROBLEMS, OR CASES, IN SPHERICAL TRIGONOMETRY, BY LOGARITHMS.

Spherical Trigonometry is that branch of the mathematics which shows how to find the measures of the unknown sides and angles of spherical triangles from some that are already known. It is divided into three parts; viz., right-angled, quadrantal, and oblique-angled.

A right-angled spherical triangle has one right angle; the sides including the right angle are called legs, and that opposite thereto the hypotenuse.

A quadrantal spherical triangle has one side equal to 90° , or the fourth part of a circle.

An oblique-angled spherical triangle has neither a side nor an angle equal to 90° .

A spherical triangle is formed by the intersection of three great circles on the surface of the sphere.

The three angles of a spherical triangle are always more than two, but less than six, right angles.

The three sides of a spherical triangle are always less than two semi-circles, or 360° .

Any two sides of a spherical triangle, taken together, are greater than the third.

The greater side subtends the greater angle; the lesser side the lesser angle, and conversely.

Equal sides subtend equal angles, and, *vice versa*, equal angles are subtended by equal sides.

The two sides or two angles of a spherical triangle, when compared together, are said to be alike, or of the same affection, when both are less or both greater than 90° ; but when one is greater and the other less than 90° , they are said to be unlike, or of different affections.

Every side of a right-angled spherical triangle exceeding 90° , is greater than the hypotenuse; but every side less than that quantity, is less than the hypotenuse.

The hypotenuse is less than a quadrant, if the legs be of the same affection; but greater than a quadrant, if they be of different affections.

The hypotenuse is, also, less or greater than a quadrant, according as the adjacent angles are of the same or of different affections.

When the hypotenuse and one leg, or its opposite angle, are of the same or of different affections, the other side, or its opposite angle, will be, accordingly, less or greater than a quadrant.

The legs and their opposite angles are always of the same affection.

The sides of a spherical triangle may be changed into angles, and conversely.

Every spherical triangle consists of six parts: viz., three sides and three angles; of which, if any three be given, the remaining three may be readily computed; but in right-angled spherical triangles, it is sufficient that two only be given, because the right angle is always known.

SOLUTION OF RIGHT-ANGLED SPHERICAL TRIANGLES, BY LOGARITHMS, AGREEABLY TO LORD NAPIER'S RULES.

In every right-angled spherical triangle there are five circular parts, exclusive of the right angle, which is not taken into consideration. These five parts consist of the *two legs*, or sides; the *complement of the hypotenuse*; and the *complements of the two angles*. They are called circular parts, because each of them is measured by the arc of a great circle.

Three of these circular parts, besides the radius, enter into every proportion; two of which are given, and the third required. One is called the *middle part*, and the other two the *extremes conjunct* or *disjunct*.

The *middle part*, and also the *extremes conjunct* or *disjunct*, may be determined by the following rules.

Rule 1.—When the three circular parts under consideration are joined together, or follow each other in successive order, the middle one is termed the *middle part*, and the other two the *extremes conjunct*, because they are directly conjoined thereto.

Rule 2.—When the three circular parts do not join, or follow each other in successive order, that which stands alone, or disjoined from the other two, is termed the *middle part*, and the other two the *extremes disjunct*, because they are separated or disjoined therefrom by the intervention of a side, or an angle not concerned in the proportion.

Note.—In determining the *middle part*, it is to be observed, that the right angle does not separate or disjoin the legs: therefore, when these are under consideration, they are always to follow each other in succession.

These things being premised, the required parts are to be computed by the two following equations; viz.,

1st.—The product of radius and the sine of the *middle part*, is equal to the product of the tangents of the *extremes conjunct*.

2d.—The product of radius and the sine of the *middle part*, is equal to the product of the co-sines of the *extremes disjunct*.

Since these equations are adapted to the complements of the hypotenuse and angles, and since the sine or the tangent of the complement of an arch is represented directly by the co-sine or co-tangent of that arch,—therefore, to save the trouble of finding the complements, let a co-sine or co-tangent be used instead of a sine or tangent, and a sine instead of a co-sine, &c. &c., when the angles or the hypotenuse are in question.

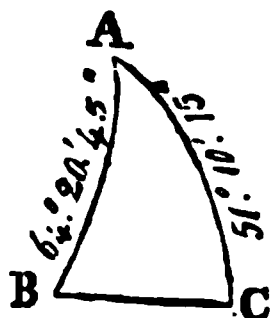
Now, the *middle part* being determined by the rules 1 or 2, as above, according as the extremes are *conjunct* or *disjunct*, the terms under consideration are then to be reduced to a proportion, as thus:—Put the unknown or required term *last*, that with which it is connected *first*, and the remaining two in the middle, in any order; this being done, the equation will then be ready for a direct solution by logarithmical numbers.

PROBLEM I.

Given the Hypothenuse and one Leg, to find the Angles and the other Leg.

Example.

Let the hypothenuse AB , of the spherical triangle ABC , be $64^{\circ}20'45''$, and the leg AC $51^{\circ}10'15''$; required the angles A and B , and the leg BC ?



To find the angle A :—

Here the hypothenuse AB , the given leg AC , and the required angle A , are the three circular parts which enter the proportion; and since the angle A evidently connects the hypothenuse and the given leg, it is therefore the *middle part*, and the other two the *extremes conjunct*, according to rule 1, page 183; therefore, by equation 1, page 183,

Radius \times co-sine of angle A = tangent of AC \times co-tangent of AB .

Now, since radius is *connected* with the required term, it is to be the first term in the proportion. Hence,

As radius = $90^{\circ} 0' 0''$ Log. sine ar. comp. = 10.000000

Is to the leg AC = . . . $51.10.15$ Log. tangent = . 10.094280

So is the hypothenuse AB = $64.20.45$ Log. co-tangent = 9.681497

To the angle A = . . . $53^{\circ}21'50''$ Log. co-sine = . 9.775777

Note.—The angle A is acute, because the hypothenuse and the given leg are both of the same affection.

To find the angle B :—

The three circular parts which enter the proportion, in this case, are the hypothenuse AB , the given leg AC , and the required angle B ; and since the leg AC is disjoined from the other two parts by the angle A , it is therefore the *middle part*, and the other two the *extremes disjunct*, according to rule 2, page 183; therefore, by equation 2, page 183,

Radius \times sine leg AC = sine hyp. AB \times sine of angle B .

Now, since the hypothenuse is connected with the required term, it is to stand first in the proportion. Hence,

As the hypotenuse $AB = 64^{\circ}20'45''$ Log. sine ar. comp. = 10.045071
 Is to radius = . . . 90. 0. 0 Log. sine = . . . 10.000000
 So is the leg $AC = . . . 51.10.15$ Log. sine = . . . 9.891548

 To the angle $B = . . . 59^{\circ}47'34''$ Log. sine = . . . 9.936619

Note.—The angle B is acute, because the hypotenuse and the given leg are of the same affection.

To find the leg BC :—

In this case the three circular parts which enter the proportion, are the hypotenuse and the two legs ; and since the hypotenuse is disjoined from the legs by the angles A and B , it is the *middle part*, and the other two are the *extremes disjunct* ; therefore,

Radius \times co-sine hyp. $AB =$ co-sine leg $AC \times$ co-sine leg BC .

Now, the leg AC , being connected with the required term, is, therefore, to stand first in the proportion. Hence,

As the leg $AC = . . . 51^{\circ}10'15''$ Log. co-sine ar. comp. = 10.202732
 Is to radius = . . . 90. 0. 0 Log. sine = . . . 10.000000
 So is hypotenuse $AB = 64.20.45$ Log. co-sine = . . . 9.636426

 To the leg $BC = . . . 46^{\circ}19'52''$ Log. co-sine = . . . 9.839158

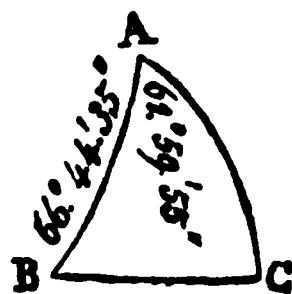
Note.—The leg BC is acute, because the hypotenuse and the given leg are of the same affection.

PROBLEM II.

Given the Hypotenuse and one Angle, to find the other Angle and the two Legs.

Example.

Let the hypotenuse AB , of the spherical triangle ABC , be $66^{\circ}44'35''$, and the angle A $61^{\circ}59'55''$; required the angle B and the legs AC and BC ?



To find the angle B :—

Here the three circular parts are connected or joined together ; therefore the hypotenuse AB is the *middle part*, and the angles A and B *extremes conjunct* (rule 1, page 183) ; therefore, by equation 1, page 183,

Radius \times co-sine hyp. $AB =$ co-tangent angle $A \times$ co-tangent angle B .

Now, the angle A , being connected with the required part, is therefore to stand first in the proportion. Hence,

As the angle $A = . \quad 61^{\circ}59'55''$ Log. co-tang. ar. comp. $= 10.274300$
 Is to radius $= . \quad 90. \quad 0. \quad 0$ Log. sine $= . \quad . \quad . \quad . \quad 10.000000$
 So is the hyp. $AB = 66.44.35$ Log. co-sine $. \quad . \quad . \quad . \quad 9.596438$

 To the angle $B = . \quad 53^{\circ}24'12''$ Log. co-tangent $= . \quad . \quad . \quad . \quad 9.870738$

Note.—The angle B is acute, because the hypotenuse and the given angle are of the same affection.

To find the leg AC :—

In this case, the three circular parts are joined together; therefore the angle A is the *middle part*, and the hypotenuse AB and required leg AC are the *extremes conjunct*; therefore,

Radius \times co-sine of angle $A =$ co-tangent $AB \times$ tangent AC .

And since the hypotenuse is connected with the required part, it is therefore to be the first term in the proportion. Hence,

As the hyp. $AB = 66^{\circ}44'35''$ Log. co-tang. ar. comp. $= 10.366756$
 Is to radius $= . \quad 90. \quad 0. \quad 0$ Log. sine $= . \quad . \quad . \quad . \quad 10.000000$
 So is the angle $A = 61.59.55$ Log. co-sine $. \quad . \quad . \quad . \quad 9.671629$

 To the leg $AC = . \quad 47^{\circ}31'42''$ Log. tangent $= . \quad . \quad . \quad . \quad 10.038385$

Note.—The leg AC is acute, because the hypotenuse and the given angle are of the same affection.

To find the leg BC :—

In this case the leg BC is the *middle part*, because it stands alone, or is disjoined from the other two circular parts concerned, by the angle B : hence the hypotenuse AB and the given angle A are *extremes disjunct*, according to rule 2, page 183; therefore, by equation 2, page 183,

Radius \times sine of leg $BC =$ sine of hyp. $AB \times$ sine of angle A .

And since radius is connected with the required part, it is to be the first term in the proportion. Hence,

As radius $= . \quad . \quad . \quad 90^{\circ} \quad 0' \quad 0''$ Log. sine ar. comp. $= 10.000000$
 Is to hypotenuse $AB = 66.44.35$ Log. sine $= . \quad . \quad . \quad . \quad 9.963194$
 So is the angle $A = . \quad 61.59.55$ Log. sine $= . \quad . \quad . \quad . \quad 9.945929$

 To the leg $BC = . \quad 54^{\circ}12'45''$ Log. sine $= . \quad . \quad . \quad . \quad 9.909123$

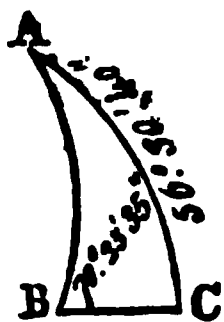
Note.—The leg BC is acute, because the hypotenuse and the given angle are of the same affection.

PROBLEM III.

Given a Leg and its opposite Angle, to find the other Angle, the other Leg, and the Hypotenuse.

Example.

Let the leg AC , of the spherical triangle ABC , be $56^\circ 30' 40''$, and the angle B $70^\circ 23' 35''$; required the angle A , the leg BC , and the hypotenuse AB ?



To find the angle A :—

Here the three circular parts which enter the proportion, are the given angle B , the given leg AC , and the required angle A ; and since the angle B is disjoined from the other two parts by the intervention of the hypotenuse AB , it is the *middle part*, and the other two are the *extremes disjunct*, according to rule 2, page 183; therefore, by equation 2, page 183,

Radius \times co-sine of the angle $B = \sin$ of the angle $A \times$ co-sine of the leg AC .

And since AC is connected with the required part, it is to be the first term in the proportion. Hence,

As the leg $AC = 56^\circ 30' 40''$	Log. co-sine ar. comp. =	10.258238
Is to radius = 90. 0. 0	Log. sine =	10.000000
So is the angle $B = 70. 23. 35$	Log. co-sine =	9.525778
<hr/>		
To the angle $A = \left\{ \begin{array}{l} 37^\circ 27' 23'' \\ 142. 32. 37 \end{array} \right\}$	Log. sine =	9.784016

Note.—The angle A is *ambiguous*, since it cannot be determined, from the parts given, whether it is acute or obtuse.

To find the leg BC :—

The three circular parts concerned in this case, are the legs AC and BC , and the given angle A ; and since the right angle *never separates the legs*, BC is the *middle part*, and AC and the angle B are the *extremes conjunct*, by rule 1, page 183; therefore, by equation 1, page 183,

Radius \times sine of the leg $BC = \tan$ leg $AC \times$ co-tangent angle B .

Now, since radius is connected with the required term, it is to stand first in the proportion. Hence,

As radius = . . . 90° 0' 0"	Log. sine ar. comp. = 10.000000
Is to the leg A C = 56.30.40	Log. tangent = . . . 10.179400
So is the angle B = 70.23.35	Log. co-tangent = . . . 9.551719

To the leg B C = $\left\{ \begin{array}{l} 32^{\circ}34'33'' \\ 147.25.27 \end{array} \right\}$ Log. sine = . . . 9.731119

Note.—The leg B C is *ambiguous*, since it cannot be determined, from the parts given, whether it is acute or obtuse.

To find the hypotenuse A B :—

Here the given leg A C is the *middle part*, because it is disjoined from the other two circular parts concerned, by the intervention of the angle A: hence the angle B and the hypotenuse A B are *extremes disjunct*; therefore,

Radius \times sine of leg A C = sine of hyp. A B \times sine of angle B.

And since the angle B is connected with the required term, it is to stand first in the proportion. Hence,

As the angle B = . . . 70°23'35"	Log. sine ar. comp. = 10.025941
Is to the leg A C = . . . 56.30.40	Log. sine = . . . 9.921162
So is radius = . . . 90. 0. 0	Log. sine = . . . 10.000000

To the hyp. A B = $\left\{ \begin{array}{l} 62^{\circ}17'30'' \\ 117.42.30 \end{array} \right\}$ Log. sine = . . . 9.947103

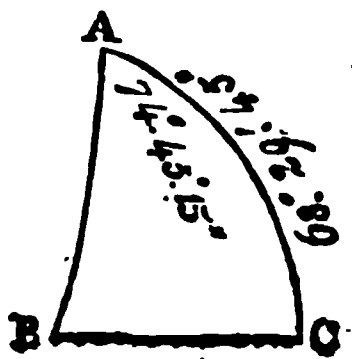
Note.—The hypotenuse A B is *ambiguous*; that is, it may be either acute or obtuse, from the parts given.

PROBLEM IV.

Given a Leg and its adjacent Angle, to find the other Angle, the other Leg, and the Hypotenuse.

Example.

Let the leg A C, of the spherical triangle A B C, be $68^{\circ}29'45''$, and the angle A $74^{\circ}45'15''$; required the angle B, the leg B C, and the hypotenuse A B?



To find the Angle B :—

Here the circular parts concerned are, the leg A C, the given angle A, and the required angle B; and since the angle B is disjoined from the other two parts by the hypotenuse A B, it is the *middle part*, and the other two are the *extremes disjunct*, by rule 2, page 183; therefore, by equation 2, page 183,

$$\text{Radius} \times \text{co-sine angle B} = \text{sine of angle A} \times \text{co-sine leg A C}.$$

Now, since radius is connected with the required term, it is to stand first in the proportion. Hence,

As radius = . . . 90° 0' 0"	Log. sine ar. comp. = 10.000000
Is to the angle A = 74.45.15	Log. sine = . . . 9.984440
So is the leg A C = 68.29.45	Log. co-sine = . . . 9.564156
To the angle B = 69°17'17"	Log. co-sine = . . . 9.548596

Note.—The angle B is acute, or of the same affection with its opposite given leg A C.

To find the Leg B C :—

In this case, since the *right angle never separates the legs*, the three circular parts are joined together: hence the leg A C is the *middle part*, and the leg B C and the angle A are the *extremes conjunct*, according to rule 1, page 183; therefore, by equation 1, page 183,

$$\text{Radius} \times \text{sine of leg A C} = \text{co-tangent angle A} \times \text{tangent of leg B C}.$$

And since the angle A is connected with the required part, it is to be the first term in the proportion. Hence,

As the angle A = 74°45'15"	Log. co-tang. ar. comp. = 10.564549
Is to radius = . . . 90. 0. 0	Log. sine = . . . 10.000000
So is the leg A C = 68.29.45	Log. sine = . . . 9.968666
To the leg B C = 73°40'20½"	Log. tangent = . . . 10.533215

Note.—The leg B C is acute, or of the same affection with its opposite given angle A.

To find the Hypotenuse A B :—

In this case, since the three circular parts which enter the proportion are joined together, the given angle A is the *middle part*, and the leg A C and the hypotenuse A B are the *extremes conjunct*: therefore,

$$\text{Radius} \times \text{co-sine of angle A} = \text{tangent of leg A C} \times \text{co-tangent hypothenuse A B}.$$

Now, the leg AC , being connected with the required part, is therefore to be the first term in the proportion. Hence,

As the leg $AC =$	$68^{\circ}29'45''$	Log. tang. ar. comp. =	9.595490
Is to radius =	$90. 0. 0$	Log. sine =	10.000000
So is the angle $A =$	$74. 45. 15$	Log. co-sine =	9.419891

To the hypotenuse $AC = 84^{\circ} 5' 6''$ Log. co-tangent = 9.015381

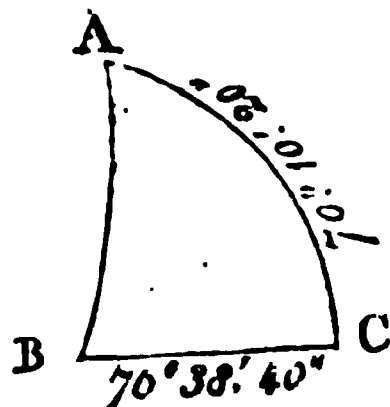
Note.—The hypotenuse is acute, because the given leg and angle are of the same affection.

PROBLEM V.

Given the two Legs, to find the Angles and the Hypotenuse.

Example.

Let the leg AC , of the spherical triangle ABC , be $70^{\circ}10'20''$, and the leg BC $76^{\circ}38'40''$; required the angles A and B , and the hypotenuse AB ?



To find the Angle A :—

Here, since the right angle *never separates the legs*, the leg AC is the *middle part*, and the leg BC and the required angle A are the *extremes conjunct*, agreeably to rule 1, page 183; therefore, by equation 1, page 183,

Radius \times sine leg $AC =$ tangent leg $BC \times$ co-tangent angle A .

Now, since the leg BC is connected with the required part, it is to be the first term in the proportion. Hence,

As the leg $BC =$	$76^{\circ}38'40''$	Log. tangent ar. comp. =	9.375506
Is to radius =	$90. 0. 0$	Log. sine =	10.000000
So is the leg $AC =$	$70. 10. 20$	Log. sine =	9.973459

To the angle $A = 77^{\circ}24'37''$ Log. co-tangent = 9.348965

Note.—The angle A is acute, or of the same affection with its opposite given leg BC .

To find the Angle B :—

In this case the leg BC is the *middle part*, and the leg AC and the

required angle B are the *extremes conjunct*, according to rule 1, page 183; therefore, by equation 1, page 183,

Radius \times sine of the leg B C = tangent of leg A C \times co-tangent angle B.

And, since the leg A C is connected with the required part, it is to be the first term in the proportion. Hence,

As the leg A C =	70° 10' 20"	Log. tangent ar. comp. =	9.556990
Is to radius =	. . 90. 0. 0	Log. sine = 10.000000
So is the leg B C =	76.38.40	Log. sine = 9.988093

To the angle B =	70° 40' 5½"	Log. co-tangent =	. . 9.545083
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Note.—The angle B is acute, or of the same affection with its opposite given leg A C.

To find the Hypothenuse A B:—

Here the hypothenuse A B is the *middle part*, because it is disjoined from the legs by the angles A and B: hence A C and B C are *extremes disjunct*, agreeably to rule 2, page 183; therefore, by equation 2, page 183,

Radius \times co-sine hypothenuse A B = co-sine leg A C \times co-sine leg B C.

And radius, being connected with the middle part, is therefore to be the first term in the proportion. Hence,

As radius =	. . 90° 0' 0"	Log. sine ar. comp. =	. . 10.000000
Is to the leg A C =	70. 10. 20	Log. co-sine =	. . . 9.530448
So is the leg B C =	76.38.40	Log. co-sine =	. . . 9.363599
To the hyp. A B =	85° 30' 22"	Log. co-sine =	. . . 8.894047

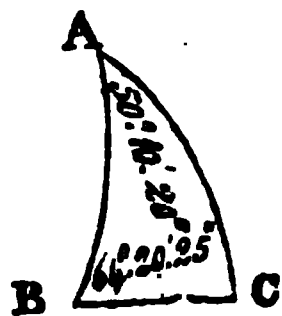
Note.—The hypothenuse A B is acute, because the given legs A C and B C are of the same affection.

PROBLEM VI.

Given the two Angles, to find the Hypothenuse and the two Legs.

Example.

Let the angle A, of the spherical triangle A B C, be 50° 10' 20", and the angle B 64° 20' 25"; required the legs A C and B C, and the hypothenuse A B?



To find the Hypothenuse A B :—

Here, because the three circular parts are joined together, the hypothenuse A B is the *middle part*, and the angles A and B are the *extremes conjunct*, agreeably to rule 1, page 183 ; therefore, by equation 1, page 183,

Radius \times co-sine hypothenuse A B = co-tangent angle A \times co-tangent angle B.

Now, since radius is connected with the required part, it is to be the first term in the proportion. Hence,

As radius =	90° 0' 0"	Log. sine ar. comp. =	10.000000
Is to the angle A =	50. 10. 20	Log. co-tangent =	9.921161
So is the angle B =	64. 20. 25	Log. co-tangent =	9.681605
To the hyp. A B =	66° 22' 52"	Log. co-sine =	9.602766

Note.—The hypothenuse A B is acute, because the given angles A and C are of the same affection.

To find the leg A C :—

Here, since the angle B is disjoined by the hypothenuse A B from the other two circular parts concerned, it is the *middle part*, and the angle A and the required leg A C are the *extremes disjunct*, agreeably to rule 2, page 183 ; therefore, by equation 2, page 183,

Radius \times co-sine angle B = sine of angle A \times co-sine of leg A C.

And because the angle A is connected with the required part, it is to stand first in the proportion. Hence,

As the angle A =	50° 10' 20"	Log. sine ar. comp. =	10.114654
Is to radius =	90. 0. 0	Log. sine =	10.000000
So is the angle B =	64. 20. 25	Log. co-sine =	9.636514
To the leg A C =	55° 40' 38"	Log. co-sine =	9.751168

Note.—The leg A C is acute, or of the same affection with its opposite given angle B.

To find the Leg B C :—

In this case the angle A is the *middle part*, because it is disjoined from the other two circular parts by the hypothenuse A B : hence the angle B and the required leg B C are *extremes disjunct* ; therefore,

Radius \times co-sine of angle A = sine of angle B \times co-sine of leg B C.

And as the angle B is connected with the required part, it is to be the first term in the proportion. Hence,

As the angle B =	64°20'45"	Log. sine ar. comp. =	. 10.045091
Is to radius =	. 90. 0. 0	Log. sine = 10.000000
So is the angle A =	50. 10. 20	Log. co-sine	9.806507
<hr/>			
To the leg B C =	44°43'11½"	Log. co-sine = 9.851598

Note.—The leg B C is acute, or of the same affection with its opposite given angle A.

SOLUTION OF QUADRANTAL SPHERICAL TRIANGLES, BY LOGARITHMS.

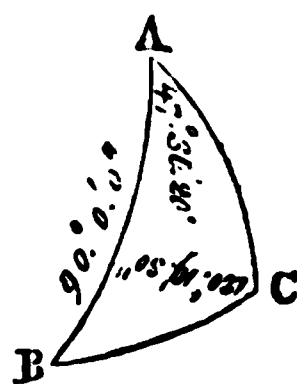
PROBLEM I.

Given a Quadrantal Side, its opposite Angle, and an adjacent Angle, to find the remaining Angle and the other two Sides.

Remark.—Since the sides of a spherical triangle may be turned into angles, and, *vice versa*, the angles into sides, all the cases of quadrantal spherical triangles may be resolved agreeably to the principles of right-angled spherical triangles; as thus: let the quadrantal side be esteemed the radius; the *supplement of the angle* subtending that side, the hypotenuse; and the other angles legs, or the legs angles, as the case may be. Then the middle part, and the extremes conjunct or disjunct, being established, the required parts are to be computed, and the affections of the angles and sides determined, in the same manner precisely as if it were a right-angled spherical triangle that was under consideration.

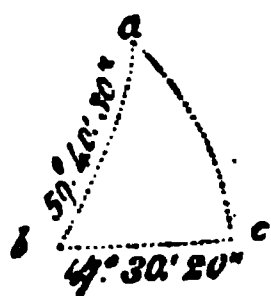
Example.

Let A B, in the spherical triangle A B C, be the quadrantal side = 90°, the angle C 120°19'30", and the angle A 47°30'20"; required the sides A C and B C, and the angle B?



Solution.—Let the supplement of the angle C (59°40'30"), subtending the quadrantal side A B, represent the hypotenuse *a b* of the dotted spherical triangle *a b c*. Let the given angle A 47°30'20" represent the leg *b c* of the said dotted triangle, and the required angle B the leg *a c*.

Then, in the right-angled spherical triangle abc , given the hypotenuse ab $59^{\circ}40'30''$, and the leg bc $47^{\circ}30'20''$, to find the leg ac = the angle B in the quadrantal triangle; the angle a = the leg BC , and the angle b = the leg AC , of the said quadrantal triangle.



To find the Leg ac = the Angle B in the Quadrantal Triangle :—

Here the hypotenuse ab is the *middle part*, and the legs bc and ac are the *extremes disjunct*; therefore,

$$\text{Radius} \times \text{co-sine hyp. } ab = \text{co-sine leg } bc \times \text{co-sine leg } ac.$$

Now, since the leg bc is connected with the required part, it is to be the first term in the proportion. Hence,

As the leg bc =	$47^{\circ}30'20''$	Log. co-sine ar. comp. =	10.170363
Is to radius =	. 90. 0. 0	Log. sine =	10.000000
So is the hyp. ab =	$59.40.30$	Log. co-sine =	9.703209
To the leg ac =	$41^{\circ}37'54''$	Log. co-sine =	9.873572

Note.—The leg ac is acute, because the hypotenuse and the given leg are of the same affection: hence the angle B (in the quadrantal triangle), represented by the leg ac , is also acute = $41^{\circ}37'54''$.

To find the Angle a = the Leg BC in the Quadrantal Triangle :—

Here the leg bc is the *middle part*, and the hypotenuse ab and angle a are the *extremes disjunct*; therefore,

Radius \times sine of leg bc = sine of hypotenuse $ab \times$ sine of the angle a .

And since the hypotenuse is connected with the required part, it is to be the first term in the proportion. Hence,

As the hyp. ab =	. $59^{\circ}40'30''$	Log. sine ar. comp. =	10.063901
Is to radius =	. . 90. 0. 0	Log. sine =	10.000000
So is the leg bc =	. $47.30.20$	Log. sine =	9.867670
To the angle a =	. $58^{\circ}40'26''$	Log. sine =	9.931571

Note.—The angle a is acute, because the hypotenuse and the given leg are of the same affection: hence the leg BC (of the quadrantal triangle), represented by the angle a , is also acute = $58^{\circ}40'26''$.

To find the angle b = the Leg A C in the Quadrantal Triangle:—

In this case the angle b is the *middle part*, and the hypotenuse $a b$ and the leg $b c$ are the *extremes conjunct*; therefore,

Radius \times co-sine of the angle b = co-tangent hypotenuse $a b \times$ tangent of leg $b c$.

And radius, being connected with the required part, is, therefore, to stand first in the proportion. Hence,

As radius =	90° 0' 0"	Log. sine ar. comp. =	10.000000
Is to the hyp. $a b$ =	59.40.30	Log. co-tangent =	9.767110
So is the leg $b c$ =	47.30.20	Log. tangent =	10.038032
				9.805142
To the angle b =	50° 19' 19"	Log. co-sine =

Note.—The angle b is acute, because the hypotenuse and the given leg are of the same affection. Hence, the leg A C (of the quadrantal triangle), represented by the angle b , is also acute = 50° 19' 19".

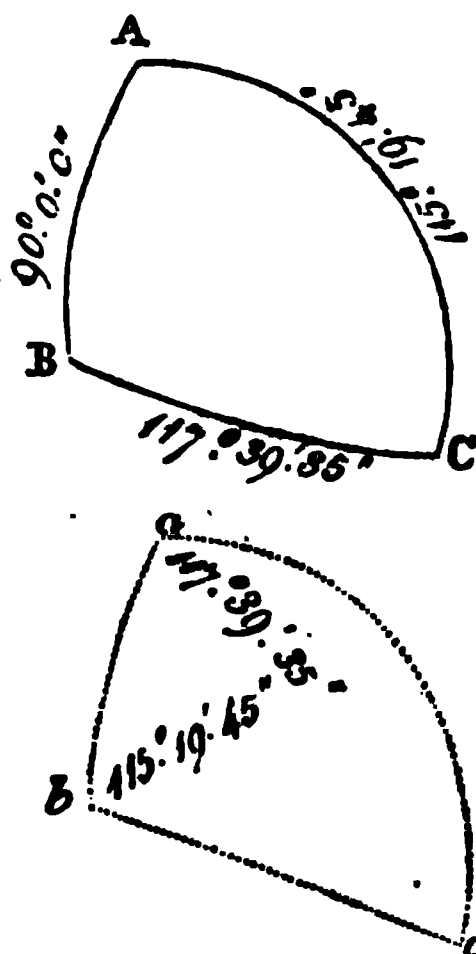
PROBLEM II.

Given the Quadrantal Side and the other two Sides, to find the three Angles.

Example.

Let A B, in the spherical triangle A B C, be the quadrantal side = 90°; the side A C, 115° 19' 45"; and the side B C, 117° 39' 35"; required the angles A, B, and C?

Solution.—Let the angle c , in the dotted spherical triangle $a b c$, be radius, and represent the side A B = 90° of the quadrantal triangle A B C. Let the angle a , of the dotted triangle, represent the side B C of the quadrantal triangle = 117° 39' 35", and let the angle b represent the side A C of the said quadrantal triangle = 115° 19' 45". Then, in the right-angled spherical triangle $a b c$, right-angled at c , given the angle a = 117° 39' 35", and the angle b = 115° 19' 45", to find the hypotenuse $a b$, the leg $a c$, and the leg $b c$; the first of which represents the supplement of the angle



C opposite to the quadrantal side AB , in the triangle ABC ; the second represents the angle B ; and the third the angle A , in the said quadrantal triangle.

To find the Hypothenuse ab = the Supplement of the Angle C , subtending the Quadrantal Side AB :—

Here the hypothenuse ab is the *middle part*, and the given angles a and b are the *extremes conjunct*; therefore,

Radius \times co-sine hypothenuse ab = co-tangent of angle $a \times$ co-tangent of angle b .—Now, since radius is connected with the required part, it is to be the first term in the proportion.—Hence,

As radius = . . .	$90^\circ 0' 0''$	Log. sine ar. compt. =	10.000000
Is to the angle a =	$117.39.35$	Log. co-tangent =	9.719427
So is the angle b =	$115.19.45$	Log. co-tangent =	: 9.675156

To the hypo. ab =	$75^\circ 38' 11''$	Log. co-sine = . . .	9.394583
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Note.—The hypothenuse ab is acute because the given angles are of the same affection :—but since it only represents the supplement of the angle C ; therefore the angle C is obtuse, or $104^\circ 21' 49''$.

To find the Leg ac = the Angle B in the Quadrantal Triangle.

The angle b , in this case, is the *middle part*, and the angle a and leg ac *extremes disjunct*.—Therefore, radius \times co-sine of angle b = sine of angle $a \times$ co-sine of leg ac .

And the angle a being connected with the required part, is, therefore, to be the first term in the proportion.—Hence,

As the angle a =	$117^\circ 19' 35''$	Log. sine ar. compt. =	10.052703
Is to radius = . . .	$90. 0. 0$	Log. sine = . . .	10.000000
So is the angle b =	$115.19.45$	Log. co-sine = . . .	9.631259

To the side ac =	$118^\circ 52' 57''$	Log. co-sine = . . .	9.683962
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Note.—The side ac is obtuse, or of the same affection with its opposite angle b :—and since ac represents the angle B ; therefore the angle B , in the quadrantal triangle, is obtuse, or $118^\circ 52' 57''$.

To find the Leg bc = the Angle A in the Quadrantal Triangle.

In this case the angle a is the *middle part*, and the angle b and leg bc *extremes disjunct*.—Therefore, radius \times co-sine of the angle a = sine of the angle $b \times$ co-sine of the leg bc .

And since the angle b is connected with the required part, it is to be the first term in the proportion.—Hence,

As the angle $b = 115^{\circ}19'45''$ Log. sine ar. compt. $= 10.043896$

Is to radius $= . . 90. 0. 0$ Log. sine $= . . . 10.000000$

So is the angle $a = 117.39.35$ Log. co-sine $= . . . 9.666723$

To the leg $bc = 120^{\circ}54'12''$ Log. co-sine $= . . . 9.710619$

Note.—The leg bc is obtuse, or of the same affection with its opposite angle a :—and since the leg bc represents the angle A , in the quadrantal triangle; therefore the angle A is obtuse, or $120^{\circ}54'12''$.

Remark.—From the ample solutions of the two preceding Problems, it must appear obvious, that all the cases of quadrantal spherical triangles may be easily resolved by the equations for right-angled spherical triangles. And if the analogies of those two Problems be well understood, all the *apparent* difficulty attending the trigonometrical solution of quadrantal triangles will entirely vanish.

SOLUTION OF OBLIQUE-ANGLED SPHERICAL TRIANGLES BY LOGARITHMS.

The most natural, and, perhaps, the easiest method of solving the *four first Problems*, or cases of oblique-angled spherical triangles, is by means of a perpendicular let fall from an angle to its opposite side, continued if necessary; and thus reducing the oblique into two right-angled spherical triangles.—The perpendicular, however, should be let fall in such a manner that *two of the given parts* in the oblique triangle may remain known in one of the right-angled triangles:—Then, the other parts may be readily computed by means of Lord Napier's analogies, as given in the equations 1 and 2, page 183.—But, since the solution of oblique-angled spherical triangles without a perpendicular is possessed of many advantages in astronomical calculations; and, besides, since the author's object is to establish the use of the Tables contained in this work by a variety of rules and formulæ which, it is hoped, may not be found quite uninteresting to persons but slightly informed on trigonometrical subjects; the different cases of oblique triangles will, therefore, be resolved independently of a perpendicular, agreeably to the propositions generally used in such cases.

PROBLEM I.

Given Two Sides of an Oblique-angled Spherical Triangle, and an Angle opposite to one of them ; to find the remaining Angles and the Third Side.

RULE.

1.—To find an angle opposite to one of the given sides.

As the log. sine of the side opposite to the given angle, is to the log. sine of the given angle ; so is the log. sine of the other given side, to the log. sine of its opposite angle.

Now, to know whether the angle thus found is determinate ; that is, whether it is ambiguous, acute, or obtuse, proceed in the following manner, viz.—To the angle so found, and its supplement, add the given angle, or that used in the proportion.—Then, if each of these sums be of the *same affection* with respect to 180° as the *sum of the two given sides*, or those used in the proportion, the angle is *ambiguous* ; that is, it may be either acute or obtuse ; and, therefore, indeterminate.—But, if those sums are of *different affections with respect to the sum of the sides*, the angle is determinate, and, therefore, *not* ambiguous :—In this case that value of the angle is to be taken, whether acute or obtuse, which, when added to the given angle, produces a quantity of the same affection with the sum of the two sides.

2.—To find the angle contained between the two given sides.

Find half the difference, and half the sum of the two given sides :—find, also, half the difference of their opposite angles. Then say,

As the log. sine of half the difference of the sides, is to the log. sine of half their sum ; so is the log. tangent of half the difference of their opposite angles, to the log. co-tangent of half the angle contained between the two given sides ; the double of which will be the angle sought.

3.—To find the third side.

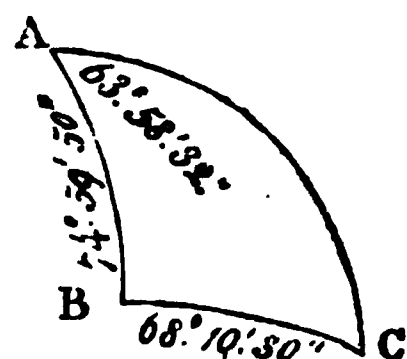
Since the sides are proportional to the sines of their opposite angles ; therefore the third side may be found by the converse of the first part of the rule ; as thus :

As the log. sine of a given angle opposite to a given side, is to the log. sine of that side ; so is the log. sine of the given angle opposite to the required side, to the log. sine of the required side.

Note.—When the angle comes out ambiguous, or indeterminate, in the first proportion ; the contained angle and the third side, found by the other proportions, will also be ambiguous.

Example.

In the oblique-angled spherical triangle ABC , let the side AB be $74^{\circ}59'50''$, the side BC $68^{\circ}10'30''$ and the angle A $63^{\circ}58'32''$; required the angles B and C , and the side AC ?



To find the Angle C :—

As the side $BC = 68^{\circ}10'30''$ Log. sine ar. compt. = 10.032301
 Is to the angle $A = 63.58.32$ Log. sine . . . = 9.953570
 So is the side $AB = 74.59.50$ Log. sine . . . = 9.984938

To the angle $C = 69^{\circ}13'37''$ Log. sine . . . = 9.970809

To determine whether the Angle C is Ambiguous, Acute, or Obtuse :—

Angle $C = 69^{\circ}13'37''$	Sup. = $110^{\circ}46'23''$	Side $BC = 68^{\circ}10'30''$
Angle $A = 63.58.32$	Angle $A = 63.58.32$	Side $AB = 74.59.50$
Sum = $133^{\circ}12'9''$	Sum = $174^{\circ}44'55''$	Sum = $143^{\circ}10'20''$

Here, since the three sums are of the same affection with respect to 180° the angle C is ambiguous; therefore it may be either $69^{\circ}13'37''$ or the supplement thereof; viz., $110^{\circ}46'23''$.

To find the Angle B :—

As the side AB —the side $BC+2=3^{\circ}24'40''$ Log. S. ar. compt. 11.225483
 Is to the S. AB +the S. $BC+2=71.35.10$ Log. sine = . . 9.977174
 So is the ang. C —the ang. $A+2=2.37.32\frac{1}{2}$ Log. tangent = 8.661426

To half the angle $B = . . . 53^{\circ}49'22''$ Log. co-tangent 9.864083

Angle $B = . . . 107.38.44$; which is ambiguous because the angle C came out indeterminate.

To find the Side AC :—

As the angle $A = . . 63^{\circ}58'32''$ Log. sine ar. compt. 10.046430
 Is to the side $BC = . 68.10.30$ Log. sine = . . . 9.967699
 So is the angle $B = . 107.38.44$ Log. sine = . . . 9.979070

To the side $AC = . 100^{\circ}6'47''$ Log. sine = . . . 9.993199

The side AC is also ambiguous because the angle C came out indeterminate.

PROBLEM II.

Given Two Angles of an Oblique Angled Spherical Triangle, and a Side opposite to one of them ; to find the remaining Angle and the other Two Sides.

RULE.

1.—*To find a side opposite to one of the given angles.*

As the log. sine of the angle opposite to the given side, is to the log. sine of the given side : so is the log. sine of the other given angle, to the log. sine of its opposite side.

Now, to know whether the side thus found is ambiguous, acute, or obtuse, proceed as follows ; viz.,

To the side so found, and its supplement, add the given side, or that used in the proportion.—Then, if each of these sums be of the *same affection* with respect to 180° as the *sum of the two given angles*, or those used in the proportion, the side is *ambiguous* ; that is, it may be either acute, or obtuse ; and, therefore, indeterminate.

But, if those sums are of *different affections* with respect to the sum of the angles, the side is *not* ambiguous : in this case that value of the side is to be taken, whether acute or obtuse, which, when added to the given side, produces a quantity of the same affection with the sum of the angles.

2.—*To find the side contained between the two given angles.*

Find half the difference, and half the sum of the two given angles :—find, also, half the difference of their opposite sides.—Then say,

As the log. sine of half the difference of the angles, is to the log. sine of half their sum ; so is the log. tangent of half the difference of their opposite sides, to the log. tangent of half the side contained between the two given angles ; the double of which will be the side sought.

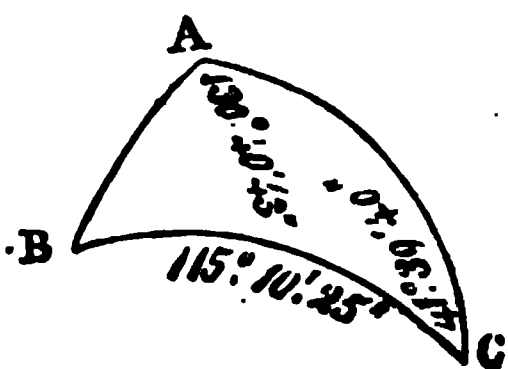
3.—*To find the third, or remaining angle.*

As the log. sine of a given side opposite to a given angle, is to the log. sine of that angle ; so is the log. sine of the side opposite to the required angle, to the log. sine of the required angle.

Note.—When the side comes out ambiguous, or indeterminate, in the first proportion ; the contained side and the third angle, found by the other proportions, will also be ambiguous,

Example.

Let the angle A, of the spherical triangle A B C, be $130^{\circ}40'43''$, the angle C $41^{\circ}39'40''$, and the side B C $115^{\circ}10'25''$; required the angle B, and the sides A B and A C?



To find the Side A B :—

As the angle A = $130^{\circ}40'43''$ Log. sine ar. compt. = 10.120114

Is to the side B C = $115.10.25$ Log. sine = 9.956660

So is the angle C = $41.39.40$ Log. sine = 9.822641

To the side A B = $52^{\circ}29'28''$ Log. sine = 9.899415

To determine whether the Side A B is Ambiguous, Acute, or Obtuse :—

Side A B $52^{\circ}29'28''$ Supplement = $127^{\circ}30'32''$ Angle A $130^{\circ}40'43''$

Side B C $115.10.25$ Side B C = $115.10.25$ Angle C $41.39.40$

Sum = $167^{\circ}39'53''$ Sum = . . $242^{\circ}50'57''$ Sum = $172^{\circ}20'23''$

Here, since the two first sums, viz. A B and B C, and the supplement of A B and B C, are of different affections with respect to 180° , the side A B is not ambiguous ;—and since the sum of the acute value of A B added to B C is of the same affection with the sum of the angles ; therefore the side A B is acute = $52^{\circ}29'28''$.

To find the Side A C :—

As the ang. A—the ang. C+2= $44^{\circ}30'31\frac{1}{2}''$ Log. S. ar. compt. 10.154271

Is to angle A + angle C + 2= $86.10.11\frac{1}{2}$ Log. sine = . . 9.999029

So is the S. B C—S. A B+2= $31.10.28\frac{1}{2}$ Log. tangent = . 9.784614

To half the side A C = . . $40^{\circ}55'6''$ Log. tangent . 9.937914

Side A C = $81^{\circ}50'12''$; which is acute, because the side A B came out determinate, and that its acute value applied to B C is of the same affection with the sum of the angles.

To find the Angle B :—

As the side B C = . . $115^{\circ}10'25''$ Log. S. ar. compt. . . 10.043340

Is to the angle A = . $130.40.43$ Log. sine = 9.879886

So is the side A C = . $81.50.12$ Log. sine = 9.995577

To the angle B = . . $56^{\circ}2'41\frac{1}{2}''$ Log. sine = 9.918803

Note.—The angle B is acute like its opposite side A C, because the side A B is not ambiguous ; and that its acute value applied to the side B C is of the same affection with the sum of the angles.

PROBLEM III.

Given Two Sides of an Oblique-angled Spherical Triangle, and the Angle contained between them ; to find the other Two Angles and the Third Side.

RULE.

1.—*To find the other two angles.*

As the log. co-sine of half the sum of the two given sides, is to the log. co-sine of half their difference ; so is the log. co-tangent of half the contained angle, to the log. tangent of half the sum of the other two angles.

Half the sum of the angles thus found, will be of the same affection with half the sum of the sides.—Again : As the log. sine of half the sum of the two given sides, is to the log. sine of half their difference ; so is the log. co-tangent of half the contained angle, to the log. tangent of half the difference of the other two angles.—Half the difference of the angles, thus found, will always be acute.

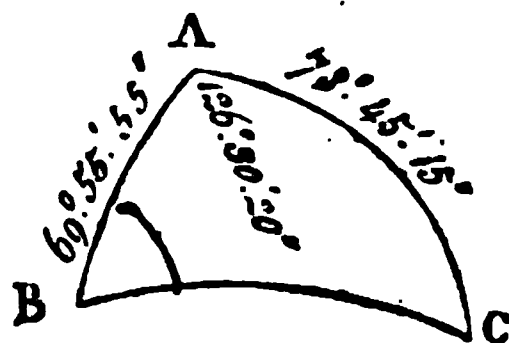
Now, half the sum of the two angles, added to half their difference, will give the greater angle ; and half the difference of the angles subtracted from half their sum will leave the lesser angle.

2.—*To find the third side.*

The angles being known, the third or remaining side is to be computed by Rule 3, Problem I., page 198.

Example.

Let the side A C, of the spherical triangle A B C, be $78^{\circ}45'15''$, the side A B $69^{\circ}55'55''$, and the contained angle $126^{\circ}30'20''$; required the angles B and C ; and the side B C ?



To find the Angle B :—

As the side $AC + AB + 2 = 74^\circ 20' 35''$ Log. co-sine ar. comp. = 10.568834
 Is to the side $AC - AB + 2 = 4.24.40$ Log. co-sine = . . . 9.998712
 So is the angle $A \div 2 = 63.15.10$ Log. co-tang. = . . . 9.702414

To $\frac{1}{2}$ the sum of the an. = $61^\circ 45' 38''$ Log. tangent = . . . 10.269960
 Half diff. of the angles = $2.18.19$, as below

Sum = . . . $64^\circ 3' 57''$ = Angle B.

To find the Angle C :—

As the side $AC + AB + 2 = 74^\circ 20' 35''$ Log. sine ar. compt. = 10.016421
 Is to the side $AC - AB + 2 = 4.24.40$ Log. sine = . . . 8.885996
 So is the angle $A + 2 = 63.15.10$ Log. co-tangent = . . . 9.702414

To half the diff. of the ang. = $2^\circ 18' 19''$ Log. tangent = . . . 8.604831
 Half sum of the angles = $61.45.38$, as above

Difference = . . . $59^\circ 27' 19''$ = Angle C

Note.—The half sum of the angles came out acute, because the half sum of the sides is acute : the half difference of the angles is *always acute*.

To find the Side B C :—

As the angle B = $64^\circ 3' 57''$ Log. sine ar. compt. = 10.046097
 Is to the side A C = $78.45.15$ Log. sine = . . . 9.991580
 So is the angle A = $126.30.20$ Log. sine = . . . 9.905148

To the side B C = $118^\circ 45' 34''$ Log. sine = . . . 9.942825

Remark 1.—The side B C may be found directly, independently of the angles B and C, by the following general Rule.

To twice the log. sine of half the contained angle, add the log. sines of the two containing sides ; from half the sum of these three logs. subtract the log. sine of half the difference of the sides, and the remainder will be the log. tangent of an arch : the log. sine of which being subtracted from the half sum of the three logs. will leave the log. sine of half the required side.

Example.

Let the side A C, of a spherical triangle, be $62^\circ 10' 25''$, the side A B $50^\circ 14' 45''$, and the included angle A $123^\circ 11' 40''$; required the side B C ?

$$\text{Half ang. A} = 61^\circ 35' 50'' \left\{ \begin{smallmatrix} \text{Twice the} \\ \text{log. sine} \end{smallmatrix} \right\} = 19.888596$$

$$\text{Side A C} = 62.10.25 \quad \text{Log. sine} = 9.946632$$

$$\text{Side A B} = 50.14.45 \quad \text{Log. sine} = 9.885811$$

$$\text{Sum} = 39.721039$$

$$\text{Diff. of Sides } 11^\circ 55' 40'' \quad \text{Half} = 19.860519\frac{1}{2} \quad . \quad . \quad 19.860519\frac{1}{2}$$

$$\text{Half ditto} = 5^\circ 57' 50'' \quad \text{Log. sine} = 9.016622$$

$$\text{Arch} = . \quad . \quad 81^\circ 50' 52'' \quad \text{Log. tang.} = 10.843897\frac{1}{2} \quad \text{Log. S.} = 9.995588\frac{1}{2}$$

$$\frac{1}{2} \text{ Side B C} = 47^\circ 6' 50'' = \text{Log. sine} = . \quad . \quad . \quad . \quad . \quad 9.864932$$

$$\text{Side B C} = 94^\circ 13' 40'', \text{ as required.}$$

Remark 2.—The side B C may be also computed by the following general rule, viz.

To twice the log. sine of half the contained angle, add the log. sines of the two containing sides, and the constant logarithm 6.301030; the sum (rejecting 40 from the index), will be the log. of a natural number:—which being added to the natural versed sine of the difference of the containing sides, the result will be the natural versed sine of the third side.

Thus, to find the side B C in the above example.

$$\text{Half included ang. A} = 61^\circ 35' 50'' \text{ twice the log. sine} = . \quad . \quad 19.888596$$

$$\text{Side A C} = . \quad . \quad . \quad 62.10.25 \quad \text{Log. sine} = . \quad . \quad . \quad . \quad . \quad 9.946632$$

$$\text{Side A B} = . \quad . \quad . \quad 50.14.45 \quad \text{Log. sine} = . \quad . \quad . \quad . \quad . \quad 9.885811$$

$$\text{Constant Logarithm} = . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 6.301030$$

$$\text{Diff. of the sides } 11^\circ 55' 40'' \text{ N.V.S.} = 021591$$

$$\text{Natural number} = . \quad . \quad . \quad . \quad . \quad . \quad 1052130 = \quad \text{Log. } 6.022069$$

$$\text{Side B C} = . \quad 94^\circ 13' 40'' \text{ N.V.S.} = 1.073721; \text{ the same as above.}$$

Note.—This formula will be found exceedingly useful on many Astronomical occasions.—And, when the index of the sum of the four Logarithms is 6, as in the above *example*, the natural number corresponding thereto may be very readily found in the second part of Table XXVII., between pages 153 and 166. *Vol II.*

PROBLEM IV.

Given Two Angles of a Spherical Triangle, and the Side comprehended between them ; to find the remaining Angle and the other Two Sides.

RULE.

1.—To find the other two sides.

As the log. co-sine of half the sum of the two given angles, is to the log. co-sine of half their difference ; so is the log. tangent of half the comprehended side, to the log. tangent of half the sum of the other two sides.

Half the sum of the sides, thus found, will be of the same affection with the half sum of the angles.

Again.—As the log. sine of half the sum of the two given angles, is to the log. sine of half their difference ; so is the log. tangent of half the comprehended side, to the log. tangent of half the difference of the other two sides.

Half the difference of the angles, thus found, will always be acute.

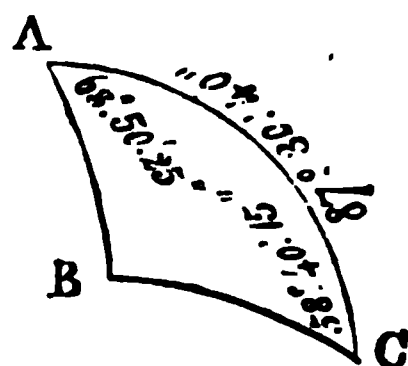
Now, half the sum of the two sides, added to half their difference, will give the greater side ; and half the difference of the two sides, subtracted from half their sum, will leave the lesser side.

2.—To find the remaining angle.

The sides and two angles being known, the remaining or third angle is to be computed by Rule 3, Problem II., page 200.

Example.

Let the angle A, of the spherical triangle A B C, be $63^{\circ}50'25''$; the angle C $58^{\circ}40'15''$, and the comprehended side A C $87^{\circ}30'40''$; required the sides A B and B C, and the remaining angle B ?



To find the Side B C :—

As the angle $A + \text{angle } C + 2 = 61^{\circ}15'20''$ L. co-sine ar.com. = 10.317942
 Is to the ang. $A - \text{angle } C + 2 = 2.35.5$ Log co-sine = . . 9.999557
 So is the side $A C \div 2 = 43.45.20$ Log. tangent = . . 9.981129

To half the sum of the sides = $63^{\circ}18'28''$ Log. tangent = . 10.298628
 Half difference of the sides = $2.49.10$, as in the next operation.

Sum = $66^{\circ}7'38''$ = the side B C.

Remark 2.—The angle B may be also very readily computed by the following general Rule ; viz.,

To twice the log. co-sine of half the given side, comprehended between the two given angles, add the log. sines of those angles, and the sum (rejecting 30 from the index), will be the log. of a natural number.—Now, the sum of twice this natural number and the natural versed sine of the difference of the angles, will be the natural versed sine of the required angle.

Thus, to find the angle B in the last example.

Half the given side A C = $43^{\circ}45'20''$ twice the log. co-sine = 19.717432
 Angle A = $69.50.25$ Log. sine = 9.953068
 Angle C = $58.40.15$ Log. sine = 9.931557

Natural number = 399998 = Log. 9.602057

Twice the natural number = . 799996

Diff. of the ang. = $5^{\circ}10'10''$ nat. versed sine = 004067

Angle B = $78^{\circ}42'2''$ nat. versed sine = 804063; the same as by the former Rule.

PROBLEM V.

Given the Three Sides of a Spherical Triangle, to find the Angles.

RULE.

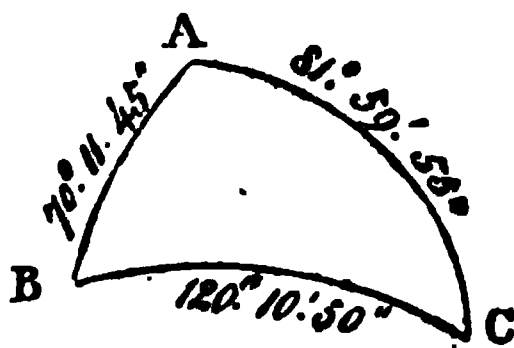
Add the three sides together and take half their sum ; find the difference between this half sum and the side opposite to the required angle, which call the remainder ; then,

To the log. co-secants, less radius, of the other two sides, add the log. sines of the half sum and the remainder :—half the sum of these four logs. will be the log. co-sine of an arch, which being doubled will be the required angle.

One angle being thus found, the remaining angles may be computed by Rule 3, Problem II., page 200.

Example.

In the spherical triangle A B C, let the side A B be $70^{\circ}11'45''$, the side A C $81^{\circ}59'55''$, and the side B C $120^{\circ}10'50''$; required the angles A, B, and C ?



To find the Angle A :—

Side B C =	. . .	120° 10' 50"	
Side A C =	. . .	81. 59. 55	Log. co-secant, less radius=0. 004248
Side A B =	. . .	70. 11. 45	Log. co-secant, less radius=0. 026477
Sum	. .	272. 22. 30	
Half sum =	136. 11. 15	Log. sine = 9. 840295
Remainder=	16. 0. 25	Log. sine = 9. 440522
		Sum = 19. 311542
Arch =	. : 63° 5' 8"	= Log. co-sine = 9. 655771
Angle A =	126° 10' 16"		

To find the Angle B :—

As the side B C =	. . .	120° 10' 50"	Log. co-secant =	10. 063262
Is to the angle A =	. .	126. 10. 16	Log. sine =	. . . 9. 907012
So is the side A C =	. .	81. 59. 55	Log. sine =	. . . 9. 995752
To the angle B =	. . .	67° 37' 52"	Log. sine =	. . . 9. 966026

To find the Angle C .—

As the side B C =	. . .	120° 10' 50"	Log. co-secant =	10. 063262
Is to the angle A =	. .	126. 10. 16	Log. sine =	. . . 9. 907012
So is the side A B =	. .	70. 11. 45	Log. sine =	. . . 9. 973523
To the angle C =	. . .	61° 28' 31"	Log. sine =	. . . 9. 943797

Remark.—The required angle of a spherical triangle (when the three sides are given), may be also found by the following general Rule ; viz.,

Add the three sides together and take half their sum : find the difference between this half sum and each of the sides containing the required angle, and note the remainders.—Then,

To the log. co-secants, less radius, of those sides, add the log. sines of the two remainders :—half the sum of these four logs. will be the log. sine of half the required angle.

Thus, to find the angle A in the last example.

Side B C = . . .	120° 10' 50"	
Side A C = . . .	81. 59. 55	Log. co-secant, less radius=0. 004248
Side A B = . . .	70. 11. 45	Log. co-secant, less radius=0. 026477
Sum =	272. 22. 30	
Half sum	136° 11' 15"	
Remainder, by A C =	54. 11. 20	Log. sine = 9. 908994
Remainder, by A B =	65. 59. 30	Log. sine = 9. 960702
		Sum = . . . 19. 900421
Half the angle A =	63° 5' 8"	Log. sine = 9. 950210½

Which being doubled, shows the angle A to be 126° 10' 16"; the same as by the former rule.

PROBLEM VI.

Given the Three Angles of a Spherical Triangle, to find the Sides.

RULE.

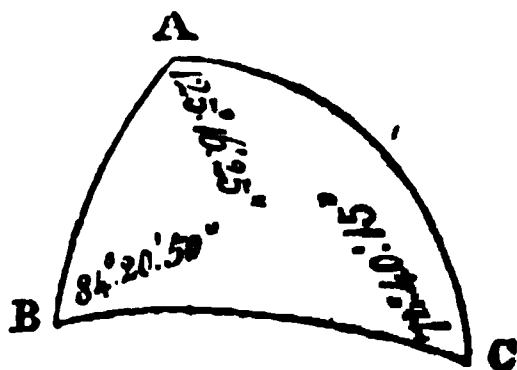
Add the three angles together and take half their sum; find the difference between the half sum and the angle opposite to the required side, which call the remainder.—Then,

To the log. co-secants, less radius, of the other two angles, add the log. co-sines of the half sum, and the remainder; half the sum of these four logs. will be the log. sine of half the required side.

One side being thus found, the remaining sides may be computed by Rule 3. Problem I., page 198.

Example.

In the spherical triangle A B C, let the angle A be 125° 16' 25"; the angle B 84° 20' 50", and the angle C 72° 40' 15"; required the sides B C, A B, and A C?



To find the side B C :—

Angle A =	. .	125° 16' 25"		
Angle B =	. .	84. 20. 50	Log. co-secant, less radius =	0. 002117
Angle C =	. .	72. 40. 15	Log. co-secant, less radius =	0. 020174
<hr/>				
Sum =	. . .	282. 17. 30		
<hr/>				
Half sum =	. .	141° 8' 45"	Log. co-sine = 9. 891395
Remainder =	. .	15. 52. 20	Log. co-sine = 9. 983118
				<hr/>
				Sum = . . 19. 896804
<hr/>				
Half the side B C =	. .	62° 37' 13"	Log. sine = 9. 948402
<hr/>				
The double of which gives 125° 14' 26", for the whole side B C.				

To find the Side A B :—

As the angle A =	. .	125° 16' 25"	Log. co-secant =	. . . 10. 088095
Is to the side B C =	. .	125. 14. 26	Log. sine = 9. 912083
So is the angle C =	. .	72. 40. 15	Log. sine = 9. 979826
<hr/>				
To the side A B =	. .	72° 44' 46"	Log. sine = 9. 980004

To find the Side A C :—

As the angle A =	125° 16' 25"	Log. co-secant =	. . . 10. 088095
Is to the side B C =	125. 14. 26	Log. sine = 9. 912083
So is the angle B =	84. 20. 50	Log. sine = 9. 997883
<hr/>			
To the side A C =	84° 35' 25"	Log. sine = 9. 998061

Remark.—The required side of a spherical triangle (when the three angles are given,) may be also found by the following general rule ; viz.,

Add the three angles together and take half their sum ; find the difference between the half sum and each of the angles comprehending the required side, and note the remainders.—Then to the log. co-secants less radius, of those angles, add the log. co-sines of the two remainders : half the sum of these four logs. will be the log. co-sine of half the required side.

Thus, to find the side B C in the last example.

Angle A = . . 125° 16' 25"

Angle B = . . 84. 20. 50 Log. co-secant, less radius = 0.002117

Angle C = . . 72. 40. 15 Log. co-secant, less radius = 0.020174

Sum = 282. 17. 30

Half sum = . . 141° 8' 45"

Remainder by B = 56. 47. 55 Log. co-sine = 9.738450

Remainder by C = 68. 28. 30 Log. co-sine = 9.564556

Sum = 19.325297

Half Side B C = 62° 37' 13" Log. co-sine = . 9.662648½

Which being doubled gives = 125° 14' 26", for the side B C; the same as by the former rule.

THE RESOLUTION OF PROBLEMS IN NAVIGATION BY LOG-ARITHMS; AND, ALSO, BY THE GENERAL TRAVERSE TABLE.

Lest the mariner should feel some degree of disappointment in not finding a regular course of navigation in this work: the author thinks it right to *remind him*, that his present intention carries him no farther than merely to show the proper application of the Tables to some of the most useful parts of the sciences on which he may touch:—it being completely at variance with the plan of this work, to enter into such parts of the sciences as could reasonably be dispensed with, without entirely losing sight of their principles.—Hence it is, that the cases of plane sailing, usually met with in books on navigation, will not be noticed in this.—However, since it is not improbable that this volume may fall into the hands of persons not very deeply versed in nautical matters; it therefore may not be deemed unnecessary to give a few introductory definitions, &c. for their immediate guidance, previously to entering upon the essentially useful parts of the sailings.

NAVIGATION is the art of conducting a ship, through the wide and pathless ocean, from one part of the world to another.—Or, it is the method of finding the latitude and longitude of a ship's place at sea; and of thence determining her course and distance from that place, to any other given place.

The *Equator* is a great circle circumscribing the earth, every point of which is equally distant from the poles; thus dividing the globe into two equal parts, called hemispheres: that towards the North Pole is called the northern hemisphere, and the other, the southern hemisphere.—The equator, like all other great circles, is divided into 360 equal parts, called degrees; each degree into 60 equal parts, called minutes; each minute into 60 equal parts, called seconds, and so on.

The *Meridian* of any place on the earth is a great circle passing through that place and the poles, and cutting the equator at right angles.—Every point on the surface of the sphere may be conceived to have a meridian line passing through it;—hence there may be as many meridians as there are points in the equator.—Since the *First Meridian* is merely an imaginary circle passing through any remarkable place and the poles of the world; therefore it is entirely arbitrary.—Hence it is that the British reckon their *first meridian* to be that which passes through the Royal Observatory at Greenwich: the French esteem their *first meridian* to be that which passes through the Royal Observatory at Paris; the Spaniards that which passes through Cadiz, &c. &c. &c.

Every meridian line may be said, with respect to the place through which it passes, to divide the surface of the sphere into two equal parts, called the eastern and western hemispheres.

The *Latitude* of any place on the earth is that portion of its meridian which is intercepted between the equator and the given place; and is named north or south, according as the given place is in the northern or southern hemisphere.—As the latitude begins at the equator, where it is nothing, and is reckoned thence to the poles, where it terminates; therefore the greatest latitude any place can have, is 90 degrees.

The *Difference of Latitude* between two places on the earth is an arc of the meridian intercepted between their corresponding parallels of latitude; showing how far one of them is to the northward or southward of the other:—The difference of latitude between two places can never exceed 180 degrees.

The *Longitude* of any place on the earth is that arc or portion of the equator which is contained between the *first meridian* and the meridian of the given place; and is denominated east, or west, according as it may be situated with respect to the *first meridian*.—As the longitude is reckoned both ways from the *first meridian* (east and west) till it meets at the same meridian on the opposite part of the equator; therefore the longitude of any place can never exceed 180 degrees.

The *difference of Longitude* between two places on the earth is an arc of the equator intercepted between the meridians of those places ; showing how far one of them is to the eastward or westward of the other :—The difference of longitude between two places can never exceed 180 degrees.

When the latitudes of two places on the earth are both north or both south ; or their longitudes both east or both west, they are said to be of the same name.—But, when one latitude is north and the other south ; or one longitude east and the other west ; then they are said to be of different names.

The *Horizon* is that great circle which is equally distant from the zenith and nadir, and divides the visible from the invisible hemisphere ; this is called the rational horizon.—The sensible horizon is that which terminates the view of a spectator in any part of the world.

The *Mariner's Compass* is an artificial representation of the horizon :—it is divided into 32 equal parts, called points ; each point consisting of $11^{\circ}15'$.—Hence the whole compass card contains 360 degrees ; for $11^{\circ}15'$ multiplied by 32 points = 360 degrees.

A *Rhumb Line* is a right line, or rather curve, drawn from the centre of the compass to the horizon, and obtains its name from the point of the horizon it falls in with.—Hence there may be as many rhumb-lines as there are points in the horizon.

The *Course* steered by a ship is the angle contained between the meridian of the place sailed from, and the rhumb-line on which she sails ; and is either estimated in points or degrees.

The *Distance* is the number of miles intercepted between any two places, reckoned on the rhumb line of the course ; or it is the absolute length that a ship has sailed in a given time.

The *Departure* is the distance of the ship from the meridian of the place sailed from, reckoned on the parallel of latitude at which she arrives ; and is named east or west, according as the course is in the eastern or western hemisphere.

If a ship's course be due north or south, she sails on a meridian, and therefore makes no departure :—hence the distance sailed will be equal to the difference of latitude.

If a ship's course be due east or west, she sails either on the equator, or on some parallel of latitude ; in this case since she makes no difference of latitude, the distance sailed will, therefore, be equal to the departure.

When the course is 4 points, or 45 degrees, the difference of latitude and departure are equal.

When the course is *less than* 4 points, or 45 degrees, the difference of latitude exceeds the departure ; but when it is *more than* 4 points, or 45 degrees, the departure exceeds the difference of latitude.

Note.—Since the distance sailed, the difference of latitude, and the departure form the sides of a right angled plane triangle ; in which the hypotenuse is represented by the distance ; the perpendicular, by the difference of latitude ; the base, by the departure ; the angle opposite to the base, by the course ; and the angle opposite to the perpendicular, by the complement of the course ; therefore any two of these five parts being given, the remaining three may be readily found by the analogies for right angled plane trigonometry.

These being premised, we will now proceed to the following *Introductory Problems*.

PROBLEM I.

Given the Latitudes of Two Places on the Earth, to find the difference of Latitude.

RULE.

When the latitudes are of the same name ; that is, both north, or both south, their difference will be the difference of latitude ; but when one is north and the other south, their sum will express the difference of latitude.

Note.—The same Rule is to be observed in finding the meridional difference of latitude between two places.

Example 1.

Required the difference of latitude between Portsmouth and Cape Trafalgar ?

Lat. of Portsmouth = $50^{\circ}47'$ N.

Lat. of C. Trafalgar = 36.10 N.

Diff. of Lat. = $\quad\quad\quad 14^{\circ}37'$

Ditto in Miles = $\quad\quad\quad 877$

Example 2.

Required the difference of latitude between Portsmouth and James Town, St. Helena ?

Lat. of Portsmouth = $50^{\circ}47'$ N.

Lat. of James Town = 15.55 S.

Diff. of Lat. = $\quad\quad\quad 46^{\circ}42'$

Ditto in Miles = $\quad\quad\quad 2802$

Note.—In finding the difference of latitude, or the difference of longitude between two places (when any of the sailings are under consideration), it will be sufficiently exact to take out the latitudes and longitudes from Table LVIII. to the nearest minute of a degree, as above.

PROBLEM II.

Given the Latitude left and the difference of Latitude, to find the Latitude in.

RULE.

When the latitude left and the difference of latitude are of the same name their sum will be the latitude; but when they are of contrary denominations, their difference will be the latitude required:—This latitude will always be of the same name with the greater quantity.

Example 1.

A ship, from a place in latitude $30^{\circ}45'$ north sailed 497 miles in a northerly direction; required the latitude at which she arrived?

Latitude left = . . . $30^{\circ}45'$ N.
Diff. of Lat. = 497 ms. or 8. 17 N.

Lat. arrived at = . . . $39^{\circ} 2'$ N.

Example 2.

A ship from a place in latitude $2^{\circ}50'$ north, sails 530 miles in a southerly direction; required the latitude come to?

Latitude left = . . . $2^{\circ}50'$ N.
Diff. of lat. = 530 ms. or 8. 50 S.

Lat. come to = . . . $6^{\circ} 0'$ S.

PROBLEM III.

Given the Longitudes of Two Places on the Earth, to find the difference of Longitude.

RULE.

When the longitudes are of the same name: that is, both east, or both west, their difference will express the difference of longitude; but when one is east and the other west, their sum will be the difference of longitude. If the sum of the longitudes exceed 180° , subtract it from 360° , and the remainder will be the difference of longitude.

Example 1.

Required the difference of longitude between Portsmouth and Fayal, one of the western islands?

Long. of Portsmouth = $1^{\circ} 6' W.$

Long. of Fayal, Horta, $28.43 W.$

Diff. of long. = $27^{\circ} 37'$

Ditto in miles = 1657

Example 2.

Required the difference of longitude between Canton and Point Venus, in the island of Otaheite?

Long. of Canton = $113^{\circ} 3' E.$

Long. of Point Venus = $149.36 W.$

Sum = $262^{\circ} 39'$

Diff. of Long. = $97^{\circ} 21'$

Ditto in miles = 5841

PROBLEM IV:

Given the Longitude left and the difference of Longitude, to find the Longitude in.

RULE.

When the longitude left and the difference of longitude are of the same name, their sum will be the longitude in; should that sum exceed 180° , subtract it from 360° ; and the remainder will be the longitude in, of a contrary name to the longitude left.—But, when the longitude left and the difference of longitude are of contrary names, their difference will be the longitude in, of the same name with the greater quantity.

Example 1.

A ship from a place in longitude $50^{\circ} 40'$ west, sails westward till her difference of longitude is 410 miles; required the longitude in?

Long. left = $50^{\circ} 40' W.$

Diff. of long. = $410 \text{ ms. or } 6.50 W.$

Longitude in = $57^{\circ} 30' W.$

Example 2.

Let the long. left be $174^{\circ} 45'$ west, and the difference of longitude $13^{\circ} 17'$ west; required the longitude in?

Longitude left = $174^{\circ} 45' W.$

Diff. of Long. = $13.17 W.$

Sum = 188.2

Longitude in = $171^{\circ} 58' E.$

Example 3.

Let the longitude left be $41^{\circ}37'$ east, and the difference of longitude $11^{\circ}20'$ west; required the longitude come to?

Longitude left = . . $41^{\circ}37'$ E.
Diff. of long. = . . 11.20 W.

Longitude in = . . $30^{\circ}17'$ E.

Example 4.

Let the longitude left be $5^{\circ}40'$ east, and the difference of longitude $10^{\circ}17'$ west; required the longitude in?

Longitude left = . . $5^{\circ}40'$ E.
Diff. of long. = . . 10.17 W.

Long. in = . . . $4^{\circ}37'$ W.

Remarks.—If a ship be in north latitude sailing northerly, or in south latitude sailing southerly, she *increases* her latitude, and therefore the difference of latitude must be *added* to the latitude left, in order to find the latitude in :—but, in north latitude sailing southerly, or in south latitude, northerly, she *decreases* her latitude; therefore the difference of latitude *subtracted* from the latitude left will give the latitude in :—should the difference of latitude be the greatest, the latitude left is to be taken from it; in this case the ship will be on the opposite side of the equator with respect to the latitude sailed from.—Again,

If a ship be in east longitude sailing easterly, or in west longitude sailing westerly, she *increases* her longitude; therefore the difference of longitude *added* to the longitude left will give the longitude in; should the sum exceed 180° , the ship will be on the opposite side of the *first meridian* with respect to the longitude sailed from.—But, in east longitude sailing westerly, or in west longitude sailing easterly, she *decreases* her longitude, and therefore the difference of longitude is to be *subtracted* from the longitude left, in order to find the longitude in;—should the difference of longitude be the greatest, the longitude left is to be taken from it; in this case the ship will, also, be on the opposite side of the *first meridian* with respect to the longitude sailed from. These remarks will appear evident on a comparison with the above Examples.

SOLUTION OF PROBLEMS IN PARALLEL SAILING.

Parallel Sailing is the method of finding the distance between two places situate under the same parallel of latitude; or of finding the difference of longitude corresponding to the meridional distance, when a ship sails due east or west.

PROBLEM I.

Given the Difference of Longitude between two Places, both in the same Parallel of Latitude, to find their Distance.

RULE.

As radius, is to the co-sine of the latitude; so is the difference of longitude, to the distance.

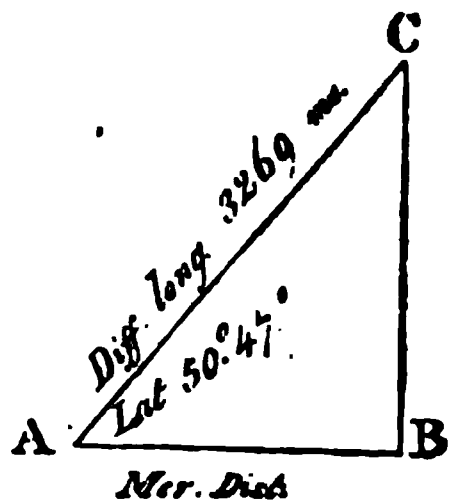
Example.

Required the distance between Portsmouth, in longitude $1^{\circ}6'$ W., and Green Island, Newfoundland, in longitude $55^{\circ}35'$ W.; their common latitude being $50^{\circ}47'$ N.?

Long. of Portsmouth = $1^{\circ}6'$ W.

Long. of Green Island = 55.35 W.

Diff. of long. = $54^{\circ}29' = 3269$ miles.

*Solution.*

In the right-angled triangle A B C, where the hypotenuse A C represents the difference of longitude between the two given places, the angle A the latitude of the parallel of those places, and the base A B their meridional distance: given the side A C = 3269 miles, and the angle A = $50^{\circ}47'$, to find the side A B. Hence, by right-angled plane trigonometry, problem I., page 171,

As radius = . . . $90^{\circ} 0' 0''$ Log. co-secant = . 10.000000

Is to the diff. of long. A C = 3269 miles Log. = . 3.514415

So is the lat. = the angle A = $50^{\circ}47' 0''$ Log. co-sine = . 9.800892

To the merid. dist. A B = 2066.8 miles Log. = . 3.315307

To find the Meridional Distance by *Inspection* in the general Traverse Table:—

Note.—This case may be solved by Problem I., page 107, as thus:

To latitude 50° as a course, and one-eleventh of the difference of longitude (*viz.* 297.2) as a distance, the corresponding difference of latitude is 190.9; and to latitude 51° , and distance 297.2, the difference of latitude

is 186.9 : hence the change of meridional distance (represented by difference of latitude,) to 1° or 60' of latitude, is 4'. Now, $4' \times 47' \div 60' = 3'.1$; this being subtracted from the first difference of latitude, because it is decreasing, gives 187.8 ; and 187.8 multiplied by 11, the aliquot part, gives 2065.8 for the meridional distance ; which comes within one mile of the result by calculation.

PROBLEM II.

Given the Distance between two Places, both in the same Parallel of Latitude, to find the Difference of Longitude between those Places.

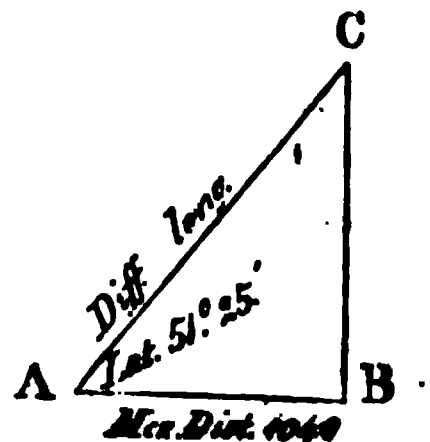
RULE.

As the co-sine of the latitude, is to radius ; so is the distance, to the difference of longitude.

Example.

A ship from Cape Clear, in latitude $51^{\circ}25'$ N. and longitude $9^{\circ}29'$ W., sailed due west 1040 miles ; required the longitude at which she then arrived ?

Solution.—In the right angled triangle A B C, let the hypotenuse A C represent the difference of longitude ; the angle A, the latitude of the parallel on which the ship sailed ; and the base A B, the meridional distance : then, in this triangle, there are given, the angle $A = 51^{\circ}25'$, and the base $A B = 1040$ miles, to find the side A C. Hence, by right angled plane trigonometry, Problem II., page 172,



As radius = $90^{\circ} 0' 0''$ Log. co-secant = 10.000000
 Is to the merid. dist. $A B = 1040$ miles. Log. = . 3.017033
 So is the lat. = the angle $A = 51^{\circ}25' 0''$ Log. secant = . 10.205057

To the difference of long. $A C = 1667.6$ miles. Log. = . 3.222090

Longitude of Cape Clear = $9^{\circ}29'$ west.
 Difference of longitude 1667.6 miles, or . . 27.48 west.
 Longitude at which the ship arrived = . . $37^{\circ}17'$ west.

To find the Difference of Longitude by *Inspection* in the general Traverse Table :—

Note.—This case falls under Problem V., page 111 : hence,

To latitude 51° as a course, and one-eighth of the meridional distance = 130, in a difference of latitude column, the corresponding distance is 207 ; and to latitude 52° , and difference of latitude 130, the distance is 211 : hence, the difference of distance to 1° of latitude, is 4 miles. Now, $4' \times 25' + 60' = 1'.6$, which being added to the first distance, because it is increasing, gives 208.6 ; this being multiplied by 8 (the aliquot part), gives 1668.8 for the difference of longitude.

PROBLEM III.

Given the Difference of Longitude and the Distance between two Places, in the same Parallel of Latitude, to find the Latitude of that Parallel.

RULE.

As the difference of longitude, is to the distance ; so is radius, to the co-sine of the latitude.

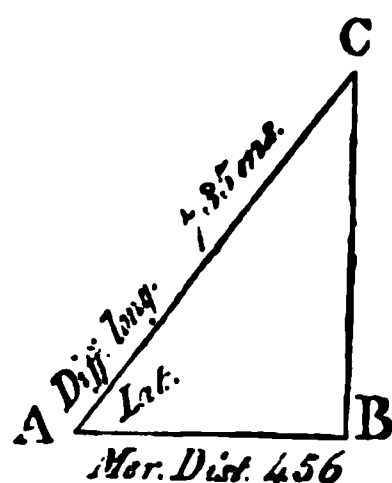
Example.

A ship, from a place in longitude $16^\circ 30'$ W., sailed due east 456 miles, and then by observation was found to be in the longitude of $4^\circ 15'$ W. ; required the latitude of the parallel on which she sailed ?

Long. sailed from = $16^\circ 30'$ W.

Long. come to = 4.15 W.

Diff. of long. = $12^\circ 15' = 735$ miles.



Solution.—In the right angled triangle ABC , let the hypotenuse AC represent the difference of longitude ; the angle A , the latitude of the parallel ; and the base AB , the meridional distance : then, there are given, the side $AC = 735$ miles, and the leg $AB = 456$ miles, to find the angle A . Hence, by right angled plane trigonometry, Problem III., page 174,

As the diff. of longitude $AC = 735$ miles.	Log. ar. comp. =	7.138713
Is to radius = $90^\circ 0' 0''$	Log. sine =	10.000000
So is the merid. distance $AB = 456$ miles.	Log. = . . .	2.658965
To lat. of parall. = ang. $A = 51^\circ 39' 14''$	Log. co-sine =	9.792678

To find the Latitude of the Parallel by *Inspection* in the general Traverse Table :—

Enter the Table with one-third the difference of longitude $= 245$ as a distance, and one-third the meridional distance $= 152$, in a difference of latitude column; and the latitude corresponding to them will be found to lie between 51° and 52° . Now, to latitude 51° , and distance 245, the corresponding difference of latitude is 154.2, which exceeds half the meridional distance by $2'.2$; and, to latitude 52° , and distance 245, the difference of latitude is 150.8, which is $1'.8$ less than half the meridional distance. Hence, $1'.8 + 2'.2 = 4'$ is the change of meridional distance to 1° of latitude. And, as $4' : 2'.2 :: 60' : 38'$; this, being added to 51° , gives $51^\circ 38'$ for the required latitude.

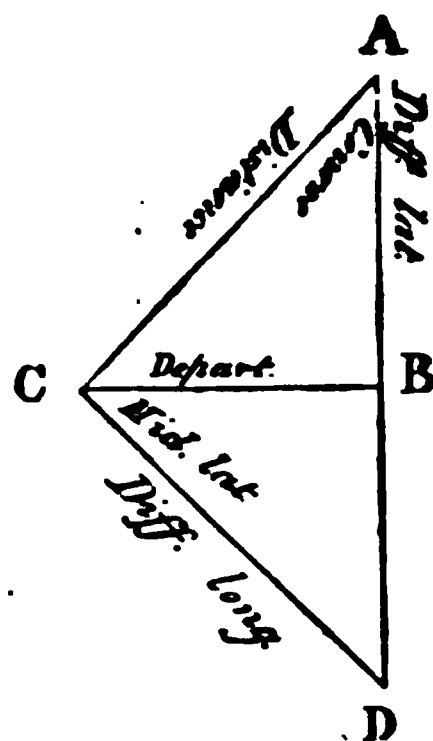
SOLUTION OF PROBLEMS IN MIDDLE LATITUDE SAILING.

Middle Latitude Sailing is the method of solving the several cases, or problems, in Mercator's sailing, by principles compounded of plane and parallel sailing. This method is founded on the supposition that the meridional distance, at that point which is a middle parallel between the latitude left and the latitude bound to, is equal to the departure which the ship makes in sailing from one parallel of latitude to the other.

This method of sailing, though not quite accurate, is, nevertheless, sufficiently so for a *single day's run*, particularly in low latitudes, or when the ship's course is not more than two or three points from a parallel. But, in high latitudes, or places considerably distant from the equator, it fails of the desired accuracy: in such places, therefore, the mariner should never employ it in the determination of a ship's place, when he wishes to draw correct nautical conclusions from his operations.

With the intention of avoiding prolixity and unnecessary repetition, in resolving the different problems in this method of sailing, we will here briefly give a general view of the principles on which the solutions of those problems are founded; as thus :—

In the annexed diagram, let the triangle ABC be a figure in plane sailing, in which AC represents the distance, AB the difference of latitude, BC the departure, and the angle A the course. Again, let DBC be a figure in parallel sailing, in which DC represents the difference of longitude, BC the meridional distance, and the angle C the middle latitude. Hence, the parts concerned form two connected right angled triangles, in which the departure or meridional distance BC is a side common to both.



Now, in one of these triangles, there will be always two terms given, and in the other one term, at least, to find the required terms. The required parts in that triangle which has two terms given, may be readily found by the analogies for right angled plane trigonometry, page 171 to 177; and, hence, the unknown terms in the other triangle.

When the departure BC is not under consideration, the two connected triangles may be considered as one oblique angled triangle, and resolved as such. In this case, if the course, distance, middle latitude, and difference of longitude, are the terms in question, any three of them being given, the fourth may be found by one direct proportion. Thus, in the oblique angled triangle ACD , the side AC is the distance; the angle A , the course; the angle BCD , the middle latitude; and, consequently, the angle D its complement, and the side DC the difference of longitude. Now, if any three of these be known, the fourth may be found by one of the following analogies; viz.,

1. As co-sine middle latitude $= C$: sine of course $= A$:: distance $= AC$: difference of longitude $= DC$.

2. As sine of course $= A$: co-sine middle latitude $= C$:: difference of longitude $= DC$: distance $= AC$.

3. As distance $= AC$: difference of longitude $= DC$:: co-sine of middle latitude $= C$: sine of course $= A$.

4. As difference of longitude $= DC$: distance $= AC$:: sine of course $= A$: co-sine of middle latitude $= C$.

Again, if the course, middle latitude, difference of latitude, and difference of longitude, be the terms under consideration, the resulting analogies will be,

5. As difference of latitude $= AB$: difference of longitude $= DC$:: co-sine of middle latitude $= C$: tangent of course $= A$.

6. As difference of longitude $= DC$: difference of latitude $= AB$:: tangent of course $= A$: co-sine of middle latitude $= C$.

7. As co-sine of middle latitude = C : tangent of course = A :: difference of latitude = A B : difference of longitude = A C.

8. As tangent of course = A : co-sine of middle latitude = C :: difference of longitude = D C : difference of latitude = A B.

In these four analogies, it is evident that the course must be a tangent, because the difference of latitude A B is concerned.

Note.—Since the sine complement of the middle latitude = the angle D, is expressed directly by the co-sine of the angle B C D, therefore, with the view of abridging the preceding analogies, the co-sine of the middle latitude has been used instead of its sine complement; and, in the operations which follow, the same term will be invariably employed.

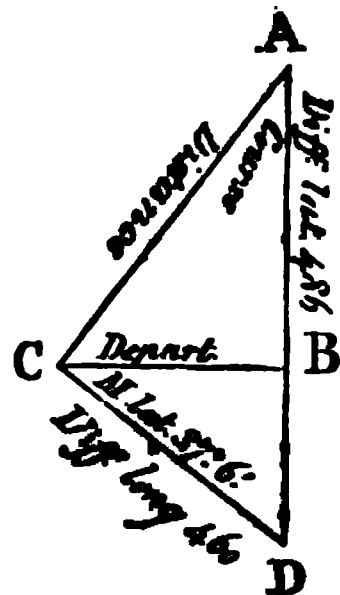
Remark.—The middle latitude between two places is found by taking half the sum of the two latitudes, when they are both of the same name, or half their difference if of contrary names.

PROBLEM I.

Given the Latitudes and Longitudes of two Places, to find the Course and Distance between them.

Example.

Required the course and distance from Oporto, in latitude $41^{\circ}9'$ N. and longitude $8^{\circ}37'$ W. to Porto Santo, in latitude $33^{\circ}3'$ N. and longitude $16^{\circ}17'$ W.?



Latitude of Oporto $41^{\circ}9'$ N. Longitude = $8^{\circ}37'$ W.
 Lat. of Porto Santo 33.3 N. Longitude = , . . . 16.17 W.

Diff. of latitude = $8^{\circ}6' = 486$ miles. Diff. of long. = $7^{\circ}40' = 460$ ms.
 Sum of latitudes = $74^{\circ}12' \div 2 = 37^{\circ}6' =$ the middle latitude.

To find the Course = Angle A :—

Here, since the departure is not in question, the parts concerned come under the 5th analogy in page 222 : hence,

As the diff. of latitude = 486 miles,	Log. ar. comp. =	7.313364
Is to the diff. of long. = 460 miles,	Log. = . . .	2.662758
So is the mid. latitude = $37^{\circ}6'$	Log. co-sine = .	9.901776
To the course = . . $37^{\circ}3'$	Log. tangent =	<u>9.877898</u>

To find the Distance = AC :—

The course being thus found, the distance may be determined by trigonometry, Problem II., page 172 : hence,

As radius = . . . $90^{\circ}0'$	Log. co-secant =	10.000000
Is to the diff. of lat. = 486 miles,	Log. = . . .	2.686636
So is the course = . $37^{\circ}3'$	Log. secant = .	<u>10.097937</u>
To the distance = . 608.9 miles,	Log. = . . .	2.784573

Hence, the true course from Oporto to Porto Santo is S. $37^{\circ}3'$ W., or S.W. $\frac{3}{4}$ S. nearly, and the distance 609 miles.

To find the Course and Distance by Inspection in the general Traverse Table :—

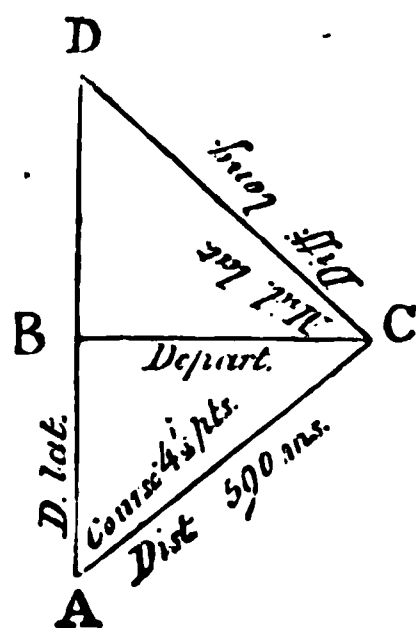
To middle latitude = 37° as a course, and one-fourth the difference of longitude = 115, as a distance, the corresponding difference of latitude is 91.8 = the meridional distance. Now, one-fourth the difference of latitude = 121.5, and the meridional distance 91.8 in a departure column, are found to agree nearest at 37° , under distance 152. Hence, the course is S. 37° W., and the distance $152 \times 4 = 608$ miles.

PROBLEM II.

Given the Latitude and Longitude of the Place sailed from, the Course, and Distance ; to find the Latitude and Longitude of the Place come to.

Example.

A ship from Corvo, in latitude $39^{\circ}41'$ N., and longitude $31^{\circ}3'$ W., sailed N.E. $\frac{1}{2}$ E., 590 miles; required the latitude and longitude come to ?



To find the Difference of Latitude = A B :—

Here the course = A, and the distance = A C, being given, the difference of latitude = A B may be found by trigonometry, Problem I., page 171; as thus :

As radius = . . . 90°0'	Log. co-secant =	10.000000
Is to the distance = 590 miles,	Log. = . . .	2.770852
So is the course = 4½ points,	Log. co-sine = .	9.802359
To the diff. of lat. = 374.3 miles,	Log. = . . .	2.573211
Latitude left = 39°41' N.		39°41' N.
Diff. of lat. = 374.3 miles, or = 6.14 N.	Half =	3. 7 N.
Latitude come to = 45°55' N.	Mid. lat. =	42°48'

To find the Difference of Longitude = C D :—

Here, since the departure is not concerned, the parts in question come under the 1st analogy in page 222 : hence,

As the mid. lat. = . . . 42°48'	Log. secant =	10.134464
Is to the course = . . . 4½ points,	Log. sine = .	9.888185
So is the distance = 590 miles,	Log. = . . .	2.770852
To the diff. of longitude = 621.6 miles,	Log. = . . .	2.793501
Longitude left = 31° 3' W.		
Diff. of longitude = 621.6 miles, or =		10.22 E.
Longitude come to = 20°41' W.		

To find the Difference of Latitude and Difference of Longitude by Inspection:—

Under or over one-fifth of the given distance = 118, and opposite to the course = 4½ points, is difference of latitude 74.9, and departure 91.2. Tabular difference of latitude $74.9 \times 5 = 374.5$, the whole difference of latitude; whence the latitude in, is 45°55' N., and the middle latitude 42°48'. Now, to middle latitude 42°, and departure 91.2, in a latitude column, the corresponding distance is 123 miles; and to middle latitude 43°, and departure 91.2, the distance is 125 miles : hence, the difference of distance to 1° of latitude, is 2 miles; and $2' \times 48' \div 60' = 1'.6$, which, added to 123, gives 124.6; this, being multiplied by 5 (the aliquot part), gives 623 miles = the difference of longitude, or 10°23' E.

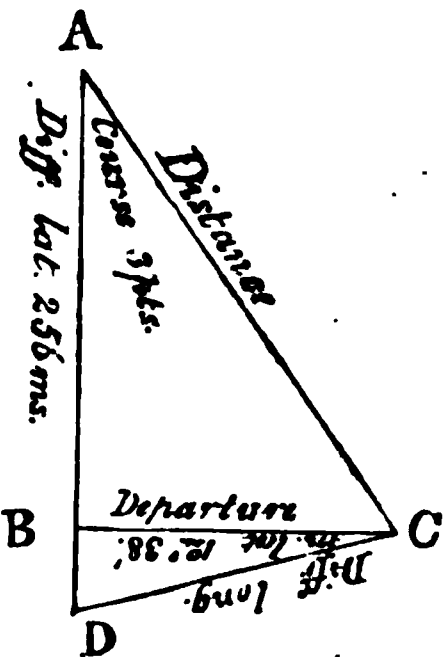
PROBLEM III.

Given both Latitudes and the Course; to find the Distance and the Longitude in.

Example.

A ship from Brava, in latitude $14^{\circ}46'$ N., and longitude $24^{\circ}46'$ W., sailed S.E. b. S., until; by observation, she was found to be in latitude $10^{\circ}30'$ N.; required the distance sailed and her present longitude?

Lat. of Brava = $14^{\circ}46'$ N. . . . $14^{\circ}46'$ N.
Lat. by obs. = 10.30 N. . . . 10.30 N.
Diff. of lat. = $4^{\circ}16' = 256$ m. Sum = $25^{\circ}16'$
Middle latitude = $12^{\circ}38'$



To find the Distance = AC:—

With the course = A, and the difference of latitude = AB, the distance is found by trigonometry, Problem II., page 172; as thus:

As radius = $90^{\circ}0'$ Log. co-secant = 10.000000
Is to the diff. of latitude = 256 miles Log. = 2.408240
So is the course = 3 points, Log. secant = 10.080154
To the distance = 307.9 miles, Log. = 2.488394

To find the Difference of Longitude = CD:—

Here, since the departure is not in question, the parts concerned fall under the 7th analogy, page 222: hence,

As the middle latitude = $12^{\circ}38'$ Log. secant = 10.010644
Is to the course = 3 points, Log. tangent = 9.824893
So is the diff. of lat. = 256 miles, Log. = 2.408240
To the diff. of long. = 175.2 miles, Log. = 2.243777

Longitude of Brava, the place sailed from = $24^{\circ}46'$ W.
Difference of longitude = 175.2 miles, or = 2.55 E.

Longitude of the ship = $21^{\circ}51'$ W.

To find the Distance sailed, and the Difference of Longitude, by Inspection :—

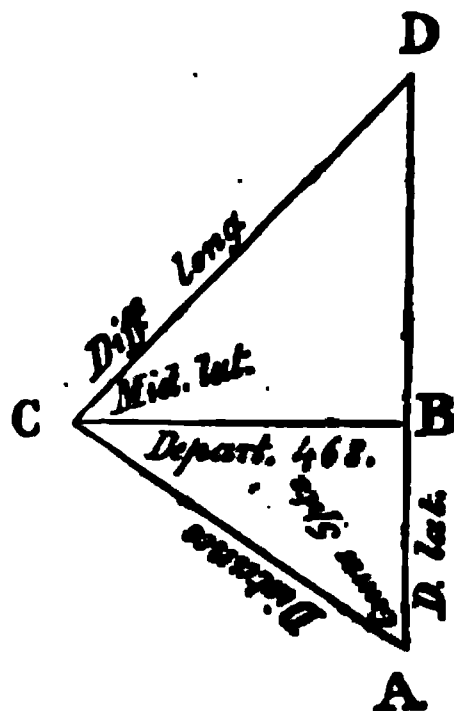
To the course 3 points, and half the difference of latitude = 128, the distance is 154, and the departure 85.5. Now, $154 \times 2 = 308$ miles, is the required distance. Again, to middle latitude 12° , and departure 85.5, in a latitude column, the corresponding distance is 87; and to latitude 13° , and departure 85.5, the distance is 88: hence, to middle latitude $12^\circ 38'$, and departure 85.5, the distance is $87\frac{1}{2}$; the double of which = 175 miles, is the difference of longitude, as required.

PROBLEM IV.

Given the Latitude and Longitude of the Place sailed from, the Course, and the Departure; to find the Distance sailed, and the Latitude and Longitude of the Place come to.

Example.

A ship from Cape Finisterre, in latitude $42^\circ 54'$ N., and longitude $9^\circ 16'$ W., sailed N.W. b. W., till her departure was 468 miles; required the distance sailed, and the latitude and longitude come to?



To find the Distance = A C :—

With the course = A. and the departure B C, the distance may be found by trigonometry, Problem II., page 172; as thus:

As radius = $90^\circ 0'$	Log. co-secant = 10.000000
Is to the departure = . . . 468 miles,	Log. = 2.670246
So is the course = 5 points,	Log. co-secant = 10.080154
To the distance = 562.9 miles,	Log. = 2.750400

To find the Difference of Latitude = A B :—

With the course = A, and the departure B C, the distance is found by trigonometry, Problem II., page 172; as thus:

As the course = 5 points,	Log. co-tangent = 9.824893
Is to the departure = . . . 468 miles,	Log. = . . . 2.670246
So is radius = 90° 0′	Log. sine = . . . 10.000000
To the diff. of latitude = . . 312.7	Log. = . . . 2.495139

Latitude of Cape Finisterre = 42° 54′ N. 42° 54′ N.
Diff. of lat. = 313 miles, or = 5.13 N.	Half diff. of lat. = 2.36 N.
Latitude of the ship = . . 48° 7′ N.	Middle lat. = 45° 30′

To find the Difference of Longitude = C D:—

With the middle latitude = B C D, and the departure B C, the difference of longitude is found by trigonometry, Problem II., page 172.—

As radius = 90° 0′	Log. co-secant = 10.000000
Is to the departure = . . . 468 miles,	Log. = . . . 2.670246
So is the mid. lat. = . . . 45° 30′	Log. secant = 10.154338
To the diff. of long. = . . 667.7 miles,	Log. = . . . 2.824584

Longitude of Cape Finisterre = 9° 16′ W.
Difference of long. = 667.7 miles, or = 11. 8 W.
Longitude of the ship = 20. 24 W.

To find the Distance, Difference of Latitude, and Difference of Longitude, by Inspection:—

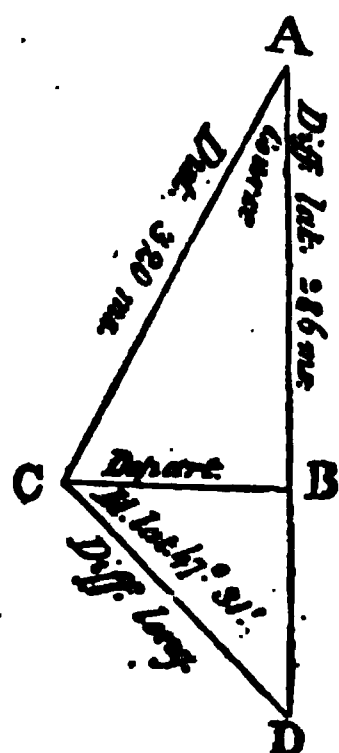
To course 5 points, and one-fourth of the departure = 117, the distance is 141, and the difference of latitude 78.3. Now, $141 \times 4 = 564$ miles, the distance, and $78.3 \times 4 = 313.2$, or 5° 13′, the difference of latitude; whence the latitude in, is 48° 7′ N., and the middle latitude 45° 30′. Again, to middle latitude 45°, and one-fourth the departure = 117, in a latitude column, the distance is 166; and to middle latitude 46°, and departure 117, the distance is 168: hence, to middle latitude 45° 30′, and departure 117, the difference of longitude is $167 \times 4 = 668$ miles; nearly the same as by calculation.

PROBLEM V.

Given both Latitudes and the Distance ; to find the Course and Difference of Longitude.

Example.

A ship from St. Agnes, Scilly, in latitude $49^{\circ}54'$ N., and longitude $6^{\circ}19'$ W., sailed 320 miles between the south and west, and then, by observation, was found to be in latitude $45^{\circ}8'$ N.; required the course, and the longitude come to?



Latitude of St. Agnes =	$49^{\circ}54'$ N.	$49^{\circ}54'$ N.
Latitude of the ship =	$45. 8$ N.	$45. 8$ N.

Difference of latitude =	$4^{\circ}46'$	= 286 miles.	Sum =	$95^{\circ} 2'$
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Middle latitude =	$47^{\circ}31'$
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To find the Course = A :—

With the distance AC, and the difference of latitude = AB, the course may be found by trigonometry, Problem III., page 174 ; as thus :

As the distance = 320 miles,	Log. ar. comp. =	7.494850
Is to radius = $90^{\circ} 0' 0''$	Log. sine =	10.000000
So is the diff. of lat. = 286 miles,	Log. =	2.456366
To the course = $26^{\circ}39' 6''$	Log. co-sine =	9.951216

To find the Difference of Longitude = CD :—

With the course, middle latitude, and distance, the difference of longitude is found by the 1st analogy, page 222 ; as thus :

As middle latitude = $47^{\circ}31' 0''$	Log. secant =	10.170455
Is to the course = $26.39. 6$	Log. sine =	9.651825
So is the distance = 320 miles,	Log. =	2.505150
To the diff. of long. = 212.5	Log. =	2.327430

Longitude of St. Agnes = . . . 6°19' W.

Diff. of long. = 212.5 miles, or = 3.33 W.

Longitude of the ship = . . . 9°52' W.

The course is S. 26°39' W., or S.S.W. $\frac{1}{4}$ W., nearly.

To find the Course and Difference of Longitude by Inspection:—

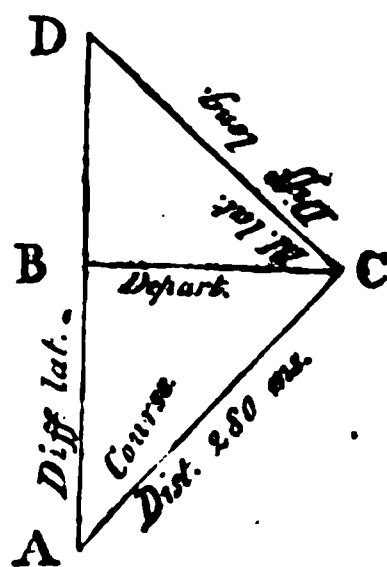
To half the distance = 160, and half the difference of latitude = 143, the course nearest agreeing is 27, and the departure 72.6. Now, to middle latitude 47° as a course, and departure 72.6, in a latitude column, the distance is 105; and to middle latitude 48°, and departure 72.6, the distance is 108; hence, the difference of distance to 1° of latitude, is 3 miles; therefore, $3' \times 31' \div 60 = 1'.5$, which, added to 105, makes 106.5: this, being multiplied by 2, gives 213 miles = the difference of longitude.

PROBLEM VI.

Given one Latitude, Distance, and Departure; to find the other Latitude, the Course, and the Difference of Longitude.

Example.

A ship from Cape Bajoli, Minorca, in latitude 40°3' N., and longitude 3°52' E., sailed 280 miles between the north and east, upon a direct course, and made 186 miles of departure; required the course, and the latitude and longitude come to?



To find the Course = A:—

The distance = AC, and the departure BC, being given, the course may be found by trigonometry, Problem III., page 174; as thus:

As the distance = . . . 280 miles,	Log. ar. comp. = 7.552842
Is to radius = . . . 90° 0' 0"	Log. sine = . 10.000000
So is the departure = . . 186 miles,	Log. = . . 2.269513
<hr/>	
To the course = . . . 41°37'39"	Log. sine = . 9.822355

To find the Difference of Latitude = A B:—

The course = A, and the distance, being thus known, the difference of latitude may be computed by trigonometry, Problem III., page 174.—

As radius =	90° 0' 0"	Log. co-secant =	10.000000
Is to the distance =	280 miles,	Log. =	2.447158
So is the course =	41° 37' 39"	Log. co-sine =	9.873599

To the diff. of lat. =	209.3 miles,	Log. =	2.320787
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Latitude of Cape Bajoli =	40° 3' N.	40° 3' N.
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Diff. of lat. = 209.3 miles, or =	3.29 N.	Half =	1.44½ N.
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Latitude come to =	43° 32' N.	Middle latitude =	41° 47½' N.
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To find the Difference of Longitude = C D:—

The middle latitude = angle B C D, and the departure B C, being given, the difference of longitude may be found by trigonometry, Problem II., page 172; as thus:

As radius =	90° 0'	Log. co-secant =	10.000000
Is to the departure =	186 miles,	Log. =	1.269518
So is the mid. lat. =	41° 47½'	Log. secant =	10.127510

To the diff. of long. =	249.5 miles,	Log. =	1.397023
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Longitude of Cape Bajoli =	3° 52' E.
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Diff. of long. = 249.5 miles, or =	4. 9 E.
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Longitude come to =	8. 1 E.
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The course is N. 41° 38' E., or N.E. ¼ N., nearly.

To find the Course, Difference of Latitude, and Difference of Longitude, by Inspection:—

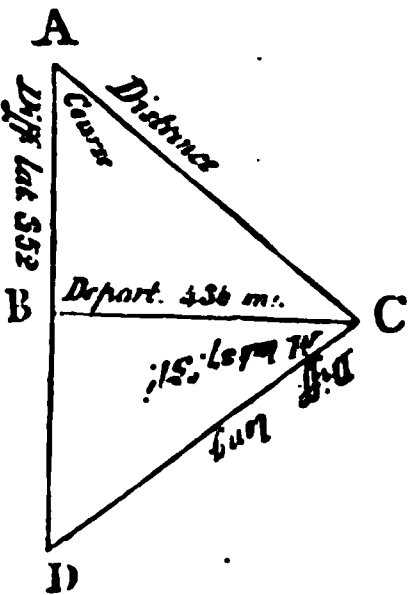
The distance 280, and departure 186, are found to agree between 41° and 42°, and the corresponding difference of latitude 208.1: whence the middle latitude is 41° 46'. Now, to middle latitude 41°, and departure 186, in a latitude column, the corresponding distance is 247; and to latitude 42°, and departure 186, the distance is 250: hence, the difference of distance to 1° of latitude, is 3 miles; and $3' \times 46 \div 60' = 2'.3$, which, added to 247, gives 249.3—the difference of longitude, as required; which nearly agrees with the result by calculation.

PROBLEM VII.

Given both Latitudes and Departure ; to find the Course, Distance, and Difference of Longitude.

Example.

A ship from Cape Agulhas, in latitude $34^{\circ}55'$ S., and longitude $20^{\circ}18'$ E., sailed upon a direct course between the south and east, till she was found, by observation, to be in latitude $40^{\circ}47'$ S., and to have made 436 miles of easting; required the course, distance, and longitude at which the ship arrived?



Latitude of Cape Agulhas =	$34^{\circ}55'$ S.	$34^{\circ}55'$ S.
Latitude of the ship =	40.47 S.	40.47 S.
Diff. of latitude =	$5^{\circ}52' = 352$ miles.								Sum =	75.42
									Middle latitude =	$37^{\circ}51'$

To find the Course = Angle A :—

Here, the difference of latitude = A B, and the departure B C, being given, the course is found by trigonometry, Problem IV., page 175 ; as thus :

As the diff. of lat. =	352 miles,	Log. ar. comp. =	7.453457
Is to radius =	$90^{\circ}0'0''$	Log. sine =	10.000000
So is the departure =	436 miles,	Log. =	2.639486
To the course =	$51^{\circ}5'5''$	Log. tangent =	10.092943

To find the Distance = A C :—

With the course, thus found, and the difference of latitude A B, the distance may be computed by trigonometry, Problem IV., page 175 : hence,

As radius =	$90^{\circ}0'0''$	Log. co-secant =	10.000000
Is to the diff. of lat. =	352 miles,	Log. =	2.546543
So is the course =	$51^{\circ}5'5''$	Log. secant =	10.201922
To the distance =	560.4 miles,	Log. =	2.748465

Hence, the course is S. $51^{\circ}5'$ E., or S.E. $\frac{1}{2}$ E., nearly, and the distance 560.4 miles.

To find the Difference of Longitude = CD :—

With the middle latitude = BCD, and the departure BC, the difference of longitude is found by trigonometry, Problem IV., page 175; as thus :

As radius = . . . $90^{\circ} 0'$ Log. co-secant = 10.000000
 Is to the departure = 436 miles, Log. = . . . 2.639486
 So is the middle lat. = $37^{\circ}51'$ Log. secant = . 10.102582

To the diff. of long. = 552.2 miles, Log. = . . . 2.742068

Longitude of Cape Agulhas = . . . $20^{\circ}18'$ E.

Diff. of longitude = 552.2 miles, or = . 9.12 E.

Longitude at which the ship arrived = . 29.30 E.

To find the Course, Distance, and Difference of Longitude, by Inspection:—

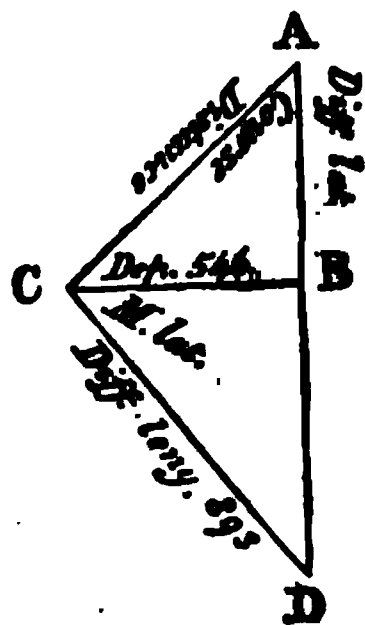
Half the difference of latitude = 176, and half the departure = 218, are found to agree nearest at 51° under or over distance 280 : hence, $280 \times 2 = 560$ miles, is the distance. Again, to middle latitude 37° as a course, and departure 218, in a latitude column, the corresponding distance is 273 ; and to latitude 38° and departure 218, the distance is 277 : hence, the change of distance to 1° of latitude, is 4 miles. Now, $4' \times 51' + 60 = 3'.4$, which, added to 273, gives 276.4 ; and this, being multiplied by 2, gives 552.8 miles ; which very nearly corresponds with the result by calculation.

PROBLEM VIII.

Given one Latitude, Departure, and Difference of Longitude; to find the other Latitude, Course, and Distance.

Example.

A ship from the Snares, New Zealand, in latitude $48^{\circ}3'$ S., and longitude $166^{\circ}20'$ E., sailed upon a direct course between the south and west, till she was found by observation to be in longitude $151^{\circ}27'$ E., and to have made 546 miles of departure ; required the latitude come to, the course steered, and the distance sailed ?



Longitude of the Snares = . . 166°20' E.

Long. of the ship by observation = 151.27 E.

Difference of longitude = . . . 14°53' = 893 miles.

To find the Middle Latitude = the Angle B C D :—

With the departure = B C, and the difference of longitude = C D, the angle of the middle latitude may be found by trigonometry, Problem III., page 174 ; as thus :

As the diff. of long. = 893 miles, Log. ar. comp. = 7.049148

Is to radius = . . . 90° 0' 0" Log. sine = . 10.000000

So is the departure = 546 miles, Log. = . . 2.737193

To the mid. lat. = 52°18'28" Log. co-sine = 9.786341

Twice mid. lat. = 104°37' 0" nearly.

Lat. of the Snares = 48. 3. 0 S.

Latitude come to = 56°34' 0" S.

Diff. of latitude = 8°31' 0" = 511 miles.

To find the Course = the Angle A :—

With the difference of latitude A B, and the departure B C, the course may be found by trigonometry, Problem IV., page 175 ; as thus :

As the diff. of lat. = 511 miles, Log. ar. comp. = 7.291579

Is to radius = . . . 90° 0' 0" Log. sine = . 10.000000

So is the departure = 546 miles, Log. = . . 2.737193

To the course = . . 46°53'48" Log. tangent = 10.028772

To find the Distance = A C :—

With the angle of the course, thus found, and the difference of latitude A B, the distance may be computed by trigonometry, Problem IV., page 175 : hence,

As radius = . . . 90° 0' 0" Log. co-secant = 10.000000

Is to diff. of lat. = 511 miles, Log. = . . . 2.708421

So is the course = 46°53'48" Log. secant = . 10.165378

To the distance = . 747.8 miles, Log. = . . . 2.873799

Hence, the course is S. $46^{\circ}54'$ W., or S.W. $\frac{1}{4}$ W. nearly, and the distance 747.8 miles.

To find the Latitude come to, Course, and Distance, by Inspection in the general Traverse Table:—

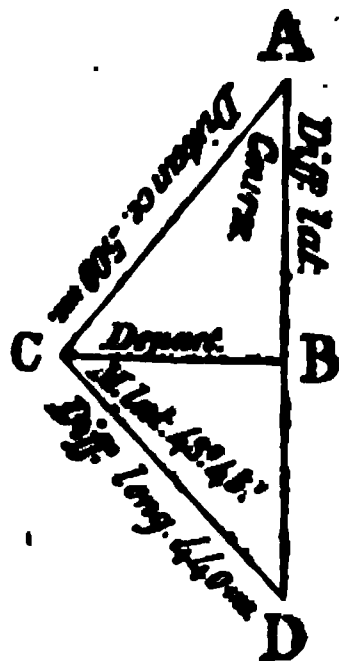
One-fourth of the difference of longitude = $223\frac{1}{4}$, taken as distance, and one-fourth of the departure = 136.5, in a latitude column, will be found to agree between 52° and 53° . Now, to latitude 52° , and distance 223, the difference of latitude is 137.3, which is $0'.8$ more than 136.5; and to latitude 53° , and distance 223, the difference of latitude is 134.2, being $2'.3$ less than 136.5: hence, the difference of meridional distance to 1° of latitude is $0'.8 + 2'.3 = 3'.1$: therefore, as $3'.1 : 0'.8 :: 60' : 16'$, which, added to 52° (proportion being made for the quarter of a mile in the distance), gives the middle latitude = $52^{\circ}18\frac{1}{2}'$: hence, the latitude come to is $56^{\circ}34'$ S., and the difference of latitude 511 miles. Again, to one-fourth of the difference of latitude = 127.75, and one-fourth of the departure = 136.5, the course is 47° , and the distance 187; which, multiplied by 4, gives 748 miles = the whole distance.

PROBLEM IX.

Given the Distance, Difference of Longitude, and Middle Latitude; to find the Course and both Latitudes.

Example.

A ship, in north latitude, sailed 500 miles upon a direct course between the south and west, until her difference of longitude was 440 miles; required the course steered, the latitude sailed from, and the latitude come to; allowing the middle latitude to be $43^{\circ}45'$ north?



To find the Angle of the Course = A:—

The course may be found by the 3d analogy, page 222, as thus:

As the distance = . . . 500 miles, Log. ar. comp. = 7.301080

Is to the diff. of longitude = 440 miles, Log. = . . . 2.648458

So is the middle latitude = $43^{\circ}45' 0''$ Log. co-sine = 9.858756

To the course = . . . S. $39^{\circ}28'14''$ W. Log. sine = 9.803289

To find the Difference of Latitude = A B :—

The difference of latitude may be found by the 8th analogy, page 222, as thus :

As the course = . . . 39°28'14" Log. co-tangent = 10.084350
 Is to the middle latitude = 43.45. 0 Log. co-sine = . . . 9.858756
 So is the diff. of long. = . 440 miles, Log. = 2.643453

To the diff. of latitude = 386 miles, Log. = 2.586559

Middle latitude = 43°45' N.

Half the diff. of lat. = 193 miles, or = . 3.13 S.

Latitude of the place sailed from = . . . 46°58' N.

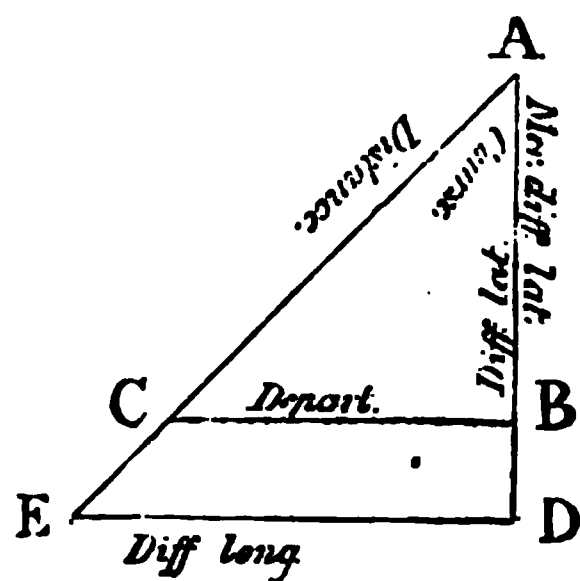
Latitude of the place come to = . . . 40.32 N.

SOLUTION OF PROBLEMS IN MERCATOR'S SAILING.

Mercator's Sailing is the method of finding, on a plane surface, the motion of a ship upon any assigned point of the compass, which shall be true in latitude, longitude, and distance sailed.

Mariners, generally speaking, solve all the practical cases in Mercator's Sailing by stated rules, called *canons*, which they early commit to memory, and, ever after, employ in the determination of a ship's place at sea. Those *canons*, certainly, hold good in most cases; but since they are destructive of the best principles of science, inasmuch as that they have a direct tendency to remove from the mind every trace of the elements of trigonometry, the very doctrine from which they were originally deduced, and on which the whole art of navigation is founded, the following observations and consequent analogies are, therefore, submitted to the attention of naval people, under the hope that they will serve as an inducement to the substitution of the rules of reason for the *rules of rote*; and thus do away with the necessity of getting *canons by heart*.

In the annexed diagram, let the triangle A B C be a figure in plane sailing, in which the angle A represents the course, A C the distance, A B the difference of latitude, and B C the departure. If A B be produced to D, until it is made equal to the meridional difference of latitude, and D E be drawn at right angles thereto, and parallel to B C; then the triangle A D E will be a figure in Mercator's sailing, in which the angle A represents the course, the side A D the meridional difference of latitude, and the side D E the difference of longitude. Now, since the two triangles A B C and A D E are right angled,



and that the angle A is common to both; therefore they are equi-angular: and because they are equi-angular, they are also similar; therefore the sides containing the equal angles of the one are proportional to the sides containing the equal angles of the other.—Euclid, Book VI., Prop. 4.

Now, from the relative properties of those two triangles, all the analogies for the solution of the different cases in Mercator's sailing may be readily deduced agreeably to the established principles of right angled trigonometry, as given in page 171, and thence to 177; as thus:—

First, in the triangle A B C, if the distance A C be made radius, the analogies will be,

1. As radius : distance A C :: sine of the course A : departure B C ;
and :: co-sine of the course A : difference of latitude A B.
2. As sine of the course A : departure B C :: radius : distance A C ;
and :: co-sine of the course A : difference of latitude A B.
3. As co-sine of the course A : difference of latitude A B :: radius : distance A C ; and :: sine of the course A : departure B C.
4. As the distance A C : radius :: departure B C : sine of the course A ;
and :: difference of latitude A B : co-sine of the course A.

Again, by making the difference of latitude A B radius, the analogies will be,

5. As the difference of latitude A B : radius :: departure B C : tangent of the course A ; and :: distance A C : secant of the course A.
6. As radius : difference of latitude A B :: tangent of the course A : departure B C ; and :: secant of the course A : distance A C.

And by making the departure B C radius, it will be,

7. As the departure B C : radius :: difference of latitude A B : co-tangent of the course A ; and :: distance A C : co-secant of the course A.
8. As radius : departure B C :: co-tangent of the course A : difference of latitude A B ; and :: co-secant of the course A : distance A C.

Now, in the triangle A D E, if the meridional difference of latitude A D be made radius, the analogies will be,

9. As the meridional difference of latitude A D : radius :: difference of longitude D E : tangent of the course A.
10. As radius : meridional difference of latitude A D :: tangent of the course A : difference of longitude D E.

And by making the difference of longitude D E radius, it will be,

11. As the difference of longitude $D E$: radius :: meridional difference of latitude $D E$: co-tangent of the course A .

12. As radius : difference of longitude $D E$:: co-tangent of the course A : meridional difference of latitude $A D$.

Finally, since the triangles $A B C$ and $A D E$ are equi-angular and similar, we have,

13. As the difference of latitude $A B$: departure $B C$:: meridional difference of latitude $A D$: difference of longitude $D E$.

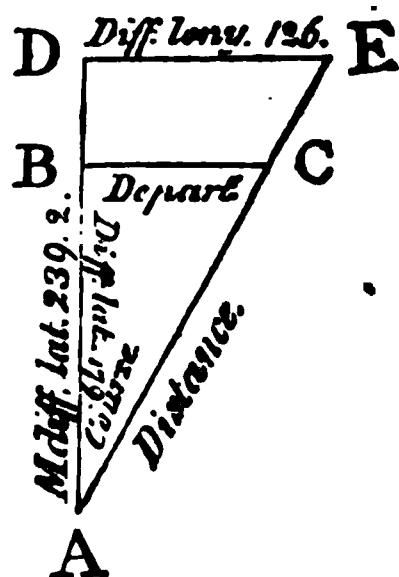
The meridional difference of latitude is found by means of Table XLIII., by the same rules as those for the difference of latitude given at page 214; as thus :—If the two given latitudes be of the same name, the difference of their corresponding meridional parts will be the meridional difference of latitude; but if the latitudes be of contrary names, the sum of these parts will be the meridional difference of latitude.

PROBLEM I.

Given the Latitudes and Longitudes of two Places ; to find the Course and Distance between them.

Example.

Required the course and distance between Cape Bajoli, in latitude $40^{\circ}3' N.$, and longitude $3^{\circ}52' E.$, and Cape Sicie, in latitude $43^{\circ}2' N.$, and longitude $5^{\circ}58' E.$?



Lat. of C. Bajoli $40^{\circ} 3' N.$ Merid. pts. 2626.6. Longitude $3^{\circ}52' E.$

Lat. of C. Sicie $43. 2 N.$ Merid. pts. 2865.8. Longitude $5. 58 E.$

Diff. of latitude $2^{\circ}59'$ Merid. diff. lat. 239.2. Diff. long. $2^{\circ} 6'$

$= 179$ miles.

$= 126$ miles.

To find the Course = Angle A :—

This comes under the 9th analogy, in page 237 : hence,

As the merid. diff. of lat. = 239.2 miles, Log. ar. comp. = 7.621239

Is to radius = $90^{\circ} 0' 0''$ Log. sine = . 10.000000

So is the diff. of long. = . 126 miles, Log. = . . 2.100371

To the course = . . . $27^{\circ}46'.42''$ Log. tangent = 9.721610

To find the Distance = AC :—

This comes under the 6th analogy, in page 237 : hence,

As radius = . . . 90° 0' 0"	Log. co-secant = . . .	10.000000
Is to the diff. of lat. = 179 miles,	Log. =	2.252853
So is the course = . 27° 46' 42"	Log. secant =	10.053176
		<hr/>
To the distance = . 202.3 miles,	Log. =	2.306029

Hence, the true course from Cape Bajoli to Cape Sicie is N. 27° 47' E., or N.N.E. $\frac{1}{4}$ E. nearly, and the distance 202.3 miles.

To find the Course and Distance, by Inspection in the general Traverse Table :—

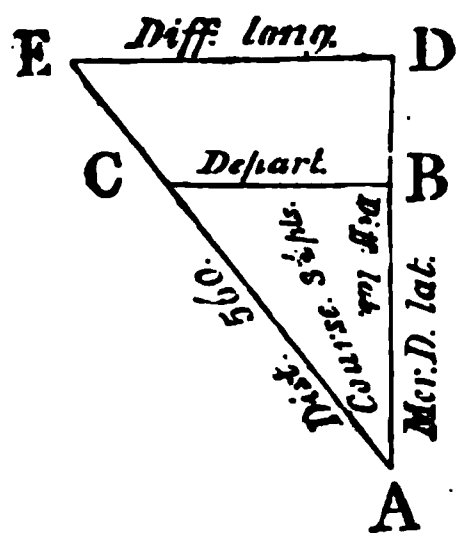
The meridional difference of latitude 239.2, and the difference of longitude 126, as departure, are found to agree nearest at 28°, which, therefore, is the course. Now, to course 28°, and difference of latitude 179, the corresponding distance is 203 miles ; which nearly agrees with the result by calculation.

PROBLEM II.

Given the Latitude and Longitude of the Place sailed from, the Course and Distance ; to find the Latitude and Longitude of the Place come to.

Example.

A ship from Cape Ortegal, in latitude 43° 47' N., and longitude 7° 49' W., sailed N.W. $\frac{1}{4}$ N. 560 miles ; required the latitude and longitude come to ?



To find the Difference of Latitude = AB :—

This comes under the 1st analogy, page 237 : hence,

As radius = 90° 0'	Log. co-secant = . . .	10.000000
Is to the distance = . 560 miles,	Log. =	2.748188
So is the course = . . 3½ points,	Log. co-sine = . . .	9.888185
		<hr/>
To the diff. of latitude = 432.9 miles,	Log. =	2.636373

To find the Difference of Longitude = 11 —

The course under the 1st analogy, is page 207 : —

As radius = . . . 90° 0' Log. co-secant = . . . 10.00000
 Is as merid. diff. of lat. = 641 miles Log. = . . . 2.80656
 So is the course = . . . $3\frac{1}{2}$ points Log. tangent = . . . 9.51173

To the diff. of long. = 526 miles Log. = . . . 2.721029

Lat. of C. Omega $42^{\circ} 45' N.$ Mer. pa 242.5. Long. of C. Omega $7^{\circ} 45' W.$
 Diff. lat. = 641 N. or $7^{\circ} 12' N.$ Diff. long. = 526 or $7^{\circ} 40' W.$

Latitude come to = $51^{\circ} 0' N.$ Mer. pa 250.5. Long. come to = $15^{\circ} 35' W.$

Merid. diff. of lat. = 641.0

To find the Difference of Latitude and Difference of Longitude, by Inspection :—

To course $3\frac{1}{2}$ points, and half the distance = 250, the difference of latitude is 216.4; the double of which, or 432.8, is the difference of latitude: hence, the latitude come to is $51^{\circ} 0' N.$, and the meridional difference of latitude 641.

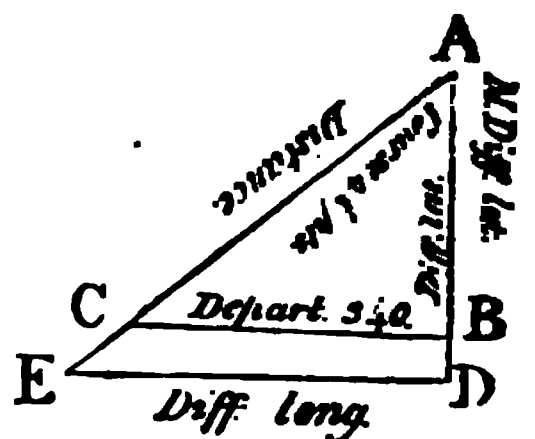
Now, to course $3\frac{1}{2}$ points, and one-third of the meridional difference of latitude = 213.7, the corresponding departure is 175.4, proportion being made for the excess of the given, above the tabular difference of latitude; then $175.4 \times 3 = 526$ miles; which, therefore, is the difference of longitude.

PROBLEM III.

Given the Latitude and Longitude of the Place sailed from, the Course, and the Departure, to find the Distance sailed, and the Latitude and Longitude of the Place come to.

Example.

A ship from Wreck Hill, Bermudas, in latitude $32^{\circ} 15' N.$, and longitude $64^{\circ} 47' W.$, sailed S.W. $\frac{1}{2} W.$, and made 340 miles of departure; required the distance sailed, and the latitude and longitude come to?



To find the Distance = A C :—

This comes under the 8th analogy, page 237 : hence,

As radius =	90° 0'	Log. co-secant =	10.000000
Is to the departure =	340 miles,	Log. =	2.531479
So is the course =	4½ points,	Log. co-secant =	10.111815
			<hr/>
To the distance =	439.8 miles,	Log. =	2.643294

To find the Difference of Latitude = A B :—

This comes under the 8th analogy, page 237 : hence,

As radius =	90°	Log. co-secant =	10.000000
Is to the departure 340 miles,	Log. =		2.531479
So is the course =	4½ points,	Log. co-tangent =	9.914173
			<hr/>
To the diff. of lat. =	279 miles,	Log. =	2.445652

Lat. of Wreck Hill, Bermudas, 32° 15' N. Merid. parts = 2046.1
 Diff. of latitude = 279 miles, or 4.39 S.

Latitude come to =	27° 36' N.	Merid. parts =	1724.0
			<hr/>
Meridional difference of latitude =	322.1		

To find the Difference of Longitude D E :—

This comes under the 10th analogy, page 237 : hence,

As radius =	90° 0'	Log. co-secant =	10.000000
Is to merid. diff. of lat. =	322.1 miles,	Log. =	2.507991
So is the course =	4½ points,	Log. tangent =	10.085827
			<hr/>
To the diff. of long. =	392.5 miles,	Log. =	2.593818

Longitude of Wreck Hill, Bermudas, = 64° 47' W.
 Difference of longitude = 392.5 miles, or = 6.32 W.

Longitude come to =	71° 19' W.
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The distance sailed is 440 miles, very nearly.

To find the Distance sailed, and the Latitude and Longitude come to,
 by Inspection :—

To the course 4½ points, and half the departure = 170, the corresponding

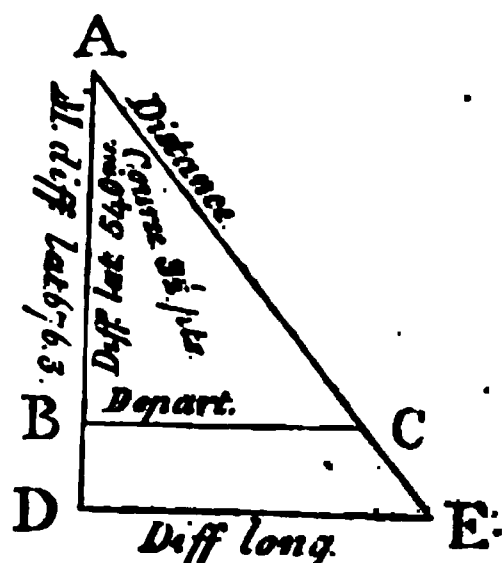
difference of latitude is 139.6, under distance 220; twice the latter, or 440 miles, is, therefore, the distance sailed; and twice 139.6 = 279.2 miles, or $4^{\circ}39'$, is the difference of latitude: whence the latitude in, is $27^{\circ}36'$ N., and the meridional difference of latitude 322.1. Now, to course $4\frac{1}{2}$ points, and half the meridional difference of latitude = 161 miles, in a latitude column, the corresponding departure is 196.3; the double of which, or 392.6 miles, is the difference of longitude: hence, the longitude come to is $71^{\circ}19\frac{1}{2}'$ W.

PROBLEM IV.

Given both Latitudes and the Course; to find the Distance and the Longitude in.

Example.

A ship from the east end of Martha's Vineyard, in latitude $41^{\circ}21'$ N., and longitude $70^{\circ}24'$ W., sailed S.E. $\frac{1}{2}$ S., and, by observation, was found to be in latitude $32^{\circ}21'$ N.; required the distance sailed, and the longitude at which she arrived?



Lat. of the east end of

Martha's Vineyard = $41^{\circ}21'$ N.

Lat. in, by observation = 32.21 N.

Merid. parts = 2729.5

Merid. parts = 2053.2

Difference of latitude = $9^{\circ} 0' = 540$ miles. Merid. diff. of lat. = 676.3

To find the Distance = AC:—

This comes under the 6th analogy, page 237; therefore,

As radius = . . $90^{\circ}0'$ Log. co-secant = . . 10.000000

Is to the diff. of lat. 540 miles, Log. = . . 2.732394

So is the course = $3\frac{1}{2}$ pts. Log. secant = . . 10.111815

To the distance = 698.6 miles, Log. = . . 2.844209

To find the Difference of Longitude = DE:—

This comes under the 10th analogy, page 237; therefore,

As radius = . . 90°0' Log. co-secant = 10.000000
 Is to mer. diff. lat. 676.3 miles, Log. = 2.830139
 So is the course = 3½ pts. Log. tangent = . 9.914173

To the diff. of long. = 555 miles, Log. = 2.744312

Longitude of east end of Martha's Vineyard = 70°24' W.

Difference of longitude = 555 miles; or = 9.15 E.

Longitude at which the ship arrived = . . 61° 9' W.

To find the Distance, and the Difference of Longitude, by Inspection.

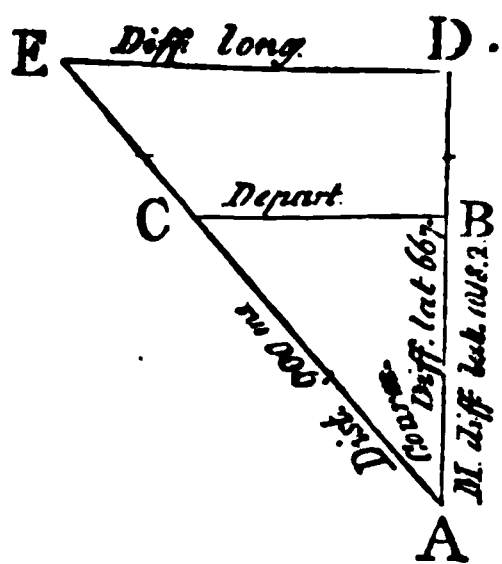
To the course 3½ points, and one third of the difference of latitude = 180, the corresponding distance is 233; which multiplied by 3, gives 699 miles, the distance.—And, to course 3½ points, and one third of the meridional difference of latitude = 225.4, in a latitude column, the corresponding departure is 185.2; now, this being multiplied by 3, gives the difference of longitude = 555.6 miles.

PROBLEM V.

Given both Latitudes and the Distance; to find the Course and Difference of Longitude.

Example.

A ship from Urris Head, Broad Haven, in lat. 54°21' north, and longitude 10°2' west, sailed 900 miles upon a direct course between the south and west, and then by observation was found to be in latitude 43°14' north; required the course steered, and the longitude come to?



Lat. of Urris Head = 54°21' N. Meridional parts = . 3900.5
 Latitude by observ. = 43.14 N. Meridional parts = . 2882.3

Difference latitude = 11° 7' = 667 miles. Merid. diff. lat. = . 1018.2

To find the Course = Angle A :—

This falls under the 4th analogy, page 237: hence,

As the distance = . . . 900 miles, Log. ar. compt. 7.045757
 Is to radius = . . . 90°0' Log. sine = .10.000000
 So is the diff. of lat. . . 667 miles, Log. = . . . 2.824126
 To the course = 42°10'26" Log. co-sine = 9.869883

To find the Difference of Longitude = D E:—

This falls under the 10th analogy, page 237 : hence,

As radius = . . . 90°0' Log. co-secant = 10.000000
 Is to mer. diff. of lat. 1018.2 miles, Log. = . . . 3.007834
 So is the course = 42°10'26" Log. tangent = 9.957087

To the diff. longitude 922.4 miles, Log. = . . . 2.964921

Longitude of Urris Head = 10° 2' W.

Difference of long. = 922.4 miles, or = 15.22 W.

Longitude come to = 25°24' W.

To find the Course, and the Difference of Longitude by Inspection.

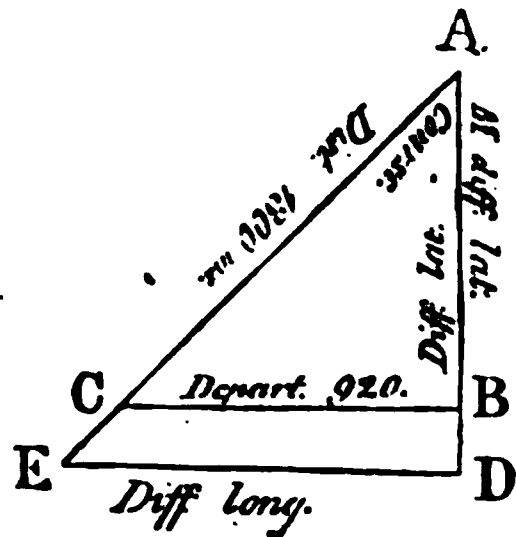
One-fourth of the distance = 225, and one-fourth of the difference of latitude = 166.7, are found to agree at $3\frac{3}{4}$ points; which, therefore, is the course.—Now, to course $3\frac{3}{4}$ points, and *one fifth* of the meridional difference of latitude = 203.6, the departure nearest agreeing is 184.7; this being multiplied by 5, gives 923.5 miles; which, therefore, is the difference of longitude:—differing about one mile from the result by calculation.

PROBLEM VI.

Given one Latitude, Distance, and Departure; to find the other Latitude, the Course, and the Difference of Longitude.

Example.

A ship from Cape St. Vincent, in latitude 37°3' north, and longitude 9°0' west, sailed 1300 miles upon a direct course between the south and west, until her departure was 920 miles; required the course steered, and the latitude and longitude at which the ship arrived?



To find the Course = Angle A :—

This comes under the 4th analogy, page 237 ; hence,

As the distance = 1300 miles,	Log. ar. compt.	6.886057
Is to radius = 90°0'	Log. sine = . .	10.000000
So is the depart. = 920 miles,	Log. = . . .	2.963788
		<hr/>
To the course = 45°2'52"	Log. sine = . .	9.849845

To find the Difference of Latitude = A B :—

This comes under the 1st analogy, page 237 ; hence,

As radius = 90°0'	Log. co-secant =	10.000000
Is to the dist. = 1300 miles,	Log. = . . .	3.113943
So is the course = 45°2'52"	Log. co-sine =	9.849125
		<hr/>
To the diff. lat. 918.5 miles,	Log. = . . .	2.963068

Latitude of Cape St. Vincent = 37° 3' N. Meridional parts = 2396.4
 Diff. of lat. = 918.5 miles, or = 15.19 S.

Lat. at which the ship arrived = 21°44' N. Meridional parts = 1336.4
 Meridional difference of latitude = . . . 1060.0

To find the Difference of Longitude = D E :—

This comes under the 10th analogy, page 237 ; hence,

As radius = . 90°0'	Log. co-secant =	10.000000
Is to mer. diff. lat. 1060 miles,	Log. = . . .	3.025306
So is the course = 45°2'52"	Log. tangent = .	10.000724
		<hr/>
To the diff. long. = 1061.8 miles,	Log. =	3.026030

Long. of Cape St. Vincent = 9° 0' W.
 Diff. of longitude = 1061.8 miles, or = . . 17.42 W.

Longitude at which the ship arrived = . . 26°42' W.

The course steered is S. 45°3' W., or S. W., very nearly.

To find the Course, and the Latitude and Longitude by Inspection.

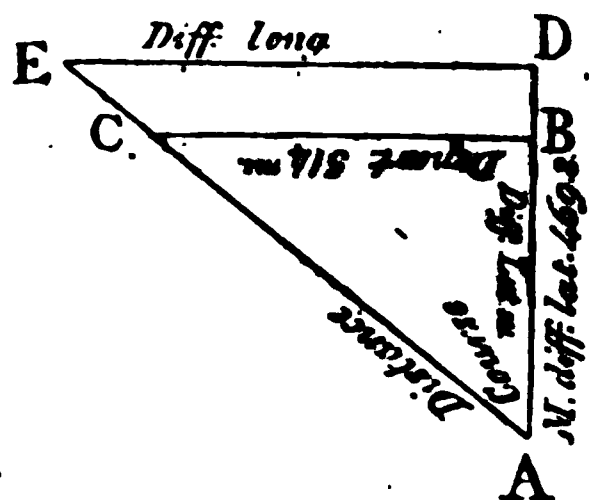
One-fifth of the distance = 260, and one-fifth of the departure = 184, are found to agree nearest at 45° ; which, therefore, is the course; and the corresponding difference of latitude is 183.8, which, multiplied by 5, gives 919 miles, the difference of latitude: whence, the latitude come to is $21^\circ 44'$ north, and the meridional difference of latitude 1060 miles. Now, to the course 45° , and one fifth of the meridional difference of latitude = 212, in a latitude column, the corresponding departure is 212.1; and this being multiplied by 5, gives the difference of longitude = 1060.5 miles; hence, the longitude at which the ship arrived is $26^\circ 41'$ west.

PROBLEM VII.

Given both Latitudes, and the Departure; to find the Course, Distance, and Difference of Longitude.

Example.

A ship from Cape Cantin, in latitude $32^\circ 33'$ north, and longitude $9^\circ 15'$ west, sailed upon a direct course between the north and west, until, by observation, she was found to be in latitude $38^\circ 54'$ N., and to have made 514 miles of departure; required the course steered, the distance sailed, and longitude come to?



Lat. of Cape Cantin = $32^\circ 33'$ N.

Merid. parts = 2067.4

Lat. in by observation = 38.54 N.

Merid. parts = 2537.2

Difference of Lat. = $6^\circ 21' = 381$ miles. Merid. diff. of Lat. = 469.8 ms.

To find the Course = the Angle A :—

This comes under the 5th analogy, page 237; hence,

As the difference latitude = 381 miles, Log. ar. compt. = 7.419075

Is to radius = . . . $90^\circ 0'$ Log. sine = 10.000000

So is the departure = 514 miles, Log. = . . . 2.710963

To the course = $53^\circ 27' 9''$ Log. tangent = 10.130038

To find the Distance = A C :

This comes under the 6th analogy, page 237 ; hence,

As radius = . . .	90°0' :	Log. co-secant =	10.000000
Is to the diff. lat.	381 miles,	Log. = . . .	2.580925
So is the course =	53°27'9" :	Log. secant =	10.225126
			<hr/>
To the distance =	639.8 miles,	Log. = . . .	2.806051

To find the Difference of Longitude = D E :—

This comes under the 10th analogy, page 237 ; hence,

As radius = . . .	90°0' :	Log. co-secant =	10.000000
Is to mer. diff. lat. =	469.8 miles,	Log. = . . .	2.671913
So is the course =	53°27'9" :	Log. tangent =	10.180039
			<hr/>
To diff. of long. =	633.8 miles,	Log. = . . .	2.801952

Longitude of Cape Cantin = 9°15' W.

Difference of long. = 633.8 miles, or = . 10.34 W.

Longitude come to = 19°49' W.

Note.—The difference of longitude D E may be readily found, independent of the course, by the 13th analogy, page 238.

To find the Course, Distance, and Difference of Longitude by Inspection :—

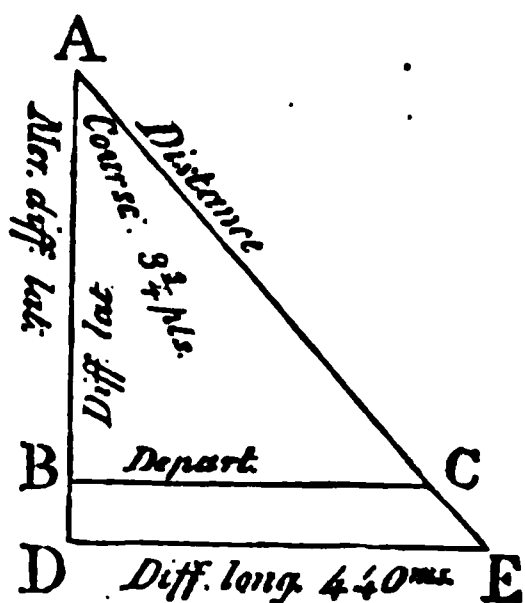
One-third of the difference of latitude = 127, and one-third of the departure = 171.3, are found to agree nearest abreast of 4½ points, the course, and under distance 213 : hence, $213 \times 3 = 639$ is the distance sailed. Again, to the course 4½ points, and one third of the meridional difference of latitude = 156.6, in a latitude column, the corresponding departure is 211.2, which, multiplied by 3, gives 633.6 ; and hence the longitude 19°49' west : being nearly the same as by calculation.

PROBLEM VIII.

Given One Latitude, Course, and Difference of Longitude ; to find the Distance and the other Latitude.

Example.

A ship from Port Dauphin, Madagascar, in latitude $25^{\circ}5'$ south, and longitude $46^{\circ}35'$ east, sailed S. E. $\frac{1}{4}$ S. till she was found by observation, to be in longitude $53^{\circ}55'$ east ; required the distance sailed, and the latitude at which the ship arrived ?



Longitude of Port Dauphin = $46^{\circ}35'$ E.

Longitude in by observation = 53.55 E.

Difference of Longitude = . . $7^{\circ}20'$ = 440 miles.

To find the Meridional Difference of Latitude = A D :—

This comes under the 12th analogy, page 238 ; hence,

As radius = . . . $90^{\circ}0'$	Log. co-secant = . . .	10.000000
Is to the diff. long. 440 miles,	Log. = . . .	2.643453
So is the course = $3\frac{1}{4}$ pts.	Log. co-tangent =	10.042705

To mer. diff. lat. = 485.5 miles,	Log. = . . .	2.686158
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Latitude of Port Dauphin $25^{\circ}5'$ S.	Mer. parts =	1555.5 S.
Meridional diff. of lat. =		485.5 S.

Latitude come to =	$32^{\circ}11'$ S.	Mer. parts =	2041.0
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Difference of lat. = $7^{\circ}6'$ = 426 miles.

To find the Distance = A C :—

This comes under the 6th analogy, page 237 ; hence,

As radius = . . . $90^{\circ}0'$	Log. co-sec. =	10.000000
Is to the diff. of lat. = 426 miles,	Log. = . . .	2.629410
So is the course = $3\frac{1}{4}$ points	Log. secant =	10.130210

To the distance = 574.9 miles,	Log. = . . .	2.759620
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To find the Latitude come to, and the Distance Sailed by
Inspection :—

To the course $3\frac{1}{2}$ points, and one third of the difference of longitude = 146. 7, in a departure column, the corresponding difference of latitude is 161. 5, which, multiplied by 3, gives the meridional difference of latitude = 484. 5 miles ; hence, the latitude come to is $32^{\circ}10'$ south, and the difference of latitude 425 miles.—Now, to one third of the difference of latitude, thus found, = 141. 7, and the course $3\frac{1}{2}$ points, the corresponding distance is 191 ; and this being multiplied by 3, gives the distance sailed = 573 miles.

PROBLEM IX.

To find the Course, Distance, Difference of Latitude, and Difference of Longitude made good, upon Compound Courses ; and also the bearing and Distance from the Ship to her intended Port, by Inspection in the General Traverse Table.

RULE.

Make a Table of any convenient size, as that to the following example, and divide it into six columns : in the first of these place the several courses, taken from the log board, corrected for lee way, if any, and variation ; and in the second, their corresponding distances. The third and fourth columns are to contain the differences of latitude, and, therefore, to be marked N. S. at top ; and the fifth and sixth, the departures, which are to be marked E. W. at top also.

Find, by Problem I., page 107, the difference of latitude and departure answering to each corrected course and distance, and place them in their respective columns : then, the difference between the sums of the north and south columns will be the whole difference of latitude made good, of the same name with the greater quantity ; and the difference between the sums of the east and west columns will be the whole departure, of the same name with the greater meridian distance.

Remark.—The courses taken from the log board are to be corrected for variation and lee-way, if any, as thus.—If the variation be easterly, it is to be allowed to the right hand of the course steered by compass ; but to the left hand if westerly.—And,

If the larboard tacks be aboard, the leeway is to be allowed to the right hand of the course steered ; but to the left hand if the starboard tacks be aboard.

To find the Course and Distance made good :—

To the whole difference of latitude and departure, so found, find the corresponding course and distance by Problem II, page 108, and thus the course and distance made good will be obtained.

To find the Latitude in, by Account, or Dead Reckoning :—

If the difference of latitude and the latitude of the place from which the ship's departure was taken, or the yesterday's latitude, be of the same name, their sum will be the latitude in, by account; but if they are of contrary names, their difference will be the latitude in, of the same name with the greater quantity.

To find the Difference of Longitude :—

With the course made good, and the meridional difference of latitude, in a latitude column, find the corresponding departure, by Problem III. page 110, and it will be the difference of longitude.

Or.—With the middle latitude as a course, and the departure, in a latitude column, find the corresponding distance, by Problem V., page 111, and it will be the difference of longitude.

To find the Longitude in, by Account, or Dead Reckoning :—

If the difference of longitude and the longitude of the place from which the ship's departure was taken, or the yesterday's longitude, be of the same name, their sum will be the longitude in, by account, when it does not exceed 180° ; otherwise, it is to be taken from 360° , and the remainder will be the longitude in, of a contrary name to that left :—but, if the difference of longitude and the longitude left are of contrary names, their difference will be the longitude in, of the same name with the greater quantity.

To find the Bearing and Distance from the Ship to the Port to which she is bound :—

By Mercator's Sailing.

With the meridional difference of latitude, in a latitude column, and the difference of longitude, as departure, find the course, by Problem IV. page 111; then, with the course, thus found, and the difference of latitude, the distance is to be obtained by the same Problem.—Or,

By Middle Latitude Sailing.

With the middle latitude between the ship and the proposed place, as a course, and the difference of longitude, as distance, find the corresponding

meridional distance, or departure, by Problem VI. page 112; then, with this departure, and the difference of latitude, the course and distance are to be obtained by the same Problem.

Note.—The true bearing or course, thus found, may be reduced to the magnetic, or compass bearing, if necessary, by allowing the value of the variation to the right hand if westerly; and to the left hand if easterly; being the converse of reducing the course steered by compass, to the true course.

And, this rule comprises the substance of that nautical operation which is generally termed *a day's work at sea*.

Example 1.

A ship from Cape Espichell, in latitude $38^{\circ}25'$ north, and longitude $9^{\circ}13'$ west, bound for Porto Santo, in latitude $33^{\circ}3'$ north, and longitude $16^{\circ}17'$ west, by reason of contrary winds was obliged to sail upon the following compass courses; viz.—W. by S. 36 miles; N. W. by W. 110 miles; W. N. W. 95 miles; S. by E. $\frac{1}{2}$ E. 50 miles; S. by W. $\frac{1}{4}$ W. 103 miles, and S. S. W. 116 miles; the variation was 2 points westerly on the three first courses, and $1\frac{1}{4}$ point on the three last: required the course, and distance made good, the latitude and longitude at which the ship arrived; with the direct course, and distance from thence to her intended port?

TRAVERSE TABLE.					
Corrected Courses.	Distances.	Difference of Latitude.		Departure.	
		N.	S.	E.	W.
S. W. by W.	56	"	31.1	"	46.6
W. by N.	110	21.5	"	"	107.9
West.	95	"	"	"	95.0
S. E. $\frac{1}{2}$ S.	50	"	40.2	29.8	"
South.	103	"	103.0	"	"
S. $\frac{1}{4}$ W.	116	"	115.9	"	5.7
		21.5	290.2	29.8	255.2
			21.5		29.8
		Diff. Lat.=	268.7	Departure=	225.4

To find the Course and Distance made good :—

Half the difference of latitude = 134.35, and half the departure = 112.7, are found to agree nearest abreast of 40° under distance 175 ;— now, $175 \times 2 = 350$ miles.—Hence, the course made good is S. 40° W. or, S. W. $\frac{1}{2}$ S. nearly, and the distance 350 miles.

To find the Latitude and Longitude come to, by Account :—

Lat. of C. Espichell=	38°25' N.	Mer. pts. 2500. 1.	Long.=9°13' W.
Diff. of lat.=269 ms., or 4. 29	S.	Diff. long.=4. 40 W.
<hr/>			
Latitude come to =	33°56' N.	Mer. pts. 2166. 7.	Long.=13°53' W.
<hr/>			
Merid. diff. of lat.	. . .	=	333. 4

To find the Difference of Longitude made good :—

To the course made good = 40° and half the meridional difference of latitude = 166.7 the corresponding departure is 140.1, which, multiplied by 2, gives the difference of longitude 280.2 miles = $4^\circ 40'$ west.—Or,

With the middle latitude = $36^\circ 10'$, and half the departure = 112.7, in a latitude column, the corresponding distance is 139.3 (proportion being made for the 10 minutes of latitude) ; hence, $139.3 \times 2 = 278.6$ miles, the difference of longitude ; being about a mile and a half less than the result by Mercator's sailing.

To find the Course and Distance from the Ship to her intended Port :—

Lat. of the ship	33°56' N.	M. pts. 2166. 7.	Longitude 13°53' W.
Lat. Porto Santo	33. 3 N,	M. pts. 2103. 1.	Longitude 16. 17 W.
<hr/>			
Diff. of Lat. =	0°53' = 53 ms.	M. diff. L. 63. 6.	Diff. Long. 2°24' = 144 [miles.

By Mercator's Sailing.

The meridional difference of latitude = 63.6 in a latitude column, and the difference of longitude = 144, in a departure column, are found to agree nearest abreast of 66° the course.—Now, to course 66° and difference of latitude 53, the corresponding distance is 130 miles.—Or,

With the middle latitude = $33^{\circ}30'$ as a course, and the difference of longitude = 144 as distance, the corresponding difference of latitude is 120.1.—Now, with 120.1 in a departure column, and the difference of latitude = 53, in its proper column, the corresponding course is 66° and the distance 131 miles.

Hence,—The course made good is S. 40° W. or S. W. $\frac{1}{2}$ S. nearly.

The distance made good is 350 miles.

The latitude by account is $33^{\circ}56'$ north.

The long. by account is 13.53 west.

And,

Porto Santo bears from the ship S. 66° W. or W.S.W. nearly.

Distance 130 miles as required.

Note.—If the latitude and longitude of the ship, or either of them, have been deduced from celestial observations, they are to be made use of, instead of those by account, in determining the course and distance between the ship and the place to which she is bound.—See the compendium of Practical Navigation near the end of this Volume.

Example 2.

A ship from Port Royal, Jamaica, in latitude $17^{\circ}58'$ north, and longitude $76^{\circ}53'$ west, got under weigh for Hayti, St. Domingo, in latitude $18^{\circ}30'$ north, and longitude $69^{\circ}49'$ west, and sailed upon the following courses, viz. ; S. 40 miles ; S. E. by S. 97 miles ; N. by E. 72 miles ; S. E. $\frac{1}{2}$ S. 108 miles ; N. by E. $\frac{1}{2}$ E. 114 miles ; S. E. 126 miles ; N. N. E. 86 miles ; and then by observation was found to be in latitude $16^{\circ}55'$ N., and longitude $72^{\circ}30'$ W. ;—the lee-way on each of those courses was a quarter of a point (the wind being between E. S. E. $\frac{1}{2}$ S. and E. by N. $\frac{1}{2}$ N.), and the variation of the compass half a point easterly :—required the true course and distance made good ; the latitude and longitude at which the ship arrived by account, with the direct course and distance between her true place by observation and the port to which she is bound ?

TRAVERSE TABLE.					
Corrected Courses.	Distances.	Difference of Latitude.		Departure.	
		N.	S.	E.	W.
S. $\frac{1}{4}$ W.	40	"	39.6	"	5.9
S. S. E. $\frac{1}{4}$ E.	97	"	87.7	41.5	"
N. by E. $\frac{1}{4}$ E.	72	69.8	"	17.5	"
S. S. E. $\frac{3}{4}$ E.	108	"	92.6	55.5	"
N. by E. $\frac{3}{4}$ E.	114	107.3	"	38.4	"
S. E. by S. $\frac{1}{4}$ E.	126	"	101.2	75.1	"
N. N. E. $\frac{1}{4}$ E.	86	77.7	"	36.8	"
		254.8	321.1	264.8	5.9
			254.8	5.9	
		Diff. Lat. =	66.3	258.9 =	Departure

To find the Course and Distance made good :—

Half the difference of Latitude = 33.15, and half the departure = 129.45, are found to agree nearest between 75° and 76° , under distance 134; and by making proportion for the difference between the given and the tabular numbers, the true course will be found = $75^{\circ}38'$; and the distance $134, \times 2 = 168$ miles.—Hence the course made good is S. $75^{\circ}38'$ E. or E. by S. $\frac{1}{4}$ S. nearly; and the distance 268 miles.

To find the Latitude and Longitude come to, by Account :—

Lat. of Port Royal = $17^{\circ}58'$ N. Mer. pts. 1096.1 Long. = $76^{\circ}53'$ W.

Diff. Lat. 66.3, or 1. 6 S. Diff. Long. = 4.30 E.

Lat. come to by ac. = $16^{\circ}52'$ N. Mer. pts. 1026.9 Long. by Acc. $72^{\circ}23'$ W.

Merid. diff. of Lat. = 69.2

To find the Difference of Longitude made good :—

To the course made good = $75^{\circ}38'$ and the meridional difference of latitude = 69.2, the corresponding departure is 270.3, proportion being made for the $38'$ in the course beyond 75° .—Hence, the difference of longitude is 270.3, or $4^{\circ}30'$ east.—Or, with the middle latitude = $17^{\circ}25'$ as a course, and half the departure made good = 129.45 in a latitude

column, the corresponding distance, at top or bottom, is 135; which, multiplied by 2, gives the difference of longitude = 270 miles.

To find the Course and Distance from the Ship to her intended Port:—

Lat. of ship by ob. $16^{\circ}55'$ N. Mer. pts. 1030.1 Long. by ob. $72^{\circ}30'$ W.

Lat. of Hayti = 18.30 N. Mer. pts. 1129.8 Lg. of Hayti 69.49 W.

Diff. of Lat. = $1^{\circ}35'$ M.D.L. = 99.7 Diff. of Long. $2^{\circ}41'$

= 95 miles.

= 161 miles.

The meridional difference of latitude = 99.7, and difference of longitude = 161, in a departure column, are found to agree nearest between 58° and 59° under distances 188 and 194; and by making proportion for the difference between the given and the tabular numbers, the true course will be found = $58^{\circ}14'$.—Now, to course $58^{\circ}14'$ and difference of latitude 95, the corresponding distance is 180 miles.—Or, with the middle latitude = $17^{\circ}42\frac{1}{2}'$ as a course, and the difference of longitude = 161 as a distance, the corresponding difference of latitude is 153.4:—now, with 153.4, in a departure column, and the difference of latitude = 95, in its proper column, the course, nearest agreeing, is 58 degrees, and the distance 181 miles.—Hence,

The Course made good is S. $75^{\circ}38'$ E. or E. by S. $\frac{1}{4}$ S. nearly.

Distance made good = 268 miles.

Latitude come to by account = $16^{\circ}52'$ N.

Latitude by observation = $16^{\circ}55'$ N.

Long. come to by account = $72^{\circ}23'$ W.

Long. by observation = $72^{\circ}30'$ W.

Hayti bears from the ship N. $58^{\circ}14'$ E. or N. E. by E. $\frac{1}{4}$ E. nearly.

Distance 180 miles, as required.

Note.—This example and the preceding exhibit all the particulars attendant on making out a *day's work at sea*.—See more of this in the compendium of Practical Navigation near the end of this Volume.

SOLUTION OF CASES IN OBLIQUE SAILING.

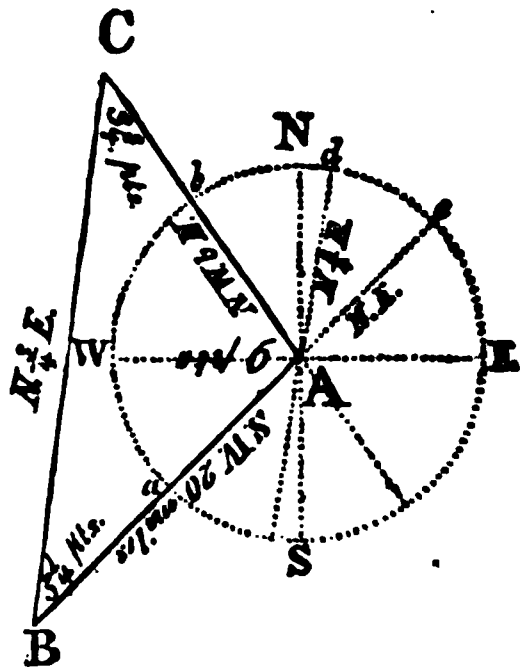
Oblique sailing is the application of oblique angled plane trigonometry to the solution of certain cases at sea: such as in coasting along shore; approaching, or leaving the land; surveying coasts and harbours, &c., where it becomes necessary to determine the distance of particular places from

the ship, and from each other.—And, also, when it is required to settle the position of any place, cape, or head-land from a ship, by observations taken on board.

Example 1.

A ship being about to take her departure from Madeira, set the Lizard Point, which bore, by azimuth compass, N. W. by N.; and after sailing S. W. 20 miles, it was again set and found to bear N. $\frac{3}{4}$ E.; required the ship's distance from the Lizard at both stations.

Solution.—In the annexed diagram let the point C represent the Lizard, and the points A and B the stations or places of the ship, whence the bearings of the point C were taken.—Now, the difference between the bearing A C = N. W. by N. and the ship's course A B = S. W. is 9 points, which is the value of the angle B A C, measured by the arc $a b$:—The difference between N. W. by N. and N. $\frac{3}{4}$ E. is $3\frac{1}{4}$ points = the angle A C B, measured by the arc $b d$; and the difference between N. $\frac{3}{4}$ E. and N. E. (the opposite point to S. W.) is $3\frac{1}{4}$ points = the angle A B C, measured by the arc $d e$.—Then, in the oblique angled triangle A B C, given the angles and the side A B, to find the sides A C and B C = the distance of the ship from the Lizard at the respective stations.—Hence, by oblique angled trigonometry, Problem I., page 177.



To find the Distance A C:—

As the angle C =	. . . $3\frac{1}{4}$ pts.	Log. co-secant =	10.172916
Is to the distance A B =	20 ms.,	Log. =	. . . 1.301030
So is the angle B =	$3\frac{1}{4}$ pts.	Log. sine =	. . . 9.775027
To the dist. A C =	17.74 ms.,	Log. =	. . . 1.248973

To find the Distance B C:—

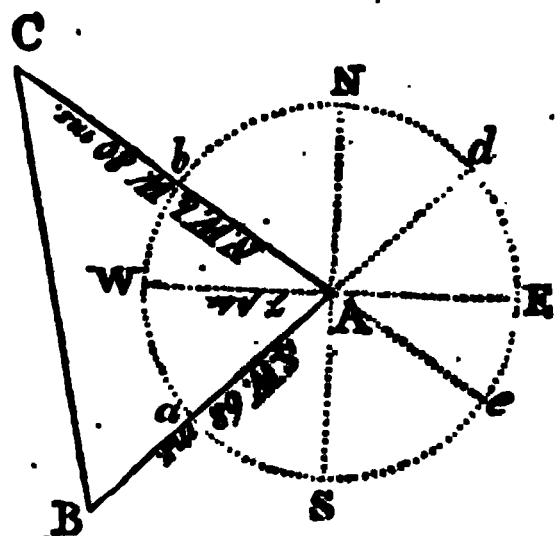
As the angle C =	. . . $3\frac{1}{4}$ pts.	Log. co-secant =	10.172916
Is to the distance A B =	20 ms.,	Log. =	. . . 1.301030
So is the angle A =	9 pts.	Log. sine =	. . . 9.991574
To the dist. B C =	29.21 miles,	Log. =	. . . 2.465520

Hence, the distance of the ship from the Lizard at the first station is $17\frac{1}{2}$ miles ; and at the second station $29\frac{1}{2}$ miles nearly.

Example 2.

Two ships sail from the same port, one N. W. by W. 80 miles, and the other S. W. 68 miles ; required the bearing and distance of those ships from each other ?

Solution.—In the annexed diagram let the side AC represent the course steered by one of the ships, and the side AB the course steered by the other ship ; and let the side BC represent the relative bearing and distance of the ships from each other.—Now, the difference between the bearing $A = \text{N. W. by W.}$ and the bearing $AB = \text{S. W.}$ is 7 points $=$ the angle BAC , measured by the arc ab .—Hence, in the oblique angled triangle ABC , given the side AC 80 miles ; the side AB 68 miles, and the included angle $A = 7$ points ; to find the other angles, and the side BC .—Therefore, by oblique angled trigonometry, Problem III., page 179,



To find the Angles B and C :—

As the sum of AB and $AC = 148$ miles, Log. ar. compt. $= 7.829736$
 Is to difference of AB and $AC = 12$ miles, Log. $= . . . 1.079181$
 So is $\frac{1}{2}$ sum of angles B and $C = 50^\circ 37' 30''$ Log. tangent $= 10.085827$

To $\frac{1}{2}$ diff. of angles B and $C = 5^\circ 38' 32''$ Log. tangent $= 8.994746$

Angle $B = . . 56^\circ 16' 2''$

Angle $C = . . 44^\circ 58' 58''$

To find the Side $BC =$ the Distance between the Ships :—

As the angle $B = 56^\circ 16' 2''$ Log. co-secant $= . . 10.080066$
 Is to the side $AC = 80$ miles, Log. $= 1.903090$
 So is the angle $A = 7$ points Log. sine $= 9.991574$

To distance $BC = 94.34$ miles, Log. $= 1.974730$

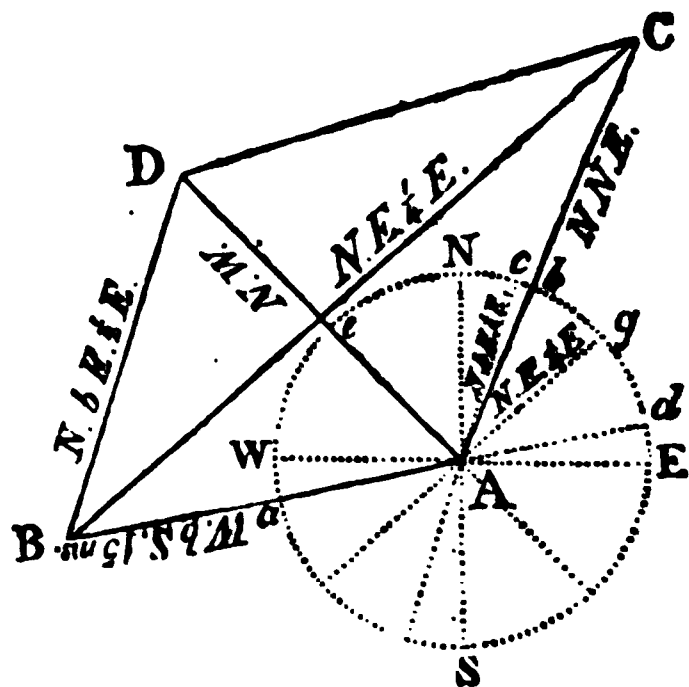
To find the relative Bearings of the Ships :—

From the angle $B = 56^{\circ}16'2''$ subtract the course from A to B $= 45^{\circ}$, and the remainder $= 11^{\circ}16'2''$ is the bearing of C from B $= N. 11^{\circ}16' W.$ or N. by W. nearly.—And from the course A C $= 56^{\circ}15'$ subtract the angle $C = 44^{\circ}58'58''$ and the remainder $= 11^{\circ}16'2''$ is the course from C to B $= S. 11^{\circ}16' E.$ or S. by E. nearly.

Example 3.

Coasting along shore two head-lands were observed ; the first bore, by azimuth compass, N.N.E., the second N.W. :—after sailing W. by S. 15 miles, the first bore N. E. $\frac{1}{2}$ E. and the second N. by E. $\frac{1}{2}$ E. ; required the relative bearing and distance of those head-lands from each other ?

Solution.—In the diagram A B D C, let the side A B represent the course steered by the ship ; A C the bearing of the first head-land, and A D the bearing of the second head-land from the place of the ship at A ; and, let B C represent the bearing of the first head-land, and B D the bearing of the second head-land from the ship's place at B.—Now, in the triangle A B D, the angles and the side A B are given, to find the side A D.—



Thus, the difference between N. W. and W. by S. is 5 points $=$ the angle B A D, measured by the arc $a e$;—the difference between N. by E. $\frac{1}{2}$ E. and E. by N. (the opposite point to W. by S. the ship's course,) is $5\frac{1}{2}$ points $=$ the angle D B A, measured by the arc $c d$, and the difference between N. by E. $\frac{1}{2}$ E. and N. W. is $5\frac{1}{2}$ points $=$ the angle A D B, measured by the arc $c e$; and the side A B $=$ 15 miles ; to find the side A D.—Hence, by oblique angled trigonometry, Problem I., page 177,

As the angle A D B $=$ $5\frac{1}{2}$ pts.	Log. co-secant $=$ 10.054570
Is to the side A B $=$ 15 miles,	Log. $=$. . . 1.176091
So is the angle A B D $=$ $5\frac{1}{2}$ pts.	Log. sine $=$. . 9.945430

To the side A D $=$ 15 miles, Log. $=$. . 1.176091

Note.—The side A D might be determined independently of calculation, as thus ; the angles B and D are equal, for each is measured by an arc of

$5\frac{1}{2}$ points; and since equal angles are subtended by equal sides, therefore the side AD is equal to the side $AB = 15$ miles.

Again.—In the triangle ABC , the angles and the side AB are given, to find the side AC ; thus, the difference between N. N. E. and W. by S. is 11 points = the angle BAC , measured by the arc ab ; the difference between N. N. E. and N. E. $\frac{1}{4}$ E. is $2\frac{1}{4}$ points = the angle ACB , measured by the arc bg , and the difference between N. E. $\frac{1}{4}$ E. and E. by N. (the opposite point to W. by S. the ship's course,) is $2\frac{1}{4}$ points = the angle ABC , measured by the arc gd :—hence, the side AC may be found by the above-mentioned Problem; as thus:

As the angle $ACB = 2\frac{1}{4}$ points Log. co-secant = 10.369008

Is to the side $AB = 15$ miles, Log. = . . . 1.176091

So is the angle $ABC = 2\frac{1}{4}$ points Log. sine = . . . 9.711050

To the side $AC = 18.03$ miles, Log. = . . . 1.256149

Now, in the triangle ADC there are given, the side $AD = 15$ miles; the side $AC = 18.03$ miles, and the included angle DAC , 6 points = the difference between N. N. E. and N. W. measured by the arc eb , to find the angles ADC and ACD , and the side DC .—Hence, by trigonometry, Problem III., page 179,

As the sum of AC and $AB = 33.03$ miles, Log. ar. compt. = 8.481091

Is to difference of AC and $AB = 3.03$. Log. = . . . 0.481443

So is $\frac{1}{2}$ sum of ang. ADC and $ACD = 56^\circ 15' 0''$ Log. tang. = 10.175107

To $\frac{1}{2}$ difference of those angles = . . . $7^\circ 49' 2''$ Log. tang. = 9.137641

Angle $ADC = . . . 64^\circ 4' 2''$

Angle $ACD = . . . 48^\circ 25' 58''$

To find the Side DC :—

As the angle $ACD = 48^\circ 25' 58''$ Log. co-secant = . . . 10.125995

Is to the side $AD = 15$ miles, Log. = . . . 1.176091

So is the angle $DAC = 6$ points Log. sine = . . . 9.965615

To the side $DC = 18.52$ miles, Log. = . . . 1.267701

Hence, the distance between the two head-lands is $15\frac{1}{2}$ miles.

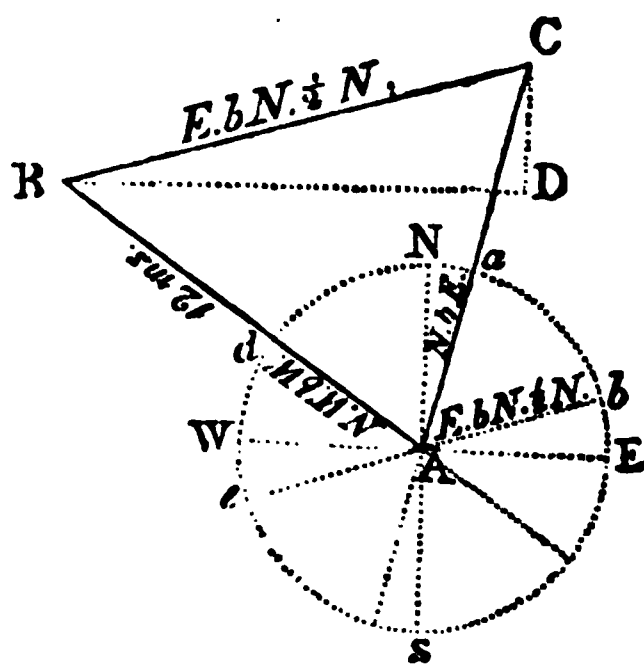
To find the relative Bearings of the two given Head-lands :—

To the angle $A C D = 48^{\circ}25'58''$ add the course or bearing from A to C = 2 points, or $22^{\circ}30'$ and the sum = $70^{\circ}55'58''$ is the bearing of D from C = S. $70^{\circ}56'$ W., or W. by S. $\frac{3}{4}$ S. nearly.—And; to the angle $A D C = 64^{\circ}4'2''$ add the bearing from A to D = 4 points, or 45 degrees, and the sum = $109^{\circ}4'2''$ being taken from 180° gives $70^{\circ}55'58''$ = the bearing of C from D = N. $70^{\circ}56'$ E. or E. by N. $\frac{3}{4}$ N. nearly.

Example 4.

Being desirous of ascertaining the exact position of a head-land, with respect to latitude and longitude, it was carefully set, by an azimuth compass, and found to bear N. b. E., and after sailing N.W. b. W. 12 miles, it was again set, and observed to bear E. b. N. $\frac{1}{2}$ N., due allowance being made for the variation of the compass. Now, the correct latitude of the ship at the last place of observation was $21^{\circ}50'21''$ N., and the longitude $85^{\circ}9'6''$ W.; required the latitude and longitude of the said head-land?

Solution.—In the oblique angled triangle ABC, where the side AC represents the first bearing of the head-land, the side BC the second bearing, and the side AB the distance sailed; given the three angles and the side AB = 12 miles, to find the side BC = the ship's distance from the headland at the second station. Thus, the difference between N. b. E., and N.W. b. W., is 6 points = the angle CAB, measured by the arc ad ; the difference between N.



W. b. W., and E. b. S. $\frac{1}{2}$ S., the opposite point to E. b. N. $\frac{1}{2}$ N., is $4\frac{1}{2}$ points = the angle ABC, measured by the arc de , and the difference between E. b. N. $\frac{1}{2}$ N., and N. b. E., is $5\frac{1}{2}$ points = the angle ACB, measured by the arc ab . Hence, by oblique angled trigonometry, Problem I., page 107, to find the side BC = the ship's distance from the head-land at the second station.

As the angle ACB = $5\frac{1}{2}$ points,	Log. co-secant =	10.054570
Is to the side AB = 12 miles,	Log. = . . .	1.079181
So is the angle CAB = 6 points,	Log. sine = .	9.965615

To the side BC = 12.57 miles,	Log. = . . .	1.099366
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Hence, the distance of the ship from the head-land at the second station is $12\frac{1}{2}$ miles, nearly.

To find the Difference of Latitude and Difference of Longitude between the Ship's Place at B, and the Head-Land C:—

In the right angled triangle B C D, given the angle C, $6\frac{1}{2}$ points = the bearing of B from C, and the distance B C = 12.57 miles, to find the difference of latitude C D, and the difference of longitude B D; therefore, by Mercator's Sailing, Problem II., page 239,

As radius = $90^{\circ}0'$ Log. secant = . . 10.000000
Is to distance B C = 12.57 miles, Log. = 1.099366
So is the course C = $6\frac{1}{2}$ points, Log. co-sine = 9.462824

To the diff. of lat. C D = 3.65 miles, Log. = 0.562190

As radius = $90^{\circ}0'$ Log co-secant = . . 10.000000
Is to mer. diff. of lat. = 3.9 miles, Log. = 0.591065
So is the course C = $6\frac{1}{2}$ points, Log. tangent = 10.518061

To the diff. of long. = 12.85 miles, Log. = 1.109126

Lat. of ship = $21^{\circ}50'21''$ N. M. pts = 1343.3 Lon. of ship = $85^{\circ}9'6''$ W.
Diff. lat. 3.65, or $3'39''$ N. Diff. lon. 12.85, or 12.51 E.

Lat. of hd. ld. $21^{\circ}54'0''$ N. M. pts = 1347.2 Lon. of hd. ld. $84^{\circ}56'15''$ W.

Meridional difference of latitude = 3.9 miles.

Hence, the latitude of the head-land is $21^{\circ}54'0''$ N., and its longitude $84^{\circ}56'15''$ W.

Note.—The foregoing examples contain all the cases in oblique sailing that are of any immediate import to the mariner. Other examples, indeed, might be given; but since they would rather tend to the exercise of the mind on trigonometrical subjects, than to any useful nautical purpose, they have therefore been intentionally omitted.

The two last examples will be found particularly useful in maritime surveying, when the operations are conducted on board of a ship or vessel.

SOLUTION OF CASES IN WINDWARD SAILING:

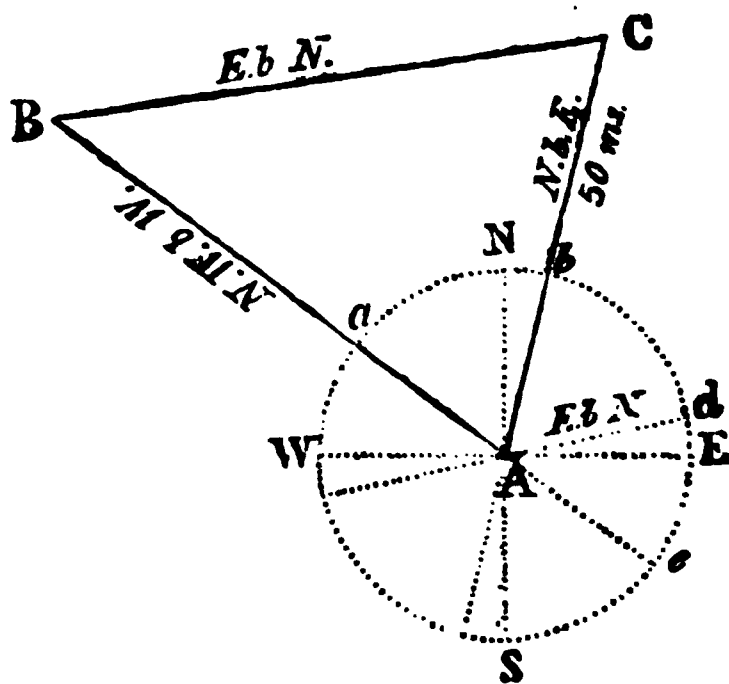
Windward Sailing is the method of reaching the port or place bound to by the shortest route, when the wind is in a direction contrary to the direct course between the ship and the place to which she is bound.

When the wind is opposed to the course which a ship should steer from any one port to another, she is obliged to sail upon different tacks, close-hauled to the wind, in order to reach the port bound to. The object, therefore, of this method of sailing, is to find the proper course to be conned on each tack, so that the ship may arrive at the place to which she is bound, in the shortest time possible.

Example 1.

A ship that can lie within 6 points of the wind is bound to a port 50 miles directly to windward, which it is intended she shall reach on two tacks; the first being on the starboard tack, and the wind steady at N. b. E.; required the course and distance to be run upon each tack?

Solution.—Since the ship can lie within six points of the wind, which is at N. b. E., the course on the starboard tack will be N.W. b. W., and that on the larboard tack E. b. N. Now, in the annexed diagram, let the side A C represent the course and distance between the ship and her intended port; A B the course and distance to be made good on the starboard tack; and B C the course and distance to be made good on the larboard tack. Then, in the triangle A B C, the



three angles are given to find the side A B or B C, which sides are mutually equal to each other, because the triangle is isosceles, and its vertex at B = the angle comprehended between those sides. Thus, the difference between N. b. E., and N.W. b. W., is 6 points = the angle B A C, measured by the arc *ab*; the difference between E. b. N., and S.E. b. E. (the opposite point to N.W. b. W.), is 4 points, measured by the arc *de*; and the difference between N. b. E., and E. b. N., is 6 points, measured by the

arc bd ; and since the distance AC is given ≈ 50 miles, the side AB , or its equal BC ,* may be readily determined by oblique angled trigonometry, Problem I., page 177; as thus:—

As the angle $B = 4$ points, Log. co-secant = 10.150515
 Is to the distance $AC = 50$ miles, Log. = 1.698970
 So is the angle $C = 6$ points, Log. sine = . 9.965615

To the distance $AB = 65.33$ miles, Log. = 1.815100

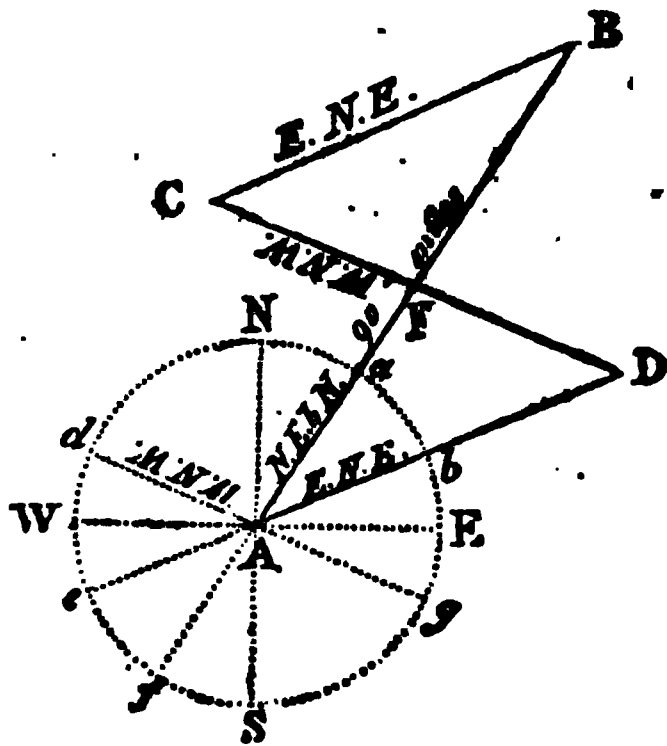
Hence, it is evident that the ship must run 65.33 miles on the starboard tack, and 65.33 miles on the larboard tack, before she can reach her intended port.

Example 2.

A ship that can lie within 6 points of the wind is bound to a port bearing N.E. b. N., distance 90 miles, which it is intended she shall reach on three tacks, with the wind steady at north; required the course and distance to be run upon each tack, the first course being on the larboard tack?

Solution.—Since the wind is at north, and that the ship can lie within 6 points thereof, the course on the larboard tack will be E.N.E., and that on the starboard tack W.N.W.

In the annexed diagram, let the N.E. b. N. line $AB = 90$ miles, represent the bearing and distance between the ship and her intended port; let the E.N.E. line AD represent the first board on the larboard tack, and, parallel thereto,



the line $BC =$ the second board on that tack. And, since the ship is to make her port in three tacks, it is evident that the board on the starboard tack, represented by the W.N.W. line CD (parallel to dg), must bisect the line AB in the point F ; and that, therefore, AF and FB are equal to one another, each being equal to 45 miles $=$ half the line, or distance AB .

Now, since the straight line AB falls upon the two parallel straight lines CB and AD , it makes the alternate angles equal to one another; there-

* Since the angles A and C are equal to one another, the sides which subtend, or are opposite to those angles (viz., BC and AB), are also equal to one another.—Euclid, Book I., Prop. 6.

fore the angle $A B C$ is equal to the angle $B A D$.—Euclid, Book I., Prop. 29. And because the straight line $C D$ falls upon the two parallel straight lines $C B$ and $A D$, it makes the angle $A D B$ equal to the angle $B C D$, by the aforesaid proposition. And because the two triangles $A D F$ and $B C F$ have, thus, two angles of the one equal to two angles of the other, viz., the angle $F A D$ to the angle $F B C$, and the angle $A D F$ to the angle $B C F$; and the side $A F$ of the one equal to the side $B F$ of the other: therefore the remaining sides $A D$ and $D F$ of the one are equal to the remaining sides $B C$ and $C F$ of the other, each to each; and the third angle $A F D$ of the one equal to the third angle $B F C$ of the other.—Euclid, Book I., Prop. 26. Now, since the two triangles $A F D$ and $B F C$ are, thus, evidently equal to one another, we have only to compute the unknown sides of one, viz., of the triangle $A F D$, where the three angles are given, and the side $A F$, to find the sides $A D$ and $F D$; thus, the difference between N.E. b. N. and E.N.E., is 3 points = the angle $F A D$, measured by the arc $a b$; the difference between E.N.E. and E.S.E. (the opposite point to W.N.W.), is 4 points = the angle $A D F$, measured by the arc $b g$; and the difference between W.N.W. and N.E. b. N., is 9 points = the angle $A F D$, measured by the arc $a d$: hence, by oblique angled trigonometry, Problem I., page 177,

To find the Side $A D$:—

As the angle $D = 4$ points,	Log. co-secant =	10.150515.
Is to the side $A F = 45$ miles,	Log. =	. 1.653213
So is the angle $F = 9$ points,	Log. sine =	9.991574
To the side $A D = 62.42$ miles,	Log. =	<u>1.795302</u>

To find the Side $F D$:—

As the angle $D = 4$ points,	Log. co-secant =	10.150515
Is to the side $A F = 45$ miles,	Log. =	. . . 1.653213
So is the angle $A = 3$ points,	Log. sine =	. 9.744739
To the side $F D = 35.35$ miles,	Log. =	<u>. . 1.548467</u>
Side $D C =$. . . 70.70 miles.

Hence it is evident that the ship must first run 62.42 miles on the larboard tack; then 70.70 miles on the starboard tack; and, again, 62.42 miles on the larboard tack, before she can reach her intended port.

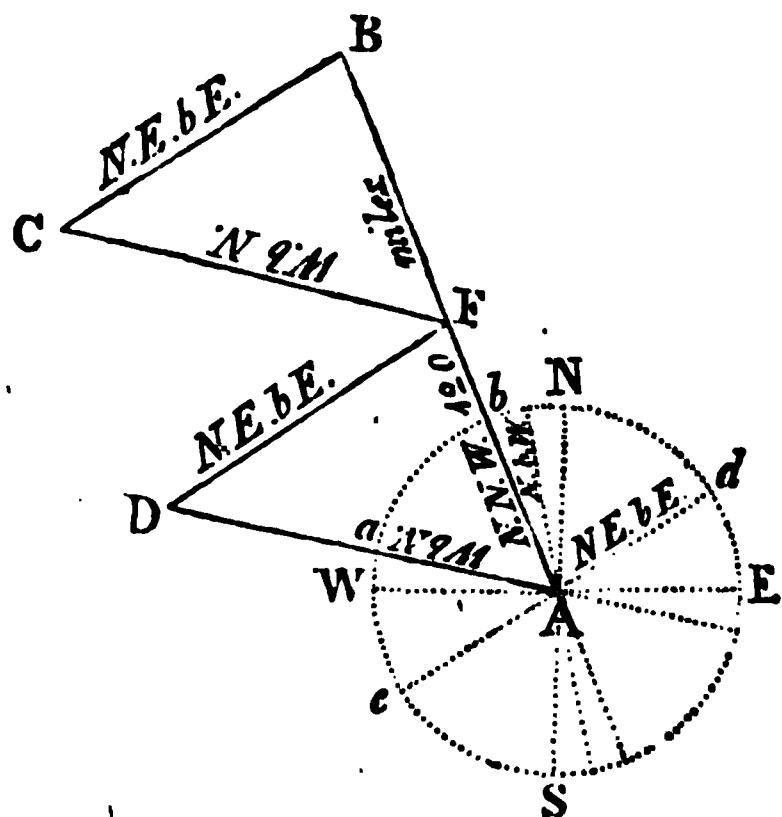
Example 3.

A ship that can lie within 6 points of the wind is bound to a port bear-

ing N.N.W., distance 120 miles, which it is intended she shall make on four tacks, with the wind at N. b. W. The coast, which is to the eastward, trends in a direction nearly parallel to the bearing of the port, so that the ship must go about as soon as she reaches the straight line joining the two ports ; required the course and distance to be run upon each tack, on the supposition that the ship's progress is not affected by either leeway or currents ?

Solution.—Since the wind is N. b. W., and the land trends in a N.N.W. direction, the first board, therefore, must be on the starboard tack ; and, as the ship can lie within 6 points of the wind, the course on the starboard tacks will be W. b. N., and that on the larboard tacks N.E. b. E.

In the annexed diagram, let the N.N.W. line A B, 120 miles, represent the bearing and distance between the ship and the port to which she is bound ; let the W. b. N. line A D represent the first board



on the starboard tack, and F C, parallel to A D, the second board on that tack ; let the N.E. b. E. line D F represent the first board on the larboard tack, and, parallel thereto, the line C B = the second board on this tack. And, since the ship is to make her port in four tacks, without going to the eastward of the line A B, therefore, at the end of the second tack, she must reach the point F, which bisects or divides the distance A B into two equal parts, of 60 miles each ; thus making A F = to A B.

Now, because the straight line A B falls upon the two parallel straight lines A D and F C, it makes the angle B F C equal to the interior and opposite angle F A D : and, because the straight line A B falls upon the two parallel straight lines F D and C B, it makes the angle A F D equal to the interior and opposite angle C B F,—Euclid, Book I., Prop. 29. And, since the two triangles A F D and F B C have, thus, two angles of the one equal to two angles of the other, viz., the angle A F D to the angle F B C, and the angle F A D to the angle B F C, and the side A F of the one equal to the side F B of the other,—therefore the remaining sides 'A D and D F of the one, are equal to the remaining sides F C and C B of the other, each to each ; and the third angle A D F of the one equal to the third angle F C B of the other.—Euclid, Book I., Prop. 26.

The two triangles A D F and F C B, being, thus, clearly equal to one

another in every respect, we have only to compute the unknown sides of one, viz., of the triangle $A F D$, where the three angles are given, and the side $A F = 60$ miles, to find the sides $A D$ and $D F$; thus the difference between N.N.W. and W.b.N., is 5 points $=$ the angle $F A D$, measured by the arc $a b$; the difference between W.b.N. and S.W.b.W. (the opposite point to N.E.b.E.), is 4 points $=$ the angle $A D F$, measured by the arc $a e$; and the difference between N.N.W. and N.E.b.E., is 7 points $=$ the angle $A F D$, measured by the arc $b d$.

Hence, by oblique angled trigonometry, Problem I., page 177,

To find the Side $A D = F C$:—

As the angle $D = 4$ points, Log. co-secant $=$	10.150515
Is to the side $A F = 60$ miles, Log. $=$. . . 1.778151
So is the angle $F = 7$ points, Log. sine $=$. . . 9.991574
	<hr/>
To the side $A D = 83.22$ miles, Log. $=$. . . 1.920240

To find the Side $D F = C B$:—

As the angle $D = 4$ points, Log. co-secant $=$	10.150515
Is to the side $A F = 60$ miles, Log. $=$. . . 1.778151
So is the angle $A = 5$ points, Log. sine $=$. . . 9.919846
	<hr/>
To the side $D F = 70.55$ miles, Log. $=$. . . 1.848512

From this it is manifest, that the ship must first run 83.22 miles upon the starboard tack; then 70.55 miles upon the larboard tack; then 83.22 miles again upon the starboard tack; and 70.55 miles upon the larboard tack, before she can reach the port to which she is bound.

SOLUTION OF CASES IN CURRENT SAILING.

Current Sailing is the method of determining the true course and distance made good by a ship, when her own motion is affected or combined with that of the current in which she sails.

A *current* is a progressive motion of the water, causing all floating bodies thereon to move in the direction to which its stream is impelled. The *setting* of a current is that point of the compass towards which the water runs; and the *drift* of a current is the rate at which it runs per hour.

When a ship sails in the direction of a current, her velocity will be equal

to the sum of her own proper motion and the current's drift; but when she sails directly against a current, her velocity will be expressed by the difference between her own proper motion and the drift of the current: in this case, the absolute motion of a ship will be a-head, if her proper velocity exceeds the drift of the current; but if it be less, she will make stern-way. When a ship's course is oblique to the direction of a current, her true course and distance will be compounded of the course and distance given by the log, and of the observed setting and drift of the current.

When a ship's course and distance by the log, and the setting and drift of the current in which she sails are given, the true course and distance made good may be found by a trigonometrical solution of the triangles forming the figure; but the easiest and most expeditious method of finding the course and distance made good, particularly when a ship sails upon different courses, is by resolving a traverse, in which the setting and drift of the current are to be esteemed as an additional course and distance to those exhibited by the log.

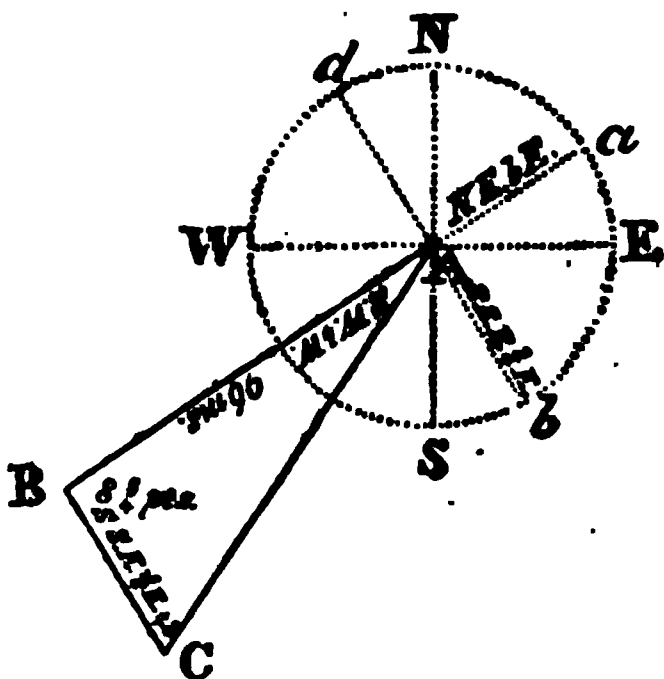
Example 1.

If a ship sails S.W. b. W., at the rate of 4 knots an hour, in a current setting S.S.E. $\frac{1}{4}$ E., at the rate of $1\frac{1}{4}$ miles an hour; required the course and distance made good in 24 hours?

Solution.— $4^{\circ} \times 24^{\text{h}} = 96$ miles, the distance sailed, by log, in 24 hours;

And $1\frac{1}{4}^{\circ} \times 24 = 42$ miles, the observed drift of the current in 24 hours.

In the annexed diagram, let the side AB of the triangle ABC represent the course and distance sailed by the log, and the side BC parallel to db the setting and drift of the current; then, the side AC will represent the course and distance made good in the given time. Now, in the triangle ABC , given the side $AB = 96$ miles, the side $BC = 42$ miles, and the included angle $B = 8\frac{1}{4}$ points, being the difference between S.S.E. $\frac{1}{4}$ E. and N.E. b. E. (the opposite point to S.W. b. W.), measured by the arc ab , to find the angles A and C , and the true distance AC . Hence, by oblique angled trigonometry, Problem III., page 179,



To find the Angles A and C :—

As $AB + BC = 138$ miles, Log. ar. comp. = 7.860121
Is to $AB - BC = 54$ miles, Log. = . . . 1.732394
So is $\frac{1}{2}$ sum of the angles $= 43^{\circ} 35' 37\frac{1}{2}''$ Log. tangent $= 9.978673$

To $\frac{1}{2}$ diff. of the angles $= 20. 25. 59$ Log. tangent $= 9.571188$

Angle C = $64^{\circ} 1' 36\frac{1}{2}''$
Angle A = $23^{\circ} 9' 38\frac{1}{2}''$

To find the true Distance = AC :—

As the angle C $= 64^{\circ} 1' 36\frac{1}{2}''$ Log. co-secant = 10.046241
Is to the side AB $= 96$ miles, Log. = 1.982271
So is the angle B $= 8\frac{1}{4}$ points, Log. sine = . . 9.999477

To the true distance $= AC = 106.7$ miles, Log. = 2.027989

To find the Course made good :—

From the angle SAB = S.W. b. W., or $56^{\circ} 15'$, subtract the angle CAB $= 23^{\circ} 9' 38\frac{1}{2}''$, and the remainder, $33^{\circ} 5' 21\frac{1}{2}''$ = the angle SAC, is the course made good.
Hence the course made good is S. $33^{\circ} 5'$ W., or S.W. b. S. nearly, and the distance $106\frac{1}{4}$ miles nearly.

To find the Course and Distance made good by the Traverse Table :—

TRAVERSE TABLE.					
Corrected Courses.	Distance.	Difference of Latitude.		Departure.	
		N.	S.	E.	W.
S.W. b. W.	96	—	53.3	—	79.8
Current S.S.E $\frac{1}{4}$ E.	42	—	36.0	21.6	—
		Diff. lat. =	89.3	21.6	79.8
				—	21.6
				Depart. =	58.2

Now, by Problem II., page 108,

The difference of latitude 89.3, and the departure 58.2, are found to agree nearest abreast of 33° , under or over distance 107.

Hence the course made good is S. 33° W., or S.W. b. S., and the distance 107 miles ; which nearly agrees with the above result.

Example 2.

Suppose a ship sails N.W. 65 miles, W.N.W. 70 miles, and N. b. E. 71 miles, in a current that sets S.E. b. S. 36 miles in the same time ; required the true course and distance made good ?

TRAVERSE TABLE.					
Corrected Courses.	Distance.	Difference of Latitude.		Departure.	
		N.	S.	E.	W.
N.W.	65	46.0	—	—	46.0
W.N.W.	70	26.8	—	—	64.7
N. b. E.	71	69.6	—	13.9	—
Current S.E. b. S.	36	—	29.9	20.0	—
		142.4	29.9	33.9	110.7
		29.9			33.9
		112.5			76.8

Solution.—With the difference of latitude and departure, thus found, the course and distance made good may be determined by Problem II., page 108 ; as thus :

The difference of latitude 112.5, and the departure 76.8, are found to agree nearest abreast of 34° under or over 136.

Hence the direct course made good is N. 34° W., or N.W. b. N. nearly, and the distance 136 miles.

To find the Course and Distance made good by Calculation :—

This may be done by means of the 5th analogy, page 237 ; as thus :

To find the true Course :—

As the diff. of lat. =	112.5	Log. ar. comp. =	7.948848
Is to radius =	. . . 90°0'	Log. sine =	. . 10.000000
So is the departure =	76.8	Log. =	. . . 1.885361
<hr/>			
To the true course =	34°19'12"	Log. tangent =	9.834209

To find the true Distance :—

As radius =	. . . 90°0'	Log. co-secant =	10.000000
Is to diff. of lat. =	. 112.5	Log. =	. . . 2.051152
So is the true course =	34°19'12"	Log. secant =	. 10.083072
<hr/>			

To the distance = 136.2 miles, Log. = . . . 2.134224

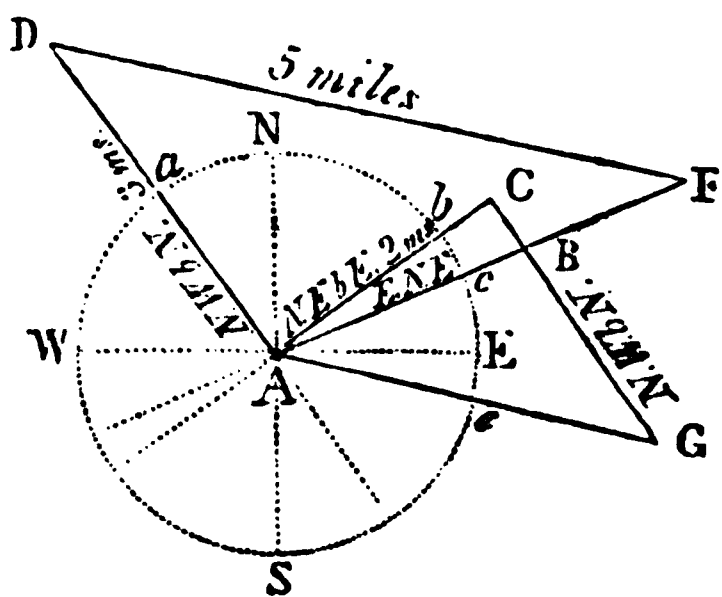
Hence the course made good is N. 34°19' W., or N.W. b. N. nearly, and the distance 136 miles.

Example 3.

There is a harbour 2 miles broad, in which the tide is running N.W. b. N. at the rate of 3 miles an hour. Now, a waterman who can pull his boat at the rate of 5 miles an hour, wishes to cross the harbour to a point on the opposite side bearing E.N.E.; required the direction in which he should pull, so as to meet with the least possible resistance from the force of the tide in gaining the intended point, and the time that it will take him to reach that point?

Solution.—Since the principles of this Problem are but little understood by the generality of young navigators, a brief account of the geometrical construction will be given, with the view of elucidating and rendering familiar the nature of the corresponding calculations. Thus,

With the chord of 60° describe the arch NESW; draw the north and south line NS, and, at right angles thereto, the east and west line WE; make the arc Na = 3 points, and draw the N.W. b. N. line Aad, which make equal to 3 miles (taken from any scale of equal parts), to represent the direction of the harbour; perpendicular thereto draw the N.E. b. E. line AbC, which make equal to 2 miles, to represent the breadth of the harbour; and, from the point C, draw the line CG parallel to AD, which lines will represent the eastern and western shores of the harbour respectively.



Make Nc equal to 6 points, and draw the E.N.E. line Acf , cutting CG in B ; then will B represent the point to which the waterman intends to cross. Take 5 miles in the compasses; place one foot on the point D ; and where the other falls upon the E.N.E. line Af , there make a point, as at F , and draw the line DF ; parallel to which, draw the line AG , and it will represent the distance and direction in which the waterman must pull to gain the point B : for in the time that he would reach the point G , by pulling at the rate of 5 miles an hour, the tide, running at the rate of 3 miles an hour, would carry him to the point B ; because BG bears the same proportion to 3 miles an hour that AG does to 5. Now, AG , being applied to the same scale of equal parts from which the other sides were taken, will measure 2.95 miles, and the angle GAe , or eAe , being applied to the line of chords, will measure $13^{\circ}33'$; hence the direction in which he should pull, is E. $13^{\circ}33'$ S, or E. b. S. $\frac{1}{4}$ S. nearly.

Now, in the triangle ADF , given the side $AD = 3$ miles, the side $DF = 5$ miles, and the angle $DAF = 9$ points (being the difference between E.N.E. and N.W. b. N., measured by the arc ac), to find the angle AFD . Hence, by oblique angled trigonometry, Problem I., page 177,

As the side $DF = 5$ miles, Log. ar. comp. = 9.301030

Is to the angle $A = 9$ points, Log. sine = . 9.991574

So is the side $AD = 3$ miles, Log. = . . . 0.477121

To the angle $AFD = 36^{\circ}2'55''$ Log. = . 9.769725

Now, because the straight line Af falls upon the two parallel straight lines DF and AG , it makes the alternate angles equal to one another; therefore the angle DFA is equal to the angle FAG ,—Euclid, Book I., Prop. 29; but the angle DFA is known to be $36^{\circ}2'55''$; therefore the angle FAG , measured by the arc ce , is also equal to $36^{\circ}2'55''$; and if to the angle FAG we add the angle $BAC = 11^{\circ}15'$ (being the difference between N.E. b. E. and E.N.E., measured by the arc bc), the sum = $47^{\circ}17'55''$ is the angle CAG , measured by the arc be . Then,

In the right angled triangle ACG , given the angle $CAG = 47^{\circ}17'55''$ and the side $AC = 2$ miles, the breadth of the harbour, to find the side AG equal to the distance which the waterman must pull before he can reach the point B . Hence, by right angled trigonometry, Problem II., page 172, making AC radius,

As radius = . . . $90^{\circ}0'$ Log. co-secant = 10.000000

Is to the side $AC = 2$ miles, Log. = . . . 0.301030

So is the angle $CAG = 47^{\circ}17'55''$ Log. secant = 10.168657

To the distance = $AG = 2.949$ miles, Log. = 0.469687

To find the Time requisite to reach the Point B:—

As distance 5 miles, Log. ar. comp. = 9.301030
 Is to 1 hour, or 60 minutes, Log. = . 1.778151
 So is A G = 2.949 miles, Log. = . . 0.469687

To the time = $35^{\circ}23'.34 = 35^{\circ}.389$ Log. = 1.548868

To find the Direction in which he should pull or steer:—

From the angle $b A e = 47^{\circ}17'55''$, take away the angle $b A E = 33^{\circ}45'$, and the remaining angle $E A e = 13^{\circ}32'55''$ is the direct course which he should steer; viz., E. $13^{\circ}33'$ S., or E. b. S. $\frac{1}{4}$ S. nearly.

Hence it is evident, that if the waterman pulls in the direction of E. $13^{\circ}33'$ S. or E. b. S. $\frac{1}{4}$ S. nearly, he will reach the intended point in the space of about 35 minutes and 23 seconds.

SOLUTION OF PROBLEMS RELATIVE TO THE ERRORS OF THE LOG-LINE AND THE HALF-MINUTE GLASS, BY LOGARITHMS.

The instruments generally employed at sea, for finding the distance run by a ship in a given time, are the log-line and the half-minute glass. Now, since a ship's reckoning is kept in nautical miles, of which 60 make a degree, the distance between any two adjacent knots on the log-line should bear the same proportion to a nautical mile that half a minute does to an hour; viz., the *one hundred and twentieth part*. And, since a nautical mile contains 6080 feet, the true length of a knot is equal to 6080 divided by 120; that is, 50 feet and 8 inches: but, because it is advisable at all times to have the reckoning a-head of the ship, so that the mariner may be looking out for the land in sufficient time, instead of his making it unexpectedly, or in an unprepared moment, 48 feet, therefore, is the customary measure allowed to a knot. And, to make up for any time that may be unavoidably lost, in turning the half-minute glass, its absolute measure should not exceed *twenty-nine seconds and a half*.

The method of finding the hourly rate of sailing, or distance run in a given time by the log-line and the half-minute glass, is subject to many errors: thus, a new log-line, though divided with the utmost care and attention, is generally found to contract after being first used;

and, after some wear, it stretches so very considerably as to be out of due proportion to the measure of the half-minute glass. Nor is the half-minute glass itself free from error: for this instrument is so very liable to be affected by various changes of weather, from moist to dry, and conversely, that notwithstanding its being perfectly correct when first taken on board, yet it alters so sensibly at sea, that at one time it will run out in the short space of 26 or 27 seconds, and at another not till it has passed the half-minute by several seconds. Hence it becomes indispensably necessary to examine those instruments frequently; and, if found erroneous, to correct the ship's run accordingly. This may be done by means of the following *rules*, which are adapted to a log-line of 48 feet to a knot, and to a glass measuring 30 seconds.

PROBLEM I.

Given the Distance sailed by the Log, and the Number of Seconds run by the Glass; to find the true Distance, the Line being truly divided.

RULE.

To the arithmetical complement of the logarithm of the number of seconds run by the glass, add the logarithm of the distance given by the log, and the constant logarithm 1.477121*; the sum of these three logarithms, abating 10 in the index, will be the logarithm of the true distance sailed.

Example 1.

Let the hourly rate of sailing be 11 knots, and the time measured by the glass 33 seconds; required the true rate of sailing?

Seconds run by the glass = 33, Log. ar. comp. =	8.481486
Rate of sailing, by log = 11 knots, Log. =	. 1.041393
Constant log. =	1.477121
	<hr/>
True rate of sailing = 10 knots, Log. =	. . . 1.000000

Example 2.

If a ship sails 198 miles by the log, and the glass is found, on examination, to run out in 26 seconds, required the true distance sailed?

Seconds run by the glass = 26, Log. ar. comp. =	8.585027
Distance sailed by log = 198 miles, Log. =	. 2.296665
Constant log. =	1.477121
	<hr/>
True distance sailed = 228.46 miles, Log. =	. 2.358813

* This is the logarithm of 30 seconds, the true measure of the half-minute glass.

PROBLEM II.

Given the Distance sailed by the Log, and the measured Length of a Knot; to find the true Distance, the Glass being correct.

RULE.

To the logarithm of the distance given by the log; add the logarithm of the measured length of a knot, and the constant logarithm 8.818759*; the sum of these three logarithms, rejecting 10 in the index, will be the logarithm of the true distance sailed.

Example 1.

Let the hourly rate of sailing be 9 knots, by a log-line which measures 53 feet to a knot; required the true rate of sailing?

Hourly rate of sailing =	9 knots,	Log. =	. 0.954248
Measured length of a knot =	53 feet,	Log. =	. 1.724276
Constant log. =		8.318759
<hr/>			
True rate of sailing =	9.937 knots,	Log. =	. . 0.997278

Example 2.

Let the distance sailed be 240 miles, by a log-line which measures 43 feet to a knot; required the true distance sailed?

Distance sailed by log =	240 miles,	Log. =	2.380211
Measured length of a knot =	43 feet,	Log. =	1.633469
Constant log. =		8.318759
<hr/>			
True distance sailed =	215 miles,	Log. =	. 2.382489

PROBLEM III.

Given the measured Length of a Knot, the Number of Seconds run by the Glass, and the Distance sailed by the Log; to find the true Distance sailed.

RULE.

To the arithmetical complement of the logarithm of the number of seconds run by the glass, add the logarithm of the measured length of a knot, the logarithm of the distance sailed by the log, and the constant

* This is the arithmetical complement of the logarithm of 48, the generally-approved length of a knot.

logarithm 9.795880*; the sum of these four logarithms, rejecting 20 from the index, will be the logarithm of the true distance sailed.

Example 1.

Let the hourly rate of sailing be 12 knots, the measured length of a knot 44 feet, and the time noted by the glass 25 seconds; required the true rate of sailing?

Seconds run by the glass = 25, Log. ar. comp.=	8.602060
Measured length of a knot=44 feet, Log. =	1.643453
Rate of sailing by log = 12 knots, Log. =	1.079181
Constant log. =	9.795880
True rate of sailing = 13.2 knots, Log. =	1.120574

Example 2.

Let the distance sailed by the log be 354 miles, the measured length of a knot 52 feet, and the interval run by the glass 34 seconds; required the true distance sailed?

Seconds run by the glass = 34, Log. ar. comp.=	8.468521
Measured length of a knot = 52 feet, Log. =	1.716003
Distance sailed by log = 354 miles, Log. =	2.549003
Constant log. =	9.795880
True distance = 338.38 miles, Log. =	2.529407

PROBLEM IV.

Given the Number of Seconds run by any Glass whatever, to find the corresponding Length of a Knot, which shall be truly proportional to the Measure of that Glass.

RULE.

To the logarithm of 10 times the number of seconds run by the glass, add the constant logarithm 9.204120, and the sum, abating 10 in the index, will be the logarithm of the proportional length of a knot, in feet, corresponding to the given glass.

* This is the sum of the two preceding constant logarithms; thus 1.477121 + 8.318759 = 9.795880.

Example 1.

Required the length of a knot corresponding to a glass that runs 27 seconds ?

$$\begin{array}{rcl}
 \text{Number of seconds } 27 \times 10 = 270 & \text{Log.} = & 2.431364 \\
 \text{Constant log.} = & & 9.204120 \\
 \hline
 \text{True length of a knot, in feet,} = 43.2 & \text{Log.} = & 1.635484
 \end{array}$$

Example 2.

Required the length of a knot corresponding to a glass that runs 34 seconds ?

$$\begin{array}{rcl}
 \text{Number of seconds } 34 \times 10 = 340 & \text{Log.} = & 2.531479 \\
 \text{Constant log.} = & & 9.204120 \\
 \hline
 \text{True length of a knot, in feet,} = 54.4 & \text{Log.} = & 1.735599
 \end{array}$$

SOLUTION OF A PROBLEM IN GREAT CIRCLE SAILING,

*Very useful to Ships going to Van Diemen's Land, or to New South Wales,
by the way of the Cape of Good Hope.*

Great Circle Sailing is the method of finding the successive latitudes and longitudes which a ship must make ; with the courses that she must steer, and the distances to be run upon such courses, so that her track may be nearly in the arc of a great circle, passing through the place sailed from and that to which she is bound.

The angle of position is an angle which a great circle, passing through two places on the sphere, makes with the meridian of one of them ; and shows the true position of each place, in relation to the intercepted arc of the great circle and the respective meridians of those places.

The polar angle is an arc of the equator intercepted between the meridians, or circles of longitude, of two given places on the sphere.

On the sphere, the shortest distance between two places is expressed by the arc of a great circle intercepted between those places : consequently the spiral, or rhumb line, passing through two places on the sphere, can never represent the shortest distance between those places, unless such rhumb line coincides with the arc of a great circle ; and this can never happen but when the places are situate under the equator, or under a

meridian. Hence, although Mercator's Sailing resolves correctly all the cases incident to a ship's course along the rhumb line passing through two places,—yet, since there is no case in which the course along the direct rhumb line indicates the shortest distance between those places, except when they both lie under the same meridian, or under the equator, the distance, therefore, obtained by that method of sailing, must always exceed the truth (the above-mentioned positions excepted); and the nearer the places are to a parallel of latitude, and the farther they are removed from the equator, the greater will be the error in distance.

Now, since it is frequently an object of the greatest importance, to the commander of a ship, to reach the port to which he is bound by the shortest route, and in the least time possible,—particularly to the commander of a ship bound from the Cape of Good Hope to Van Diemen's Land, or to His Majesty's Colony at New South Wales, where the length of the voyage generally occasions a great scarcity of fresh water,—the following Problem is, therefore, given, by which all the particulars connected with the shortest possible route between those places will be fully and clearly exhibited.

Were a ship to sail exactly in the arc of a great circle (not under the equator or upon a meridian), the navigator would be obliged to keep continually altering her course; but, as this would be attended with more trouble and inconvenience than could be reasonably admitted into the general practice of navigation, it has been deemed sufficiently exact to determine a certain number of latitudes and longitudes through which a ship should pass, with the relative courses and distances between them; so that the track, thus indicated, though not exactly in the arc of a great circle may, notwithstanding, approximate so very near thereto, as not to produce any sensible difference between it and the true spherical track.

PROBLEM.

Given the Latitudes and Longitudes of two Places on the Globe, to determine the true spherical Distance between them; together with the angular Position of those Places with respect to each other, and the successive Positions at which a Ship should arrive when sailing on or near to the Arc of a great Circle, agreeably to any proposed Change of Longitude.

RULE.

1. Find the true spherical distance between the two given places, by oblique angled spherical trigonometry, Problem III., page 202.
2. Find the highest latitude which the great circle touches that passes through the two given places; that is, find the perpendicular from the pole

to that circle by right angled spherical trigonometry, Problem II., page 185; and find, also, the several polar angles (made by the proposed alterations of longitude,) contained between the perpendicular, thus found, and the several meridians corresponding to the successive changes of longitude.

3. With the co-latitude or perpendicular, so found, and the several polar angles, compute as many corresponding co-latitudes by right angled spherical trigonometry, Problem IV., page 188.

4. With the several latitudes and longitudes through which the ship is to pass, compute the corresponding courses and distances by Mercator's Sailing, Problem I., page 238; and they will indicate the path along which a ship must sail, so as to keep nearly in the arc of a great circle.

Note.—The smaller the alterations are in the longitude, the nearer will the track, thus determined, approximate to the truth; because, in very small arcs, the difference between the arc and its corresponding chord, sine, or tangent, is so very trifling, that the one may be substituted for the other, in most nautical calculations, without producing any sensible difference in the result.

Example 1.

A captain of a ship bound from the Cape of Good Hope (in latitude $34^{\circ}24'$ S., and longitude $18^{\circ}32'$ E.) to New South Wales, being desirous of making the north point of King's Island, at the western entrance to Bass' Strait (in latitude $39^{\circ}37'$ S., and longitude $143^{\circ}54'$ E.), by the shortest possible route, proposes, therefore, to sail as near to the arc of a great circle as he can, by altering the ship's course at every 5 degrees of longitude; required the latitude at each time of altering the course, and, also, the respective courses and distances between those several latitudes and longitudes made by the proposed changes?

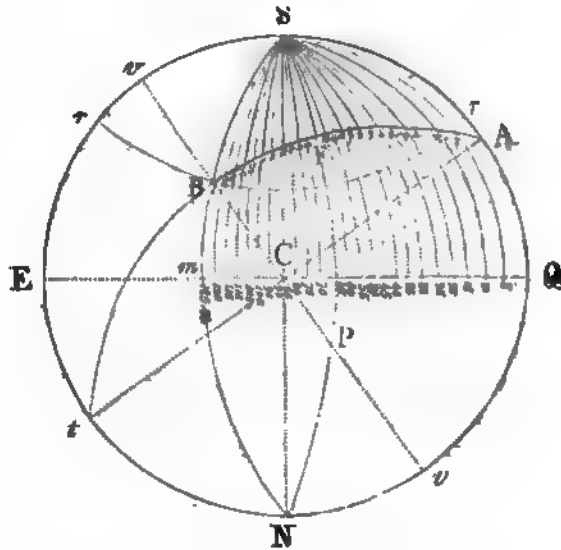
Cape of Good Hope, Latitude = $34^{\circ}24'$ S. Longitude = $18^{\circ}32'$ E.
King's Island, N. point, Latitude = 39.37 S. Longitude = 143.54 E.

Difference of longitude = $125^{\circ}22'$

Stereographic Projection.

With the chord of 60 degrees, describe the primitive circle $SENQ$ on the plane of the meridian, or circle of longitude passing through the Cape of Good Hope; draw the line EQ to represent the equator, and, at right angles thereto, the line SN for the earth's axis; then S represents the south, or elevated pole, and N the north, or depressed pole. Take the latitude of the Cape of Good Hope in the compasses from the line of chords = $34^{\circ}24'$, and lay it off from Q to A ; draw the diameter AC ,

and, at right angles thereto, the diameter $u c v$. Take the latitude of King's Island $= 39^{\circ}37'$ in the compasses from the line of chords, and lay it off from Q to r , and also from E to r ; and, with the tangent of its complement $= 50^{\circ}23'$ draw the parallel circle rr . Take the difference of longitude $125^{\circ}22'$ from the scale of semi-tangents, and lay it off on the equator from Q to m : thus 90° will reach from Q to C ; then the



excess above 90° , viz., $35^{\circ}22'$, will reach from C to m . With the secant of the complement of the excess of the difference of longitude above $90^{\circ} = 54^{\circ}38'$ (being the supplement of the difference of longitude to 180°), describe the great circle $S m N$; the intersection of which with the parallel circle rr at B shows the position of King's Island. Then, the great circle $S B m N$ represents the meridian of King's Island. Through the three points $A B t$ describe a great circle, and then will the arc $A B$ represent the true spherical distance between the Cape of Good Hope and King's Island; in which A represents the place of the former, and B that of the latter. Through P , the pole of the great circle $A B t$, draw the great circle $S F P N$; then the arc $S F$ will be perpendicular to the arc $A B$. Hence, $S F$ represents the least co-latitude at which the ship should arrive in her spherical passage from the Cape of Good Hope to King's Island; which, being reduced to the primitive circle, and measured on the scale of chords, gives about $31\frac{1}{2}$ degrees. The arc $A B$, reduced to the primitive circle, and measured on the line of chords, shows the true spherical distance to be about $90\frac{1}{2}$ degrees. The angle $S A B$ is the angle of position which the meridian of the Cape of Good Hope makes with King's Island; and the angle $S B A$ is the angle of position which the meridian of King's Island makes with the Cape of Good Hope. These angles, being reduced to the primitive circle, and measured on the line of chords, give about 39° for the former, and $42\frac{1}{2}^{\circ}$ for the latter.

Note.—The remaining parts of the projection will be explained hereafter,

Calculation.

In the oblique angled spherical triangle ASB , there are given two sides and the included angle, to find the remaining angles and the third side; viz., the side $AS = 55^\circ 36'$, the co-latitude of the Cape of Good Hope; the side $BS = 50^\circ 23'$, the co-latitude of King's Island; and the angle $ASB = 125^\circ 22'$, the difference of longitude between those places, to find the true spherical distance AB , and the respective angles of position SAB and SBA . The distance may be readily found by Remark 1 or 2, to Problem III., page 203 or 204; as thus:

Diff. of long. $ASB =$

$$125^\circ 22' + 2 = 62^\circ 41' \text{ Twice log. sine } 19.897300$$

Co-lat. of Cape of

$$\text{Good Hope} = AS \ 55.36 \text{ Log. sine } = 9.916514$$

Co-lat. of King's

$$\text{Island} = BS \ . \ 50.23 \text{ Log. sine } = 9.886676$$

$$\text{Sum} = 39.700490$$

$$\text{Diff. of co-lats.} = 5^\circ 13' \quad \text{Half} = 19.850245 \quad . \ . \ 19.850245$$

$$\text{Half diff. of do.} = 2^\circ 36' 30'' \text{ Log. sine} = 8.658090$$

$$\text{Arch} = . \ . \ . \ . \ 86^\circ 19' 27'' \text{ Log. T.} = 11.192155 \text{ Log. sine } 9.999106$$

$$\text{Half the side } AB = 45.13. \ 8\frac{1}{2} \quad \text{Log. sine} = 9.851139$$

Side $AB = . \ . \ 90^\circ 26' 17''$; which is the true spherical distance between the two given places.

To find the Angle of Position at Cape of Good Hope = Angle SAB :—

This is found by Problem I., page 198; as thus:

$$\text{As the distance } AB \ . \ 90^\circ 26' 17'' \text{ Log. co-secant} = 10.000013$$

$$\text{Is to diff. of long. } ASB \ 125.22.0 \text{ Log. sine} = . \ 9.911405$$

$$\text{So is the co-lat. } BS \ . \ 50.23.0 \text{ Log. sine} = . \ 9.886676$$

$$\text{To the ang. of posit. } SAB \ 38^\circ 55' 1'' \text{ Log. sine} = . \ 9.798094$$

To find the Angle of Position at King's Island = Angle SBA :—

This is found by Problem I., page 198; as thus:

$$\text{As the distance } AB \ . \ 90^\circ 26' 17'' \text{ Log. co-secant} = 10.000013$$

$$\text{Is to diff. of long. } ASB \ 125.22.0 \text{ Log. sine} = . \ 9.911405$$

$$\text{So is the co-lat. } AS \ . \ 55.36.0 \text{ Log. sine} = . \ 9.916514$$

$$\text{To the ang. of pos. } SBA \ 42^\circ 17' 20\frac{1}{2}'' \text{ Log. sine} = . \ 9.827932$$

To find the Perpendicular FS = the Complement of the highest southern Latitude at which the Ship should arrive in the proposed Route :—

Here we have a choice of two right angled spherical triangles, viz., ASF and BSF ; in each of which the hypotenuse and the angle at the base are given, to find the perpendicular. Thus, in the triangle ASF , given the hypotenuse AS , $55^{\circ}36'$ = the co-latitude of the Cape of Good Hope, and the angle at the base, SAF $38^{\circ}55'1''$ = the angle of position at that place, to find the perpendicular FS = the complement of the highest latitude at which the ship should arrive. Hence, by right angled spherical trigonometry, Problem II., page 185,

As radius =	$90^{\circ} 0' 0''$	Log. co-sec. =	10.000000
Is to co-lat. C. Good Hope	$AS = 55.36. 0$		Log. sine =	9.916514
So is the ang. of position	$SAF = 38.55. 1$		Log. sine =	9.798094

To the perpendicular	$FS =$	<u>$31.13.13\frac{1}{2}$</u>	Log. sine =	<u>9.714608</u>
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Highest lat. at which the ship

should arrive = $58^{\circ}46'.46\frac{1}{2}''$ south.

Hence the true spherical distance between the Cape of Good Hope and the north point of King's Island, is $90^{\circ}26'.17''$, or 5426.3 miles; the angle of position at the Cape of Good Hope, is $38^{\circ}55'.1''$; and that at King's Island, $42^{\circ}17'.20''$; and the highest southern latitude at which the ship should arrive, $58^{\circ}46'.46''$. Now, by Mercator's Sailing, the course from the Cape of Good Hope to King's Island is $S. 87^{\circ}1' E.$, or $E. \frac{1}{4} S.$ nearly, and the distance 6011.2 miles; whence it is evident, that if a ship sails on the direct rhumb line indicated by Mercator's Sailing, she will have to run a distance of no less than 585 miles more than if her course had been shaped along the arc of a great circle passing through the two given places.

Now, since it is extremely difficult for persons unacquainted with the doctrine of spherics to reconcile a route to their senses, as the shortest distance between two places, which carries them nearly 22 degrees to the southward of the middle latitude between the two given places; and since, in sailing on the arc of a great circle, the course ought to be changing constantly, with the view of keeping the side of the polygon on which the ship sails as near to the arc of its circumscribing circle as possible, or that the difference between the arc and its chord may be so small that the one may be substituted for the other without sensibly affecting the result in nautical operations,—I shall, therefore, show the successive latitudes at which the ship should arrive at every 5 degrees of longitude, as proposed (which is sufficiently near to preserve the desired ratio between the arc and its chord); together with the respective courses and distances, by Merca-

tor's Sailing, between those several successive latitudes and longitudes: then, if the sum of the several distances coincide, or nearly so, with the true spherical distance found as above, the senses must become reconciled to the propriety of adopting that high southern route at which they originally seemed to recoil.

In order to determine the several successive latitudes at which the ship must arrive, we must previously compute the vertical or polar angles ASF and BS : then, if the sum of these angles makes up the whole difference of longitude, or polar angle between the two given places, it will be a convincing and satisfactory proof that, for so far, the operations will have been properly conducted. Now, in the right angled spherical triangle ASF , given the hypotenuse AS , $55^{\circ}36'$ = the co-latitude of the Cape of Good Hope, and the perpendicular FS , $31^{\circ}13'13\frac{1}{2}''$ = the complement of the highest latitude at which the ship should arrive, to find the vertical or polar angle $FS A$. And, in the right angled spherical triangle BSF , given the hypotenuse BS , $50^{\circ}23'$ = the co-latitude of King's Island, and the perpendicular FS , $31^{\circ}13'13\frac{1}{2}''$, to find the vertical or polar angle BSF . Hence, by right angled spherical trigonometry, Problem I., page 184,

To find the Polar Angle ASF .—

As radius =	$90^{\circ} 0' 0''$	Log. co-secant =	10.000000
Is to the co-latitude AS =	55.36. 0		Log. co-tangent =	9.835500
So is the co-latitude FS =	31.13. $13\frac{1}{2}$		Log. tangent =	9.782550
To the polar angle ASF =	65°28'48"		Log. co-sine =	<u>9.618059</u>

To find the Polar Angle BSF :—

As radius =	$90^{\circ} 0' 0''$	Log. co-secant =	10.000000
Is to the co-latitude BS =	50.23. 0		Log. co-tangent =	9.917906
So is the co-latitude FS =	31.13. $13\frac{1}{2}$		Log. tangent =	9.782550
To the polar angle BSF =	59°53'12"		Log. co-sine =	<u>9.700456</u>

And, since the sum of the polar angles, thus obtained, viz., ASF $65^{\circ}28'48''$ + BSF $59^{\circ}53'12''$ = $125^{\circ}22'0''$, makes up the whole difference of longitude between the two given places expressed by the whole angle ASB , it shows that thus far the work is right.

Now, on the equator, from Q to m , lay off the proposed changes of longitude, viz., 5° , 10° , 15° , 20° , 25° , &c. These are to be taken respectively, in the compasses, from the scale of semi-tangents, reckoning *backwards* from 90° towards 0° , till the proposed changes of longitude reach the centre C ; and then *forwards* on that scale, or from 0° towards 90° , till those changes of longitude meet the point m ; thus, the extent from 90°

to 85° will reach from Q to 5° ; the extent from 90° to 80° , will reach from Q to 10° , and so on to the centre C; then, the extent from 0° to 5° , will reach from C to 95° ; the extent from 0° to 10° , will reach from C to 100° , and so on to the point *m*. Through the points S and N, and the several points made by the proposed changes of longitude on the equator, draw arcs of great circles, viz., S 1, 5° ; S 2, 10° ; S 3, 15° ; S 4, 20° , &c. &c.; and then the arcs S 1, S 2, S 3, &c. &c., will represent the respective complements of the several latitudes at which the ship should arrive at the given changes of longitude; the true values of which may be found in the following manner, viz.,

From the polar angle A S F, subtract the proposed changes of longitude continually; and the several polar angles made by those changes, and contained between the perpendicular F S and the co-latitude of the Cape of Good Hope = S A, will be obtained. Thus, from the polar angle A S F = $65^\circ 28' 48''$, let 5° be continually *subtracted*, and the results will be F S 1 = $60^\circ 28' 48''$; F S 2 = $55^\circ 28' 48''$; F S 3 = $50^\circ 28' 48''$, &c. &c. And, since the last subtraction in this triangle leaves the remainder, or polar angle, F S 12 = $5^\circ 28' 48''$, which is $28' 48''$ greater than the proposed alteration of longitude, therefore, in the triangle B S F, where the polar angle S is $59^\circ 53' 12''$ (and where the several polar angles contained between the perpendicular F S and the co-latitude of King's Island are to be determined by a *contrary process* to that which was observed in the preceding triangle), the first polar angle is expressed by $5^\circ - 28' 48'' = 4^\circ 31' 12'' =$ the angle F S *a*; to which let the proposed alterations of longitude be continually *added*, and the sums will be F S *b* = $9^\circ 31' 12''$; F S *c* = $14^\circ 31' 12''$, &c. &c. Those various results are to be arranged agreeably to the form exhibited in the first column of the following Table; and, since they respectively express the true measures of the several polar angles contained between the meridians of the given places and those of the several co-latitudes to which they correspond, it is, therefore, manifest that those results reduce the two right angled spherical triangles (A S F and B S F) into a series of right angled spherical triangles; to each of which the perpendicular F S is common. Then, in each of these triangles, we have the perpendicular and the angle adjacent, to find the hypotenuse or co-latitude. Thus, in the right angled spherical triangle F S 1, right angled at F, given the perpendicular F S = $81^\circ 13' 13\frac{1}{2}''$, and the polar angle F S 1 = $60^\circ 28' 48''$, to find the hypotenuse or co-latitude S 1; in the right angled spherical triangle F S 2, given the perpendicular F S = $81^\circ 13' 13\frac{1}{2}''$, and the polar angle F S 2 = $55^\circ 28' 48''$, to find the hypotenuse or co-latitude S 2, &c. &c. Hence, by right angled spherical trigonometry, Problem IV., page 188,

To find the Hypothenuse, or Co-Latitude = S 1 :—

As the perpendicular FS = $31^{\circ}13'13\frac{1}{2}''$ Log. co-tangent = 10.217450*
 Is to the radius = . . 90. 0. 0 Log. sine = . . 10.000000
 So is the angle FS 1 = . 60.28.48 Log. co-sine = 9.692607

To the co-latitude S 1 = 50.53.28 Log. co-tangent = 9.910057

First latitude = . . . $39^{\circ}6'32''$ S., at which the ship should arrive.

To find the Hypothenuse, or Co-Latitude = S 2 :—

As the perpendicular FS = $31^{\circ}13'13\frac{1}{2}''$ Log. co-tangent = 10.217450*
 Is to the radius = . . 90. 0. 0 Log. sine = . . 10.000000
 So is the angle FS 2 = . 55.28.48 Log. co-sine = . 9.753349

To the co-latitude S 2 = 46.55.29 Log. co-tangent = 9.970799

Second latitude = . . $43^{\circ}4'31''$ S., at which the ship should arrive.

Hence, the first latitude at which the ship should arrive, is $39^{\circ}6'32''$ S.; and the second latitude $43^{\circ}4'31''$ S.: and, since it is the latitude itself, and not its complement, that is required, if the log. tangent of the sum of the three logarithms be taken, it will give the latitude direct; and, by rejecting the radius, the work will be considerably facilitated. Proceeding in this manner, the several successive latitudes corresponding to the proposed alterations of longitude will be found, as in the third column of the following Table.

Now, let the several successive longitudes be arranged (agreeably to the proposed change, and to the measure of the corresponding polar angles,) as given in the second column of the following Table; and find the difference between every two adjacent longitudes, as shown in the fourth column of that Table. Find the difference between every two successive latitudes, and place them in the fifth column of the Table. Take out from Table XLIII. the meridional parts corresponding to the several successive latitudes, as given in column 6, and find the difference between every two adjacent numbers, as given in the seventh column. Then find, by Mercator's Sailing, Problem I., page 238, the respective courses and distances between the several successive latitudes and longitudes; and let those courses and distances, so found, be arranged as in the two last columns of the following Table : viz.,

* The log. co-tangent is used, so as to avoid the trouble of finding the arithmetical complement of the log. tangent.

A TABLE,
Exhibiting, at Sight, all the Principal Elements attendant on the Computation of the Approximate Spherical Route from the Cape of Good Hope to the North Point of King's Island, at the Western Entrance to Bass' Strait.

Polar Angles.	Successive Longitudes.	Successive Latitudes.	Differences of Longitude.	Differences of Latitude.	Meridional Parts.	Meridional Difference of Latitude.	By Mercator's Sailing.	
							Courses.	Distances.
F 8 A = 63° 28' 48"	180° 32' 0" E.	34° 24' 0" S.	Miles. 300.0	Miles. 282.53	2200.50	Miles. 352.84	S. 40° 27' E.	370.85
F 8 1 = 60.28.48	23.32. 0	39. 6.32	300.0	282.53	2353.34	315.94	43.31	328.17
F 8 2 = 55.28.48	28.32. 0	43. 4.31	300.0	237.98	2869.28	280.43	46.56	291.60
F 8 3 = 50.28.48	33.32. 0	46.23.39	300.0	199.13	3149.71	246.87	50.33	261.01
F 8 4 = 45.28.48	38.32. 0	49. 9.30	300.0	165.85	3396.58	215.39	54.19	235.81
F 8 5 = 40.28.48	43.32. 0	51.27. 2	300.0	137.53	3611.97	185.74	58.14	215.27
F 8 6 = 35.28.48	48.32. 0	53.20.21	300.0	113.32	3797.71	157.86	62.15	198.70
F 8 7 = 30.28.48	53.32. 0	54.52.53	300.0	92.53	3955.57	131.40	66.21	185.54
F 8 8 = 25.28.48	58.32. 0	56. 7.19	300.0	74.44	4086.97	106.34	70.29	175.10
F 8 9 = 20.28.48	63.32. 0	57. 5.49	300.0	58.50	4193.31	82.16	74.41	167.26
F 8 10 = 15.28.48	68.32. 0	57.50. 0	300.0	44.18	4275.47	58.71	78.56	161.57
F 8 11 = 10.28.48	73.32. 0	58.21. 2	300.0	31.03	4334.18	35.90	83.11	157.80
F 8 12 = 5.28.48	78.32. 0	58.39.47	300.0	18.75	4370.08	13.47	87.39	170.53
F 8 = 0. 0. 0	84.00.48	58.46.46½	328.8	6.98	4383.55	5.32	88.53	140.50
F 8 a = 4.31.12	88.32. 0	58.44. 1	271.2	2.75	4378.23	35.37	83.17	157.40
F 8 b = 9.31.12	93.32. 0	58.25.35	300.0	18.43	4342.86	54.32	79.44	160.75
F 8 c = 14.31.12	98.32. 0	57.56.57	300.0	28.64	4288.54	77.61	75.30	166.06
F 8 d = 19.31.12	103.32. 0	57.15.22	300.0	41.59	4210.93	101.59	71.18	173.47
F 8 e = 24.31.12	108.32. 0	56.19.44	300.0	55.64	4109.34	126.53	67. 8	183.29
F 8 f = 29.31.12	113.32. 0	55. 8.30	300.0	71.23	3982.81	152.68	63. 2	195.93
F 8 g = 34.31.12	118.32. 0	53.39.38	300.0	88.87	3830.13	180.26	59. 0	211.79
F 8 h = 39.31.12	123.32. 0	51.50.33	300.0	109.08	3649.87	209.52	55. 4	231.53
F 8 i = 44.31.12	128.32. 0	49.37.59	300.0	132.57	3440.35	240.69	51.16	255.79
F 8 k = 49.31.12	133.32. 0	46.57.55	300.0	160.07	3199.66	273.85	47.37	285.28
F 8 l = 54.31.12	138.32. 0	43.45.35	300.0	192.33	2925.81	333.06	44. 2	345.76
F 8 B = 59.53.12	143.54. 0	39.37. 0	322.0	248.58	2592.75			
			7522. 0	2612.33		3973.85		5426.46

Now, the sum of the several successive differences of longitude = 7522 miles, makes up the whole difference of longitude between the two given places; the sum of the successive differences of latitude = 2612.53 miles, is equal to the whole difference of latitude comprehended under the highest latitude at which the ship should arrive, and the latitudes of the two given places, viz. $34^{\circ}24'0''$ S., $58^{\circ}46'46\frac{1}{2}''$ S., and $39^{\circ}37'0''$ S.—And, the sum of the several meridional differences of latitude = 3973.85 miles, coincides exactly with the whole meridional difference of latitude corresponding to the highest latitude, and the latitudes of the two given places; which several agreements, form an incontestable proof that the work has been carefully conducted.

The sum of the several distances measured on the consecutive rhumb lines intercepted between the successive latitudes and longitudes, as exhibited in the last column of the Table, is 5426.46 miles;—but the true spherical distance on the arc of a great circle is 5426.30 miles; the difference, therefore, is only $0'.16$; or, about $\frac{1}{8}$ of a mile; which is very trifling, considering the extent of the arc.—The distance by Mercator's sailing is 6011.2 miles; which is 585 miles more than by great circle sailing.

Hence, it is evident that the shortest and most direct route from the Cape of Good Hope to King's Island is by the latitude of $58^{\circ}46'46\frac{1}{2}''$ S.; and that the ship must make, successively, the several longitudes and latitudes contained in the 2nd and 3rd columns of the Table, in the same manner, precisely, as if they were so many headlands, or places of rendezvous, at which she was required to touch.—The first course, therefore, from the Cape of Good Hope is S. $40^{\circ}22'$ E. distance 371 miles, which will bring the ship to longitude $23^{\circ}32'$ E. and latitude $39^{\circ}6'32''$ S.;—the second course is S. $43^{\circ}31'$ E. distance 328 miles, which brings the ship to longitude $28^{\circ}32'$ E. and latitude $43^{\circ}4'31''$ S.; the third course is S. $46^{\circ}56'$ E. distance 292 miles, which brings the ship to longitude $33^{\circ}32'$ E. and latitude $46^{\circ}23'39''$ S.;—and so on of the rest.—Whence, it is evident that if the ship sails upon the several courses, and runs the corresponding distances respectively set forth in the two last columns of the Table, she will, most assuredly, arrive at the several successive longitudes and latitudes pointed out in the 2nd and 3rd columns of that Table; and thus will she reach King's Island, the place which it is intended she shall make, by a track 585 miles shorter than if such track had been determined agreeably to the principles of Mercator's sailing.

And, in a long voyage, like the present, in which ships generally experience a great scarcity of fresh water, particularly those bound to His Majesty's colony at New South Wales with troops, or convicts, the saving of 585 miles run at sea becomes a consideration of no inconsiderable importance.

Nor is there any more difficulty in sailing on the arc of a great circle,

thus determined, than there is in sailing on a parallel of latitude ; for, if the ship's compass be but tolerably good, the variation thereof carefully attended to, and proper attention paid to the steerage, the courses and distances expressed in the two last columns of the Table will, undoubtedly, carry the ship direct from the Cape of Good Hope to the north point of King's Island, without ever referring to celestial observation for either latitude or longitude ; provided, indeed, that the ship's way is not affected by currents :—but, since the courses contained in the 8th column of the Table, express the true bearings between the several successive latitudes and longitudes through which the ship must pass ; these must, therefore, be reduced to the magnetic, or compass course, by allowing the observed variation to the right hand thereof if it be westerly, but to the left hand if easterly ; this being the converse process of reducing the magnetic, or course steered by compass, to the true course.—And, if the spherical track, so determined, be delineated on a Mercator's chart, it will, perhaps, not only simplify the navigation, but also point out to the mariner any *known land* that may be adjacent thereto * ; and thus enable him to alter his course as occasion may require.—The spherical track may be readily delineated on a chart by means of the angles of meeting made by the several latitudes and longitudes, which show the places or points where the ship is to alter her course :—Now, those points being joined by right lines will indicate the true courses and distances, or the absolute route on which the ship must sail from the Cape of Good Hope to King's Island ; then, if each day's run be carefully measured on the track, so delineated, the navigator can always know his distance from the place to which he is bound, without resorting to the trouble of calculation.

I have dwelt at considerable length upon this Problem for the express purpose of simplifying a subject which is but very little understood by the generality of maritime people :—and, with the view of rendering it still more familiar, another example will be given by which the approximate spherical route, as performed by His Majesty's ship *Dauntless*, under the command of George Cornish Gambier, esq. on her voyage from Port Jackson to Valparaiso, in the year 1822, will be clearly illustrated.

Example 2.

His Majesty's ship *Dauntless* being bound from Port Jackson, in latitude $33^{\circ}52'$ S. and longitude $151^{\circ}16'$ E. to Valparaiso, in latitude $33^{\circ}1'$ S. and longitude $71^{\circ}52'$ W., the captain, G. C. Gambier, Esq., proposed to navigate her as near to the arc of a great circle as he could, by altering her course at every 5 degrees of longitude ; required the latitude at each

* It is presumed that there is not any land to intercept a ship's progress in this track.

time of altering the course, together with the respective courses and distances between those several latitudes and longitudes, occasioned by the proposed changes ?

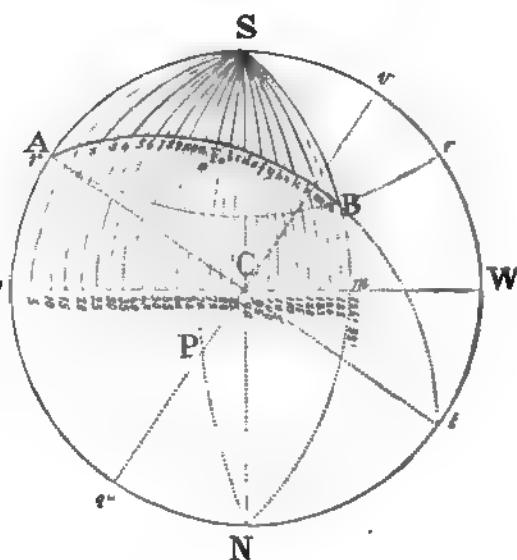
Port Jackson, . . Latitude $33^{\circ}52'$ S. Longitude = $151^{\circ}16'$ E.
 Valparaiso, . . Latitude 33.1 S. Longitude = 71.52 W.

Sum = . . . $223^{\circ} 8'$

Difference of longitude between the two given places = $136^{\circ}52'$

Calculation.

Since the elements of this Example are analogous to those of the last; it is not, therefore, deemed necessary to repeat the mode of projection; the only difference in the construction being that, in the preceding diagram, because the ship is bound to a place to the eastward of that from which she is to sail; the latter is, therefore, for the sake of uniformity, placed on the primitive circle in the western hemisphere:—and, in the present diagram, because the ship is bound to a port to the westward of that from which she is to sail, the latter (for the sake of uniformity also) is placed on the primitive circle in the eastern hemisphere:—the letter Q representing the western hemisphere in the former case, and the eastern hemisphere in the latter.



Now, the figure being thus constructed on the plane of the meridian passing through Port Jackson; let the point A represent that place; the point B, the place of Valparaiso, and the arc AB, the true spherical distance between those places;—then, AS represents the co-latitude of Port Jackson; BS, that of Valparaiso; SAB, the angle of position at the former place, and SBA, the angle of position at the latter place.—The arc FS, which is drawn perpendicular to AB, represents the complement of the highest latitude at which the ship should arrive; and the several arcs S I;

S 2; S 3; S 4; &c. &c. &c., represent the complements of the successive latitudes through which the ship must pass.—Hence, in the oblique angled spherical triangle A B S, two sides and the included angle are given to find the third side and the remaining angles; viz., the side A S = $56^{\circ}8'$ the co-latitude of Port Jackson; the side B S = $56^{\circ}59'$ the co-latitude of Valparaiso, and the angle A S B = $136^{\circ}52'$ the difference of longitude between those places; to find the spherical distance A B, and the respective angles of position = S A B and S B A :—the distance may be readily found by Remark 1, or 2, to Problem III., page 203 or 204; as thus :

Diff. long. A S B $136^{\circ}52'$ + 2 =

$68^{\circ}26'$ Twice log. S. = 19.936958

Co-latitude of Port

Jackson = A S $56^{\circ}8'$ Log. sine = 9.919254

Co-lat. of Valpa-

raiso = B S $56^{\circ}59'$ Log. sine = 9.923509

Sum = 39.779721

Diff. of co-lat. = $0^{\circ}51'$ Half = 19.889860 $\frac{1}{2}$ 19.889860 $\frac{3}{4}$

Half diff. of ditto = $0^{\circ}25'30''$ Log. S. = 7.870262

Arch = . . . $89^{\circ}25'47''$ Log. tang. = 12.019598 $\frac{1}{2}$ Log. S. = 9.999979

Half the arc A B = $50^{\circ}53'56''$ Log. sine = 9.889881 $\frac{1}{2}$

Side A B = . . $101^{\circ}47'52''$ = the true spherical distance between the two given places.

To find the Angle of Position at Port Jackson = Angle S A B :—

This is found by Problem I., page 198; as thus :

As the distance A B = $101^{\circ}47'52''$ Log. co-secant = 10.009273

Is to diff. long. A S B = $136.52.0$ Log. sine = . . 9.834865

So is the co-latitude B S = $56.59.0$ Log. sine = . . 9.923509

To angle of pos. = S A B = $35^{\circ}50'59''$ Log. sine = . . 9.767647

To find the Angle of Position at Valparaiso = Angle S B A :—

This is found by Problem I., page 198; as thus :—

As the distance $A B = 101^{\circ}47'52''$ Log. co-secant = 10.009273
 Is to diff. long. $A S B = 136.52.0$ Log. sine = . . 9.834865
 So is the co-lat. $A S = 56.8.0$ Log. sine = . . 9.919254

To angle of pos. $S B A = 35^{\circ}26'50''$ Log. sine = . . 9.763392

To find the Perpendicular $F S =$ the Complement of the highest Southern Latitude at which the Ship should arrive:—

Here we have a choice of two right angled spherical triangles, viz. $A S F$ and $B S F$; in each of which the hypotenuse and the angle at the base are given to find the perpendicular;—thus, in the triangle $A S F$, given the hypotenuse $A S = 56^{\circ}8'$ the co-latitude of Port Jackson; and the angle at the base $S A B = 35^{\circ}50'59''$ the angle of position at that place, to find the perpendicular $F S =$ the complement of the highest southern latitude at which the ship should arrive:—

Hence, by right angled spherical trigonometry, Problem II., page 185,

As radius = $90^{\circ}0'0''$ Log. co-secant = 10.000000
 Is to co-lat. Port Jackson = $A S 56.8.0$ Log. sine = . . 9.919254
 So is ang. of position = $S A F 35.50.59$ Log. sine = . . 9.767647

To the perpendicular $F S = 29^{\circ}5'51''$ Log. sine = . . 9.686901

Highest lat. at which the
 ship should arrive = $60^{\circ}54'9''$ south.

From the above calculations it appears evident that the true spherical distance between Port Jackson and Valparaiso is $101^{\circ}47'52''$, or 6107.87 miles; the angle of position at Port Jackson = $35^{\circ}50'59''$, and that at Valparaiso = $35^{\circ}26'50''$, and the highest southern latitude at which the ship should arrive = $60^{\circ}54'9''$.—Now, by Mercator's sailing, the course from Port Jackson to Valparaiso is $N. 89^{\circ}34' E.$ and the distance 6853.16 miles;—whence it is manifest, that if a ship sails on the direct rhumb-line between those places, as indicated by that mode of sailing, she will have to run $745\frac{1}{4}$ miles more than by shaping her course along the arc of a great circle.

To compute the vertical, or Polar Angles $A S F$, and $B S F$:—

In the right angled spherical triangle $A S F$, given the hypotenuse $A S 56^{\circ}8' =$ the co-latitude of Port Jackson, and the perpendicular $F S 29^{\circ}5'51'' =$ the complement of the highest latitude at which the ship should arrive; to find the vertical, or polar angle $A S F$.—And, in the right angled triangle $B S F$, given the hypotenuse $B S, 56^{\circ}59' =$ the co-

latitude of Valparaiso, and the perpendicular F S, $29^{\circ}5'51''$; to find the polar angle B S F.—Hence, by right angled spherical trigonometry, Prob. I., page 184,

To find the Polar Angle A S F :—

As radius =	$90^{\circ}0'0''$	Log. co-secant =	10.000000
Is to the co-latitude A S = .	$56.8.0$	Log. co-tangent =	9.826805
So is the co-latitude F S = .	$29.5.51$	Log. tangent =	9.745493
<hr/>			
To the polar angle A S F =	$68^{\circ}4'5''$	Log. co-sine =	9.572298
<hr/>			

To find the Polar Angle B S F :—

As radius =	$90^{\circ}0'0''$	Log. co-secant =	10.000000
Is to the co-latitude B S =	$56.59.0$	Log. co-tangent =	9.812794
So is the co-latitude F S =	$29.5.51$	Log. tangent =	9.745493
<hr/>			
To the polar angle B S F =	$68^{\circ}47'55''$	Log. co-sine =	9.558287

Now, since the sum of the polar angles, thus obtained, viz. A S F, $68^{\circ}4'5''$ + B S F, $68^{\circ}47'55''$ = $136^{\circ}52'$, makes up the whole difference of longitude between the two given places, expressed by the whole angle A S B, it shows that, thus far, the work is right.

To find the several successive Polar Angles made by the proposed changes of Longitude.

From the polar angle A S F, subtract the proposed alteration of longitude continually, as far as subtraction can be made; and the several polar angles occasioned by those alterations, and contained between the perpendicular F S, and the co-latitude of Port Jackson = A S, will be obtained.—Thus, from the polar angle A S F = $68^{\circ}4'5''$, let 5° be continually *subtracted*, and the results will be F S 1 = $63^{\circ}4'5''$ F S 2 = $58^{\circ}4'5''$; F S 3 = $53^{\circ}4'5''$, &c. &c., the last remainder being $3^{\circ}4'5''$ = the polar angle F S 13.—Now, the polar angles contained between the perpendicular F S, and the co-latitude of Valparaiso = B S, are to be determined by a contrary process; and, since the last subtraction in the triangle F S A, left the remainder, or polar angle F S 13 = $3^{\circ}4'5''$, which is $1^{\circ}55'55''$, less than the proposed alteration of longitude; therefore, the first polar angle in the triangle F B S, must be $1^{\circ}55'55''$ = the polar angle F S a; to which, let 5° be continually added, as far as the measure of the angle F S B will allow, and we shall have F S b = $6^{\circ}55'55''$; F S c = $11^{\circ}55'55''$; F S d = $16^{\circ}55'55''$, and so on; as expressed in the first column of the following Table.

To compute the successive Latitudes at which the Ship should arrive :—

Since the several successive polar angles, obtained as above, evidently reduce the two right angled spherical triangles A F S and B F S, into a series of right angled spherical triangles, to each of which the perpendicular F S is common; therefore, in each triangle of this series we have the perpendicular and the angle adjacent, to find the hypotenuse, or co-latitude.—Thus, in the right angled spherical triangle F S 1, right angled at F, given the perpendicular F S = $29^{\circ} 5' 51''$, and the polar angle F S 1 = $63^{\circ} 4' 5''$; to find the hypotenuse, or co-latitude S 1;—In the right angled spherical triangle F S 2, given the perpendicular F S = $29^{\circ} 5' 51''$, and the polar angle F S 2 = $58^{\circ} 4' 5''$; to find the hypotenuse, or co-latitude S 2, &c. &c. &c.

Hence, by right angled spherical trigonometry, Problem IV., page 188,

To find the Hypotenuse, or Co-latitude S 1 :—

As the perpendicular F S =	$29^{\circ} 5' 51''$	Log. co-tang. =	10.254507*
Is to the radius =	. . . 90. 0. 0	Log. sine =	. 10.000000
So is the polar angle F S 1 =	63. 4. 5	Log. co-sine	. 9.656033

To the co-latitude S 1 = $50.51.36$ Log. co-tangent = 9.910540

First latitude = . . $39^{\circ} 8' 24''$ S. at which the ship should arrive.

To find the Hypotenuse, or Co-latitude S 2 :—

As the perpendicular F S =	$29^{\circ} 5' 51''$	Log. co-tang =	10.254507*
Is to the radius =	. . . 90. 0. 0	Log. sine =	. . 10.000000
So is the polar angle F S 2 =	58. 4. 5	Log. co-sine =	9.723383

To the co-latitude S 2 = $46.27.28$ Log. co-tang. = 9.977890

Second latitude = $43^{\circ} 32' 32''$ S. at which the ship should arrive.

Hence, the first latitude at which the ship should arrive is $39^{\circ} 8' 24''$ S.; and the second latitude $43^{\circ} 32' 32''$ S.—And since it is the latitude, and not its complement that is required; therefore, if the log. tangent of the sum of the three logs. be taken, it will give the latitude direct; and, by rejecting the radius from the calculation, the work will be considerably facilitated.—Proceeding in this manner, the several successive latitudes cor-

* The log. co-tangent is used, so as to save the trouble of finding the arithmetical complement of the log. tangent.

responding to the proposed alterations of longitude will be found as shown in the 3d column of the following Table.

Now, let the several successive longitudes be arranged (agreeably to the proposed change, and to the measure of the corresponding polar angles), as exhibited in the 2d column of the following Table; and find the difference between every two adjacent longitudes, as shown in the 4th column of that Table.—Find the difference between every two adjacent latitudes, and place those differences in the 5th column.—Find the meridional parts corresponding to the several successive latitudes, which place in the 6th column; and find the difference between every two adjacent meridional latitudes, as shown in the 7th column.—Then find, by Mercator's sailing, Problem I., page 238, the respective courses and distances between the several successive latitudes and longitudes; and, let the courses and distances, so found, be arranged in regular succession, as exhibited in the two last columns of the Table.—Then, will this Table be duly prepared for navigating a ship on the arc of a great circle, agreeably to the proposed alterations of longitude.—And, should the sum of the several successive differences of longitude, contained in the Table, coincide with the whole difference of longitude between the two given places;—the sum of the several successive differences of latitude be found to agree with the whole difference of latitude comprehended under the mean, or highest latitude, and its corresponding extremes;—the sum of the several meridional differences of latitude to be equal to the whole meridional difference of latitude corresponding to the mean, or highest latitude, and its respective extremes,—and the sum of the several successive distances to make up the whole spherical distance (or nearly so,) between the two given places; then, those several concurring equalities will be so many satisfactory proofs that the work is right.

Note.—In the spherical track laid down in the following Table, it is presumed that there is not any land to intercept a ship's progress: but since this track will take the navigator into high southern latitudes, it will be indispensably necessary to keep a sharp look-out at all times, particularly during the night, so as to guard against any of the ice-bergs that may be floating to the northward of the Antarctic circle;—though, if the track be made in the months of November, December, January, or February, there will be no real night or darkness to experience; for during these months there will be a strong twilight between the latitudes of 53, and 61 degrees south; and thus the navigating at night will be attended with very little more danger than that by day.

A TABLE,

Exhibiting, at Sight, all the Particulars attendant on the Computation of the Approximate Spherical Route from Port Jackson, in New South Wales, to Valparaiso on the Coast of Chili.

Polar Angles.	Successive Longitudes.	Successive Latitudes.	Differences of Longitude.	Differences of Latitude.	Meridional Parts.	Meridional Difference of Latitude.	By Mercator's Sailing.	
							Courses.	Distances.
FS A = 68° 4' 5"	151° 16' 0" E.	33° 52' 0" S.	Miles.	Miles.	2161.84	Miles.	S. 37° 18' E.	397.71
FS 1 = 63. 4. 5	156. 16. 0	39. 8. 24	300. 00	316. 40	2555.75	393.91	40. 26	347.04
FS 2 = 58. 4. 5	161. 16. 0	43. 32. 32	300. 00	264. 14	2907.79	352.04	43. 53	303.92
FS 3 = 53. 4. 5	166. 16. 0	47. 11. 36	300. 00	219. 06	3219.75	311.96	47. 33	268.26
FS 4 = 48. 4. 5	171. 16. 0	50. 12. 41	300. 00	181. 09	3494.24	274.49	51. 23	239.17
FS 5 = 43. 4. 5	176. 16. 0	52. 41. 58	300. 00	149. 29	3733.92	239.68	55. 22	215.70
FS 6 = 38. 4. 5	178. 44. 0 W.	54. 44. 35	300. 00	122. 60	3941.17	207.25	59. 26	196.91
FS 7 = 33. 4. 5	173. 44. 0	56. 24. 42	300. 00	100. 12	4118.31	177.14	63. 36	181.90
FS 8 = 28. 4. 5	168. 44. 0	57. 45. 35	300. 00	80. 89	4267.26	148.95	67. 51	170.31
FS 9 = 23. 4. 5	163. 44. 0	58. 49. 48	300. 00	64. 21	4389.38	122.12	72. 7	161.20
FS 10 = 18. 4. 5	158. 44. 0	59. 39. 19	300. 00	49. 52	4486.22	96.84	76. 26	154.50
FS 11 = 13. 4. 5	153. 44. 0	60. 15. 33	300. 00	36. 23	4558.59	72.37	80. 46	149.85
FS 12 = 8. 4. 5	148. 44. 0	60. 39. 35	300. 00	24. 04	4607.35	48.76	85. 8	147.06
FS 13 = 3. 4. 5	143. 44. 0	60. 52. 3	300. 00	12. 46	4632.86	25.51	88. 39	89.50
FS = 0. 0. 0	140. 39. 55	60. 54. 9	184. 08	2. 10	4637.18	4.32	N. 89. 9	56.26
FS a = 1.55.55	138. 44. 0	60. 53. 19	115. 92	0. 83	4635.47	1.71	86. 8	146.71
FS b = 6.55.55	133. 44. 0	60. 43. 25	300. 00	9. 91	4615.16	20.31	81. 46	149.01
FS c = 11.55.55	128. 44. 0	60. 22. 4	300. 00	21. 34	4571.75	43.41	77. 24	153.06
FS d = 16.55.55	123. 44. 0	59. 48. 40	300. 00	33. 40	4504.67	67.08	73. 7	159.65
FS e = 21.55.55	118. 44. 0	59. 2. 17	300. 00	46. 37	4413.61	91.06	68. 48	167.95
FS f = 26.55.55	113. 44. 0	58. 1. 34	300. 00	60. 73	4297.26	116.35	64. 34	179.04
FS g = 31.55.55	108. 44. 0	56. 44. 40	300. 00	76. 90	4154.57	142.69	60. 17	193.38
FS h = 36.55.55	103. 44. 0	55. 8. 50	300. 00	95. 84	3983.39	171.18	56. 21	210.88
FS i = 41.55.55	98. 44. 0	53. 12. 0	300. 00	116. 83	3783.75	199.64	52. 18	233.51
FS k = 46.55.55	93. 44. 0	50. 49. 12	300. 00	142. 80	3551.88	231.87	48. 23	260.99
FS l = 51.55.55	88. 44. 0	47. 55. 51	300. 00	173. 35	3285.33	266.55	44. 42	295.19
FS m = 56.55.55	83. 44. 0	44. 26. 0	300. 00	209. 85	2982.09	303.24	41. 12	336.55
FS n = 61.55.55	78. 44. 0	40. 12. 46	300. 00	253. 23	2639.39	342.70	37. 59	385.58
FS o = 66.55.55	73. 44. 0	35. 8. 50	300. 00	303. 94	2255.07	384.32	35. 58	157.94
FS B = 68.47.55	71. 52. 0	33. 1. 0	112. 00	127. 83	2100.72	154.35		
			8212. 00	3295. 30		5011.80		6108.73

Now, the sum of the several successive differences of longitude, viz. 8212 miles, coincides exactly with the whole difference of longitude between the two given places; the sum of the successive differences of latitude = 3295.30 miles, agrees with the whole difference of latitude comprehended under the highest latitude at which the ship should arrive, and the latitudes of the two given places; viz. $33^{\circ}52'0''$ S; $60^{\circ}54'9''$ S, and $33^{\circ}1'0''$ S:—and, the sum of the several meridional differences of latitude = 5011.80 miles, makes up the whole difference of latitude corresponding to the highest latitude and the latitudes of its respective extremes:—these several concurrences or agreements, form, therefore, the most satisfactory and indisputable proofs that the work has been properly conducted.

The sum of the several distances, measured on the respective rhumb-lines intercepted between the successive longitudes and latitudes, as given in the last column of the Table, is 6108.73 miles;—but the true spherical distance on the arc of a great circle is 6107.87 miles; the difference, therefore, is only 0.86, or a little more than three-fourths of a mile; which is a very close approximation in the measure of so great an arc.

The distance by Mercator's sailing is 6853.16 miles; which is 745.29, or about $745\frac{1}{4}$ miles more than by great circle sailing.—Hence, it is evident that the shortest and most direct route from Port Jackson to Valparaiso is by the latitude of $60^{\circ}54'9''$ S; and that the ship must make, successively, the several longitudes and latitudes contained in the 2nd and 3rd columns of the Table, in the same manner precisely, as if they were so many ports or places of rendezvous, at which she was directed to touch.

The first course, therefore, from Port Jackson to Valparaiso, is S, $37^{\circ}18'$ E. distance 398 miles; which will bring the ship to longitude $156^{\circ}16'0''$ E. and latitude $39^{\circ}8'24''$ S;—the second course is S. $40^{\circ}26'$ E. distance 347 miles; which brings the ship to longitude $161^{\circ}16'0''$ E. and latitude $43^{\circ}32'32''$ S;—the third course is S. $43^{\circ}53'$ E. distance 304 miles, which brings the ship to $166^{\circ}16'0''$ E. and latitude $47^{\circ}11'36''$ S. &c. &c. &c.

Whence it is evident that Captain Gambier saved a distance of $745\frac{1}{4}$ miles in that judicious and well-planned route: And this saving of distance should be an object of the highest consideration to every captain who wishes to recruit the strength and spirits of his ship's company by a generous supply of fresh provisions after a fatiguing and tedious voyage; the measure of which falls very little short of being equal to one-fourth of the earth's circumference as taken under the equator, or to the one-third of that circumference if taken under the given parallel of latitude.

SOLUTION OF PROBLEMS IN NAUTICAL ASTRONOMY.

NAUTICAL ASTRONOMY is the method of finding, by celestial observation, the latitude and longitude of a ship at sea; the variation of the compass; the mean time at ship; the altitudes of the heavenly bodies, &c. &c. &c.—Or, it is that branch of mathematical astronomy which shows how to solve all the important Problems in navigation by means of spherical operations, when the altitudes, or distances of the celestial objects are under consideration.

Before entering upon the Astronomical part of this work, it appears to be indispensably necessary that a few observations should be made relative to the *new* and *most important element* called “Sidercal Time,” which is given in page II. of the month in the Nautical Almanac: for this essentially useful element enters so generally into all the calculations in which the moon, stars, and planets are concerned, that, without a previous knowledge of the principles upon which it is founded, and of the uses to which it may be applied, it would be in vain to attempt the solution of a Problem which has any relation to *mean time*.—Indeed, unless the nature of “Sidercal Time” be clearly understood, even the most simple of the preliminary problems will appear to be incomprehensible.

Although it is my intention to associate a concise explanation with every problem that may appear to demand a specific illustration, yet, with the view of elucidating the elementary expressions in the Nautical Almanac, and of giving the young navigator a competent knowledge of the above-named *important element*, I shall here place before his view the following

General Definitions.

1. The grateful phenomenon of *Day* and *Night* is produced by the circumrotation of the earth from *west* to *east*, round an imaginary line called the axis, the extremities of which are named Poles: the extremity which lies towards the most northern part of Europe is denominated the North Pole, and its opposite, the South Pole.

2. *The Celestial Poles* are two immovable points, round which the heavens seem to *turn*, or move *apparently* from east to west: they may be considered as the extremities of the earth's axis-produced to the firmament, or to the sphere of the fixed stars. The *apparent motion* of the heavens from east to west, is occasioned by the *actual diurnal motion* of the earth round its axis from west to east, which gives an *illusive* motion, in a contrary direction, to all the heavenly bodies.

3. *The Equator* is a great circle on the earth, every point of which is

equally distant from the poles. It divides the globe into two equal parts, called hemispheres :—the half which has the North Pole in its centre is named the northern hemisphere, and the other, the southern hemisphere.

4. *The Equinoctial* is a great circle in the heavens, every part of which is equally distant from the celestial poles, and corresponding with the equatorial circle on the earth :—hence, it may be considered as being the plane of the earth's equator, extended to the starry firmament.—The equator, or equinoctial, as well as all the other great circles of the spheres, is divided into 360 equal parts, called degrees ; each degree is divided into 60 equal parts, called minutes, and each minute into 60 equal parts, called seconds.

5. *The Meridian* of any place on the earth, is a semicircle which passes through that place, cuts the equator at right angles, and terminates at the poles of the world :—and, as every part of the globe has a meridian passing through it, there may be as many meridians as there are points in the equator. The meridians divide the great equatorial circle of 360 degrees into 24 equal parts, each consisting of 15 degrees ; and since those equal parts correspond to the exact measure of a natural day of 24 hours, therefore, one hour is equal to 15 degrees of the equator, or of *longitude*.—When the plane of the meridian is extended to the sphere of the fixed stars, it then becomes a circle of right ascension, or an hour circle. And circles of right ascension divide the equinoctial into as many parts as the terrestrial meridians, or *circles of longitude*, divide the equator.

6. *The First Meridian* is an imaginary semicircle, passing through any remarkable place and the poles of the world : hence it is *arbitrary*. The English esteem that circle of longitude to be *the first meridian*, which passes through the Royal Observatory at Greenwich ; the French esteem that to be the first meridian which passes through the Royal Observatory at Paris ; the Spaniards, that which passes through Cadiz, &c. &c. &c.

7. *The Zenith* of any place is the point of the heavens which is directly above, or perpendicular to such place ; and is, therefore, the *elevated pole* of the horizon. The *Nadir* is the depressed pole of the horizon, or the point diametrically opposite to the zenith.

8. *A Vertical* is a great circle passing through the zenith and nadir, intersecting the horizon at *right angles*.—A *vertical* is frequently called an *azimuth circle* ; and when it cuts the horizon in the east or west points, it is then called *the prime vertical* :—it may also be called a *circle of altitude*.

9. *The Horizon* is a great circle which is equally distant from the zenith and nadir, and which divides the upper from the lower hemi-

sphere; this is called the *true*, or the *rational* horizon, because it passes through the centre of the earth. The *apparent* or *visible* horizon is the utmost apparent view which the eye of an observer can take either at sea or on land. The *sensible horizon* is that which terminates our view; and it is represented by that great circle which we see in fine clear weather, when the sky, or the azure vault of heaven, seems to rest upon the earth or sea:—and as it may be represented by a plane, passing through the eye of an observer, perpendicular to a plumb-line hanging freely; it is therefore parallel to the plane of the *true* or rational horizon which passes through the earth's centre.

10. *The Ecliptic* is a great circle in the heavens, through which the sun appears to move amongst the fixed zodiacal stars; but, in reality, it is the great circle in which the annual revolution of the earth is performed: hence it represents the earth's orbit. It is denominated the *ecliptic*, because the solar and lunar eclipses cannot take place unless the moon is either in or adjacent thereto; viz. except the moon be either in or very contiguous to the circle of the earth's annual path round the sun.—The ecliptic cuts the equinoctial in an angle of $23^{\circ}27'38''$, which is called the obliquity of the ecliptic:—this angle is constantly diminishing, at the rate of about 50 seconds of a degree in a century. Some of the modern astronomers have endeavoured to set limits to the diminution of the angle of obliquity, by asserting that it will stop at $20^{\circ}34'$; after which it will begin to increase at the rate of 50 seconds in a century, until it arrives at an angle of $27^{\circ}48''$:—this, however, is but a *mere fanciful hypothesis*.

The ecliptic, or *the earth's orbit*, like all other great circles, is divided into 360 degrees; and it is further divided into twelve equal parts, called signs; the names and characters of which are as follow, viz.

0. ♈ Aries, the Ram.	6. ♎ Libra, the Balance.
1. ♉ Taurus, the Bull.	7. ♏ Scorpio, the Scorpion.
2. ♊ Gemini, the Twins.	8. ♐ Sagittarius, the Archer.
3. ♋ Cancer, the Crab.	9. ♑ Capricornus, the Goat.
4. ♌ Leo, the Lion.	10. ♒ Aquarius, the Waterbearer.
5. ♍ Virgo, the Virgin.	11. ♓ Pisces, the Fishes.

The signs in the first, or *left-hand* division of the above, are called northern signs, because they are to the northward of the equinoctial:—when the sun is in any of these, his declination is *north*.—Those in the second, or *right-hand* division, are denominated southern signs, because they lie on the south side of the equinoctial:—when the sun is in any of these, he is said to have *south declination*.

11. The points in which the ecliptic and the equinoctial intersect each other, are called *the Equinoctial Points*, or, the equinoxes;—the sun

enters one of these on the 21st March, and the other on the 23rd September.—Those points of the ecliptic which are equidistant from the equinoxes, are called the *solstitial points*, or the solstices;—the sun enters one of these on the 21st June, and the other on the 21st December: hence, the first is named the *summer*, and the other the *winter*, solstice. The great circle which passes through the equinoctial points and the poles of the earth or heavens, is called *the equinoctial colure*; and that which passes through the solstitial points and the said poles, is called *the solstitial colure*.

12. *The Declination* of the sun, moon, star, or planet, is an arc of the celestial meridian intercepted between the equinoctial and the centre of the sun, moon, star, &c.; and it is called north or south, according as the sun, &c. &c. is situate with respect to the equinoctial. And the celestial meridian may be called a circle of declination; because it is a semi-circle which passes through the centre of a heavenly body, cuts the equinoctial at right angles, and terminates at the poles.

13. *The Tropics* are two small circles, or parallels of declination; they are parallel to the equinoctial, from which they are distant $23^{\circ}27'38''$.—The tropic of Cancer is on the north side of the equinoctial; and the tropic of Capricorn on its south side.

14. *Polar Circles* are two small circles, parallel to the equinoctial; from which they are distant $66^{\circ}32'22''$; or, at the distance of $23^{\circ}27'38''$ from each pole of the heavens or earth.

15. *The Right Ascension* of a heavenly body is an arc of the equinoctial, intercepted between the circle of declination passing through such body and the first point of Aries, measured according to the order of the signs: or, it is the angle at the pole of the heavens which is contained between the vernal equinox and the celestial meridian, or circle of declination, which passes through the celestial body.

16. *The Right Ascension of the Meridian*, or mid-heaven, signifies that point of the equinoctial which comes to the meridian of any place at a given time.

17. *The Culminating Point* of a star, or planet, is that part of its orbit which, on any given day, is the most elevated. Hence, a star is said to culminate when it comes to the meridian of any given place; because then its altitude is the greatest. The culminating, time of transit, and meridional passage of a celestial object, are synonymous terms; each of which signifies the passage of a heavenly body over the meridian of any given place.

18. *The Geocentric Latitudes* and longitudes, right ascensions, and declinations of the planets, are their latitudes and longitudes, right ascensions, &c., as seen from the centre of the earth.

19. *The Aphelion* is that point in the orbit of the earth, or of any

other planet, which is *most remote* from the sun :—this point is called the *higher Apsis*.

20. *The Perihelion* is that point in the orbit of the earth, or of any other planet, which is *most adjacent* to the sun. This point is called the lower *Apsis*, being diametrically opposite to the aphelion or higher *Apsis*.

21. *The Line of the Apsides* is a straight line joining the higher and the lower apsis of the earth, or other planet ;—or, it is the imaginary straight line which is drawn between the centres of the sun and the earth :—hence, it may be supposed to attach the earth to the sun ;—it lies evenly between the aphelion and the perihelion points of the earth's orbit.

22. *The Radius Vector* of the earth is an imaginary right line, which extends from the centre of the earth to that *focus* in the transverse diameter of its orbit, in which the sun is posited at any given time ;—or, it may be considered as being the true line of distance between the centres of the earth and sun :—hence, it may be called the *semi-line of the apsides*.

23. *An Occultation* is the obscuration or hiding from our view, any fixed star or planet by the interposition of the moon, or by that of the body of some other planet.

24. *The Transit of the first Point of Aries*, signifies the instant in *mean time* when the true point of intersection, which is made by the ecliptic and the equinoctial *at the vernal equinox*, is on the meridian of the Royal Observatory at Greenwich.

25. *The Nodes*.—These are the two opposite points in the ecliptic, that are intersected by the orbit of the moon or of a planet. The point where the moon &c. appears to ascend from the south to the north side of the ecliptic, is called *the ascending Node*, and is marked thus ♌ : and the opposite point, or that where the moon &c. appears to descend from the north to the south side of the ecliptic, is called *the descending Node*, and is marked ♎.

26. *The Aspect of the Stars or Planets*.—This signifies their situation with respect to each other. There are five aspects, viz. ♌, *conjunction*, when they are in the same sign and degree ; ♌, *sextile*, when they are two signs, or 60 degrees distant ; □, *quartile*, when they are three signs, or 90 degrees, viz. the fourth part of a circle, distant ; Δ, *trine*, when they are four signs, or 120 degrees, or the third part of a circle, asunder ; and ♌, *opposition*, when they are six signs, or 180 degrees, or the measure of a complete semicircle from each other.

EXPLANATORY ARTICLES: for *illustrating* and reducing to *familiarity* all the essential points and expressions in the New Nautical Almanac.

Of Time.

1. *Time* is divided into periods, cycles, years, months, weeks, days, hours, minutes, and seconds. But as it is the *year* and the *day* that chiefly concern the navigator; it is of these that we shall now particularly treat.

2. *A Year* is that period or portion of time, in which a revolution of the earth round its orbit, with respect to some fixed point in the heavens, is completed.

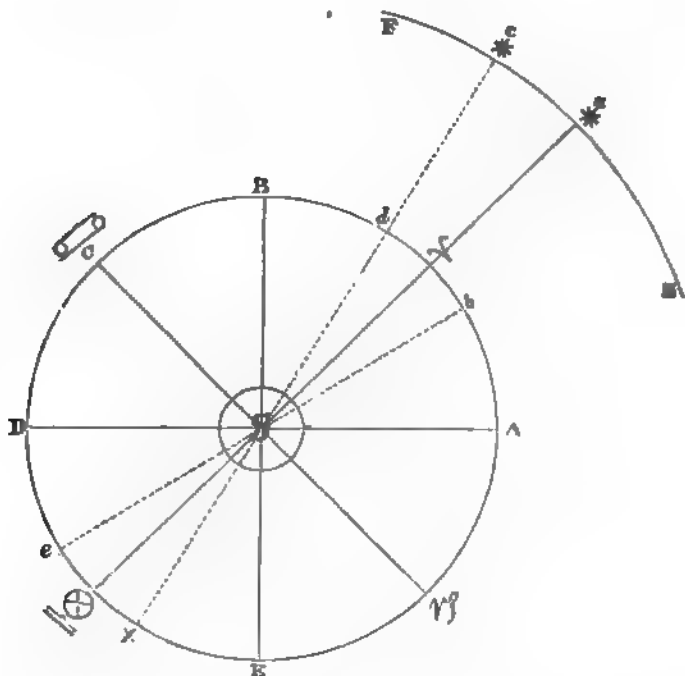
3. *A Solar, or Tropical Year*, is the interval of time between two consecutive returns of the earth to the same equinoctial, or to the same solstitial, point;—or, it signifies the time that the earth takes in moving round the ecliptic, from the first point of Aries, till its periodical return to the same point again:—this year consists of 365 days, 5 hours, 48 minutes, and 48 seconds.

4. *A Sidereal Year* is the interval of time between two consecutive returns of the earth to the imaginary straight line, which is conceived to pass through the centres of the earth and sun, and continued to a given fixed star in the firmament of heaven:—this year consists of 365 days, 6 hours, 9 minutes, $11\frac{1}{2}$ seconds; being 20 minutes and $23\frac{1}{2}$ seconds longer than the solar year.

5. As the length of the sidereal year cannot be clearly comprehended without a competent knowledge of *the recession of the equinoctial points*, commonly called “the precession of the equinoxes,” I shall therefore endeavour to present the reader with a familiar view of the *retrograde motion* of the equinoctial points: for it is upon this that the absolute length of the sidereal year is entirely dependent.

In the following diagram let A B C D represent the orbit of the earth; \oplus , the place of the earth at any given time; \odot , sun, near the centre of the earth's orbit:—let the arc, F H, represent a portion of the firmament of heaven, beyond the sun, and $\star a$, the place of a fixed star. Let the black line, or *radius vector* of the earth, $\oplus S$, be extended to the equinoctial colure γ , and thence to the fixed star at a ; and let the three objects, viz. the earth, sun, and star, be in this line or colure on the 21st of March in any year. Now, while the earth is progressing round its orbit from *west* to *east*, viz. from the point \oplus according to the direction of the letters and characters $\alpha E \gamma \Delta b$, &c., &c., the equinoctial points γ and Δ are moving slowly in a contrary direction, viz. from *east* to *west*; and thus, by the time that the earth has arrived at the point of the ecliptic, $\oplus \Delta$, from which it set out; the equinoctial point γ will have fallen back to b , and Δ to e ; which

will cause the equinoctial colures to be in the plane of the dotted line eSb :—Hence, this actual retrograde motion of the equinoxes gives an



apparent progressive motion to the fixed star at *a* in the firmament, and causes it to appear at the point *c*, a little to the *eastward* of where it was seen the year before: and therefore the earth must travel farther, or move on from \oplus to *x*, before it can bring the sun and star in the same right line, viz. in the dotted line *x S d c*, so as to complete the sidereal year.

The arc of the ecliptic, which is intercepted between the points \oplus and $x = \gamma b$, is the actual *recession of the equinoctial points*, Aries and Libra:—and the arc γd , which is similar to $\oplus x$, and equal to the arc $a c$, in the starry firmament, is what is called “the precession of the equinoxes.”

Precession is evidently an incorrect denomination ; but, it is in perfect harmony with that optical illusion, under which, to an observer on the earth at $\oplus \Delta$, the fixed star, a , will appear to have advanced to the eastward, or to have described the arc ac :—The annual value of this arc, deduced from numberless observations, is 50 $\frac{1}{2}$ seconds of a degree.

Now, because the recession of the equinoxes has no reference to the sun, S, but to the fixed star, α , which is at an infinite distance beyond it; therefore the earth will revolve from the point \oplus to the point \oplus again, and thus go through the orbital circle of 360 degrees in its an-

nual revolution round the sun, the same as if the equinoctial points \triangle and γ were immovable, or had no retrograde motion. Hence, the solar year will be completed in every^d return of the earth to the same relative point of the ecliptic; at which point it will always arrive, the instant that it has performed 365.24222 diurnal revolutions round its axis. But, with respect to the fixed star, a ; while the earth is moving round the ecliptic from west to east, the line of the equinoctial colures, marked $\triangle \oplus S \gamma * a$, is moving in an opposite direction, or from east to west; and therefore the earth and the equinoctial point \triangle will meet at e , before the sidereal year is completed.—The fixed star, a , will then be, *apparently*, at the point C, in the firmament; and thus the earth will have to move from e to x , so as to be in the same right line (the dotted line $x s d c$) with the sun and the fixed star, at c ; which completes the sidereal year.

The earth, in moving round its orbit, so as to accomplish the sidereal year, must *always perform something more than one diurnal* revolution on its axis, *beyond the number of diurnal* turns which it takes to complete the solar year. This, though seemingly paradoxical, will be easily comprehended, by reflecting that, since the equinoctial point \triangle is moving from east to west, or advancing, as it were, *to meet the earth*; this retrograde motion of the equinox shortens the measure of the earth's daily advance in the ecliptic by a certain portion of a degree, which bears the same proportion to the annual value of the equinoctial recession $\triangle e$, that the earth's daily motion in its orbit does to the great circle of 360 degrees:—and, hence, by the time that the earth has arrived at the point $\triangle \oplus$, it will have made 366.25222 revolutions upon its axis; that is, it will reckon *one complete revolution and a small fraction more* upon its axis, than it did in performing the same circuit with respect to the sun. But, as it must move from \oplus to x , before the sidereal year is completed; therefore, it will reckon the *fractional part* .00420 beyond the above expression: thus making in the whole 366.25642 revolutions.

The earth, in this respect, may be likened to a *ship*, which, by sailing round the globe in an *easterly* direction, would gain one complete day by the time that she returned to the port from which she set out:—for, when a ship sails easterly, she *advances towards the sun*, and therefore shortens the interval of time between every two returns of the sun to *her* noon, or 12 o'clock meridian line, in proportion to the meridional distance *made good* during that interval. Hence, being farther advanced towards the east every evening than in the preceding morning, she will cause the sun to set below her western horizon, something sooner than if she had not so advanced:—and, therefore, by curtailing each diurnal arc in proportion to the *east longitude* made good, she will

register *one day more* in her log-book at her return (let the period of her circumnavigation be ever so short or ever so long), than will be reckoned by the inhabitants of the port or place from whence she sailed.

6. Although the illustration of the diagram is sufficient to show that it is the recession of the equinoctial points which causes the sidereal year to be longer than the solar; yet, with the view of elucidating this curious subject in the most ample manner, I shall arrange a few simple proportions, which will not only confirm the above fact, but also prove that it is owing to the same cause, that the earth takes *a little beyond one diurnal revolution more* upon its axis to complete the sidereal year, than it does to finish the solar year.

The earth moves round the ecliptic, and completes the tropical year, in 365 days, 5 hours, 48 minutes, 48 seconds, or in 31556928 seconds of mean solar time. The ecliptic circle contains 360 degrees, or 1296000 seconds of motion. A *natural day*, viz. while the earth is turning once round upon its axis, consists of 24 hours, or 86400 seconds, mean solar time. Then—As 31556928 : 1296000 :: 86400 : to 3548° 33018 (=59° 8' 33;) which, therefore, is the arc of the ecliptic, that the earth describes every day, or the *mean rate* at which it moves round the ecliptic, during the period of making one complete revolution round its axis. Now, as the arc of the ecliptic, thus described, is to one diurnal revolution, so is the great circle of the ecliptic, to the number of diurnal revolutions which the earth takes to complete the solar year :—Hence,

As 3548° 33018 : 1 Rev. :: 1296000° to 365.24222 revolutions; which is the correct number of times that the earth must revolve round its axis, in performing its annual circuit round the sun.

It has been determined by numberless observations, as stated at bottom of page 302, that the recession of the equinoxes amounts to 50½ seconds of a degree every year; and we have seen, as above, that the earth advances 3548° 33018 in the ecliptic, while it is describing the great circle of its diurnal revolution; and since this circle, like all others, contains 360 degrees or 1296000 seconds, we are thus furnished with the necessary *data* for determining the absolute length of the sidereal year :—Hence,

As 3548° 33018 : 1296000 :: 50° 25, to 18353° 42; which being converted into time, gives 20 minutes 23½ seconds for the excess of the sidereal year above the solar :—then, this excess being *added* to the solar year, gives 365 days, 6 hours, 9 minutes, 11½ seconds, or 31558151½ seconds; which, therefore, is the absolute length of the sidereal year.

Now, observation shows that the earth performs one diurnal revolution round its axis, viz. that it turns completely round from any *fixed star to the same star again*, in 23 hours, 56 minutes, 4.0966 seconds, or in 86164.0906 seconds of mean solar time.—Hence,

As $86164:0906 : 1 \text{ Rev.} :: 31558151 : .5$, to 366.25642 ; which, therefore, is the actual number of revolutions that the earth must make round its axis, to complete the sidereal year; which is *one* complete revolution, and the fraction $.01420$ of *another*, more than it takes to finish the solar year.

Having thus shown that the excess of the sidereal year above the solar, is entirely owing to the retrograde motion of the equinoxes, we shall now resume the subject relating to the consideration of *time*, from which we broke off at the end of Article 4, page 301.

7. *An Apparent Solar Day* is the interval of time between two consecutive transits of the sun over *the same meridian*, as shown by a correct sun-dial:—this species of day is subject to an incessant variation, arising from the obliquity of the ecliptic, and the unequal motion of the earth round its orbit.

8. *A Natural Day*.—This consists of 24 hours, as shown by a well-regulated clock, or chronometer; being exactly equal to the time that the earth takes to turn once round upon its axis.

9. *A Mean Solar Day* is equal to the average length of all the days in a tropical year, and consists of 24 hours, 3 minutes, 56.5554 seconds, in sidereal time; but of 24 hours, exactly, in *mean* or equable time.

10. *A Sidereal Day* is the interval of time between two consecutive returns of any fixed star to the *same meridian*; or, rather, it is the absolute time in which the earth performs *one* revolution round its axis, in relation to a fixed star:—this consists of 23 hours, 56 minutes, 4.0906 seconds, in *mean solar time*, or 24 hours in sidereal time.

11. If the earth had not an annual motion round the sun, the solar day and the sidereal day would be precisely of the same length; but, while the earth revolves once round its axis, it advances $59^{\circ}8'33''$ in its orbit (paragraph 2, Article 6); and, therefore, should the sun and a fixed star be on the meridian of a place on any given day, the star will come to the *same meridian* the next day, when the sun is $59^{\circ}8'33''$ short of it:—Hence, the earth must perform *something more than one complete turn* on its axis; or go through an arc of its diurnal circle, equal to the measure of its daily advance in the ecliptic, before it can bring the sun to the same meridian again. The value of this diurnal *rotatory* arc of excess, may be determined in the following manner, viz.:

As 360 degrees : 1 Rev. :: $59^{\circ}8'33''$ to $.00273$;^{*} which is the absolute value of the arc that the earth must describe *beyond* one diurnal rotatory turn upon its axis, before it can cause the sun to be upon the same meridian that it was the day preceding. But, as to the fixed stars; they will always return to the same meridian at the end of every 23 hours, 56 minutes, 4.0906 seconds, *mean solar time*, or 24 hours

* Or,—As $24h. : 1 \text{ diurnal turn} :: 3m. 56s. 5554 \text{ to } .00273 \text{ of a diurnal turn.}$

sidereal time :—for the distance at which they are placed in the firmament of heaven, *beyond the sun*, is so immeasurably great, that the diameter of the earth's orbit, compared therewith (though upwards of 190 millions of miles in extent), *dwindles* into a mere *dimensionless point*; and therefore the earth will bring the meridian of any given place to the same star again, at the end of every complete revolution round its axis, the same as if it had no annual motion round the ecliptic.

12. The equable motion of the earth, *with respect to the fixed stars*, will cause any given star in the heavens to return to the same meridian again, at the end of every complete diurnal revolution round its axis;—and since this is performed in $23^{\circ}56'4''.0906$, mean solar time, or 24 hours sidereal time; the fixed stars, therefore, anticipate $3^{\circ}55'9094$ in *mean solar time*, upon the sun, every day, or $3^{\circ}56'5554$ in sidereal time. The *never-failing uniformity* of this measure of time, furnishes us with an infallible standard for proving the correctness of clocks and watches, as pointed out in the explanation of Table XLV. in pages 117 and 118.

13. *A Lunar Day* is the interval of time between two consecutive returns of the moon to the same meridian: this, upon an average, is about 24 hours, 48 minutes, 46 seconds. The lunar day being so much longer than the solar, is owing to the combined motions of the earth and moon: for, while the earth turns once round upon its axis, in 24 hours, it advances $59^{\circ}8'33''$ in its orbit; but, during that time, the moon, in going through a portion (betwixt the 29th and 30th part) of her periodical revolution round the earth, advances, at a *mean rate*, $13^{\circ}10'35''.02$ in the ecliptic: hence, she gains, on an average, $12^{\circ}11'26''.69$ upon the earth every day; and therefore the earth cannot bring the moon upon the same meridian that she passed the day before, until it has described an arc of $12^{\circ}11'26''.69$ over and above the great circle of 360 degrees, which measures the diurnal circuit round its axis. This arc, reduced to time, is $48^{\circ}45'77''$, or, rejecting fractions, 48 minutes and 46 seconds; and, as the arc of excess, thus described, bears the same proportion to a great circle, that the *fraction* over a diurnal turn of the earth does to one complete revolution round its axis; therefore,

As 360 degrees : $12^{\circ}11'26''.69$:: 1 Rev. to .03386; which is the value of the *fraction of excess* in relation to one diurnal turn:—hence, the earth must make one complete revolution round its axis and .03386, or rather more than *the third part of another turn*, before it can cause the moon to transit over the same meridian again.

14. *A Synodical Luration* is the interval of time between two consecutive new, or full moons: this consists of 29 days, 12 hours, 44 minutes, and 3 seconds, mean solar time.

15. *A Periodical Luration* is the time which the moon takes to finish

her revolution round the earth, with respect to some fixed star: this consists of 27 days, 7 hours, 43 minutes, 5 seconds, mean solar time.

16. The synodical lunation is longer than the periodical; because, while the moon is revolving round the earth from syzygia to syzygia again, she is advancing in the ecliptic at the mean rate of $13^{\circ}10'35''.02$ every 24 hours:—for;

As $27^{\circ}7'43''.5 : 360^{\circ} :: 1 \text{ day to } 13^{\circ}10'35''.02$:—but, during the time that the moon is describing this arc of the ecliptic, the earth describes an arc of $59'8''.33$:—Hence, $13^{\circ}10'35''.02 - 59'8''.33 = 12^{\circ}11'26''.69$, is the diurnal excess of the moon's motion in the ecliptic, over that of the earth. Now, as $12^{\circ}11'26''.69 : 1 \text{ day} :: 360^{\circ} \text{ to } 29^{\circ}12'44''.3$; which is the correct measure of a synodical lunation, or the true value of the interval of time between new moon and new moon; and therefore it is 2 days, 5 hours, 0 minutes, 58 seconds, longer than the periodical lunation or time that the moon takes to revolve round the earth, with respect to any given fixed star. From what is stated in the four last articles, it will appear manifest to the reader that if the earth had not an annual motion round its orbit, the moon would revolve round it, so as to complete her synodical and her periodical revolutions in the same exact measure of time, viz. in 27 days, 7 hours, 43 minutes, and 5 seconds.

17. *A Lunar Year* consists of twelve synodical revolutions:—hence, $29^{\circ}12'44''.3 \times 12 = 354 \text{ days, 8 hours, 48 minutes, 36 seconds}$, is the correct length of the lunar year; which is 10 days, 21 hours, 0 minutes, 12 seconds *shorter* than the solar year. This difference is estimated at 11 days in *round numbers*; and it is upon this that *the epact* is founded.

As the epact and certain subjects connected therewith are prefixed to the Nautical Almanac, it may not be unnecessary to make a few observations relative to “the principal articles of the calendar.”

18. *The Lunar Cycle*, or golden number, is a revolution of 19 years; in the course of which, the conjunctions, oppositions, and other aspects of the moon, will *fall upon the same days* that they did nineteen years before.

19. *The Solar Cycle* is a revolution of 28 years; in the course of which, the days of the month return again to the same days of the week on which they fell 28 years before; and the *leap-years* begin the same course over again, with respect to the days of the week on which the days of the month fall:—all the variations of the dominical letters will also take place, and then return in the same order that they did twenty-eight years before.

20. *The Dominical Letter* is the Sunday letter of the year; A, being *always taken* as the first of January, or the representative of New-year's Day. The first seven letters of the alphabet are placed in the calendar opposite to the seven days of the week, and that which answers to Sun-

day is called *the dominical letter*. If the year consisted of 365 days exactly, a period of the dominical letters would be completed in 7 years; but, because every fourth year is a *bissextile*, and contains 366 days, the period cannot be completed in a less time than four times seven (7×4), or twenty-eight years, agreeably to the revolution of the solar cycle.

21. *The Epact* signifies the moon's age at the beginning of the year, viz. the interval of time between the *first minute* of the first of January, and the *first minute* of the *last* new moon in the preceding month.

22. *The Roman Indiction* is a cycle of 15 years. It was used by the Romans, when masters of the *then* known world, for the purpose of indicating the times for levying a periodical tax upon the inhabitants of the conquered countries.

23. *The Julian Period* is a cycle of 7980 years, being the product arising from the multiplication of the cycles of the sun, moon, and Roman indiction, viz. $28 \times 19 \times 15 = 7980$ years.

24. *The Grand Celestial Period, or the Platonic Day*, is a revolution of 25791 years; in which the annual *retrograde motion of the colures* (see pages between 301 and 305), at the rate of 50.25 seconds of a degree, will cause the equinoxes to move completely round the ecliptic:—for,

As $50''25 : 1 \text{ year} :: 360^\circ : 25791 \text{ years}$. At the end of this long period, *and not sooner*, all the fixed stars in the firmament will return to the same precise places that they now occupy; and again describe the same circles, with respect to the equator and the poles of the earth, that they describe at present in the ethereal vault of heaven.

The respective values of the cycles, &c. mentioned in the above articles, may be easily determined; as thus:—

25. To find the *Solar Cycle*:—Add 9* to the given year of our Lord, and divide the sum by 28: the quotient will be the number of cycles since the epoch of Christianity, and the remainder the *solar cycle* for the given year:—should *nothing remain*, the cycle is to be estimated at 28.

26. To find the *Golden Number*:—Add 1† to the given year of our Lord, and divide the sum by 19; then the quotient is the number of *lunar cycles* since the birth of Christ, and the remainder is the *golden number*: if *nothing remain*, the golden number will be 19.

27. To find the *Julian Period*:—Increase the given year of our Lord by 1, and add it to 4712; or, since Christ was born in the 4713th year of the Julian period; therefore, to the given year add the common number 4713, and the result will be the Julian period corresponding to the given year of the Christian era.

28. To find the *Epact*:—Subtract 1 from the *golden number* found as above, multiply by 11 and divide by 30; the result will be the *epact for the year*, or the moon's age on the first of January.

* The Solar Cycle was 9 at the birth of Christ.

† The Golden Number was 1 at the birth of Christ.

29. To find the *Dominical Letter* :—If the day of the week on which the first day of the year falls be known ; let A, be taken as the first of January ; B, the 2d ; C, the 3d, &c. &c. ; then, the letter answering to Sunday will be the *dominical letter* for the year. But, if the day of the week on which the year commences be *not* known, proceed as thus :—To the given year *add its fourth part*, rejecting fractions ; then divide the sum by 7, and the remainder will be the number of the dominical letter ; calling G, 1 ; F, 2 ; E, 3 ; D, 4 ; C, 5 ; B, 6 ; and A, 0 :—should there be no remainder, then A is the Sunday letter for the year. As the 52 weeks into which the year is divided contain but 364 days, instead of 365, the *dominical letter*, therefore, retrogrades or falls back *one* in every succeeding common year, and *two*, when the year consists of 366 days :—hence, all *leap-years* are noted by *two Sunday letters*, viz. the letter which is peculiar to the intercalated year, and that which precedes it in the order of the first seven letters of the alphabet, as above.

I think it right to observe, that the subjects which are contained in the articles from 18 to the above, are of such minor importance, that they would not be noticed in this work, were it not for the purpose of keeping it *in unison* with the Nautical Almanac. We must now return to the further consideration of *time*, from which we broke off at the end of Article 17.

30. *Apparent Time* signifies the sun's *horary distance* from the meridian, reckoned *westward* from the time of transit ; or, it simply expresses the hour of the day, shown by a correct sun-dial :—hence, it only relates to the *true sun*, and *not* to any other celestial object.

31. *Mean Time* is the hour which is shown by an *equable-going* clock or chronometer, adjusted to go 24 hours in an average *solar* day of 24 hours, 3 minutes, 56.5554 seconds, measured in *sidereal time* : it is reckoned westward from the transit of the *mean* sun's centre over the meridian.

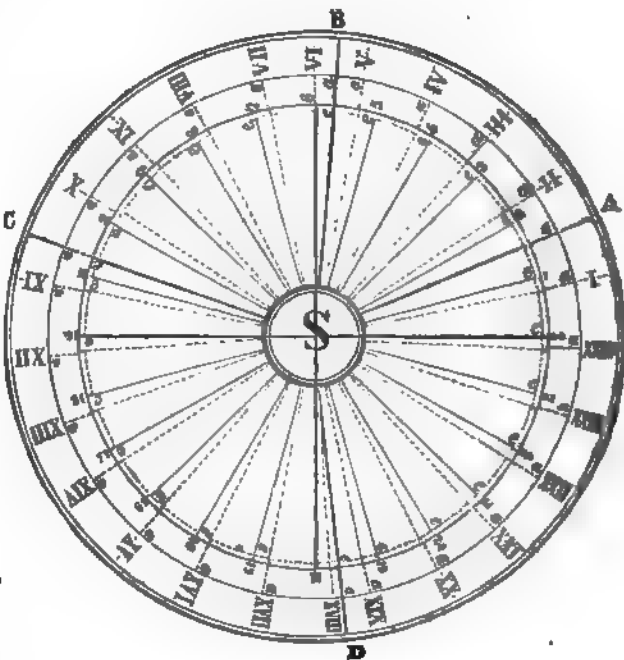
32. *Sidereal Time* is the hour which is shown by a well-regulated clock, adjusted to go 24 hours in a sidereal day of 23 hours, 56 minutes, 4.0906 seconds, measured in *mean solar time*. This *time* is reckoned *westward*, from the transit of the first point of Aries over the meridian ; and since it is always equal to the sum of the right ascension of the *mean* sun and the *mean time* at any given place on the earth, it is, therefore, the same as *the right ascension of the meridian* ; (defin. 16, page 299) ; and thus its value is quite different from the *sidereal time*, which is given in page II. of the month in the Nautical Almanac, as will be shown presently.

33. *The Equation of Time* is the difference between the sun's *true* right ascension and his mean longitude (in time), corrected by the equation of the equinoxes in right ascension. This *equation* implies a correction which is *additive* to, or *subtractive* from, the apparent time, deduced

from an observation of the sun, in order to reduce it to equable or *mean time*, such as that shown by a perfect chronometer.

34. The diurnal motion of the earth upon its axis being perfectly uniform at all times throughout the year, the sidereal days are therefore always of the same exact length. But, owing to the inequalities of the earth's annual motion in an elliptic orbit, combined with the obliquity of that orbit to the plane of the equator, the length of the solar day is constantly varying. And because the ecliptic is inclined to the equator in an angle of $23^{\circ}27'38''$, the equable motion of the earth on its axis brings *unequal* portions of the ecliptic to the same meridian, in *equal portions* of time;—and thus it is that the *apparent time*, deduced from an observation of the sun, never corresponds with the *mean time* shown by a well-regulated clock or chronometer, except on *four days* of the year, viz. about the 15th of April, the 15th of June, the 31st of August, and the 24th of December. At all other times of the year, the unequal velocity of the earth in moving round its orbit, will cause the sun to appear upon the *same meridian* a little earlier or a little later every day, than the time indicated by an equable-going clock or chronometer: and, hence, the sun will appear to be *fast*, or before a perfect time-keeper on some days of the year, and *slow*, or behind it, on other days. This may be familiarly explained in the following manner, viz.:—

35. In the annexed diagram, let us suppose that there are two globes, a real one and an *imaginary* one, moving round the sun S, at the centre:—let the real globe be noted by *a*, and the imaginary one by *e*. Let the former move in the plane of the equator, *a*, *a*, &c. with an unequal motion, according to the order of right ascension indicated by the *Roman numerals*; and, the latter in the plane of the



interior equator, represented by the dotted circle *e, e, e, &c.*, with an uniform motion, according to the order of right ascension indicated by the *common numerals* 1, 2, 3, &c., on to 24. The black straight lines answering to the *common numerals* are the meridians to which the *imaginary* globe comes every day at mean noon. The *dotted* straight lines corresponding to the *Roman numerals* I., II., III., &c., are the *apparent meridians* to which the real globe comes every day at *apparent noon*, in its annual revolution round the sun S. The letters A, B, C, D, represent the four days in the year, viz. the 15th April, the 15th June, the 31st August, and the 24th December; on which the right ascensions of the real globe and the *imaginary one* are equal, and on which the equation of time vanishes.

Let the two globes *a* and *e* be on the meridian, in the line S A, on the 15th of April. On this day, the right ascension of each globe will be about $1^{\text{h}}33^{\text{m}}$: hence, as there is no difference in the right ascension, the *mean noon*, shown by a well-regulated clock, that is adjusted to go exactly 24 hours in a *mean solar day*, will correspond with the *apparent noon* indicated by a correct sun-dial. Now, whilst the *imaginary* globe *e*, moves at an uniform rate round the dotted equator, increasing its right ascension by the diurnal fixed quantity $3^{\text{m}}56^{\text{s}}.5554$, the real globe *a*, will move with an unequal degree of velocity round the plane of the equator, *a, a, a, &c.*, increasing its right ascension by unequal increments, which will be at certain times of the year *greater*, and at others *less*, than the above invariable quantity. While the two globes are moving from A to B, the right ascension of *a*, being *less* than that of *e*, the true globe comes to the meridian *before* the imaginary one; and thus its meridians, marked by the dotted lines S *a* II., S *a* III., &c. fall to the left hand of the black meridian lines S *e* 2, S *e* 3, &c. belonging to the imaginary globe*: and therefore *the equation of time*, or the solar angle *e S a*, contained between the two meridian lines, is *subtractive* from *apparent* time, and *additive* to *mean* time. The two globes will be in the line S B on the 15th June; on which day, as their right ascensions will be equal, each being about $5^{\text{h}}33^{\text{m}}$, the hour shown by a well-regulated clock will correspond with the hour indicated by a correct sun-dial, and therefore there will be *no* equation of time.

During the time that the two globes are moving from B to C, the right ascension of *a*, being *greater* than that of *e*, the real globe will not come to the meridian until *after* the imaginary one; and thus its dotted meridians S *a* VII., S *a* VIII., &c. fall to the right hand of the black meridian lines, S *e* 7, S *e* 8, &c. belonging to the imaginary globe*; and therefore the equation of time, indicated by the solar angle *e S a*, or the difference between the two meridian lines, is to be applied by *addition* to *apparent* time, and by *subtraction* to *mean* time.

* The eye of the reader is to be directed from the centre S to the circumference.

About the 31st August the two globes will be in the line S C, on which day their right ascensions will be equal, each being about $10^{\text{h}}37^{\text{m}}$; and because of this equality, there will be *no* equation of time: hence, on that day the hour-hand of a well-adjusted clock will correspond with the horary shadow of a correct sun-dial. As the two globes advance from C to D, the right ascension of *a* will be *less* than that of *e*; and therefore the real globe will come to the meridian *earlier* than the imaginary one; and thus its dotted meridian lines S *a* XI., S *a* XII., &c. fall to the left hand of the black meridian lines S *e* 11, S *e* 12, &c. belonging to the imaginary globe*:—hence, the equation of time, indicated by the solar angle comprehended between the two meridian lines, is *subtractive* from *apparent time*, and *additive* to *mean time*.

The two globes will be in the line S D about the 24th December; on which day their right ascensions will be equal, each being about $18^{\text{h}}10^{\text{m}}$; and because of this equality the equation of time vanishes.

During the time that the two globes are moving from D to A, the right ascension of *a* being now *greater* than that of *e*, the real globe will not come to the meridian until *after* the imaginary one:—hence, its dotted meridian lines S *a* XIX., S *a* XX., &c. fall to the right hand of the black meridian lines S *e* 19, S *e* 20, &c. belonging to the imaginary globe; and therefore the equation of time, expressed by the solar angle *e* S *a*, is *additive* to *apparent time*, and *subtractive* from *mean time*. From the above it is evident, that the *equation of time* is the angular distance in time, or the solar angle which is contained between the places of the real globe and the imaginary one.

In the above diagram, the spaces comprehended between the points A and B, B and C, C and D, and D and A, are as proportional to the intervening times as the nature of the projection would admit of:—Thus, since A represents the 15th April, B, the 15th June, C, the 31st August, and D, the 24th December: therefore, the first space comprehends an interval of 61 days; the second, an interval of 77 days; the third, an interval of 115 days; and the fourth, an interval of 112 days: making in the whole 365 days. During the 61 days that are included between A and B, and the 115 days betwixt C and D, viz. between the right ascension $1^{\text{h}}33^{\text{m}}$, and $5^{\text{h}}33^{\text{m}}$, and between the right ascension $10^{\text{h}}37^{\text{m}}$ and $18^{\text{h}}10^{\text{m}}$, the true globe *precedes* the imaginary one; and, therefore, as it comes to the meridian *first*, the equation of time is *subtractive* from *apparent time*. But, during the 77 days that are included between B and C, and the 112 days betwixt D and A, viz. between the right ascension $5^{\text{h}}33^{\text{m}}$, and $10^{\text{h}}37^{\text{m}}$, and between the right ascension $18^{\text{h}}10^{\text{m}}$ and $25^{\text{h}}33^{\text{m}}$ (viz. $1^{\text{h}}33^{\text{m}}$), the real globe is *behind* the

* The eye of the reader is to be directed from the centre S to the circumference.

imaginary one; and, therefore, since it comes to the meridian *later*, the equation of time is *additive* to *apparent time*;—hence, it is manifest that the equation of time is *subtractive* for $61 + 115 = 176$ days, and *additive* for $77 + 112 = 189$ days in every year.

36. As the above expressions are adapted to *apparent time*, therefore, whenever *mean time* is under consideration, the equation of time is to have a contrary sign; that is, for *subtractive* read *additive*, and *vice versa*, for *additive* read *subtractive*. Hence it will appear evident, that between the 15th April and the 15th of June, and between the 31st August and the 24th of December, the equation of time is *additive* to *mean time*. But, between the 15th June and the 31st of August, and between the 24th December and the 15th of April, the equation of time is *subtractive* from *mean time*.

37. Since the generality of astronomers have adapted their language to the senses, or to the ideas immediately resulting from celestial appearances; it is therefore usual in *astronomical expressions*, to apply that motion to the sun which nature has impressed upon the earth. This is to be regretted, because it tends to perpetuate that optical illusion under which the uninformed in astronomy have ever laboured; and which has been the means of leading them into numberless extravagant speculations in relation to the heavenly bodies. However, whether the earth be in motion and the sun at rest (as *they actually are*); or the earth at rest and the sun in motion, the appearance of the heavens will be always the same: for, in whatever part of the ecliptic the earth moves, the sun will be posited in that point of the firmament which is diametrically opposite; and therefore if we notice what *point* of the ecliptic comes to the meridian at midnight on any given day, the sun (then *apparently under the earth*), will be exactly 180 degrees distant from that point:—Hence, so far as calculation is in question, it is perfectly immaterial whether the sun be at *rest* or in motion, because the result will be always the same. And, in consequence of this, we may change the terms used in the diagram, Article 35, at pleasure:—And, therefore, if we allow S to represent the earth;—*a* I., *a* II., *a* III., &c., to represent the real sun moving with a *variable* degree of velocity round the earth along the circular plane of the equinoctial indicated by the points *a*, *a*, *a*, &c., and *e* 1, *e* 2, *e* 3, &c. to represent an *imaginary sun* moving with an *invariable* degree of velocity along the *dotted* equinoctial marked *e*, *e*, *e*, &c.; we will have the *true meaning* of the second paragraph in page 497 of the Nautical Almanac for 1836, where it is said, “An imaginary sun, called the *mean sun*, is conceived to move uniformly in the equator with the real sun’s *mean* motion in right ascension.” Now, since the *mean* motion of the real sun is at the rate of 59′ 8″ 33 per day (Article 6, second paragraph), which in *time* answers

to $3^{\circ}56'5554$; this, therefore, is the *equable diurnal rate* at which the imaginary sun is conceived to move round the equator: and it is the accumulation of this equable rate that constitutes the element called "Sidereal Time" which is given in page II. of the month in the Nautical Almanac.

38. The imaginary, or *mean sun*, and the *imaginary first point of Aries* may be esteemed as the synonymies of each other; they are precisely of the same import; and, therefore, the right ascension of the *mean sun* expresses the right ascension of the *first point of Aries*:—And hence it is that the right ascension of the *mean sun* is called *sidereal time* in the Nautical Almanac; and hence, also, that the interval of time between two consecutive returns of the *imaginary* or *mean sun* to the same meridian, which consists of 24 hours, 3 minutes, 56.5554 seconds, is called *a mean solar day in sidereal time*, Article 9.

39. A *mean solar day* is $3^{\circ}56'5554$ longer than a natural day, which consists of 24 hours in mean time: and therefore *mean solar time* is converted into *sidereal time* by the *addition* of an equation; and *vice versa*, *sidereal time* is converted into *mean solar time* by the *subtraction* of an equation; as particularly explained in the description of Tables XLV. and XLVI., between pages 117 and 119; to which the reader is requested to refer.

40. Having thus touched upon the conversion of *solar* into *sidereal* time, &c., it may be advisable to notice the "Tables of Equivalents," which are given between pages 486 and 489 of the Nautical Almanac for 1836:—The first of these is for the conversion of mean solar time into sidereal time; the construction of which is as follows, viz.:

As $360^{\circ} : 24^{\text{h}} :: 360^{\circ}59'8''33018$ to $24^{\text{h}}3^{\circ}56'5554$; which is the correct length of a mean solar day in *sidereal time*, Article 9. Now, the *twenty-fourth* part of this gives the value of 1 mean solar hour = $1^{\text{h}}0^{\text{m}}9^{\text{s}}8565$, *sidereal time*: hence, 2 mean solar hours = $2^{\text{h}}0^{\text{m}}19^{\text{s}}7130$, *sidereal time*;—3 mean solar hours = $3^{\text{h}}0^{\text{m}}29^{\text{s}}5694$ *sidereal time*, &c. &c.

Note.—It is the *excess* of the minutes and seconds *over the hours*, obtained in the above manner, that is contained in the second and following columns of Table XLVI., volume II., page 597.

41. The second "Table of Equivalents" in the Ephemeris, or that for converting *sidereal time* into *mean solar time*, may be constructed in the following manner, viz..

As $360^{\circ}59'8''33018 : 24^{\text{h}} :: 360^{\circ}$ to $23^{\text{h}}56^{\text{m}}4^{\text{s}}0906$; which is the correct length of a *sidereal day*, or the absolute space of time that the earth takes to revolve *once* round its axis in *mean solar time*, Article 10. Now, the *twenty-fourth* part of this gives the value of 1 sidereal hour = $0^{\text{h}}59^{\text{m}}50^{\text{s}}1704$ in *mean solar time*:—hence, 2 sidereal hours =

1^h59^m40^s.3409 mean solar time ;—3 sidereal hours = 2^h59^m30^s.5113 mean solar time, &c. &c , as in the Ephemeris, page 490.

Note.—If the *equivalents* thus found be subtracted from 24 hours, the remainder will be the equations in *mean time*, which are contained in the second and following columns of Table XLV., volume II., page 597.

42. The mean sun's right ascension (given in page II. of the month in the Nautical Almanac under the head "Sidereal Time"), and the "Mean Time of Transit of the First Point of Aries," page XXII. of the month in the Ephemeris, may be deduced from each other in the following manner, viz. :—Let the mean sun's right ascension be *subtracted* from 24 hours, *diminish* the remainder by the corresponding equation in Table XLV., volume II., page 597, and the result will be the *mean time of transit of the first point of Aries*. And, let the *mean time of transit of the first point of Aries*, in the Ephemeris, be *increased* by the corresponding equation in Table XLVI.; then, this being subtracted from 24 hours, the result will be "the sidereal time," or the *mean sun's right ascension*.

43. Since the equation of time is measured by the solar angle, which is contained between the two meridian lines that flow from the centre of the sun to the centres of the real globe and the imaginary one, as appears evident by the diagram, Article 35; and since those meridian lines express the right ascensions of two objects (a real one and a *fictitious*), which, by the substitution of terms, we may now call the true sun, and an *imaginary* or *mean sun*; therefore the equation of time, which is given in page II. of the month in the Nautical Almanac, is simply, and *bona fide*, the difference between the *mean sun's* right ascension, viz., "Sidereal Time," and the *true sun's* right ascension, as given in the same page.

44. Having thus shown the nature of the new and important element called "Sidereal Time;" having demonstrated that it is simply the *mean sun's* right ascension; and, moreover, having shown that it is essentially different from *the sidereal time* which is deduced from the diurnal revolution of the earth in relation to the fixed stars (Articles 10 and 32); I shall therefore conclude this article by observing that, as the element in question corresponds with the angular distance of *the first point of Aries* from the instant of the vernal equinox, it ought, by analogy, to be denominated either *the right ascension of the first point of Aries*, or *the mean sun's right ascension*: but, as the latter denomination is evidently the most appropriate (Article 37); therefore, throughout the rest of this work * the "Sidereal Time," which is given in

* After arriving at the astronomical calculations.

page II. of the month in the Nautical Almanac, shall be called *the mean sun's right ascension*: because this will conduce to obviate the perplexity which young computers experience in consulting the *last column* of the above-mentioned page in the Ephemeris.

ON THE ADJUSTMENT AND USE OF NAUTICAL INSTRUMENTS.

45. In the foregoing explanatory articles the young navigator is presented with all the points of information that have any relation to the essentially important element called "Sidereal Time," as well as to the other species of time, viz. *Apparent* and *Mean*, that are familiar to nautical astronomers: and, on the supposition that he has a competent knowledge of the whole, we will now make a few remarks relative to the *adjustment* and *use* of the nautical instruments that are used for the purposes of *celestial observation*.

46. The instruments made use of at sea for determining the latitude and longitude of a ship, are quadrants and sextants.* But since space cannot be afforded in this work for giving descriptions of those *well-known* instruments, the reader is, therefore, respectfully referred to an ocular inspection thereof, and to a few explanatory hints from some person who is practically acquainted with them. A few words from an experienced navigator will convey more substantial information to a young gentleman, than if he were to spend a whole year in poring over the many treatises that have been written by different authors, relative to "the description and use of the quadrant and sextant."

Every midshipman in the Royal Navy who has been about three weeks at sea, is just as well acquainted with the description of the quadrant and sextant as he is with that of his *cocked-hat and sword*; and full as familiar with the adjusting-screws of those instruments, as he is with the steps of the *quarter-deck ladder*. But there are few, even amongst the more experienced officers, who are so thoroughly acquainted with the nature of *the adjustments* as to be able to determine the absolute value of the *index error*: for there is a peculiarity in the index-bar which seems to have escaped the notice of its makers; and of this I shall endeavour to satisfy the reader in a subsequent article. Knowing, from the long experience of thirty years, that such descriptions are as useless and unnecessary as "the examination of a young sea officer" in certain books on navigation; I shall therefore skip over them, and enter at once upon the *principal rectifications* of which these instruments are susceptible: and this becomes the more necessary

* And sometimes reflecting circles.

since it frequently happens, and *not* improperly so, that their *adjustments* constitute a part of the examination which all mates and midshipmen must undergo before obtaining a lieutenant's commission; which is the *golden step* to promotion in the Royal Naval Service of His Majesty.

ADJUSTMENT I.

To Set the Index-Glass, or Moveable Reflector, perpendicular to the Plane of the Sextant, &c.

47. Move the index to about 60° , viz., to near the middle of the arc or *limb*. Hold the sextant with its face up, the index-glass being next to the observer, and the arc, or limb, turned from him; and keep it so that its plane may be nearly parallel to that of the horizon. Direct the sight, in an *oblique manner* to the index-glass or moveable reflector;—(this is frequently called *the speculum*;)—then, if the reflected limb seen in the glass be exactly in the same plane, or *unbroken arc*, with that seen by direct vision, the index-glass is truly perpendicular to the plane of the instrument. But, should the reflected limb appear to be raised above, or depressed below the plane of the real limb, the glass is *not* perpendicular: in this case, the screws at the back of the index-glass must be gradually, but most *cautiously*, turned till both limbs appear to be in the same plane, or to form but one continued arc. & In the case of a quadrant, the index is to be set about at 45° , then proceed as above. Here it may be noted, that the present adjustment is more applicable to the quadrant than to the sextant: because, in all *well-made* instruments of the latter denomination, the index-glass is rendered so *very secure* by the maker, that it can never lose its perpendicularity, so long as the sextant is handled in a proper manner.

ADJUSTMENT II.

To Set the Horizon-Glass, or Fixed Reflector, perpendicular to the Plane of the Sextant, &c.

48. The index-glass, or moveable reflector, being adjusted as above; set the O on the vernier or dividing scale of the index to O or *zero* on the limb of the instrument; and make the *coincidence* of these points quite *perfect* by means of the *tangent-screw*, using for this purpose the magnifying lens;—when *perfect*, clamp the index by means of the screw at its back. Hold the sextant with its *face up*, or so that its plane may be parallel to that of the horizon; look through the sight-vane, or through the socket which receives the telescope, and direct the sight to the horizon-glass, or fixed reflector; then, if the reflected horizon appear to be in the same uniform plane, or *continued straight*

line with that seen by direct vision, the horizon-glass is truly perpendicular to the plane of the sextant. But, should the horizon seen by reflection appear to be either above or below that seen directly, the upper adjusting-screw at the back of the horizon-glass is to be carefully turned, till the coincidence of the reflected and real horizons is quite perfect. In making this adjustment for a quadrant, it is to be observed, that if the horizon seen by reflection be *higher* than that seen by direct vision, the screw which is nearest to the glass in the pedestal is to be *eased*, and that which is farthest from it to be *screwed up*, till the two horizons appear to be in the same plane. But, if the reflected horizon be *lower* than that seen directly, the screw which is farthest from the horizon-glass is to be *eased*, and that which is nearest to it *screwed up*, till the coincidence of the reflected and real horizons appear to be quite perfect: taking care, however, to leave *both screws equally tight*.

49. The above adjustment may be conveniently made in the following manner, viz.:—Screw the telescope into its place; adjust it to *distinct vision*, and turn the tube or eye-part thereof until two of the *cross wires* are parallel to the plane of the sextant. Move the index, only *loosely clamped*, to about *zero* on the limb: arrange the *shades* so that *one dark glass* may intervene above, and another below the fixed reflector. Hold the instrument with its *face up*, and direct the sight, through the telescope, to the sun, when its altitude is not more than about 20 or 30 degrees, or when it is not so great as to inconvenience the observer; taking care to hold it so that its plane may be parallel to the horizontal diameter of the sun. Move the index slowly, by *hand*, backwards and forwards; then, if the reflected image of the sun pass exactly over the face of the real sun, the horizon-glass is perpendicular to the plane of the sextant. But should the reflected sun appear to pass a little *above* or *below* the upper or the lower points of the real sun's disc, the glass is *not* perpendicular to the plane of the instrument, and therefore it must be adjusted, as directed in Article 48.

ADJUSTMENT III.

To set the Horizon-Glass, or Fixed Reflector, parallel to the Index-Glass, or Moveable Reflector.

50. The two preceding adjustments being completed, set the *O* on the vernier or dividing scale of the index, to *O* or *zero* on the limb: clamp the index in this position, and make the coincidence of the points perfect by means of the tangent-screw; using for this purpose the microscope or magnifying lens; screw the telescope into its socket, adjust it to distinct vision, and turn the tube or eye-part thereof, until two of the *cross-wires* are parallel to the plane of

the sextant. Raise the socket of the telescope by means of the milled screw at back of the collar, until the field of view of the telescope is fairly bisected by the line which separates the silvered and the transparent parts of the horizon-glass. Hold the sextant in a *vertical* position; that is, with its arch or limb *downwards*. Look through the telescope, and direct the sight to the horizon: then, if the reflected horizon, and that seen by direct vision be exactly in the same plane, the horizon-glass is truly parallel to the index-glass. But, should the horizons appear to be *broken*, or *not* to be in the same continued straight line, the lower adjusting-screw at the back of the horizon-glass is to be *very carefully* turned, till the coincidence of the two horizons appears to be perfect.

It may be right to observe that the adjusting-screws are placed differently in different instruments, according to the fancy of the makers. In sextants of Berge's construction, the screw for the above adjustment is placed towards the *lower part* of the back of the horizon-glass; and that for the preceding adjustment towards its *upper* part. *

In making the present adjustment for a quadrant, if the horizon seen by reflection does not coincide with that seen by direct vision, *ease* the milled-screw at the back of the fixed reflector, and turn the nut at the end of the *lever* till the two horizons appear to be in the same continued straight line; then, fix the *lever* in this position by tightening the milled-screw. In tightening this screw it frequently happens that the coincidence of the two horizons becomes *imperfect*; in this case, the adjustment is to be *repeated* until both horizons exactly coincide.

51. The above adjustment may be very correctly made in the following manner, viz.: Arrange the shades so that one dark glass may intervene above, and the other below the fixed reflector: hold the sextant in a vertical position, as before, direct the sight to the sun, through the telescope; ease the clamp of the index, and then move the latter gently, *by hand*, backwards and forwards, so as to cause the reflected image of the sun to pass up and down, in a *vertical* manner, over the face of the real sun. Then, if the reflected sun pass so exactly over the true sun as not to project beyond either its west or its east limb; that is, either to the *right* or *left*, the fixed reflector is truly parallel to the moveable reflector. But, should the reflected image project anything either to the right or left of the real sun, the adjusting-screw must be carefully turned, till the two suns appear to pass over each other in a direct vertical manner.

ADJUSTMENT IV.

To make the Line of Collimation parallel to the Plane of the Sextant.

52. *Note.*—The *Line of Collimation* is an imaginary straight line joining the centre of refractions of the object-glass of the telescope and the centre or middle point between its parallel wires; at which central point the contact of the limbs of the two objects must be made in the act of taking a lunar observation.

The mode of making the present adjustment is as follows, viz.:—Screw the telescope into its place, adjust it to distinct vision, and turn the tube, or eye-part thereof, until two of the *cross-wires* are perfectly parallel to the plane of the sextant. Select two celestial objects whose angular distance is *not less* than a right angle: but, for this purpose the sun and moon are the fittest and most eligible objects, particularly when they are so situated in the heavens that the arc comprehended between them is nearly equal to *a third part* of the whole zodiac; that is, when their distance does not differ much from 120 degrees. Turn down all the coloured glasses, or shades that are above and below the fixed reflector, except one of the darkest, or *deepest* red, which is to be left standing *betwixt the fixed reflector* and the index-glass, so as to protect the eye of the observer from the effects of the solar rays. Look through the telescope; direct the sight to the sun, and cause its darkened image, by moving the index forward, to touch the moon. Clamp the index, and make the contact of the limbs, by means of the tangent-screw, as perfect as possible at the wire *nearest* to the plane of the sextant: then, without a moment's loss of time, bring the point of contact (taking great care not to move the index) to the other wire, or that which is furthest from the plane of the instrument. Now, if the contact appears to be perfect at this wire, *the axis of the telescope* is truly adjusted; that is, its line of collimation is perfectly parallel to the plane of the sextant. But, should the limbs of the objects appear to be either separated, or to partly cover each other, one of the screws of the *collar* in which the telescope is fixed must be *eased*, and the other *screwed up*, until the contact of the limbs appears to be perfect at both wires.

53. Instead of meddling with the screws of *the collar*, which should *never be touched, except in extreme cases*, the following method may be adopted, viz.:—When the contact is made at the wire *nearest* to the plane of the sextant, let an assistant note down, per watch that shows seconds, the exact moment of contact, and the corresponding angular distance; then, bring the point of contact to the other, or *distant* wire, and make it perfect by means of the tangent-screw; the assistant not-

ing down, as before, the exact moment of its being perfected, and the value of the corresponding angular distance. Now, the difference of the times shows the interval between the moments of contact, and that of the corresponding angular distances, shows *the approximate error* in the line of collimation: which *error* is to be corrected on account of the *change of distance*, during the interval in the following manner, viz. :—Reduce the time of observation to the meridian of Greenwich; with which enter the Nautical Almanac, and take out, amongst *the distances* for the given day, the proportional logarithm answering to the hour which immediately *precedes* and *follows* the Greenwich time; to this add the proportional logarithm of the interval between the moments of contact; the sum will be the proportional logarithm of a *correction*; which, being *subtracted* from the difference of the angular distances when the limbs appear to be separated at the *distant* wire, or added thereto when they overlap or partly cover each other; the result will be the correct value of the error in the line of collimation. Now, this being known, its absolute effect on any other lunar distance may be readily found by means of Table XXIII.; as thus, enter that Table with the error, or “inclination of the line of collimation,” at top, and the observed distance in the side column; in the angle of meeting stands a correction which is always *subtractive* from the observed distance: and which may be used afterwards as a *constant quantity* to be applied to all lunar distances *taken with the same instrument*; so long as the *error* or inclination of the line of collimation, *remains unaltered*.

54. In sextants of Berge’s construction, the above adjustment very rarely, if ever, becomes necessary; because in those, the collars, or *sockets* for the telescope are so well fortified, and so firmly supported that *the parallelism of the line of collimation* must always remain perfect so long as the instrument does not receive some violent shock: for, the line of vision is so well guarded, that nothing but *foul play*, such as an unlucky blow, or a fall from some height, can possibly turn the axis of the telescope from the true plane of the sextant.

To find the Index Error of a Sextant.

55. When a sextant is duly rectified agreeably to the four preceding *adjustments*; it is evident that it will then be perfectly free from errors, and therefore in a fit state for either measuring the angular distances, or for taking the altitudes of the heavenly bodies. But, since the adjusting screws are liable to be injured by much turning or screwing; and, moreover, since frequent adjustments have a tendency to derange the correctness of the best instrument; it is, therefore, highly advisable to *tighten* all the adjusting screws at once, *when the sextant is perfect*, and never to touch them again except in cases of absolute necessity;

that is, unless the perpendicularity and parallelism of the two reflectors be palpably defective. Hence, instead of making the above adjustments, the judicious observer will, in all cases, and at all times, prefer finding the index error of his sextant: this may be conveniently done, as thus,—

56. Screw the telescope into its socket, adjust it to distinct vision, and turn the tube or eye-part thereof, until two of the cross wires are parallel to the plane of the sextant. Raise the collar or socket for the telescope, by means of the milled screw at its back, till the field of view of the telescope is bisected by the line which separates the silvered and the transparent parts of the horizon-glass. Make the O on the vernier of the index correspond with the O or zero on the limb; then clamp the index *sufficiently tight for observation*. Hold the sextant in a vertical position, and direct the sight, through the telescope, to the horizon of *the sea*, when it is clear and well defined:—then, if the reflected horizon and the real horizon be exactly in the same plane, or form *one continued straight line*, there will not be any error in the index; but, should the two horizons *not coincide*, which very frequently will be the case *with inferior instruments*, make their coincidence perfect by means of the tangent-screw of the index; then, the angle indicated by the vernier on the arch or limb, will be the index error of the sextant; which will be *subtractive* when the angle is *on the arch*; that is, when the O on the vernier is to the *left of zero* on the limb; but *additive* when it is *off the arch*, viz., when the O on the vernier is to the *right of zero* on the limb.

57. The index error may be found with more exactness than as above, by adopting the following method, viz.:—Place one or two of the dark screens, according to the brightness of the sun, so as to intervene on each side of the horizon-glass; bring the index to zero on the limb, and tighten its clamp *sufficient for observation*: hold the sextant with its *face up*, and so that its plane may be parallel to the *horizontal diameter of the sun*; direct the sight to the sun, through the telescope, move the index forward, by means of the tangent-screw, till the *right-hand limb* of the reflected sun makes a perfect contact with the *left-hand limb* of the real sun, which is seen by direct vision through the transparent part of the horizon-glass. Read off the angle by means of the magnifying lens; note down its value, and it will express the measure of the sun's diameter to the left of zero, or of O *on the arch*. Ease the clamp and bring the index back to zero on the limb; then, tighten the clamp *sufficient for observation*, as directed above, and move the index backward, by means of the tangent-screw, till the *left-hand limb* of the reflected sun makes a perfect contact with the *right-hand limb* of the real sun. Read off the angle by means of

the microscope; note down its value, and it will express the measure of the sun's diameter to the right of zero, or of *O off the arch*. Now, if both angles, or horizontal diameters of the sun be of the *same value*, there is no index error in the sextant; but, if the diameters are of *unequal* value, *half their difference* will be the index error of the instrument; which will be *subtractive* when the diameter to *the left* of zero is greater than that measured to *the right* of zero; otherwise, it will be additive.

Note.—In reading off the value of the diameter to the right of zero, or *off the arch*, it must be remembered that this is a *retrograde* operation; and, therefore, it is *the complement* of the minutes and seconds shown by the *vernier* that is to be added to the angle indicated by the first point of the index:—hence, should the vernier (one extended to 15 minutes) cut, or *coincide* at 12'40"; the complement of this, or 2'20" is the true *arc of excess* to be added to the angle, or divisions of a degree, pointed out by O on the index.

58. The above would be a very correct way of determining the index error of a sextant, provided the index-bar was inflexible or *non-elastic*, and that it had no play round its centre of motion: but, because the index is an *elastic* bar that must be made so as to *move freely* round its centre of motion beneath the moveable reflector; it is, therefore, invariably bent, or forced from the true line of the radius of the sextant, whenever it is moved backward or forward by means of the tangent-screw; *its clamp being duly tightened for observation*: and thus, as the O on the *vernier* part of the index is forced beyond the extremity of the direct line of the radius, the angle measured by the *progressive motion* of the tangent-screw will be *something too much*; whilst that measured by the *retrograde* motion of the screw will be *too little*. Of this singular fact the reader can easily satisfy himself in the following manner, viz.—Move the index to any degree on the limb, no matter which; but let us say, for the sake of perspicuity, 60°; then, fasten the clamp, and move the index *forward* by means of the tangent-screw, to 60°30', keep the sight, by means of a good magnifying lens, fixed upon this point;—now, *suddenly release* the clamp, and *the spring* of the index-bar will become perceptible; for it will be seen to *fall back* something upon the arch or limb. The converse of this will take place if the index be moved *backward* to the same extent; for, the instant its clamp is *released* (this being done *suddenly*), it will be seen to *spring forward* a little upon the arch or limb. Hence, it is clearly manifest that the preceding method of finding the index error of a sextant is subject to a *certain degree of incorrectness*. And, as this is a subject of vast importance to the practical navigator, I shall therefore discourse of it more generally in the following article.

The true Method of finding the Index Error of a Sextant, so as to guard against the Errors arising from the Flexibility and the Friction of the Index-bar.

59. The customary method of finding the index error of a quadrant or sextant (as directed by writers on the use of these instruments,) is by measuring the vertical diameter of the sun to the right and left of O on the arch, with a motion of the index in contrary directions (that is, by bringing the reflected image to touch the lower and upper limbs of the direct object alternately), and then taking half the difference of those measures for the index error of the instrument.—This method, it must be observed, is very far from being correct; because it is the horizontal diameter of the sun, and not its vertical diameter, that should be measured; for while the former remains invariably the same, the latter is subject to continual alterations, owing to the effects of atmospherical refraction, as will appear evident by an inspection of the last column of Table V.—Moreover, since the index is not an inflexible bar, and since it does not turn upon its centre without suffering some slight degree of friction; it is therefore evident that the measure of the sun's diameter taken by the progressive motion of the index will, in most cases, be *more* than the truth: whilst that taken by the contrary or retrogressive motion will, in general, be *less* than the truth:—hence, the index error established upon the above principles must frequently mislead the mariner by rendering inaccurate what, otherwise, might be a very correct observation. And this accounts for the result of the evening observations, taken on shore by means of an artificial horizon, so very seldom agreeing with the result of those taken in the morning; even though all imaginable care be used, and though the observer keeps the same plane and roof of the horizon directed to him during the time of both observations.

Now, to guard against the errors arising from the bending and the friction of the index-bar, as well as that proceeding from the contraction of the sun's vertical diameter; let the following observations be attended to, and the joint effects of the whole will be obviated.

First.—To find the Error for a Progressive Motion of the Index:—

Screw the inverting telescope into its place. Arrange the shades for observation. Slack the clamp. Turn the tangent-screw *backward* to nearly as far as it will go. Put the *vernier* to about $1^{\circ}15'$ to the *right* of O on the arch, and then fasten the clamp sufficiently tight for observation.—Hold the sextant so that its plane may be parallel to the horizontal diameter of the sun: direct the sight to that object, and

turn the tangent-screw *forward* until the limbs of the sun seen by reflection and direct vision make a perfect contact.—Note down the angle and it will express the measure of the sun's diameter to the right of O on the arch.—Direct the sight again to the sun, and turn the tangent-screw *still forward* until the opposite limbs are in perfect contact: note down the angle, and it will be the measure of the sun's diameter to the left of O on the arch.—Now, if both measures of the diameter are the same, there is no error in the angles shown by the progressive motion of the index; but if those measures do not correspond, half their difference is to be taken as the index error of the instrument, which error will be additive when the diameter measured to the right of O exceeds that measured to the left; otherwise, subtractive.—Then, this error is to be considered as a constant quantity (so long as the instrument does not meet with any accident), and to be applied to all *increasing angles*, either of altitude or distance, which may be taken by the progressive motion of the index.

Again.—To find the Error for a Retrogressive Motion of the Index:—

Slack the clamp. Turn the tangent-screw *forward* to nearly as far as it will go. Put the vernier to about $1^{\circ}15'$ to the *left* of O on the arch, and then fasten the clamp sufficiently tight for observation.—Hold the sextant as before; direct the sight to the sun, and turn the tangent-screw *backward* until the limbs of the sun seen by reflection and direct vision make a perfect contact:—note down the angle, and it will express the measure of the sun's diameter to the left of O on the arch.—Direct the sight again to the sun, and turn the tangent-screw *still backward* until the opposite limbs are in perfect contact; read off the angle, and it will be the measure of the sun's diameter to the right of O on the arch.—Now, if both measures of the diameter are the same, there is no error in the angles shown by the retrogressive motion of the index: but if those measures do not correspond, half their difference is to be taken as the index error of the instrument; which error will be additive when the diameter measured to the right of O exceeds that measured to the left; otherwise, subtractive. Then, this error is to be considered as a constant quantity (so long as the instrument does not meet with any accident), and to be applied to all *decreasing angles*, either of altitude or distance, which may be taken by the backward or retrogressive motion of the index.

Hence it is very probable that *two errors* may be established for the same instrument; the one for increasing, and the other for decreasing angles. The true values of those errors should be noted down (for the

future guidance of the observer), with a black-lead pencil on the inside of his sextant-case in the following manner, viz. :—

Error for the forward or progressive motion of the index $0^{\circ}10'$ subtractive.

Error for the backward or retrogressive motion of the index $1^{\circ}40'$ additive.

Or whatever the errors may be.

And thus the correct values of the index error will be properly determined, whilst the errors arising from the spring and the friction of the bar, together with that proceeding from the contraction of the sun's vertical diameter will be all safely provided against.

60. Now the index error of the sextant being thus truly established, the instrument is properly adapted for making celestial observations: that is, for taking the altitudes, and measuring the angular distances of the heavenly bodies.

I. *To take the Sun's Altitude at Sea.*

61. Prepare the sextant as directed in Article 50, omitting the telescope, and let one of the darkest shades intervene between the index-glass and the fixed reflector. Slacken the clamp, and bring the index to *zero* on the limb. Hold the sextant in a vertical position. Look through the sight-vane, or *socket*, direct the sight to the sun, and his reflected image will be seen in the silvered part of the horizon-glass: move the index forward by *hand*, till the lower limb of the image is seen *near* to the horizon; then tighten the clamp sufficient for observation; screw the telescope into its place, and make the *contact* of the limb and the horizon perfect by means of the tangent screw: taking care that it be made at that point of the horizon which is exactly under the sun, and which would be touched by a plumb-line let fall from its centre. And to be certain that the contact is so made, give the sextant *an immediate* vibratory motion, and the reflected image will appear to describe an arch which will be above the horizon at all points except *that* to which the real sun is perpendicular. Now, the sun's lower limb being thus made to touch the horizon, the degrees, minutes, and seconds indicated by the vernier, and read-off by means of the magnifying lens, will be its observed altitude.

Note.—In rough weather, or when the sea runs high and the ship pitches, the *plain tube* may be substituted for the telescope.

II. *To take the Moon's Altitude at Sea.*

62. Prepare the sextant as in last Article :—if the time of observation be at night, let the green shade, or one of the lightest red screens, intervene betwixt the two reflectors (by day this becomes unnecessary):—then proceed as directed for the sun ; observing that it is the *round* or *well-defined* limb of the moon, whether it be the *lower* or the *upper*, that is to be brought in contact with that point of the horizon on which a plumb-line would fall if dropped from the moon's centre. Now, the angle indicated by the vernier, read-off by means of the microscope, will be the observed altitude of the moon's lower or upper limb, according as it may be enlightened.

III. *To take the Altitude of a Star at Sea.*

63. Set the index of the quadrant or sextant to *zero* on the limb.—Hold the instrument in a direct vertical position: look through the sight-vane of the quadrant, or the socket of the sextant, as the case may be, and direct the sight to the star through the transparent part of the horizon-glass ; then, by a slight motion of the instrument to *the left hand*, the reflected image of the star will be seen in the silvered part of that glass. Move the index forward, by hand, and the star will appear to *descend*: continue the motion of the index, in a gentle manner, and follow the reflected image of the star with the eye, directing the sight lower and lower till the star seems to touch the horizon. Screw a telescope, with a *good field of view* and *properly adapted to the purpose*, into the socket ; clamp the index sufficiently tight for observation ; and make the contact of the star and the horizon perfect by means of the tangent-screw. Give the instrument an immediate vibratory motion, of which the eye is to be the centre, so as to be satisfied that the reflected image of the star touches the horizon exactly in a point which is perpendicular to the real star ; which, of course, will always be the shortest distance between the real star and the horizon. Then, the angle indicated by the vernier, and read-off by means of the microscope, will be the observed altitude of the star's centre. Should the horizon be ill-defined, and the star not very bright, *the telescope must be dispensed with*: in this case the observer's line of vision to the point of contact is to be guided by the sight-vane.

When the altitude of the star is considerable, some other star, besides the required one, may appear by reflection in the speculum of the horizon-glass ; when this happens, the false star may be readily distinguished by its tremulous motion : for it will *dance* in the specu-

lum as the index is moved forward : and thus, there can be no danger of its ever being mistaken for the real star ; because this will appear to remain stationary about the middle of the fixed reflector.

IV. *To take the Altitude of a Planet at Sea.*

64. As the diameters of Mars and Saturn are but of small value, the altitudes of those planets may be taken the same as if they were fixed stars, as directed in the last Article :—observing, however, that it is *the centre* of the object that is to be brought in contact with the horizon. But, as Venus and Jupiter have very sensible diameters, it is the *lower limb* of either of these that is to be brought in contact with the horizon :—and, although Venus is subject to all the various phases of the moon, yet, in *the absence of the sun*, it is her *enlightened limb* that will be always *next to the horizon* ; and therefore it is the altitude of her *lower limb* that is to be taken *at night*. However, for the ordinary purposes of navigation, it will, in general, be quite sufficient to bring *the centre* of the planet down to the horizon of the sea :—then, the angle indicated by the vernier will be the observed altitude of the planet's centre, or of its lower limb, as the case may be.

V. *To take the Altitude of a Celestial Object on Shore.*

65. Altitudes are taken on shore by means of an artificial horizon. And, in settling the positions of places in-land in an astronomical manner, or in ascertaining the error and the rate of a chronometer on shore, the observer must, in all cases, have recourse to an artificial horizon for the purpose of taking the necessary angles of altitude.—But, as this instrument, *unlike* the quadrant and the sextant, is not to be found in the hands of all nautical persons, I shall, therefore, make a few observations relative to its description and use.

66. Although there is a great variety of artificial horizons now extant, yet, for the sake of conciseness, I shall only treat of the two that are in my own possession. The first of these consists of a plane speculum, or polished plate of *dark glass* (4 inches long by 3 broad), fixed in a brass frame, and standing upon three adjusting screws : by means of these and a spirit-level, placed in different positions on its surface, it may be made perfectly parallel to the plane of the horizon : observing that the adjusting screws are to be turned until the air-bubble rests in the middle of the spirit-level on the surface of the speculum.—The other is the common, or quicksilver horizon ;—this simply consists of a small wooden trough, about half an inch deep, $3\frac{1}{2}$ inches long, and $2\frac{1}{2}$ inches broad ;—into this trough a few pounds of mercury or quicksilver are

poured; the surface of which assumes when settled, agreeably to the nature of fluids, an exact horizontal plane. To prevent the mercury from being ruffled or agitated by the action of the wind, a roof is placed over it, in which are fixed two plates of glass, the two sides of each plate being ground mathematically plane and parallel to one another:—And, of all artificial horizons an instrument of this description is the very best that can be employed in taking the altitudes of the heavenly bodies.

Of the Use of the Artificial Horizon; that is, to observe the Altitude of the Sun, or other Celestial Object, with a Sextant, and an Artificial Horizon.

67. In taking the altitude of the sun, or other luminary, the observer is to place his artificial horizon betwixt him and the object selected for observation; and at such a convenient distance as to see the image of that object reflected from the middle of the quicksilver as well as the real object in the heavens:—then, having screwed the plain tube, or the natural telescope of the sextant into its place in the socket; and placed one or two of the dark screens, according to the brightness of the sun, to intervene on each side of the horizon-glass; the lower limb of the reflected image of the sun, as seen through the erect or natural telescope, is to be brought into contact with the upper limb of the image reflected from the artificial horizon:—but, if the altitude of the upper limb of the object be required, it must be brought into contact with the lower limb of the image as seen in the artificial horizon.—Now, the angle on the arch of the sextant being read off, and *the index error*, if any, applied to it, the result will be the double of the sun's, or other object's altitude above the horizontal plane: to the half of which, if the object be the sun, let the semi-diameter, refraction and parallax be applied, and the true central altitude will be obtained. This shall be shown hereafter.

68. Since neither the plain tube, nor the natural or erect telescope, can be depended upon in taking observations when rigorous exactness is required; the inverting telescope should, therefore, be invariably made use of, in all cases where angles of altitude are to be measured with astronomical precision:—and here, perhaps, it may not be unnecessary to state that when the inverting telescope is used, the lower limb of the sun, or moon, will appear to be the upper limb, and conversely.—Hence, in observing the altitude of the lower limb of the sun or moon, the *apparent upper limb of the object*, as seen in the horizon-glass through the inverting telescope, is to be brought into contact with

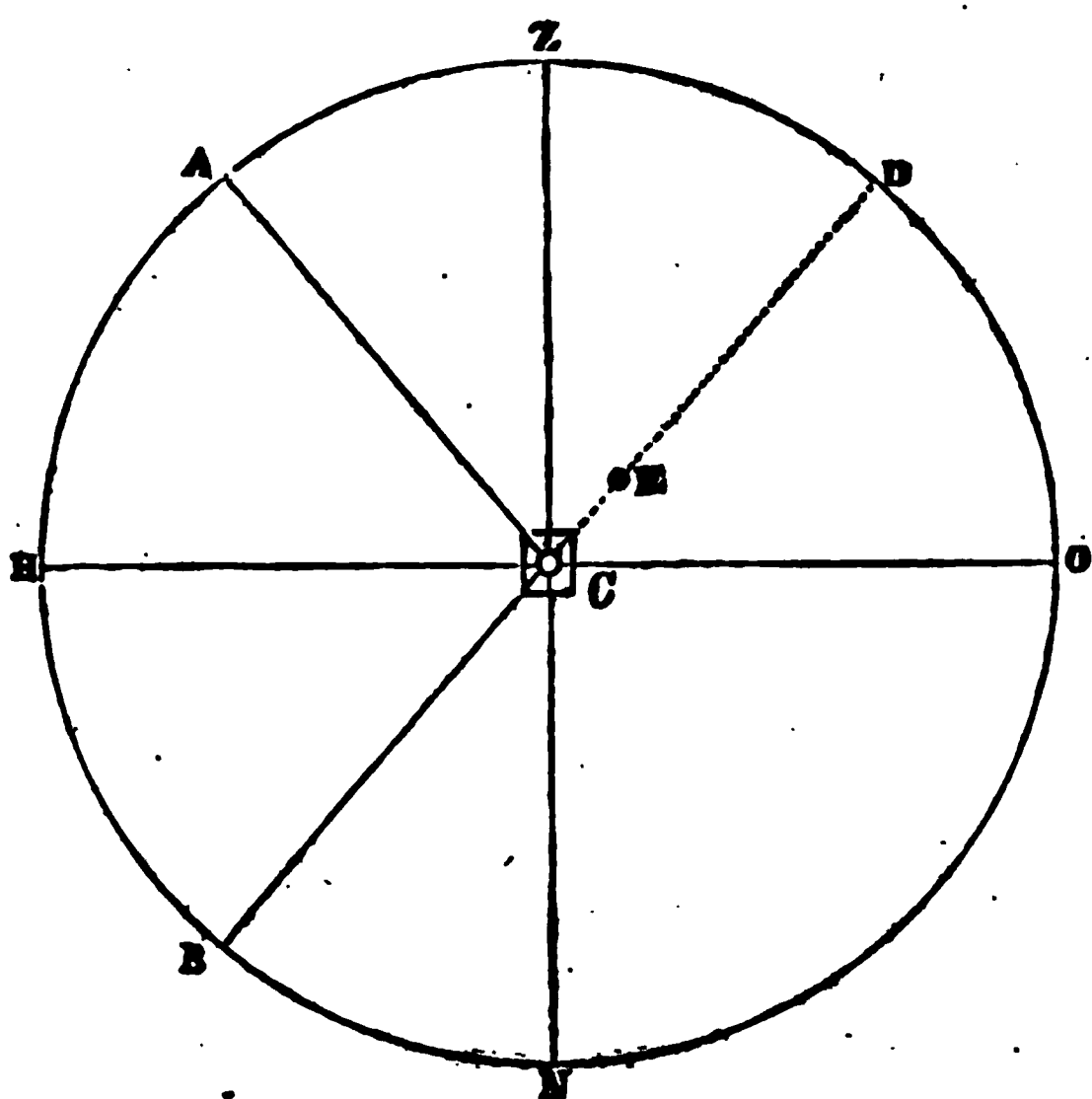
the *lower limb of the image in the artificial horizon* :—in this case the reflected image in the artificial horizon will appear to be uppermost.—Again, in observing the altitude of the upper limb of the sun or moon, the *apparent lower limb of the object*, as seen in the horizon-glass of the sextant through the inverting telescope, is to be brought into contact with the *upper limb of the image in the artificial horizon* :—in this case the reflected image in the artificial horizon will appear to be undermost.

69. If an observer be placed as remote from, or as near to, an artificial horizon as possible, the rays of light passing from the sun or other celestial object to his eye, and from that object to the surface of the artificial horizon, will, on account of the immense distance of such object from the earth, be physically equal and parallel in every respect to each other :—hence, it is easy to perceive that it is immaterial whether the artificial horizon be placed high or low, remote or near, with respect to the observer, provided he can but see the object's reflected image therein.

70. When an angle of altitude is taken by means of an artificial horizon, its measure on the limb of the sextant will always be double of the true value thereof above the horizontal plane :—this will appear evident by considering that if a person places himself at any distance before a plane mirror, or common looking-glass, his reflected image will appear just as far behind such looking-glass as he is before it :—and, upon this simple principle it is that the reflected image of the sun, or other object, will appear to be as far below the surface of the artificial horizon as the real object is above it ;—but since the limb of the real object, as reflected from the index-glass of the sextant, is to be brought into contact with that of the image apparently reflected below the surface of the artificial horizon, it is therefore manifest that the contained angle, as expressed on the arch of the sextant, must be equal to twice the measure of the observed angle of altitude above the plane of the horizon :—and from this we may readily perceive that angles of altitude taken in the above manner are not affected by the angle of horizontal depression, commonly called “the dip of the horizon.”

71. The principles of the artificial horizon may be more clearly explained in the following manner, viz.—When a ray of light flows from the sun, or any other celestial body, and falls upon the surface of a mirror placed horizontally, it will be reflected in such a manner, that the angle which is contained betwixt the reflected ray and the zenith will be equal to the angle which is contained between the direct ray and the zenith.—Hence, let a celestial object be at any degree of elevation above the horizon ; the angle of reflection $Z C D$, will be always equal to the angle of incidence $Z C A$, as in the annexed diagram.

Let $H O$ represent the visible horizon; Z , the zenith; N , the nadir; and A , the place of the sun, moon, or other celestial object:—and, let the small square at the centre, C , represent the surface of an artificial horizon. — Now, to an observer at the point E , the image of the sun A , which is received in the quicksilver at C , will be reflected to his eye in



the direction of $C E$:—then, by directing his sight to the surface of the quicksilver, the solar image will appear to be transferred to the point B ; that is, it will be seen in the line of reflection $D E C$, continued to B . But when two straight lines cut one another, the opposite angles are equal; therefore, the angle $B C N$, is equal to the angle $Z C D$; and since this is the angle of reflection, which is equal to the angle of incidence, therefore the angle $B C N$, is also equal to the angle of incidence $Z C A$:—Hence, the reflected image of the sun at B , will always appear to be as far from the nadir N , as the real sun at A is from the zenith Z ; and, consequently, the reflected sun at B must invariably appear to be as far below the horizon H , as the real sun at A is above it.—Now, the true altitude of the celestial object is expressed by the arc $H A$; but, since the round or well-defined limb of the object at A must be brought in contact with the well-defined limb of the reflected image at B ; it is thus manifest that the arc $A B$, is double the value of the arc $H A$; and, therefore, it is equal to twice the altitude of the celestial object.—Hence, it is clearly evident that an angle of altitude taken by means of an artificial horizon, must, after *being duly corrected for the index error of the sextant*, be divided by 2, in order to obtain the correct value of the observed altitude.

72. When the altitude of a celestial object exceeds 60 degrees, it cannot be taken by means of a sextant and an artificial horizon; because, in this case, the measure of the double angle of altitude would exceed the limits of the graduated arch of the sextant.

73. In observing equal altitudes by means of an artificial horizon, or

in taking a continued series of altitudes for the purpose of determining the error and the rate of a chronometer; it will be essentially necessary to keep the same plane of the glass roof of the horizon towards the observer in each observation; so that in the event of there being any trifling defect in the parallelism of the surfaces of the two plates of polished glass, which form the roof of the horizon, the error arising therefrom may equally affect each observed altitude. To be certain of always observing through the same side of the roof, it will be advisable to make a small mark in the wooden part thereof: then, this mark being kept towards the observer, in every observation, the altitudes will thus be prevented from being unequally affected by any want of parallelism that may chance to be in the planes of the glass part of the roof.

74. In calm weather the altitudes may be taken by reflection from the quicksilver without making use of the glass roof:—in like manner they may be taken, during such weather, by reflection from a basin of water; or, by reflection from a cup of tar, treacle, oil, or other fluid and viscous substance.

75. Mariners frequently supply themselves with, what may be termed, a *home-made*, or *ship-built* artificial horizon; the quicksilver in which they shelter under a roof formed by two squares of the thick glass with which ships are usually furnished:—this is, to say the least of it, a poor substitute:—it is vainly endeavouring to accomplish that, by means of a couple of squares of common glass, which can scarcely be effected by the most highly-finished and parallel planes that can possibly be produced by the labour and ingenuity of the most eminent optician, or mathematical-instrument maker:—for, since the surfaces of those squares are not rendered mathematically accurate by being ground perfectly plane and parallel to one another, the rays of light will be bent from the true line of refraction in passing through the glass from the sun, and again in going out of it to the eye of the observer. Hence, the angle of reflection cannot be equal to the angle of incidence; and thus the altitudes observed in such a *make-shift and defective* horizon will be always *erroneous*.

To take a Lunar Distance; that is, to measure the Angular Distance between the Moon and a given Celestial Object.

I. To observe the Distance between the Sun and Moon.

76. Turn down all the coloured glasses, or shades, except one of the darkest, which is to be left up so as to intervene between the two reflectors, viz. between the index-glass and the horizon-glass. *Ease* the

clamp, and bring the index to *zero* on the limb. Hold the sextant so that its plane may be parallel to the imaginary line joining the centres of the two objects ; with its *face up* when the sun is to *the right of the moon* ; otherwise, with its face downwards. Direct the sight to the sun through the socket, or hole for the telescope ; and move the index forward, by hand, till the reflected image of the sun is brought close to the moon. Then clamp the index sufficiently tight for observation ; screw *the inverting telescope* into its place ; adjust it to distinct vision ; turn the tube or eye-part thereof until two of the cross wires are parallel to the plane of the instrument ; and raise the telescope by means of the milled screw at the back of its collar, till the field of view is bisected by the line which separates the silvered and the transparent parts of the horizon-glass.—Then direct the sight, through the telescope, to the objects ; and turn the tangent-screw of the index until the limbs of the sun and moon make a perfect *contact*, viz., until they appear to *touch*, like the arcs of two circles, *without cutting each other*.—Now, the contact of the limbs being thus duly perfected, the angular distance is to be read-off by means of the magnifying lens.

Note.—The contact of the limbs must be made perfect at the *middle point* between the two wires that are parallel to the plane of the sextant : because any deviation from that point, with *respect to the plane of the instrument*, would cause the observed distance to be something more than the truth.

II. To observe the Distance between the Moon and a Fixed Star.

77. If the moon be very bright, allow one of the light-coloured glasses to remain up so as to intervene betwixt the two reflectors : then proceed as directed for the sun, in the above article, until the image of the moon's enlightened limb is brought close to the star.—Screw the telescope into its place, and adjust it as pointed out in the preceding article.—Direct the sight, through the telescope, to the objects ; and turn the tangent-screw until the moon's bright and well-defined limb appears to *bisect the star*, or until the star seems to be *half off* and *half on* the enlightened edge of the moon :—This constitutes the *contact* of the moon and a fixed star : which contact ought to be made, if possible, at the middle point between the two parallel wires, as stated in the *Note* to Article 76.—But as those wires cannot be distinctly seen at night, except about the time of the opposition or full moon, the observer must endeavour to make the contact as near to the centre of the field of view of the telescope as he possibly can.—Now, the angle indicated by the vernier on the limb of the sextant being read-off, the result will be the

observed distance between *the nearest point* of the moon's enlightened limb and the *centre* of the star.

Note.—When the moon is to the *right* of the star, the sextant is to be held with its face upwards; but, if to the left, with its face downwards.

III. *To observe the Distance between the Moon and a Planet.*

78. As the semi-diameters of Mars and Saturn are but insignificant in point of measure, the moon's distance from *either* of these may be observed in the same manner as if it were a fixed star: that is, the *nearest point* of the moon's enlightened limb is to be brought in contact with the centre of the planet, so as to *bisect* its disc. But, since the semi-diameters of Venus and Jupiter are of considerable values, the contact must be made with one of their limbs.—Since Jupiter assumes the appearance of a perfect circle, all parts of its circumference being equally well defined; therefore, the *nearest point* of the moon's enlightened limb may be brought in contact with the *nearest limb* of that planet:—This will cause the semi-diameter of Jupiter to be *always additive*; whilst that of the moon will be additive or subtractive, according as her enlightened limb may be directed to, or turned from the planet.—But, since Venus assumes all the various phases of the moon, from the crescent to the enlightened hemisphere, except that of *never* being quite full or completely round; and as her enlightened limb may be turned *to*, or *from*, the moon according to circumstances; therefore a degree of caution must be observed, so as to guard against falling into an error in bringing the moon in contact with her *enlightened limb*; as thus:—When Venus is an *evening star*, it is her *western limb*; but when a *morning star*, it is her *eastern limb*, that is enlightened and well-defined; and therefore it is with such limb that the enlightened one of the moon must be brought in contact. Hence, at any time between the new moon and the full, Venus being an *evening star*, if the moon be *west* of the planet, it is her remote limb that is to be brought in contact with the *nearest limb* of Venus: in this case, the moon's semi-diameter becomes *subtractive* from, and that of Venus *additive* to, the observed distance;—but, if the moon be east of the planet, her nearest limb is to be brought in contact with the *remote limb* of Venus; and this will cause the moon's semi-diameter to be *additive* and that of Venus *subtractive*.—Again, at any time between the full moon and the new, Venus being a *morning star*, the converse of the above takes place; viz. if the moon be west of the planet, it is her nearest limb that is to be brought in contact with the *remote limb* of Venus; but if the moon be

east, her remote limb is to be brought in contact with the nearest limb of Venus :—in the first part of *this case*, the moon's semi-diameter becomes *additive*, and that of Venus *subtractive* ; and in the second part, the moon's semi-diameter becomes *subtractive* from, and that of Venus *additive* to the observed distance.—This subject shall be treated more amply in a subsequent page.

The Method of taking a Complete Set of Lunar Observations.

79. In taking a regular set of lunar observations, three assistants become necessary ; two of whom are to observe the altitudes of the objects at the same time that the distance is measured by the principal observer ; the other, having a watch that shows seconds, is to note the time when the observations are made. Now, the index errors of the sextant and quadrants being accurately determined, the assistants are to place themselves in the most convenient situation with respect to the principal observer : then, all are to begin observing at the same time ; and when the principal observer has brought the limbs of the sun and moon, or the enlightened limb of the moon and a star or planet into contact, he is to ask the assistants if they are all ready ; then, on being answered in the affirmative, as soon as he has made a perfect contact of the limbs of the objects, (as directed in Articles 76, 77, and 78), he is to make it known to his assistants by calling out *stop*. The person having the watch, and being provided with a pencil and paper, is to mark down the exact moment that the word *stop* is uttered :—then the principal observer is to read off the angular distance ; and the other two observers the altitudes, which are to be written down in succession after the *time*, and on the same horizontal line. In this manner let the observations be repeated till about *five* sets are taken.—Now, the sums of the times, the distances, and the altitudes, being divided by the number of sets ; the result will be the mean time, mean distance, and mean altitude respectively : and thus, *a complete set of lunar observations will be obtained*.—This will be practically illustrated in a subsequent page on the longitude.

80. As reference will have to be made occasionally to the three new Tables (given in pages 610* and 611* of the second volume), on various important points connected with the longitude, it may be advisable to speak of them here before entering into the introductory problems.

TABLE A.*

Equation of Second Differences, for correcting the Approximate Mean Time at Greenwich.

When the moon either approaches or recedes from a fixed star, or other celestial object that is exactly situate in the line of her path round the zodiac, the Greenwich time, corresponding to a computed lunar distance, will, in general, be sufficiently correct:—but, when her motion is an oblique direction with respect to the position of a fixed star, &c., the time deduced from the computed distance will *not* be quite correct; for, in some cases it will differ more than 50 seconds from the truth.—Hence, whenever a rigid degree of exactness becomes necessary, a *correction* must be applied to the Greenwich time answering to the computed distance, on account of the irregularity of the moon's motion, in relation to the object with which she may be compared. This correction is contained in Table A, Volume II., page 610* ; the arguments of which are, at top, the *difference* between the proportional logarithms that stand opposite to the two lunar distances, in the Nautical Almanac, which are *next greater* and *next less* than the computed distance ; and, in either the left or right-hand column, the *portion of time* : in the angle of meeting stands the corresponding correction.—Thus, let the *difference* between the proportional logarithms answering to two adjacent distances in the Ephemeris be 84, and the *portion of time* $1^{\text{h}}20^{\text{m}}15^{\text{s}}$; then, under 84, at top, and opposite to $1^{\text{h}}20^{\text{m}}$ in one of the side columns, stands 26 seconds ; which is the *correction* of the Greenwich time.—Although, when the moon is in a favourable position for observation with respect to the fixed stars, the *differences* of the proportional logarithms in the Nautical Almanac will seldom exceed 90 ; yet cases frequently occur in which those *differences* will far exceed the limits of the Table. When this happens, take the sum of the equations answering to *any two tabular differences* that will make up, or come nearest the given one ; and it will be the required correction.—Thus, the difference between the proportional logarithms corresponding to the distances between the moon and Fornalhaut at midnight, and XV. hours on the 24th August, 1836, per Nautical Almanac, is 186 :—now, let *the portion* of time be $1^{\text{h}}20^{\text{m}}15^{\text{s}}$;—then, since 90 and 96 will make up the given difference ; therefore, under those numbers and opposite to $1^{\text{h}}20^{\text{m}}$ stand 28 and 30 respectively ; the sum of which, or 58 seconds, is the required correction. The

* See Nautical Almanac for 1836, page 484.

equation thus found is to be applied by *addition* to the computed time at Greenwich, when the proportional logarithms are *decreasing*; but by *subtraction* when they are increasing; as will be shown hereafter.

81. The above-mentioned equations may be computed in the following manner, viz.:—Let the *difference* of the proportional logarithms, answering to the lunar distances in the Ephemeris, be esteemed as a whole number: then, to the logarithm of this, increasing the index by unity or 1, add the logarithm of the interval in time; the logarithm of its complement to 3 hours, and the constant logarithm 8. 142667* ;—the sum, abating 10 in the index, will be the logarithm of the equation in *seconds*.

Example.

Let the *difference* between the proportional logarithms answering to two lunar distances in the Ephemeris, be 80, and the interval or *portion of time* 1^h 10^m; required the corresponding equation?

Difference of proportional Logarithms 80.	Logarithm = 2.903090
Portion of time 1 ^h 10 ^m = 1 ^h 16 ^m 67 ^s	Logarithm . 0.066959
Complement of ditto to 3 hours = . 1.8333	Logarithm . 0.263233
Constant Logarithm	8.142667

Equation, as required = 23^m 77^s Logarithm . 1.375949
Hence, the correction answering to 80 and 1^h 10^m is 24 seconds in *whole numbers*.

82.—Table B, Volume II., page 611.*

Reduction of Latitude on account of the Oblate Spheroidal Figure of the Earth; or, to reduce the Geographical to the Geocentric Latitude.

In the explanation of Table XLI., which is given in page 105, the ratio of the equatorial semidiameter of the earth to its polar semi-axis, has been assumed at 230 to 229, agreeably to the Newtonian hypothesis; and therefore the excess of the spherical above the elliptic arch, in the latitude of 45 degrees, north or south of the equator, has been estimated at 11^m 53^s:—this, however, does not exactly correspond with modern calculations; for, since the ratio of the earth's equatorial radius to its polar semi-axis is now admitted to be as 305 to 304, as established by the French philosophers; therefore, the excess of the spherical above the elliptic arch in the mean parallel of latitude is no more than 11^m 17^s. The spherical excess signifies *the angular distance*

* The log. arithmetical complement of 72, viz. 24×3 ,—three hours being the common interval between the lunar distances in the Nautical Almanac.

between the centre and the apparent zenith of an observer in the parallel of 45 degrees from the equator.—This excess, which is 36' less than the latitude, is determined in the following manner, viz.:

To the logarithm of the equatorial semidiameter of the earth, add the logarithm of its polar semi-axis: the log. tangent of 1', and the constant logarithm of 2.000000.—From the sum of these four terms subtract the logarithm of the sum of the two semidiameters; then, the arithmetical complement of the remainder will be the logarithm of the spherical excess in seconds: as thus:—

Equatorial semidiameter 3950. Logarithm =	2. 484300
Polar semidiameter = 3940. Logarithm	2. 482874
Co-tangent of 1 degree = 1' Log. tangent	4. 685575
Constant Logarithm, viz. Log. of 2	0. 301080
		<hr/>
Sum	9. 953779
Sum of the semidiameters = 7890. Logarithm	2. 784617
		<hr/>
Remainder	7. 169162

Spherical excess = 6, 7'—Logarithm 2. 830838, or, 11' 17"; which, therefore, is the correct *angular distance between the central and the apparent zeniths of an observer at the mean parallel*; or, the true reduction of latitude on account of the oblate spheroidal figure of the earth, in the parallel of 45 degrees north or south of the equator.—Now this being known, the reduction corresponding to any given latitude betwixt the equator and the Poles may be determined in the following manner, viz.:

The geographical latitude at the *mean parallel* being diminished by the spherical excess, the result will be the geocentrical latitude:—hence, $45^\circ - 11' 17'' = 44^\circ 48' 43''$; the log. co-tangent of which, rejecting radius, is 0. 002850.—Now, this (*taken as a constant quantity*) being subtracted from the log. tangent of any given latitude, the remainder will be the log. tangent of such latitude reduced to the oblate spheroidal figure of the earth: the difference between which and the given latitude will be the *tabular* reduction of latitude; as thus:—

To reduce the Geographical Latitude 50 degrees to the Geocentrical Latitude.

Given geographical latitude = 50: —	Log. tangent	10. 076186
Constant quantity, as above	0. 002850
		<hr/>
Geocentrical latitude . . .	49: 48: 53" Log. tangent	10. 073336
		<hr/>
Difference 11' 7"; which, therefore, is the	

reduction of latitude in the parallel of 50° north or south of the equator;—and in the same manner were all the equations in Table B determined.

Note.—In taking out the equations from the present Table, proportion must be made for the *excess* of the minutes above the given degree: this may be readily done; viz.:—As 60 minutes are to the difference of correction between the two degrees that are next greater and next less than the given degree; so are the minutes of latitude to the required equation:—but, in general, the equation may be taken out at sight, without the trouble of making any proportion; and it is always to be applied by *subtraction* to the geographical latitude.

83.—Table C, Volume II., page 611.*

Logarithmical Radius of the Earth, for reducing the Moon's Horizontal Parallax to the Earth's Oblate Spheroidal Figure.

In treating of the reduction of the moon's horizontal parallax at pages 104 and 105, the ratio of the polar axis to the equatorial diameter of the earth was assumed at 229 to 230, as established by Newton, and the compression of the Poles as $\frac{1}{300}$:—Agreeably to that ratio, taking the equatorial diameter of the earth at 7917.5 English miles, its polar axis would be 7883.7 miles; which is 33.8, or, in round numbers, 34 miles less than the diameter at the equator.—Modern philosophers however, have given up Newton's ratio, and substituted in its stead *the mean of the two ratios* that were determined by *Le Lande* and *Delambre*. This *mean ratio* makes the polar axis be to the equatorial diameter as 304 to 305:—Therefore, taking the equatorial diameter of the earth at 7917.5 English miles; its polar diameter will be 7891.5 miles; which is but 26 miles less than the diameter at the equator; hence, the polar radius is only 13 miles less than the equatorial radius; and this is found to correspond with the phenomena which should arise from the precession of the equinoxes, and the nutation of the earth's axis.—And since it thus appears that the equatorial radius of the earth is to the polar semi-axis in the ratio of 305 to 304; therefore, the compression of the Poles is $\frac{305-304}{304} = \frac{1}{304}$, instead of $\frac{1}{300}$; as in page 105.

Now, since the greater the diameter of the earth, the greater must be the value of the moon's horizontal parallax; and, *vice versa*, the less the diameter of the earth, the less will be the value of the moon's horizontal parallax;—therefore, since the polar semi-axis of the earth is 13 miles less than the equatorial semidiameter; and, consequently, since the *radius* of the earth must diminish at every point, from the equator to the Poles, so must the value of the moon's horizontal parallax.—The

diminution of the moon's horizontal parallax may be computed agreeably to the formula in page 105, by substituting the constant logarithm 7.517126 (the log. arith. comp. of 304), for that 'given' in the rule. But, since it is more convenient, in general practice, to deduce the value of the moon's horizontal parallax from the earth's radius, therefore, assuming the equatorial semidiameter of the earth as unity or 1, the relative value thereof at any point of the meridian, betwixt the equator and the Poles of the world, may be readily determined in the following manner, viz. :—

From the logarithm of the polar radius, subtract *half* the logarithm of the sum of the polar and equatorial radii :—Then; let the remainder be subtracted from the log. sine of the latitude ; and the result will be the log. tangent of an arc : the log. co-sine of which will be the logarithm of the radius, or of the semidiameter of the earth in the given parallel of latitude.

Example.

Let the equatorial radius of the earth be 1 ; the ratio of the equatorial to the polar semidiameter as 305 to 304, and the latitude of the place 50° , north or south ; to find the logarithm of the earth's radius in that latitude :

Polar radius	304—	Logarithm . . .	2.482874
Polar radius + equatorial radius = 609	half its logarithm .		1.392308
		Remainder . . .	1.090566
Given parallel of latitude = 50° .	Log. sine . . .		9.884254
Arc =	$3^\circ 33' 30''$	Log. tangent . .	8.793688

Now, the log. co-sine of this arc is 9.999162 ; which, therefore, is the logarithm of the earth's radius in the given parallel of latitude ;—and in this manner were all the logarithms in Table C computed.

Now, to the log. sine of the moon's horizontal parallax (*reduced to the meridian of Greenwich*), add the tabular log. radius of the earth corresponding to the given degree of latitude (*reduced to the oblate figure*) ; the sum, abating 10 in the index, will be the log. sine of the moon's diminished equatorial horizontal parallax.—Or, to the common logarithm of the moon's horizontal parallax *in seconds*, add the tabular log. radius of the earth ; the sum, abating 10 in the index, will be the logarithm of the moon's horizontal parallax reduced to the oblate figure of the earth.

Note.—The elementary principles upon which the logarithmical

rules in Articles 82 and 83 are founded, may be seen by referring to Mr. W. S. B. Woolhouse's admirable Paper on *Eclipses*; which is given as an Appendix to the Nautical Almanac for 1836, between pages 55 and 58.

84. Having thus explained, in the preceding Articles and Definitions, every point of importance in the new Nautical Almanac, we shall now proceed to the consideration of such problems as may be deemed introductory to the Science of Nautical Astronomy.

INTRODUCTORY PROBLEMS TO THE SCIENCE OF NAUTICAL ASTRONOMY.

It may be necessary to premise, that throughout the astronomical part of this work the time will be reckoned agreeably to the mean solar day, viz. from *mean noon* to *mean noon*, or from 0 to 24 hours, the same as in the Nautical Almanac,* without paying any regard to the nautical or civil division of time at midnight:—this will conduce much to simplicity, as well as to uniformity; and do away with that confusion which frequently arises from the nautical distinctions of A. M. and P. M., when the time at a given place is to be reduced to the meridian of the Royal Observatory at Greenwich.

PROBLEM I.

To convert Longitude or Parts of the Equator into Time.

RULE.

Multiply the given degrees by 4, and the product will be the corresponding time:—observing that seconds multiplied by 4 produce thirds; minutes multiplied by 4 produce seconds, and degrees multiplied by 4 produce minutes, which, divided by 60, give hours, &c.

Example 1.

Required the time corresponding to $12^{\circ}40'45''$?

Given degrees = $12^{\circ}40'45''$

Multiply by 4

Corresp. time = $0^{\text{h}}50^{\text{m}}43^{\text{s}}0^{\text{t}}$

Example 2.

Required the time corresponding to $76^{\circ}20'30''$?

Given degrees = $76^{\circ}20'30''$

Multiply by 4

Corresp. time = $5^{\text{h}}5^{\text{m}}22^{\text{s}}0^{\text{t}}$

* See the fourth paragraph, page 498 of the Nautical Almanac for 1836.

PROBLEM II.

To convert Time into Longitude, or parts of the Equator.

RULE.

Reduce the hours to minutes, to which add the odd minutes, if any; then, the minutes divided by 4 give degrees; the seconds divided by 4 give minutes, and the thirds divided by 4 give seconds.

Example 1.

Required the degrees corresponding to $0^h 47^m 36^s$?

Given time = $0^h 47^m 36^s$

Divide by $4) 47^m 36^s$

Corresp. deg. = $11^{\circ} 54' 0''$

Example 2.

Required the degrees corresponding to $9^h 25^m 37^s$?

Given time = $9^h 25^m 37^s$

Divide by $4) 565^m 37^s$

Corresp. deg. = $141^{\circ} 24' 15''$

The above problems may be readily solved by means of Table I:—the principles upon which they are founded are set forth in the paragraph which follows the Examples in page 2.

PROBLEM III.

Given the Time under a known Meridian, to find the corresponding Time at Greenwich:—or, to reduce the Mean Time at any place to the Meridian of Greenwich.

RULE.

Let the given mean time be always reckoned from *the preceding noon*, to which apply the longitude of the place *in time* (reduced by Problem I.), by *addition*, if it be *west*, or *subtraction*, if *east*; the *sum*, or *difference*, will be the corresponding mean time at Greenwich.

Note.—*This Rule is general*; but it is subject to certain conditions, viz. :—In *west* longitude, when the *sum* is less than 24 hours, it will express the mean time at Greenwich past noon of the given day; but, when it is *greater*, then the *sum*, *minus* 24 hours, will be the mean time at Greenwich past noon of the *day following the given one*.

Again.—In *east* longitude, the *difference* will be the mean time at Greenwich past noon of the given day: but when the time at ship is *less* than the longitude *in time*, it is to be increased by 24 hours:—in this case, the increased time *minus* the longitude *in time* will be the mean time at Greenwich past noon of the *day preceding the given one*.

Example 1.

January 8th, 1836,—Required the corresponding times at Greenwich when it is $4^h 40^m 13^s$ and $21^h 50^m 45^s$, at a ship in longitude $80^\circ 53' 15''$ West?

First Time.

Mean time at ship . $4^h 40^m 13^s$
 Long. $80^\circ 53' 15''$ W.
 in time = . . . $5. 23. 33$

Mn. time at Greenw. $10^h 3^m 46^s$
 past noon of the given day.

Second Time.

Mean time at ship . $21^h 50^m 45^s$
 Long. $80^\circ 53' 15''$ W.
 in time = . . . $5. 23. 33$

Mn. time at Greenw. $3^h 14^m 18^s$
 past noon, Jan. the 9th, or the
 day *following* the given one.

Example 2.

February 10th, 1836,—Required the corresponding times at Greenwich, when it is $20^h 11^m 41^s$ and $2^h 40^m 35^s$, at a ship in longitude $98^\circ 14' 45''$ East?

First Time.

Mean time at ship . $20^h 11^m 41^s$
 Long. $98^\circ 14' 45''$ E.
 in time = . . . $6. 32. 59$

Mn. time at Greenw. $13^h 38^m 42^s$
 past noon of the given day.

Second Time.

Mean time at ship . $2^h 40^m 35^s$
 Long. $98^\circ 14' 45''$ E.
 in time . . . $6. 32. 59$

Mn. time at Greenw. $20^h 7^m 36^s$
 past noon, Feb. the 9th, or the
 day *preceding* the given one.

PROBLEM IV.

Given the Time at Greenwich, to find the corresponding Time under a known Meridian.

RULE.

Let the given mean time be always reckoned from the *preceding noon*, to which apply the longitude of the place *in time* (reduced by Problem I. as above), by addition if it be east, or subtraction if west; and the sum, or difference, will be the corresponding time under the given meridian.

Example 1.

When may the emersion of the first satellite of Jupiter be observed at Trincomalee, in longitude $81^\circ 22'$ E., which, by the Nautical Almanac, happens at Greenwich, March 6th, 1836, at $7^h 6^m 19^s$?

Mean time of emersion at Greenwich = . . . 7^h 6^m 19^s

Longitude of Trincomalee 81° 22' E., in time = 5. 25. 28

Mean time of emersion at Trincomalee = . . . 12^h 31^m 47^s

Example 2.

When may the immersion of the first satellite of Jupiter be observed at Port Royal, Jamaica, in longitude 76° 52' 30" W., which, by the Nautical Almanac, happens at Greenwich, Nov. 6th, 1836, at 15^h 49^m 55^s?

Mean time of immersion at Greenwich = . . . 15^h 49^m 55^s

Longitude of Port Royal 76° 52' 30" W., in time = 5. 7. 30

Mean time of immersion at Port Royal = . . . 10^h 42^m 25^s

As the above problem is the converse of Problem III., the longitude *in time* is therefore applied to the given mean time with a *contrary sign*, viz., by *subtraction* in *west*, and *addition* in *east* longitude. It is particularly useful in looking out for the approximate times of observing the eclipses of Jupiter's satellites.

PROBLEM V.

To Reduce the MEAN Sun's Right Ascension (viz. the "Sidereal Time," which is given in page II. of the Month in the Nautical Almanac,) to any given Meridian, and to any Time under that Meridian.

RULE.

Turn the longitude into time (Problem I., page 341), and *add* it to the mean time at ship or place, if it be *west*, but *subtract* it if *east*; the *sum*, or *difference*, will be the corresponding mean time at Greenwich, subject to the conditions in Problem III., page 342.—To the mean time at Greenwich, thus found, take out the corresponding equations in Table XLVI., volume II., page 597; which being *added* to the *mean sun's* right ascension at the *noon preceding* the Greenwich time; the *result* will be the correct right ascension of the *mean sun* at the given time and place.

Should the result exceed the measure of a day, it is to be diminished by 24 hours.

Example 1.

Required the *mean sun's* right ascension, January 1st, 1836, at 20^h 40^m 36^s mean time; the longitude being 120° 45' East?

Mean time at given place 20^h 40^m 36^s :
 Longitude 120° 45' East, in time = . . . -8. 3. 00

Mean time at Greenwich 12^h 37^m 36^s : past noon
 of the given day.

Mean sun's R. A. at noon of given day, per Naut. Alm. 18^h 40^m 43^s : 04
 Equation answering to 12 hours, in Table XLVI. = . . . 1. 58. 28
 Ditto 37 minutes, in ditto 6. 08
 Ditto 36 seconds, in ditto 0. 10

Mean sun's correct right ascension, as required = . . . 18^h 42^m 47^s : 50

Example 2.

Required the *mean* sun's right ascension, January 10th, 1836, at
 21^h 10^m 30^s : mean time ; the longitude being 140° 35' west ?

Mean time at given place 21^h 10^m 30^s :
 Longitude 140° 35' West, in time = . . . +9. 22. 20

Mean time at Greenwich = 6^h 32^m 50^s : past noon
 of the *day following* the given one, viz. of January 11th.

Mean sun's R. A. at noon January 11th, per Naut. Alm. 19^h 20^m 8^s : 62
 Equation answering to 6 hours, in Table XLVI. = . . . 59. 14
 Ditto 32 minutes, in ditto 5. 26
 Ditto 50 seconds, in ditto 0. 14

Mean sun's correct right ascension, as required = . . . 19^h 21^m 13^s : 16

Note.—See Explanatory Article 37, page 313, relative to the “*Sidereal Time*,” in page II. of the month in the Ephemeris, being called •
 the *MEAN Sun's Right Ascension*.

PROBLEM VI.

*Given the Mean Time at Ship or Place, and the Longitude, to find the
 Right Ascension of the Meridian ; or, to Reduce Mean Solar Time
 to Sidereal Time.*

RULE.

Turn the longitude into time (Problem I., page 341), and *add* it to
 the mean time at ship, if it be *west*, but *subtract* it if *east* ; the sum, or
 difference will be the mean time at Greenwich, subject to the condi-
 tions in Problem III., page 342. To the mean time at Greenwich,
 take out the corresponding equations in Table XLVI., Vol. II., page

597 ; which, being added to the mean sun's right ascension at the noon *preceding* the Greenwich time, the sum will be, the mean sun's correct right ascension. Now, this being *added* to the given mean time at ship, the sum (diminished by 24 hours if necessary) will be the right ascension of the meridian.

Example.

Required the right ascension of the meridian, May 6th, 1836, at 14^h55^m19^s mean time ; the longitude being 45°45'15" West ?

Mean time at ship or place 14^h55^m19^s

Longitude 45°45'15" West, in time = +3. 3. 1

Mean time at Greenwich 17^h58^m20^s past noon, May 6th.

Mean sun's R. A. at noon, May 6th, per Nautical Alm. 2^h57^m29^s00

Equation answering to 17 hours in Table XLVI. = 2. 47. 56

Ditto 58 minutes, in ditto 9. 03

Ditto 20 seconds, in ditto 0. 05

Mean sun's correct right ascension 3^h 0^m25^s64

Given mean time at ship or place 14. 55. 19.—

Right ascension of the meridian as required 17^h55^m44^s64

Remark.—This problem solves the Example marked "*vice versa*," in page 502 of the Nautical Almanac for 1836 ; viz. :—

To convert 2^h24^m19^s73 mean solar time, January 2nd, 1836, into sidereal time for the meridian of Greenwich :—

Mean sun's R. A. at noon, January 2nd = 18^h44^m39^s60

Equation answering to 2 hours in Table XLVI. 0. 19. 71

Ditto 24 minutes, in ditto 3. 94

Ditto 19.73 seconds in ditto 0. 05

Mean sun's correct right ascension 18^h45^m 3^s90

Given mean time 2. 24. 19.73

Sidereal time = 21^h 9^m23^s03

In the same manner may be solved the *Example* which is given in the first "Table of Equivalents," page 489 of the Ephemeris.

Note.—From the above it is clearly manifest that the right ascension of the meridian at any given place is *the sidereal time* at that place ; and *conversely*.—See Explanatory Article 32, page 309.

PROBLEM VII.

Given the Right Ascension of the Meridian and the Longitude, to find the Mean Time at Ship, or Place ;—or, to Reduce the Sidereal Time at Ship to Mean Solar Time.

RULE.

Since the mean time at ship may be always estimated within two or three minutes of the truth ; if, therefore, the longitude *in time* be applied thereto by *addition* when it is *west*, or by *subtraction* when *east*, the sum, or difference, will be the assumed time at Greenwich ; subject to the conditions in Problem III., page 342.

Take, from page II. of the Month in the Nautical Almanac, the *mean* sun's right ascension for the noon *preceding* the Greenwich time : which subtract from the given right ascension of the meridian (increased by 24 hours if necessary), and the remainder will be the approximate mean time at ship. Reduce this to the meridian of Greenwich, by Problem III. ; and find the corresponding equation in Table XLV. (the first in page 597 of the second volume). Now, the equation thus found, being subtracted from the approximate mean time ; the result will be the correct mean time at ship or place.

Example.

May 6th, 1836, in longitude $45^{\circ}45'15''$ west, the right ascension of the meridian was $17^{\text{h}}55^{\text{m}}44^{\text{s}}.64$, and the estimated mean time at ship $14^{\text{h}}53^{\text{m}}56^{\text{s}}$; required the correct mean time at ship.

Estimated mean time at ship = . . . $14^{\text{h}}53^{\text{m}}56^{\text{s}}$

Longitude $45^{\circ}45'15''$ west, in time . +3. 3. 1

Assumed time at Greenwich . . . $17^{\text{h}}56^{\text{m}}57^{\text{s}}$ past noon, May 6th.

—The only use that is made of this time is to indicate the *proper noon* for taking out the *mean* sun's right ascension from the Nautical Almanac.

Given right ascension of the meridian $17^{\text{h}}55^{\text{m}}44^{\text{s}}.64$

Mean sun's R. A. at noon, May 6th . $2.57.29.00$

Approximate mean time at ship . . . $14^{\text{h}}58^{\text{m}}15^{\text{s}}.64$. $14^{\text{h}}58^{\text{m}}15^{\text{s}}.64$

Longitude $45^{\circ}45'15''$ west, in time = 3. 3. 1. —

Approximate time at Greenwich . . . $18^{\text{h}}1^{\text{m}}16^{\text{s}}.64$

Equation to 18 hours, in Table XLV. = $2^{\text{m}}56^{\text{s}}.93$

Ditto 1 minute, in ditto . . . 0.16 } Sum = — $2^{\text{m}}57^{\text{s}}.14$

Ditto 16.64 seconds, in ditto . . . 0.05 } —————

Correct mean time at ship, as required $14^{\text{h}}55^{\text{m}}18^{\text{s}}.50$

Remark.—This problem solves the *Example* in page 502 of the Nautical Almanac for the year 1836, viz. :—

To convert 21^h 9^m 23^s.04 sidereal time, January 2nd, 1836, into mean solar time for the meridian of Greenwich.

Given sidereal time = 21^h 9^m 23^s.04

Mean sun's R. A. at noon, Jan. 2nd . 18. 44. 39. 60

Approximate mean time at Greenwich 2^h 24^m 43^s.44 . . 2^h 24^m 43^s.44

Equation to 2 hours, in Table XLV. = 0^m 19^s.66

Ditto 24 minutes, in ditto . 3. 93 } Sum = . — 0^m 23^s.71

Ditto 43. 44 seconds, in ditto 0. 12 } —————

Mean solar time, as required 2^h 24^m 19^s.76

And in the same manner may the *example* in the second “Table of Equivalents,” page 491 of the Ephemeris be solved.

From the above it appears evident that *the sidereal time is the same as the right ascension of the meridian* ; and *vice versa*, that the right ascension at any place is the same as *the sidereal time at such place*. —See Explanatory Article 32, page 309.

PROBLEM VIII.

To find the Mean Time of a Star's Transit, or Passage over the Meridian of any known Place.

Since the plane of the meridian of any given place may be conceived to be extended to the sphere of the fixed stars,—therefore, when the diurnal motion of the earth round its axis brings the plane of that meridian to any particular star, such star is then said to transit, or pass over the meridian of that place. This observation is applicable to all other celestial objects.

The mean time of transit of a known fixed star is to be computed by the following

RULE.

Reduce the given mean time at ship or place to the meridian of Greenwich by Problem III., page 342 :—to the Greenwich time, thus found, let the *mean sun's* right ascension be corrected by Problem V., page 344. Reduce the right ascension of the star, as given in Table XLIV., to the given day ; or, *take it at once out of the Nautical Almanac* : from which, increased by 24 hours, if necessary, subtract the *mean sun's* correct right ascension ; and the remainder will be the mean time of the star's transit over the meridian of the given place.

Example.

At what time on the 2nd of January, 1836, will the star *Rigel* transit, or come to the meridian of a place $165^{\circ}30'$ east of Greenwich, the mean time at such place being about $10^{\text{h}}21^{\text{m}}$?

Given mean time at ship or place . . . $10^{\text{h}}21^{\text{m}}$

Longitude $165^{\circ}30'$ east, in time = . . . — 11. 2

Mean time at Greenwich, *about* . . . $23^{\text{h}}19^{\text{m}}$ past noon of the *preceding* day; viz., of the 1st of January.

Mean sun's R. A. at noon Jan. 1st, per Nautical Almanac $18^{\text{h}}40^{\text{m}}43^{\text{s}}.04$

Equation answering to $23^{\text{h}}19^{\text{m}}$ in Table XLVI. = . . . + 3. 49. 82

Mean sun's correct right ascension $18^{\text{h}}44^{\text{m}}32^{\text{s}}.86$

Right ascension of the star *Rigel* on the given day . . . 5. 6. 40. 06

Mean time of the star's transit, as required $10^{\text{h}}22^{\text{m}} 7^{\text{s}}.20$

Remarks.—1. In the solution of the present problem, it will be quite sufficient, if the mean time at ship be known within 3 or 4 minutes of the truth; because an error to that amount in the estimated time will not affect the computed mean time of transit more than about *half a second*.

2. Should the mean time of the star's transit *below the pole* be required:—Let the constant quantity $11^{\text{h}}58^{\text{m}}1^{\text{s}}.72$ (viz. 12 hours diminished by $1^{\text{h}}58^{\text{m}}28^{\text{s}}$, or *half* the diurnal increase of the *mean sun's* right ascension) be *added* to the mean time of transit found as above; the sum, abating 24 hours, if necessary, will express the mean time of the star's transit *below the pole*.*

PROBLEM IX.*

To find what Stars will be on, or nearest to, the Meridian of a Ship or Place at any given Time.

RULE.

Since the right ascension of a fixed star is measured eastward, according to the order of the signs, from the first *point of Aries* to the point of the *equinoctial*, which is intersected by its circle of declination; and since the right ascension of the meridian signifies the point of the equinoctial which comes to the meridian of a place at any given time (Definitions 15 and 16); therefore, the right ascension of the meri-

* The author has invented a *stellarium*, which shows (without the trouble of calculation) the solution of Problems VIII. and IX. in all parts of the world.

dian expresses the correct value of *the sidereal* right ascension of the heavens at any given time and place :—Hence,—

Let the right ascension of the meridian be found by Problem VI, page 345 ; then look for this among the right ascensions of the stars in the Nautical Almanac, or amongst the right ascensions in Table XLIV. ; and it will show the stars that are either on or nearest to the meridian at the given mean time. The stars whose right ascensions are *less* than the computed right ascension of the meridian, will have passed, or be to the *westward* of the ship's meridian ; but those whose right ascensions are *greater*, will be to the eastward of, and apparently approaching, the meridian of the ship or place.

Example.

What star will be nearest to the meridian, December 31st, 1836, at 10^h 12^m 41^s mean time ; the longitude being 70° 45' West ?

Mean sun's right ascension, reduced by Problem V. 18^h 42^m 13^s 11

Given mean time at ship or place + 10. 12. 41.—

Right ascension of the meridian 4^h 54^m 54^s 11

Now, this being looked for among the right ascensions of the stars in Table XLIV., it will be seen that the star whose right ascension corresponds *the nearest thereto* is β Eridani ; which, therefore, is the star that is most contiguous to the meridian at the given time. Capella is 9^h 41^m to the eastward of the meridian ; and Aldebaran 28^m 22^s to the westward of it :—hence, the former is approaching, and the latter receding from, the meridian.

Note.—In general, the correction of the *mean* sun's right ascension may be dispensed with. The right ascension at *noon* of the given day, *added* to the mean time at ship, will show the value of the right ascension of the meridian to a sufficient degree of exactness for nautical purposes.

The above problem will be found useful when the latitude is to be deduced from the meridional altitude of a fixed star.

PROBLEM X.

To compute the Mean Time of the Moon's Transit over the Meridian of Greenwich.

Since the moon's transit over the Royal Observatory at Greenwich is only given to the *nearest tenth* of a minute in the Nautical Almanac ; and since it becomes absolutely necessary, on many astronomical occa-

sions, to have it more strictly determined; the following method is therefore given, by which the mean time of the moon's transit over the meridian of Greenwich may be obtained true to the decimal part of a second.

RULE.

From the moon's right ascension at *noon* of the given day (increased by 24 hours if necessary), subtract the *mean* sun's right ascension at that noon; the remainder will be the approximate time of transit.

Find the *difference* between the moon's right ascension at noon, and at the hour which is *next greater* than the approximate time of transit: diminish this *difference* by the equation, in Table XLVI., corresponding to the hour which is *next greater* than the approximate time; the result will be the excess of the moon's motion over that of the *mean* sun in the interval betwixt noon and the approximate time of transit: then say,—as the *next greater hour* diminished by this excess, is to the *next greater hour*, so is the approximate time of transit, to the correct mean time of transit over the meridian of Greenwich.

Reduce all the terms to *seconds*; and then the proportion may be easily worked by logarithms, as in the following

Example.

Required the mean time of the moon's transit over the meridian of Greenwich on the 1st day of January, 1836?

☾'s R. A. at noon . . . 4 ^h 43 ^m 5 ^s 83	☾'s R. A. at noon = 4 ^h 43 ^m 5 ^s 83
Mean sun's ditto . . . 18. 40. 43. 04	Ditto, at 11 hours . . . 5. 6. 26. 77
<hr/>	<hr/>
Approx. time of tran. 10 ^h 2 ^m 22 ^s 79	Difference = . . . 23 ^m 20 ^s 94
<hr/>	Equation to 11 hours
Do. reduced to seconds = 36142. 79	Table XLVI. = . . . -1. 48. 42
	<hr/>
	Excess of ☾'s motion over mean sun's 21 ^m 32 ^s 52

Next greater hour is 11^h 0^m 0^s 0

Excess of ☾'s motion

over mean sun's . . . = 21. 32. 52

Difference = . . . 10^h 38^m 27^s 48 = 38307^s 48 Log. ar. comp. 5. 416717

Next greater hour 11, in seconds = 39600 — Logarithm . . . 4. 597695

Approx. time of transit, in seconds 36142. 79 Logarithm . . . 4. 558021

Mean time of transit, in seconds . . . 37362^s 24 Logarithm . . . 4. 572433

Ditto, raised to hours, &c. = . . . 10^h 22^m 42^s 24; which, therefore, is

the correct mean time of the moon's transit over the meridian of Greenwich on the given day.—The time of transit in the Nautical Almanac is 10^h 22^m 7^s.—It is much to be regretted that *tenths* have been made use of in the Ephemeris, on such an important occasion as this. The mean time of the moon's transit, as well as that of each of the *bright planets*, ought to have been given to seconds, instead of to the *nearest tenth* of a minute: because, when the practical navigator reduces the *tenths* to seconds, the result may differ, at times, three or four seconds from the truth; which, of course, will affect the time of transit over the meridian for which he may be calculating; and, in *low latitudes*, or in places where the moon approaches *near the zenith*, an error of three or four seconds in the mean time of transit would sensibly affect her meridional altitude.

PROBLEM XI.

Given the Mean Time of the Moon's Transit over the Royal Observatory at Greenwich, to find the Mean Time of her Transit over the Meridian of any other place.

RULE.

Take, from page IV. of the month in the Nautical Almanac, the moon's transit over the meridian of Greenwich on the given day, and also on the day *following*, if the longitude be *west*; but on the day *preceding* if *east*; find the difference, and it will be the *retardation* of transit, or the *excess* of the *lunar day* above the solar day. Now, 24 hours augmented by this *excess* will be the *length of the lunar day*:—Then say,—As the lunar day, so found, is to the *retardation* of transit; so is the longitude of the given place, *in time*, to a correction; which being applied by *addition* to the mean time of transit over the meridian of Greenwich, if the longitude be *west*, but by *subtraction* if *east*; the sum, or difference, will be the mean time of transit over the given meridian.

Note.—This proportion may be readily performed by proportional logarithms, esteeming the hours and minutes in the *first* and *third* terms as *minutes* and *seconds*; as in the following

Example.

Required the mean time of the moon's transit, January 1st, 1836, over a meridian 94° 30' 30" *west*; the computed mean time of transit at Greenwich being 10 hours, 22 minutes, 42 seconds?

♃'s transit at Greenwich on given day, per Ephemeris . 10^h22^m7^s
 Ditto, on the day following 11. 12. 7

Retardation of the moon's transit 0^h50^m0^s; or,
 the *excess* of the lunar above the solar day :—hence,

As 24 hours + 0^h50^m = 24^h50^m Prop. log. ar. comp. . 9. 1398
 Is to the retardation . . . 50 Prop. logarithm . . 0. 5563
 So is the longitude in *time* . 6^h18^m 2^s Prop. logarithm . . 1. 4559

Correction + 12^m41^s Prop. logarithm . . 1. 1520
 Comp. mean time of transit = 10. 22. 42, at Greenwich.

Mean time of transit = . 10. 35. 23, at the given meridian.

See Explanatory Article 13, page 306, relative to the length of the lunar day; and to the *arc of excess* which the earth must describe *beyond a complete revolution on its axis*, to bring the moon upon the same meridian that it was on the day before.

PROBLEM XII.

To Compute the Mean Time of a Planet's Transit over the Meridian of Greenwich.

RULE.

From the planet's geocentric right ascension at noon of the given day (increased by 24 hours if necessary) subtract the *mean* sun's right ascension at the same noon; the remainder will be the approximate time of transit.—Take the *difference* of the *mean* sun's diurnal motion in right ascension (viz. 3^m56^s55), and the planet's daily variation in right ascension, if the planet's motion be *progressive*;* which *difference* apply by subtraction to 24 hours, or 86400 seconds, if the planet's diurnal motion be the *greatest*; but by addition, if it be the *least*:—Then say,—As 24 hours, or 86400 seconds, diminished or augmented by the aforesaid *difference*, is to 86400 seconds; so is the approximate time of transit, to the mean time of the planet's transit over the meridian of Greenwich. But, if the planet have a *retrograde* motion,* the *sum* of its daily variation, and that of the *mean sun's* (viz. 3^m56^s55) is

* When the planet's motion is *progressive*, the difference of right ascension between the given day and the *day following*; but when it is *retrograde*, the difference of right ascension between the given day and the *preceding* day, will be its daily variation in right ascension.

always to be applied by addition to 24 hours:—then 86400 seconds, augmented by that *sum*, will be the first term in the proportion; with which proceed as above directed.

Example 1.

Required the mean time of Venus's transit over the meridian of Greenwich on the 1st of January, 1836?

Venus's geocentric	Venus's daily variation
R. A. 20 ^h 17 ^m 44 ^s 45	in R. A. 5 ^m 13 ^s 42
Mean sun's R. A. 18. 40. 43. 04	Mean sun's ditto . . . 3. 56. 55
Ap. time of transit 1 ^h 37 ^m 1 ^s 41	Difference = . . . -1 ^m 16 ^s 47
Ditto in seconds = 5821. 41.	Ditto in seconds = -76. 47.

As 24 hours or 86400^s : -76^s 47 = 86323^s 53, Log. ar. comp. 5. 063871
: 24 hours, or 86400^s —, Logarithm . . 4. 936514
:: approximate time of transit . 5821. 41, Logarithm . . 3. 765028

Mean time of transit in seconds 5827^s Logarithm . . 3. 765413

Ditto raised to hours, &c. . . 1^h 37^m 7^s; which, therefore, is the correct mean time of the planet's transit over the meridian of Greenwich on the given day;—in the Nautical Almanac it is 1^h 37^m 1^s, or 1^h 37^m 6^s.

Note.—Had the planet's diurnal motion been *less* than that of the *mean* sun, the difference of the variations in right ascension, viz. 76^s 47 would have been additive to 86400 seconds; because, in this case, the planet would come to the meridian in *less* than 24 hours after its preceding time of transit.

Example 2.

Required the mean time of Venus's transit over the meridian of Greenwich on the 30th day of July, 1836?

Venus's geocentric	Venus's daily variation
R. A. 8 ^h 3 ^m 42 ^s 69	in R. A. 2 ^m 28 ^s 26
Mean sun's R. A. 8. 32. 36. 41	Mean sun's ditto . . . 3. 56. 55
Ap. time of transit 23 ^h 31 ^m 6 ^s 28	Sum + 6 ^m 24 ^s 81
Ditto, in seconds = 84666. 28.	Ditto in seconds = + 384. 81.

As 24 hours, or 86400: + 384'81 = 86784'81, Log. ar. comp.	5.061556
: 24 hours, or 86400 —, Logarithm .	4.936514
:: approximate time of transit 84666.28, Logarithm .	4.927710
Mean time of transit in seconds . 84291 —, Logarithm	4.925780

Ditto raised to hours, &c. . . 23^h24^m51^s; which, therefore, is the correct mean time of the planet's transit over the meridian of Greenwich on the given day:—The time of transit in the Ephemeris is 23^h24^m8^s, or 23^h24^m48^s; which is 3 seconds less than the above: this is owing to the *time* in that work being only given to the *nearest tenth* of a minute.

Note.—When the planet is *stationary*, the mean time of transit over the meridian of Greenwich is found at once, by simply subtracting the *mean* sun's right ascension from the geocentric right ascension of the planet: *diminishing* the remainder by the corresponding equation in Table XLVI.

PROBLEM XIII.

Given the Mean Time of a Planet's Transit over the Meridian of Greenwich; to find the Mean Time of Transit over any other Meridian.

RULE.

Find, in the Nautical Almanac, the difference between two consecutive transits, agreeably to the following precepts, *viz.*:—If the longitude be *west*, and the *times* of transit *increasing*, take the difference between the transits on the given day, and the day following; but if *decreasing*, between the transits on the given day and the day *preceding*. Again:—If the longitude be *east*, and the times of transit *increasing*, take the difference between the transits on the given and the *preceding* days; but if *decreasing*, between the transits on the given and *following* days.—Find, also, the *interval* between the two transits: this, when the times are *increasing*, will be expressed by 24^h + the difference of transit; but when *decreasing*, by 24 hours—the difference of transit.—Then, to the proportional Logarithm *Ar. Comp.* of this expression, add the proportional logarithm of the difference of transit, and the proportional logarithm of the longitude *in time*; the sum, abating 10 in the index, will be the proportional logarithm of a correction: which, in *west* longitude, is to be applied, by *addition*, to the “meridian passage” at Greenwich on the given day, when the transits are *increasing*, or, by subtraction, if *decreasing*.—But, if the longitude be

east, the correction is to be applied conversely, viz., by *subtraction* when the transits are *increasing*, and by *addition* when *decreasing*:—In either case the result will be the correct mean time of transit over the given meridian.

Example 1.

Required the mean time, on the 2nd January, 1836, that the planet Venus will pass the meridian of a place, which is $175^{\circ}30'$ east of the Royal Observatory at Greenwich?

Venus's meridian passage, January 1st. = $1^h 37^m 6^s$ per Ephemeris.
Ditto ditto 2nd. = $1.38.24$ ditto.

Difference of transit $0^h 1^m 18^s$: — Then, —
 $24^h + 1^m 18^s = 24^h 1^m 18^s$, is the *interval* of time between the two transits.

Note.—The *difference* is taken between the transits on the given day and the day *preceding*, because the longitude is *east*, and the *time increasing*. Now,

Interval between the transits = $24^h 1^m 18^s$ Prop. log. ar. comp. 9. 1253
Difference of transit 1. 18 Prop. logarithm . 2. 1413
Long. $175^{\circ}30'$ east, in *time* = 11. 42. 0 Prop. logarithm . 1. 1871

Correction = — $0^h 38^m$ Prop. logarithm . 2. 4537
Mn. time of tran. on given day $1.38.24$ at Greenwich.

Mn. time of tran. on given day = $1^h 37^m 46^s$, at the given meridian.

Remark.—The *correction* is *subtractive*, because the longitude is *east*, and the times of transit *increasing*; had they been *decreasing*, the *correction* would be *additive* to the meridian passage at Greenwich on the given day.

Example 2.

Required the mean time, on the 23rd of July, 1836, that the planet Venus will pass the meridian of a place, which is $140^{\circ}45'$ west of Greenwich?

Venus's meridian passage, July 22nd . = $0^h 23^m 12^s$ per Ephemeris.
Ditto ditto 23rd . = $0.16.42$ ditto

Difference of transit $0^h 6^m 30^s$: — Then, —
 $24^h - 6^m 30^s = 23^h 53^m 30^s$, is the interval between the two transits.

Note.—The difference is taken between the transits on the given day and the day *preceding*, because the longitude is *west*, and the times of transit *decreasing*; this being the converse of the above example with respect to *longitude* and the *motion* of the transits.—Now,

Interval between the transits	= 23 ^h 53 ^m 30 ^s	Prop. log. ar. comp.	9.1230
Difference of transit 6.30	Prop. logarithm	.1.4424
Long. 140° 45' west, in time	= 9.23. 0	Prop. logarithm	.1.2829

Correction =	— 2 ^m 33 ^s	Prop. logarithm	.1.8483
Mn. time of tran. on given day	.0.23. 12, at Greenwich.		

Mn. time of tran. on given day = 0^h 20^m 39^s at the given meridian.

Remark.—The *correction* is *subtractive*, because the longitude is *west*, and the times of transit *decreasing*; had they been *increasing*, the *correction* would be *additive* to the meridian passage at Greenwich on the given day.

The young navigator must bear in mind, that in *west* longitude when the times of transit are *increasing*, a planet will not come to the meridian of any place until *after* it has passed the meridian of Greenwich; but, when the times are *decreasing*, it will come to the meridian of such given place *before* its transit at Greenwich.—Again, in *east* longitude, when the times of transit are *increasing*, a planet will come to the meridian of any given place *before* it passes the meridian of Greenwich; but, when the times are *decreasing*, it will not come to the meridian of such place until *after* it has passed the meridian of Greenwich. This observation will conduce to the *elucidation* of Article 3, page 517 of the Nautical Almanac for 1836.

PROBLEM XIV.

To Reduce the Sun's Right Ascension and Declination; the Equation of Time, and the Sun's Longitude, as given in the Nautical Almanac, to any given Mean Time under a known Meridian.

RULE.

Let the given mean time at ship or place be *always* reckoned from the *preceding noon*; to which apply the longitude in *time* (reduced by Problem I., page 341,) by *addition* if it be *west*, or *subtraction* if east; the sum, or difference, will be the corresponding mean time at Greenwich, subject to the conditions in Problem III., page 342.

Take, from page II. or III. of the month in the Nautical Almanac, the sun's right ascension and declination, the equation of time, or longitude, as the case may be, for the mean noons immediately preceding and following the Greenwich time, and find their difference; then,

To the proportional log. of this difference, add the proportional log. of the Greenwich time (reckoning the hours as minutes, and the minutes as seconds), and the constant log. 9.1249:* the sum of these three logs. rejecting 10 from the index, will be the proportional log. of a correction, which is always to be *added* to the sun's longitude, or right ascension, at the mean noon preceding the Greenwich time; but to be applied by addition or subtraction to the sun's declination, or the equation of time at that noon, according as these elements may be increasing or decreasing.

Example.

Required the sun's right ascension, and declination, the equation of time, and the sun's longitude, January 1st, 1836, at 3^h 40^m mean time, in longitude 35° 40' 45" west of the meridian of Greenwich?

$$\begin{array}{rcl}
 \text{Mean time at ship or place} & = & \dots\dots\dots 3^h 40^m 0^s \\
 \text{Longitude } 35^\circ 40' 45'' \text{ west, in time} & = & \dots\dots\dots +2. 22. 43 \\
 \hline
 \text{Mean time at Greenwich} & \dots\dots\dots & 6^h 2^m 43^s
 \end{array}$$

To find the Sun's Right Ascension :—

$$\begin{array}{rcl}
 \text{Difference in 24 hours} & = & \dots\dots\dots 4^m 25^s \quad \text{Prop. log.} = \dots\dots\dots 1.6102 \\
 \text{Greenwich time} & = & \dots\dots\dots 6^h 2^m 43^s \quad \text{Prop. log.} = \dots\dots\dots 1.4738 \\
 \text{Constant log.} & = & \dots\dots\dots \dots\dots\dots \dots\dots\dots 9.1249 \\
 \hline
 \text{Correction of sun's right ascension} & = & + 1^m 7^s \quad \text{P. log.} = 2.2089 \\
 \text{Sun's right asc. at noon Jan. 1st.} & = & 18^h 44^m 18^s \\
 \hline
 \text{Sun's right ascension as required} & = & 18^h 45^m 25^s
 \end{array}$$

To find the Sun's Declination :—

$$\begin{array}{rcl}
 \text{Difference in 24 hours} & = & \dots\dots\dots 4' 55'' \quad \text{Prop. log.} = \dots\dots\dots 1.5636 \\
 \text{Greenwich time} & = & \dots\dots\dots 6^h 2^m 43^s \quad \text{Prop. log.} = \dots\dots\dots 1.4738 \\
 \text{Constant log.} & = & \dots\dots\dots \dots\dots\dots \dots\dots\dots 9.1249 \\
 \hline
 \text{Correction of sun's declination} & = & \dots\dots\dots - 1' 14'' \quad \text{P. log.} = 2.1623 \\
 \text{Sun's declination at noon Jan. 1st.} & = & 23^\circ 4' 16'' \text{ south.} \\
 \hline
 \text{Sun's declination, as required} & = & 23^\circ 3' 2'' \text{ south.}
 \end{array}$$

* The arith. comp. of the prop. log. of 24 hours, esteemed as minutes.

To find the Equation of Time :—

Difference in 24 hours =	28'	Prop. logarithm	2.5863
Greenwich time	6 ^h 2 ^m 43'	Prop. logarithm	1.4738
Constant logarithm			9.1249

Correction of equation	+ 7'	Prop. logarithm	3.1850
Equation of time at noon, Jan. 1st	= 3 ^m 35'	additive.	

Equation of time, as required . . = 3^m42' additive.

To find the Sun's Longitude :—

Difference in 24 hours =	1 ^h 1' 9"	Prop. logarithm . .	0.4689
Greenwich time =	6 ^h 2 ^m 43'	Prop. logarithm . .	1.4738
Constant logarithm			9.1249

Correction =	+ 15'24"	Prop. log.	1.0676
Sun's long. at noon, Jan. 1st = . .	280°10'45"		

Sun's true longitude, as required . . 280°26' 9"

To Reduce the Sun's Declination at Mean Noon to a given Meridian ; commonly called, "Correcting the Declination."

RULE.

If the equation of time be marked *additive*, in page I. (*not* page II.) of the month in the Ephemeris, it will express the *approximate* mean time of the sun's transit over the meridian of the ship past the *apparent* noon of the given day ; but, if it be marked *subtractive*, 24 hours *minus* the equation will be the *approximate* mean time of the sun's transit past the *apparent* noon of the preceding day.

To the *approximate* mean time of transit, thus found, apply the longitude, in time, by *addition* if *west*, or by subtraction if *east* ; and the sum, or difference, will be the *approximate* mean time at Greenwich ; with which, and the difference of declination for the mean noons immediately preceding and following it, proceed as in the second of the above operations.

Example.

Required the sun's declination at *mean noon*, March 26th, 1836, at a ship in longitude 173°45' west, and also at another ship in longitude 167°15' east of the meridian of Greenwich ?

Since the equation of time *viz.* $5^{\circ}44'$ taken to the nearest second is *marked above*; therefore the mean time of noon on the given day is $0^{\circ}5^{\circ}44'$; now, the *mean longitude* in time, *viz.* $11^{\circ}35'$, being added to that, shows the Greenwich time at the western meridian to be $11^{\circ}45^{\circ}44'$, past noon of the given day.—And, the east longitude, in time, *viz.* $11^{\circ}9'$, being subtracted from $0^{\circ}5^{\circ}44'$ increased by 24 hours, shows the Greenwich time at the eastern meridian to be $12^{\circ}56^{\circ}44'$ past noon of the preceding day.

To Correct for the Ship at the Western Meridian.

Difference of declination in 24 hours =	$23^{\circ}29'$	Prop. log.	0.8845
Greenwich time	$11^{\circ}45^{\circ}44'$	Prop. log.	1.1879
Constant logarithm			9.1249

Correction of declination =	$-11^{\circ}26'$	Prop. log.	1.1973
Sun's declination at noon, March 25th .	$2^{\circ}20' 7''$	north.	

Sun's corrected declination = $2^{\circ}31^{\circ}33'$ north.

To Correct for the Eastern Meridian.

Difference of declination in 24 hours =	$23^{\circ}32'$	Prop. log. .	0.8836
Greenwich time =	$12^{\circ}56^{\circ}44'$	Prop. log. .	1.1432
Constant logarithm			9.1249

Correction of declination =	$+12^{\circ}42'$	Prop. log.	1.1517
Sun's declination at noon, March 25th =	$1^{\circ}56^{\circ}35'$	north.	

Sun's corrected declination = $2^{\circ} 9^{\circ}17'$ north.

Note.—In strictness, the equation of time at Greenwich ought to be reduced to the given meridian:—but, since the greatest diurnal difference in that element *never* exceeds 30 seconds; which, at *the utmost extent of longitude*, would only affect the Greenwich time to the value of 15 seconds; and since a difference of this value cannot affect the sun's declination, (even at the time of the equinoxes, when its diurnal variation is the greatest,) more than the small fraction $0^{\circ}25$, or *the fourth part of a second*; therefore, for the above purpose, the reduction of the equation of time may be dispensed with.

Remark.—The young navigator must bear in mind that the sun's right ascension, and declination, and, also, the equation of time *are always* to be taken from page II. of the month in the Nautical Almanac: the elements, of the same denominations, in page I. of the month, and

which are adapted to "Apparent Time," are entirely for the use of astronomers, and *not* for the purposes of navigating a ship over the boundless ocean.

Note.—The sun's right ascension is given in the Ephemeris to *hundredths* of a second, and his declination to *tenths* of a second; and the equation of time to *hundredths* of a second :—but, since, for *practical purposes at sea*, the *nearest second* in either of these elements will be sufficiently near the truth; if, therefore, the decimals in the right ascension, and in the equation of time be 50 or *under*, reject them; but if they be *more* than 50, *increase* the seconds in the right ascension, and equation of time by *unity* or 1. In like manner, should the decimal in the declination be 5, or *under*, reject it; but, if it be *more than* 5, *increase* the seconds of declination by unity or 1; as in the preceding example.

PROBLEM XV.

To Reduce the Moon's Semidiameter; Horizontal Parallax; Longitude, and Latitude, as given in the Nautical Almanac, to any given Mean Time under a known Meridian.

Let the given mean time at ship or place be *always* reckoned from the *preceding noon*; to which, apply the longitude, in *time* (reduced by Problem I., page 341), by *addition*, if it be *west*; but by *subtraction* if *east*; the sum, or difference, will be the corresponding mean time at Greenwich; with reference to the conditions in Problem III., page 342; noting whether it be past noon or midnight.

Take from pages III. and IV. of the month in the Nautical Almanac, the moon's semidiameter, horizontal parallax, longitude, and latitude for the noon and midnight immediately *preceding* and *following* the Greenwich time; and find the difference :—Then, to the proportional log. of this difference, add the proportional log. of the Greenwich time *past the preceding noon or midnight* (reckoning the hours as *minutes*, and the minutes as *seconds*), and the constant log. 8.8239;* the sum of these three logs. abating 10 in the index, will be the proportional log. of a correction, which is always to be *added* to the moon's longitude, at the noon or midnight preceding the Greenwich time; but to be applied by addition or subtraction to her latitude, semidiameter, or horizontal

* This is the arithmetical complement of the proportional log. of 12 hours, esteemed as *minutes*.

parallax, at that noon or midnight, according as it may be increasing or decreasing.

Note.—Since the difference of the moon's longitude in 12 hours will always exceed the limits of the Table ; if, therefore, an aliquot part, as $\frac{1}{2}$ or $\frac{1}{3}$ of such difference be taken, and the correction resulting therefrom be multiplied by 2 or 3, as the case may be, the required correction will be obtained.

Example.

Required the moon's semidiameter, horizontal parallax, longitude, and latitude, January 10th, 1836, at 2^h40^m10^s mean time, under a meridian which is 70°10'45" west of Greenwich.

Mean time at ship or place 2^h40^m10^s
 Given longitude 70°10'45", in time = . + 4. 40. 43

Greenwich time past noon of the given day 7^h20^m53^s:

To find the Moon's Semidiameter :—

Difference in 12 hours = . . + 5" Prop. log. 3. 3345
 Greenwich time 7^h20^m53^s!, or 7^h21^m Prop. log. 1. 3890
 Constant logarithm 8. 8239

Correction + 3" Prop. log. 3. 5474
 Moon's semidiameter at noon = 15'.41"

Moon's correct semidiameter . 15'.44", as required.

To find the Moon's Horizontal Parallax* :—

Difference in 12 hours = . . + 20" Prop. log. 2. 7324
 Greenwich time 7^h20^m53^s!, or 7^h21^m Prop. log. 1. 3890
 Constant logarithm 8. 8239

Correction + 12" Prop. log. 2. 9453
 Moon's hor. par. at noon . . 57'.33"

Moon's correct hor. parallax . 57'.45", as required.

* See the Article relating to the moon's horizontal parallax, between pages 29 and 35.

To find the Moon's Longitude :—

Difference in 12 hours	= 6° 42' 55" + 3 = 2° 14' 18½"	Prop. log.	0. 1272
Greenwich time 7° 20' 53"	Prop. log.	1. 3891
Constant logarithm		8. 8239

One third of the correction	= 1° 22' 14"	Prop. log. 0. 3402
Multiply by 3		

Whole correction = 4° 6' 42"
Moon's longitude at noon	184. 29. 37

Moon's approximate long. 188° 36' 19", as required.

To find the Moon's Latitude:—

Difference in 12 hours -23' 9"	Prop. log. 0. 8907
Greenwich time 7° 20' 53"	Prop. log. 1. 3891
Constant logarithm 8. 8239

Correction — 14' 11"	Prop. log. 1. 1037
Moon's latitude at noon 4° 6' 53"		

Moon's approximate latitude 3° 52' 42", as required.

Remarks.—1. When much accuracy is required, the proportional part, or correction of the moon's longitude and latitude, found as above, must be corrected by the equation of second difference contained in Table XVII, as explained between pages 35 and 37.—And, in all cases, the moon's semidiameter must be increased by the augmentation given in Table IV., as explained between pages 8 and 11.

2. The above problem may be very correctly solved by means of Table XVI. (See the explanation thereof in page 27.)

3. The moon's semidiameter, horizontal parallax, &c., are given in the Nautical Almanac to *tenths* of a second; but, since the nearest second in those elements will be sufficiently near the tenth for the common purposes of navigation; if, therefore, the decimal be 5, or under, reject it; but if it be *more* than 5, increase the seconds of the semidiameter, horizontal parallax, &c., by *unity* or 1; as in the above example.

PROBLEM XVI.

To Reduce the Moon's Right Ascension and Declination, as given in the Nautical Almanac, to any other Meridian, and to any given Time under that Meridian.

RULE.

To the mean time at ship or place, *always reckoned from the preceding noon*, add the longitude in *time* if it be west, but subtract it if east, the sum, or difference, will be the mean time at Greenwich.

Enter the Nautical Almanac, between pages V. and XII. of the month, and take out, under the given day, the right ascensions and declinations answering to the hours which are *next less* and *next greater* than the hour in the Greenwich time; and find the difference of each:—Then,

To the proportional logarithm of the difference *thus found*; add the proportional logarithm of the *minutes and seconds* in the Greenwich time, and the constant logarithm 9.5229;* the sum, abating 10 in the index, will be the proportional logarithm of a correction, which is always to be *added* to the right ascension at the *next less* hour; but, to be applied by addition or subtraction to the declination, at that hour, according as it may be increasing or decreasing.

Example.

Required the moon's right ascension, and declination, on the 1st day of July, 1836, at 9^h 30^m 36^s mean time; the longitude being 70° 14' 30" east?

Mean time at ship or place 9^h 30^m 36^s:

Longitude 70° 14' 30" east, in time = . . . —4. 40. 58

Mean time at Greenwich 4^h 49^m 38^s:

To find the Right Ascension:—

Moon's right ascension at 4 hours = 21^h 28^m 21^s:

Ditto at 5 hours = 21. 30. 48

Difference of right ascension . . . + 2^m 27^s: Prop. log. . 1. 8661

Minutes, &c., in Greenwich time . . 49. 38 Prop. log. . 0. 5595

Constant logarithm (Prop. log. Ar. Comp. of 60^m) 9. 5229

Correction +2^m 2^s: Prop. log. . 1. 9485

Moon's right ascension at 4 hours = 21. 28. 21

Moon's correct right ascension . . 21^h 30^m 23^s!, as required.

* This is the prop. log. ar. comp. of 60 minutes, or 1 hour.

To find the Declination :—

Moon's declination at 4 hours = $20^{\circ}19'30''$ south.

Ditto at 5 hours = 20. 8. 0 ditto

Difference of declination . . .	— $11'30''$	Prop. log. . .	1. 1946
Minutes, &c., in Greenwich time	49. 38	Prop. log. . .	0. 5595
Constant logarithm			9. 5229

Correction — $9'31''$ Prop. log. . . 1. 2770

Moon's declination at 4 hours = 20. 19. 30

Moon's correct declination . . $20^{\circ} 9'59''$, south as required.

Note.—The moon's right ascension is given in the Nautical Almanac to the *hundredths* of a second, and her declination to *tenths* of a second: however, for the common purposes of navigation, the *nearest second* in either of those elements will always be sufficiently near the truth; and, therefore, if the decimals in the right ascension be 50, or *under*, reject them; but if they be *more than* 50, increase the seconds of R. A. by *unity* or 1.—And, in like manner, should the decimal in the declination be 5, or *under*, reject it; but if it be *more than* 5, increase the seconds in this element by *unity* or 1; as in the above example.

Remarks.—In cases of great nicety, or when the moon's right ascension is required to *hundredths* of a second; let the following method be adopted, and the value of that element will be obtained to an extreme degree of exactness.

Rule.—Multiply the decimals or *hundredths* of a second by 6, and they will be converted into *thirds*; observing to cut off the right-hand figure in the product. Then, to the proportional logarithm of the difference of right ascension, in minutes, seconds, and thirds, esteemed as *hours, minutes, and seconds*, add the proportional logarithm of the minutes and seconds in the Greenwich time, and the constant logarithm 9. 5229; the sum, abating 10 in the index, will be the proportional logarithm of a correction in *hours, minutes, and seconds*, which are to be considered as minutes, seconds, and thirds:—annex a cipher to the thirds, then divide by 6, and they will be reduced to decimals, or to *hundredths* of a second. In illustration of this, we will compute the correction of the moon's right ascension at the Greenwich time in the last example.

Moon's right ascension at 4 hours **21°28'20".68**
 Ditto at 5 hours **=21.30.48.03**

Difference of right ascension . . **2°27'35"**
 Multiply the decimals by . . . 6

Diff. of R. A. esteemed as hours, &c. **2°27'21"** Prop. log. **0.0609**
 Minutes &c., in Greenwich time . **49.38** Prop. log. **0.5596**
 Constant logarithm **9.5229**

Correc. to be esteemed as minutes, &c. **2° 1'54"** Prop. log. **0.1602**

Annex a cipher to the 54; divide by 6, and the result is 90: hence,
 the true correction of the moon's right ascension is = **2° 1'90"**

Moon's right ascension at 4 hours **21.28.20.68**

Moon's true right ascension **21°30'22".58**; which
 is strictly correct.

PROBLEM XVII.

To Reduce the Geocentric Right Ascension and Declination of a Planet, as given in the Nautical Almanac, to any given Time under a known Meridian.

RULE.

Let the given mean time at ship be *always* reckoned from the *preceding noon*; to which, apply the longitude, in *time*, (reduced by Problem I., page 341,) by *addition* if it be *west*, but, by *subtraction* if *east*: the sum, or difference will be the corresponding mean time at Greenwich; subject to the conditions in Problem III., page 342.

Take, from between pages 279 and 302, or from between pages 323 and 346 of the Nautical Almanac, the given planet's geocentric right ascension and declination for the noons immediately preceding and following the Greenwich time, and find their difference; then,

To the proportional logarithm of this difference, add the proportional logarithm of the Greenwich time, (reckoning the hours as *minutes*, and the minutes as *seconds*,) and the constant logarithm 9.1249;* the sum, abating 10 in the index, will be the proportional logarithm of a correction; which, being applied by *addition* or *sub-*

* The arith. comp. of the prop. log. of 24 hours esteemed as *minutes*.

traction to the geocentric right ascension and declination of the planet at the noon preceding the Greenwich time, according as these elements may be *increasing* or *decreasing*; the result will be the correct right ascension and declination of the given planet.

Example.

Required the geocentric right ascension and declination of the planet Venus, January 3rd, 1836, at $5^h 20^m 36^s$ mean time; the longitude being $150^\circ 40' 15''$ east of Greenwich?

Given mean time $5^h 20^m 36^s$
Longitude $150^\circ 40' 15''$ east, in *time* $= -10. 2. 41$

Greenwich mean time $19^h 17^m 55^s$ past noon of the preceding day, viz., of January 2nd.

The difference of R. A. between the noons of Jan. 2nd and 3rd is $5^m 12^s$, *increasing*.

Ditto, of declination, ditto $16' 59''$ *decreasing*.

To find the Right Ascension:—

Greenwich mean time $19^h 17^m 55^s$ Prop. log. 0.9697
Variation of R. A. in 24 hours . . + $5^m 12^s$ Prop. log. 1.5393
Constant logarithm 9.1249

Correction + $4^m 11^s$ Prop. log. 1.6339
Venus's R. A. at noon, January 2nd = $20^h 22^m 58^s$

Venus's correct right ascension = $20^h 27^m 9^s$, as required.

To find the Declination:—

Greenwich mean time $19^h 17^m 55^s$ Prop. log. 0.9697
Variation of declination in 24 hours . . — $16' 59''$ Prop. log. 1.0252
Constant logarithm 9.1249

Correction = — $13' 40''$ Prop. log. 1.1198
Venus's declination at noon, January 2nd $21^\circ 1' 4''$ south.

Venus's correct declination $20^\circ 47' 24''$ south.

In the same manner may the heliocentric longitude and latitude of a planet be reduced to any given time under a known meridian.

Remarks.—1. In the Nautical Almanac, between pages 267 and 358, the geocentric right ascensions of the planets are given to *hundredths* of a second, and their declinations to *tenths* of a second ; but, since the *nearest* second in either of these elements will *always* be sufficiently exact for the ordinary purposes of navigation ; therefore, the *decimals* may be dispensed with, as in the above example :—see the *note* which is given at the end of the solution to Problem XIV., in page 361.

2. It is only the four bright planets, viz. Venus, Mars, Jupiter, and Saturn, that concern the practical navigator. The other planets, viz., Mercury, the Georgian, and the asteroids, Vesta, Juno, Pallas, and Ceres, are but very rarely visible to the naked eye; the first, because its proximity to the sun causes it to be generally lost in the splendour of the solar rays ; the second, because of its immense distance from the earth ; and the asteroids, because of their comparative minuteness ; and therefore they are of no manner of use whatever for nautical purposes on the *high seas*.

PROBLEM XVIII.

Given the observed Mean Time, per Watch, of the Sun's Transit over the Meridian of a Given Place ; to find the correct Mean Time of Transit.

Since the time of the sun's transit over the meridian of any place is indicated by the instant that a *correct* sun-dial projects its shadow along the plane of the meridian, either north or south, according to its position with respect to the equator, viz., when the shadow shows $0^{\text{h}}0^{\text{m}}0^{\text{s}}$; therefore the solution of this problem is *simply to convert apparent noon into mean noon* : this may be done by the following

RULE.

To the given observed mean time of transit apply the longitude in *time*, as directed in Problem III., page 342 ; the result will be the corresponding mean time at Greenwich : to which, let the equation of time, in *page II. of the month* in the Nautical Almanac, be reduced by Problem XIV., page 357. Now, when the equation of time is noted for *addition* in page I. of the month, (the sign in this page being always the reverse of what it is in page II.), let the *apparent* noon be called $0^{\text{h}}0^{\text{m}}0^{\text{s}}$; but, when it is noted for *subtraction*, call the *apparent* noon $24^{\text{h}}0^{\text{m}}0^{\text{s}}$:—Then, in the first case, $0^{\text{h}}0^{\text{m}}0^{\text{s}}$ + the reduced equation of time ; and, in the second, $24^{\text{h}}0^{\text{m}}0^{\text{s}}$ — the reduced equation of time will

be the correct mean time of the sun's transit or passage over the meridian of any given place.

Example.

September 29th, 1836, at $23^{\circ}50'7''$ mean time, per watch, the sun was observed to pass the meridian of a place $90^{\circ}45'$ west of Greenwich; required the correct mean time of transit?

Mean time, per watch $23^{\circ}50'7''$

Longitude $90^{\circ}45'$ west, in time 6. 3. 0

Greenwich time past noon, Sept. 30 $5^{\circ}53'7''$ Prop. log. 1.4855

Variation of equation in 24 hours = $19'15''$ Prop. log. 2.7514

Constant logarithm 9.1249

Correction = $4'72''$ Prop. log. 3.3618

Equation of time at noon, Sept. 30 = $10^{\circ}5'52''$

Reduced equation of time $10^{\circ}10'24''$.—Now, since the equation of time is marked *subtractive* in page I. of the month in the Ephemeris; therefore, $24^{\circ} - 10^{\circ}10'24'' = 23^{\circ}49'49''.76$, is the correct mean time of the sun's transit over the given meridian.

Note.—Since this is merely the conversion of *apparent noon* into *mean noon*; it does not require any further illustration.

See Explanatory Articles 35 and 36, relative to the *Equation of Time*, between pages 310 and 313.

PROBLEM XIX.

Given the Mean Time at Ship, per Watch, and the Sun's Horary Distance from the Meridian; to find the Correct Mean Time:—or, to convert Apparent Time into Mean Time.

RULE.

To the given mean time at ship, or place, apply the longitude, in *time*, as directed in Problem III., page 342; the result will be the corresponding mean time at Greenwich:—to which let the equation of time, in *page II. of the month* in the Nautical Almanac, be reduced by Problem XIV., page 357. Now, this reduced equation being applied to the sun's horary distance, viz., the *apparent time*, with a *contrary sign* to what it is marked in page II.; or, which may be something

plainer, according to its sign in page I. of the month; the result will be the correct mean time at the given place.

Example.

February 1st, 1836, at 3^h 10^m 20^s: mean time, per watch, in longitude 50° 40' west, the sun's horary distance from the meridian, or *the apparent time*, was 3^h 6^m 35^s:; required the correct mean time?

Given mean time =	3 ^h 10 ^m 20 ^s :	
Longitude 50° 40' west, in time	3. 22. 40	
<hr/>			
Greenwich time =	6 ^h 33 ^m 0 ^s :	Prop. log. 1. 4390
Variation of equation of time in 24 hours	28' 34"	Prop. log.	2. 5812
Constant logarithm =		9. 1249
<hr/>			
Correction =	7' 75"	Prop. log. 3. 1451
Equation of time at noon, Feb. 1st =	+ 3 ^m 34 ^s 91 ^m		
<hr/>			
Reduced equation of time =	+ 3 ^m 42 ^s 66 ^m	; the sign as in page I.
Given apparent time =	3 ^h 6 ^m 35 ^s :—	
<hr/>			
Correct mean time =	3 ^h 10 ^m 17 ^s 66 ^m	, as required.

Remarks—1. Although the above *example* is worked out to the *hundredths* of a second; yet, it is not absolutely necessary to do so: because, for all practical purposes at sea, the *nearest second* will be found quite sufficient; as pointed out in the *note* which is appended to Problem XIV., in page 357:—This will appear manifest by reflecting that an error of *one second* in the time will only affect the longitude to the value of *a quarter of a mile*; which, in nautical operations, *on the ocean*, may well pass unnoticed, as not worthy of a consideration.

2. The young navigator must bear in mind that *apparent time* only relates to the sun; it has nothing whatever to do with the rest of the heavenly bodies. See Articles 35 and 36, between pages 310 and 313, relative to the Equation of Time, &c. &c.

PROBLEM XX.

Given the Mean Time, and the Horary Distance of the Moon, a Fixed Star, or a Planet, from the Meridian; to find the Correct Mean Time.

RULE.

To the given mean time at ship, or place, apply the longitude, in time, as directed in Problem III., page 342; the result will be the

corresponding mean time at Greenwich : to which time let the *mean* sun's right ascension (viz., the "Sidereal Time," which is given in page II. of the month in the Nautical Almanac), be reduced by Problem V., page 344.—Reduce the star's right ascension, as given in the pages between 368 and 407 of the Nautical Almanac, to the given day ; but, the right ascension of the moon, or a planet, must be reduced to *the Greenwich time* by Problem XVI., page 364, or XVII., page 366 :—Then, if the given celestial object be *west* of the meridian, let its horary distance be applied by *addition* ; but if *east* by *subtraction* to its corrected right ascension :—the result will be the right ascension of the meridian.—From this, increased by 24 hours if necessary, subtract the reduced *mean* sun's right ascension ; and the remainder will be the correct mean time.

Example 1.

May 1st, 1836, at $10^{\text{h}}21^{\text{m}}24^{\text{s}}$ mean time, per watch, in longitude $70^{\circ}36'$ east, the horary distance of Antares, *east* of the meridian, was $3^{\text{h}}20^{\text{m}}13^{\text{s}}$; required the correct mean time ?

Given mean time =	$10^{\text{h}}21^{\text{m}}24^{\text{s}}$	Mean sun's R. A. at
Longitude $70^{\circ}36'$ east,		noon, May 1st =
in time =	$-4.42.24$	Equation to $5^{\text{h}}39^{\text{m}}$ in
		Table XLVI. =
Greenwich time =	$5^{\text{h}}39^{\text{m}}0^{\text{s}}$	$+0.55.69$
		Mean sun's cor. R. A. = $2^{\text{h}}38^{\text{m}}41^{\text{s}}.89$

Right ascension of Antares, per Ephemeris =	$16^{\text{h}}19^{\text{m}}23^{\text{s}}.26$
Horary distance of ditto,— <i>east</i> of meridian	$-3.20.13$
Right ascension of the meridian =	$12^{\text{h}}59^{\text{m}}10^{\text{s}}.26$
Mean sun's corrected right ascension	$-2.38.41.89$
Correct mean time, at given meridian =	$10^{\text{h}}20^{\text{m}}28^{\text{s}}.87$

Example 2.

May 3rd, 1836, at $17^{\text{h}}44^{\text{m}}37^{\text{s}}$ mean time, per watch, in longitude $76^{\circ}42'$ east, the horary distance of the moon, *west* of the meridian, was $3^{\text{h}}12^{\text{m}}33^{\text{s}}$; required the correct mean time ?

Given mean time =	$17^{\text{h}}44^{\text{m}}37^{\text{s}}$	Mean sun's R. A. at
Longitude $76^{\circ}42'$ east,		noon, May 3rd =
in time =	$-5.6.48$	Equation to $12^{\text{h}}37^{\text{m}}49^{\text{s}}$
		in Table XLV. =
Greenwich time	$12^{\text{h}}37^{\text{m}}49^{\text{s}}$	$+2.4.49$
		Mean sun's cor. R. A. = $2^{\text{h}}47^{\text{m}}43^{\text{s}}.80$

Moon's right ascension reduced to Greenwich time =	. 17 ^h 19 ^m 42 ^s 17
Moon's horary distance, <i>west</i> of meridian	+ 3. 12. 33. —
Right ascension of the meridian =	. 20 ^h 32 ^m 15 ^s 17
Mean sun's corrected right ascension	— 2. 47. 43. 80
Correct mean time at the given meridian	17 ^h 44 ^m 31 ^s 37

Note.—In the same manner may the mean time at ship, or place, be deduced from the horary distance of a planet.

PROBLEM XXI.

Given the Mean Time under a known Meridian; to find the Sun's Horary Distance from the Meridian:—or, to convert Mean Time into Apparent Time.

RULE.

This problem is merely the converse of Problem XIX., page 369;—hence, let the equation of time, page II. of the month in the Nautical Almanac, be reduced to the mean time at Greenwich:—then, this reduced equation being applied to the given mean time according to its sign in *that page*; the result will be the sun's horary distance from the meridian;—or, in other words, the apparent time at the given meridian.

Example.

February 1st, 1836, at 3^h 10^m 18^s mean time, in longitude 50° 40' west; required the sun's horary distance from the meridian?

Given mean time 3^h 10^m 18^s

Longitude 50° 40' west, in time = + 3. 22. 40

Greenwich time = 6^h 32^m 58^s—Now, the equation of time, page II. of the Naut. Alm., being reduced to the time at Greenwich; the result will be, to *the nearest second* 3^m 43^s *subtractive*.

Given mean time = 3^h 10^m 18^s

Sun's horary distance from the meridian = 3^h 6^m 35^s; or the *apparent* time from noon.

Note.—The apparent time only relates to the sun; it does *not* apply to either the moon, the fixed stars, or the planets.

See Explanatory Article 36, page 313, relative to applying the equation to mean time, and conversely.

PROBLEM XXII.

Given the Mean Time under a known Meridian; to find the Meridional Horary Distance of the Moon, a Fixed Star, or a Planet.

RULE.

To the given mean time at ship, or place, apply the longitude *in time*, agreeably to the directions contained in Problem III., page 342: the result will be the corresponding mean time at Greenwich; to which time, let the *mean* sun's right ascension (viz., the "Sidereal Time," in page II. of the month in the Ephemeris), be reduced by Problem V., page 344.

To the *mean* sun's reduced right ascension, add the mean time at ship, or place, and the sum, abating 24 hours, if necessary, will be the right ascension of the meridian.—(Problem VI., page 345.)—Now, the difference between this and the corrected right ascension of the given celestial object, will be the required horary distance from the meridian: which will be *east* when the right ascension of the given object is *greater* than the right ascension of the meridian; otherwise it will be *west*.

It must be remembered that the right ascension of the moon is to be reduced by Problem XVI., and that of a planet by Problem XVII.—The right ascension of a star is to be taken from the Nautical Almanac (between pages 368 and 407), and reduced to the given day.

Example 1.

May 1st, 1836, at $10^{\text{h}}20^{\text{m}}28^{\text{s}}$ correct mean time, in longitude $70^{\circ}36'$ east; required the horary distance of Antares from the meridian?

Mean time at ship = . $10^{\text{h}}20^{\text{m}}28^{\text{s}}$	Mean sun's R. A. at
Longitude $70^{\circ}36'$ east,	noon, May 1st = . $2^{\text{h}}37^{\text{m}}46^{\text{s}}20$
in time = . . . $-4.42.24$	Equation to $5^{\text{h}}38^{\text{m}}4^{\text{s}}$
	in Table XLVI. . . + $0.55.53$
Greenwich time = . $5^{\text{h}}38^{\text{m}}4^{\text{s}}$	Mean sun's cor. R. A. $2^{\text{h}}38^{\text{m}}41^{\text{s}}73$
Given mean time at ship	10. 10. 28 —
Right ascension of the meridian, or <i>sidereal time</i> . . .	$12^{\text{h}}49^{\text{m}}9^{\text{s}}73$
Right ascension of Antares	16. 29. 23. 26
Star's horary distance from the meridian	$3^{\text{h}}20^{\text{m}}13^{\text{s}}53$
which is <i>east</i> , because the star's right ascension is <i>greater</i> than the right ascension of the meridian.	

Example 2.

May 3rd, 1836, at $17^{\circ}44'31''$ correct mean time, in longitude $76^{\circ}42'$ east; required the moon's horary distance from the meridian?

Mean time at ship = $17^{\circ}44'31''$	Mean sun's R. A. at
Longitude $76^{\circ}42'$ E.	noon, May 3rd = $2^{\circ}45'39'31''$
in time . . . — 5. 6. 48	Eqn. to $12^{\circ}37'43''$
Greenwich time = . $12^{\circ}37'43''$	in Table XLVI. = +2. 4. 48
	Mean sun's cor. R.A. $2^{\circ}47'43'79''$

Given mean time at ship = $17:44.31$ —

Right ascension of the meridian, or *sidereal time* = . $20^{\circ}32'14'79''$

Moon's corrected right ascension = $-17.19.41.92$

Moon's horary distance from the meridian = $3^{\circ}12'32'87''$;
which is *west*, because the moon's right ascension is *less* than the right ascension of the meridian.

The above is simply the converse of Problem XX., page 370.

Remark.—The reason of the horary distance being called *east*, in the first example, and *west* in the second, will appear obvious by reflecting that as the right ascension of the meridian signifies that point of the Equinoctial which comes to the meridian of a place at any given time; and, as that point expresses the right ascension of the heavens, reckoned in *sidereal time* from the first point of Aries; therefore, any star or other celestial object, whose right ascension is greater than the right ascension of the said point of the Equinoctial at a given time, is to the *eastward* of the meridian which it represents; and *vice versa*, any star, or other celestial object, whose right ascension is *less* than the right ascension of the said point, is to the *westward* of the same meridian. Let the young navigator attend to this remark, and he will never be at a loss to know the true position of a heavenly body at any time, with respect to the meridian of a given place.

PROBLEM XXIII.

*Given the Observed Altitude of the Lower, or Upper Limb of the Sun;
to find the true Altitude of its Centre.*

RULE.

To the observed altitude of the sun's lower limb * (corrected for

* See Articles 60 and 61, page 326.

index error, if any), add the difference between its semidiameter* and the dip of the horizon; † the sum will be the apparent altitude of the sun's centre: or, from the corrected observed altitude of the sun's upper limb subtract the sum of the semidiameter* and the dip of the horizon; † and the remainder will be the apparent central altitude.

Now, from the apparent altitude of the sun's centre, thus found, subtract the difference between the refraction ‡ corresponding thereto, and the parallax in altitude, § and the remainder will be the true altitude of the sun's centre.

Note.—The difference between the refraction and parallax constitutes the correction of the sun's apparent altitude in the lunar observations.

Example 1.

Let the observed altitude of the sun's lower limb, by a *fore observation*, be $16^{\circ}29'$, the height of the eye above the level of the sea 24 feet, and the sun's semidiameter $16'.18''$; required the sun's true central altitude?

$$\begin{array}{rcl} \text{Observed altitude of the sun's lower limb} & = & . . 16^{\circ}29' 0'' \\ \text{Sun's semidiameter} & = & . . . 16'.18'' \\ \text{Dip of the horizon for 24 feet} & = & 4.42 \end{array} \left. \vphantom{\begin{array}{l} \text{Observed altitude of the sun's lower limb} \\ \text{Sun's semidiameter} \\ \text{Dip of the horizon for 24 feet} \end{array}} \right\} \text{Diff.} = + 11.36$$

$$\begin{array}{rcl} \text{Apparent altitude of the sun's centre} & = & . . . 16^{\circ}40'36'' \\ \text{Refraction} & = & 3'.8'' \\ \text{Parallax} & = & 0.8 \end{array} \left. \vphantom{\begin{array}{l} \text{Apparent altitude of the sun's centre} \\ \text{Refraction} \\ \text{Parallax} \end{array}} \right\} \text{Difference} = . . . - 3. 0$$

$$\text{True altitude of the sun's centre} = . . . 16^{\circ}37'36''$$

Example 2.

Let the observed altitude of the sun's upper limb, by a *fore observation*, be $18^{\circ}37'$, the height of the eye above the surface of the water 30 feet, and the sun's semidiameter $15'.46''$; required the true central altitude?

$$\begin{array}{rcl} \text{Observed altitude of the sun's upper limb} & = & . 18^{\circ}37' 0'' \\ \text{Sun's semidiameter} & = & . . . 15'.46'' \\ \text{Dip of the horizon for 30 feet} & = & 5.15. \end{array} \left. \vphantom{\begin{array}{l} \text{Observed altitude of the sun's upper limb} \\ \text{Sun's semidiameter} \\ \text{Dip of the horizon for 30 feet} \end{array}} \right\} \text{Sum} = -21. 1$$

$$\begin{array}{rcl} \text{Apparent altitude of the sun's centre} & = & . . . 18^{\circ}15'59'' \\ \text{Refraction} & = & 2'.51'' \\ \text{Parallax} & = & 0. 8 \end{array} \left. \vphantom{\begin{array}{l} \text{Apparent altitude of the sun's centre} \\ \text{Refraction} \\ \text{Parallax} \end{array}} \right\} \text{Difference} = . . . - 2.43$$

$$\text{True altitude of the sun's centre} = . . . 18^{\circ}13'16''$$

* Page II. of the month in the Nautical Almanac. † Table II. ‡ Table VIII.
§ Table VII.

Remark.—As the *back observations* (given in the first impression of this work) are more curious than useful; and, on the whole, subject to so many *inconveniences in practice* as to be very rarely, if ever, resorted to for the purpose of finding the latitude of a ship; they are, consequently, omitted in the present edition, as points not deserving the mariner's attention.

The reader is now requested to observe that throughout the rest of this work the altitudes of the heavenly bodies will be adapted to the *fore observations*, the same as in the above examples; because *these observations* are the most natural, the most simple, and the most convenient to the practical navigator.

PROBLEM XXIV.

Given the Observed Altitude of the Upper, or Lower Limb of the Moon; to find her true Central Altitude.

RULE.

To the given mean time at ship, or place, apply the longitude, in *time*, agreeably to the directions contained in Problem III., page 342; the result will be the mean time at Greenwich; to which let the moon's semidiameter and horizontal parallax be reduced by Problem XV., page 361 (or by Table XVI., as explained between pages 27 and 29), and let the reduced semidiameter be increased by the correction contained in Table IV., answering to it and the observed altitude:—then,

To the observed altitude of the moon's lower limb* (corrected for index error, if any), add the difference between the true semidiameter and the dip of the horizon; or, from the observed altitude of the upper limb subtract the sum of the semidiameter and dip, and the apparent central altitude of the moon will be obtained; to which let the correction (Table XVIII.) answering to the moon's reduced horizontal parallax and apparent central altitude be added, and the sum will be the altitude of the moon's centre.

Example 1.

In a certain latitude, March 10th, 1836, at 13^h 40^m 20^s, mean time, the observed altitude of the moon's *lower limb* was 20° 10' 40", and the height of the eye above the level of the sea 24 feet; required the true altitude of the moon's centre; the longitude being 35° 40' west?

* See Article 62, page 327.

Mn. time of observation $13^h 40^m 20^s$	\mathfrak{D} 's reduced semidiameter = $16' 8''$
Longitude $35^\circ 40'$ west,	Augmentation, Table IV. = 6
in time . . . + $2.22.40$	
Greenwich time = . $16^h 3^m 0^s$	\mathfrak{D} 's true semidiameter . $16' 14''$
	\mathfrak{D} 's red. hor. par. . . $59' 10''$
Observed altitude of \mathfrak{D} 's lower limb = $20^\circ 10' 40''$	
\mathfrak{D} 's true semidiameter = . . . $16' 14''$	} Difference = . $+11.32$
Dip of the horizon for 24 feet = 4.42	
Apparent altitude of the moon's centre = $20^\circ 22' 12''$	
Correction answering to ditto and hor. par. in Table XVIII. = $+52.55$	
True altitude of the moon's centre = $21^\circ 15' 7''$	

Example 2.

In a certain latitude, March 26th, 1836; at $3^h 30^m 47^s$ mean time, the observed altitude of the moon's upper limb was $30^\circ 17' 30''$, and the height of the eye above the level of the sea 30 feet; required the true altitude of the moon's centre; the longitude of the place of observation being $94^\circ 15' 30''$ east of the meridian of Greenwich?

Mn. time of observation $3^h 30^m 47^s$	\mathfrak{D} 's reduced semidiameter = $14' 53''$
Longitude $94^\circ 15' 30''$	Augmentation, Table IV. = $+7$
east, in time = . $-6.17.2$	
Greenwich time = . $9^h 13^m 45^s$	\mathfrak{D} 's true semidiameter . $15' 0''$
Past midnight of the 25th March.	\mathfrak{D} 's reduced horizontal parallax $54' 37''$
Observed altitude of \mathfrak{D} 's upper limb $30^\circ 17' 30''$	
\mathfrak{D} 's true semidiameter = . . $15' 0''$	} Sum = . . . $-20' 15''$
Dip of the horizon for 30 feet = 5.15	
Apparent altitude of the moon's centre $29^\circ 57' 15''$	
Correction answering to ditto and hor. par. in Table XVIII. = $+45.40$	
True altitude of the moon's centre $30^\circ 42' 55''$	

PROBLEM XXV.

Given the observed Altitude of a Planet's Centre, to find its true Altitude.

RULE.

From the planet's observed central altitude* (corrected for index

* See Article 64, page 328.

error, if any), subtract the dip of the horizon, and the remainder will be the apparent central altitude.

Find the difference between the planet's parallax in altitude* (Table VI.) and its refraction in altitude (Table VIII.) ; now, this being subtracted from the apparent altitude, the remainder will be the true central altitude of the planet.

Note.—The difference between the refraction and parallax constitutes the *correction* of a planet's apparent altitude in the lunar observations.

Example.

Let the observed central altitude of Mars be $17^{\circ}29'40''$, the index error $3'45''$ additive, and the height of the eye above the surface of the water 26 feet ; required the true central altitude of that planet, allowing his horizontal parallax to be 14 seconds ?

$$\begin{array}{rcl} \text{Observed central altitude of Mars} & = & 17^{\circ}29'40'' \\ \text{Index error} & = & + 3.45 \\ \text{Dip of the horizon for 26 feet} & = & - 4.52 \end{array}$$

$$\begin{array}{rcl} \text{Apparent central altitude of Mars} & = & 17^{\circ}28'33'' \\ \left. \begin{array}{l} \text{Refraction, Table VIII.} = 2'59'' \\ \text{Parallax, Table VI.} = 0.13 \end{array} \right\} \text{Diff.} & = & - 2.46 \end{array}$$

$$\text{Apparent central altitude of Mars} = 17^{\circ}25'47''$$

Remark.—In taking the altitude of a planet at sea, its centre is to be brought down to the horizon : in this case its semidiameter need not be taken into account. But, if the altitude be taken on shore, by means of an artificial horizon, it is the *lower limb* of the object, particularly of Venus and Jupiter, whose semidiameters are considerable, that should be brought in contact with the *apparent* upper limb, seen by reflection in the artificial horizon :—in this instance, the semidiameter of the planet is to be *added* to *half* the altitude observed in the artificial horizon ;—this shall be shown presently.—See Explanatory Article 64, page 328, and 67, page 329.

PROBLEM XXVI.

Given the observed Altitude of a fixed Star, to find the true Altitude.

RULE.

To the observed altitude of the star† apply the index error, if any ;

* The parallaxes and semidiameters of the planets are now given in the Nautical Almanac, between pages 359 and 361. † See Article 63, page 327.

from which subtract the dip of the horizon, and the remainder will be the star's apparent altitude.

From the apparent altitude, thus found, let the refraction corresponding thereto be subtracted, and the remainder will be the true altitude of the star.

Note.—The *refraction* constitutes the *correction* of a star's apparent altitude in the lunar observations.

Example.

Let the observed altitude of Spica Virginis be $18^{\circ}30'$, the index error $3'20''$ subtractive, and the height of the eye above the level of the water 18 feet; required the true altitude of that star?

Observed altitude of Spica Virginis =	$18^{\circ}30' 0''$
Index error =	$- 3, 20$
Dip of the horizon for 18 feet =	$- 4, 4$
<hr/>	
Apparent altitude of Spica Virginis =	$18^{\circ}22'36''$
Refraction =	$- 2, 50$
<hr/>	
True altitude of Spica Virginis =	$18^{\circ}19'46''$

Note.—The fixed stars do not exhibit any apparent semidiameter, nor any sensible parallax; because the immense and inconceivable distance at which they are placed from the earth's surface causes them to appear, at all times, as so many mere luminous indivisible points in the heavens.

PROBLEM XXVII.

To deduce the true Altitude of a Celestial Object from its DOUBLE Altitude, observed by means of an Artificial Horizon.

GENERAL RULE.

First.—To find the sun's true central altitude,

Correct the observed angle of altitude for the index error of the sextant, if any; to the *half* of which apply the sun's semidiameter by *addition*, if the *lower limb* be observed, but by *subtraction* if it be the *upper limb*; the result will be the apparent central altitude; from which let the difference between the refraction and parallax corresponding thereto be subtracted (Table VII. and VIII.), and the remainder will be the correct altitude of the sun's centre,

Second.—To find the Moon's true Central Altitude.

Find the moon's apparent central altitude in the same manner as if it were the *sun* that was under consideration ; observing to correct her semidiameter by the equation contained in Table IV. ;—then, to the apparent altitude, thus found, let the correction in Table XVIII. be added, and the sum will be the true altitude of the moon's centre.

Third.—To find the true Altitude of a Star.

Correct the observed angle for the index error of the sextant, if any ; from the *half* of which subtract the refraction corresponding thereto (Table VIII.), and the remainder will be the star's true altitude.

Fourth.—To find a Planet's true Central Altitude.

Correct the observed angle for the index error of the sextant, if any ; to the *half* of which let the planet's semidiameter be added, and the sum will be its apparent central altitude ; from which let the difference between the refraction and parallax (Tables VI. and VIII.) be *subtracted*, and the remainder will be the correct altitude of the planet's centre.

Example.

July 24th, 1836, the measure of the observed angle between the lower limb of the planet Venus, and the *apparent* upper limb thereof, reflected from an artificial horizon, was $33^{\circ}20'10''$, the index error of the sextant was $2'30''$ subtractive ; required the true central altitude of the planet ; her semidiameter, on the given day, being $28''$, and her horizontal parallax $30''$?

Given observed angle =	$33^{\circ}20'10''$	
Index error of the sextant =	$- 2.30$	
	<hr/>	
Corrected observed angle =	$33^{\circ}17'40''$	
	<hr/>	
The half of which, or	$16^{\circ}38'50''$	is the
observed altitude of the planet's lower limb.		
Semidiameter of Venus =	$+ 28''$	
	<hr/>	
Planet's apparent central altitude =	$16^{\circ}39'18''$	
Refraction answering to do. $= 3'8''$	} Difference = $- 2.40$	
Parallax, in altitude, Tab. VI. $= 0.28$		
	<hr/>	
True altitude of Venus's centre	$16^{\circ}36'38''$	

Note.—It is presumed that the examples to the three preceding problems render any further illustration unnecessary respecting the altitudes of the sun, moon, and stars.

PROBLEM XXVIII.

To find the Obliquity of the Ecliptic.

The obliquity, or *inclination* of the ecliptic, signifies the angle which is made by the intersection of the planes of the ecliptic and the equinoctial: the two points of intersection are diametrically opposite to each other, or 180° asunder, one being at the *first point of Aries*, and the other at the *first point of Libra*.

The solstices are those two points of the ecliptic which are equidistant, or 90° from the above-mentioned points of intersection (Definitions 10 and 11, pages 298 and 299). When the sun enters a solstitial point which is on the same side of the equator with any given place, his meridional altitude will be the *greatest* possible at that place; and when he enters the opposite solstitial point, his meridional altitude will be the *least* at the same place. From these premises the obliquity of the ecliptic may be readily determined by the following

RULE.

Let the sun's meridional altitude be carefully observed on the 21st of June, and again on the 22nd of December, or on the days on which that great luminary will be the *nearest to the solstices at noon*.—Reduce the observed meridional altitudes to the true central altitudes by Problem XXIII., page 374.—Now, half the difference of the true meridional altitudes, thus found, will be the obliquity of the ecliptic.

Example.

June 21st, 1834, in latitude $50^\circ 47' 27''$ north, the meridional altitude of the sun, duly *corrected*, was $62^\circ 40' 9''$; and on the 22nd December following, it was $15^\circ 44' 54''$, required the obliquity of the ecliptic?

Sun's <i>greatest</i> meridional altitude at given place =	$62^\circ 40' 9''$
Sun's <i>least</i> ditto at ditto =	$15. 44. 54.$

Difference =	$46^\circ 55' 15''$
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Half the difference =	$23^\circ 27' 37\frac{1}{2}''$
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which is the obliquity of the ecliptic, as required.

Note.—This differs *one or two seconds from the truth*; because the sun was not exactly in the first points of Cancer and Capricorn at the moments of observing his meridional altitudes.

PROBLEM XXIX.

Given the Latitude of a Place, and the Meridional Altitude of the Sun; to find his Declination.

RULE.

Reduce the observed meridional altitude of the sun to the true central altitude; subtract this from 90° , and the remainder will be its correct meridional zenith distance; which is to be called *North*, when the object is *South* of the observer; and *South*, when it is *North* of him. Now, when the zenith distance and the latitude are of the *same name*, their *difference*, but if of *contrary names*, their *sum*, will be the sun's declination; which will be of the same denomination as the latitude when the zenith distance is the *least* of the two terms; otherwise, it will be of a different denomination.

Example.

July 1st, 1834, in latitude $50^\circ 47' 27''$ north, the sun's correct meridional altitude was $62^\circ 21' 29''$; and, on the 1st October, it was $36^\circ 6' 21''$; required the sun's declination at the noons of those days.

Sun's Mer. altitude, July 1st = . . . $62^\circ 21' 29''$ S.	Sun's Mer. altitude, Oct. 1 = . . . $36^\circ 6' 21''$ S.
<hr/>	<hr/>
Sun's Mer. zenith distance = . . . $27^\circ 38' 31''$ N.	Sun's Mer. zenith distance = . . . $53^\circ 53' 39''$ N.
Given latitude = . $50. 47. 27.$ N.	Given latitude = . $50. 47. 27.$ N.
<hr/>	<hr/>
Sun's declination = $23^\circ 8' 56''$; which is <i>North</i> , of the same name as the latitude; because the zenith distance is the <i>least</i> of the two terms.	Sun's declination = $3^\circ 6' 12''$; which is <i>South</i> , of a contrary name to the latitude, because the zenith distance is the greatest of the two terms.

Note.—In the same manner may the declination of the moon, a fixed star, or a planet, be determined.

PROBLEM XXX.

Given the Mean Time and the computed Central Distance between the Moon and Sun, a Fixed Star, or a Planet ; to find the Mean Time at Greenwich, and the Longitude of the Place of Observation.

RULE.

If the computed central distance can be found in the Nautical Almanac, on the same horizontal line with the given day, the required mean time at Greenwich will be found standing over it at the top of the page ; but if it cannot be exactly found, which in general will be the case, take out *the nearest distance (whether greater or less)* which *immediately precedes* the computed central distance, according to the *numerical order of the hours* at top of the page ; and also the proportional logarithm standing opposite thereto on *the right-hand*. — Find the difference between the said *nearest preceding distance* and the computed one ; — then, from the proportional logarithm of this difference, subtract the proportional logarithm taken from the Ephemeris ; the remainder will be the proportional logarithm of a *portion of time* ; which being added to the hour corresponding to the said *nearest preceding distance*, the result will be the *approximate* mean time at Greenwich.

Note.—In taking out the proportional logarithm from the Ephemeris, take also the *difference* between the proportional logarithms standing opposite to the two distances which immediately *precede* and follow the computed central distance ; noting whether it be *increasing* or *decreasing*. With this *difference* and the *portion of time* enter Table A, page 610,* and take out the corresponding equation, which being applied to the *approximate* time at Greenwich by addition or subtraction, as directed at the foot of that Table ; the sum, or difference, will be the correct mean time at Greenwich.—Now, the difference between the correct mean time at Greenwich, thus found, and the mean time at ship, or place of observation, being turned into degrees, by Problem I., page 341, or by Table I. ; the result will be the longitude of the place of observation ; which will be *east*, if the mean time at ship or place be *greater*, but *west* if it be *less* than the mean time at Greenwich.

Example 1.

At sea, January 8th, 1836, in longitude, by account $54^{\circ}38'$ east, at $23^{\circ}8'22''$ mean time, *let* the true central distance between the moon and sun be $119^{\circ}13'14''$; required the corresponding mean time at Greenwich, and the longitude of the place of observation ?

Central distance at ship = $119^{\circ}13'14''$

Near.*preced.* dist., at 18^h = 119. 58. 7 P. L. 3021 Diff. 12 *decreasing*.*

Difference = $0^{\circ}44'53''$ P. L. 6032

Portion of time = $1^{\text{h}}29^{\text{m}}59^{\text{s}}$ P. L. 3011

Time at near.*preced.* dist. = 18. 0. 0

Approx. time at Greenwich = $19^{\text{h}}29^{\text{m}}59^{\text{s}}$

Equation in Table A = . . . + 4 ans. to $1^{\text{h}}30^{\text{m}}$ and diff. 12.

Correct mean time at Greenw. $19^{\text{h}}30^{\text{m}} 3^{\text{s}}$

Mean time at ship 23. 8. 22

Longitude, in time = . . . $3^{\text{h}}38^{\text{m}}19^{\text{s}}$ = $54^{\circ}34'45''$ east.

Example 2.

At sea, September 21st, 1836, in longitude by account $49^{\circ}30'$ west, at $10^{\text{h}}12^{\text{m}}16^{\text{s}}$ mean time, let the true central distance between the moon and α Pegasi be $38^{\circ}43'4''$; required the corresponding mean time at Greenwich, and the longitude of the place of observation?

Central distance at ship = . $38^{\circ}43' 4''$

Near. *preced.* dist. at 12 hours = 39. 24. 25 P. L. 3378 diff. 97 *increasing*.*

Difference = $0^{\circ}41'21''$ P. L. 6388

Portion of time = $1^{\text{h}}30^{\text{m}} 0^{\text{s}}$ P. L. 3010

Time at near. *preced.* dist. = 12. 0. 0

Approx. time at Greenwich = $13^{\text{h}}30^{\text{m}} 0^{\text{s}}$

Equation in Table A = . . . - 30 ans. to $1^{\text{h}}30^{\text{m}}$ and diff. 97.

Correct mean time at Greenw. = $13^{\text{h}}29^{\text{m}}30^{\text{s}}$

Mean time at ship = 10. 12. 16

Longitude, in time = . . . $3^{\text{h}}17^{\text{m}}14^{\text{s}}$ = $49^{\circ}18'30''$ west.

Note.—The last example shows the necessity of correcting the *approximate* Greenwich time deduced from the computed central distance :—the overlooking of that correction would, in the present instance, produce an error of $7\frac{1}{2}$ miles in the longitude; and cases occur in which the error, arising from the same cause, might affect the

* See explanation of Table A, page 336.

longitude to the value of 14 or 15 miles.—See Explanatory Article 80, between pages 335 and 337.

The reader will do well to peruse, with great attention, the various subjects which are contained between pages 296 and 375 : for in them he will find *the master-key* which unlocks and places in a familiar point of view all the articles in the *new Nautical Almanac* that are of any importance to *the naval world* ; excepting that which relates to the “Occultations” between pages 452 and 465 of the Ephemeris for 1836 :—This, however, shall be discussed in the most ample manner in a subsequent part of the work ; in which all *the hidden mysteries* attending the *grand method of deducing the longitude from the occultations of the fixed stars by the moon* shall be unveiled, rendered clearly perceptible, and so perfectly simplified as to be reduced to the comprehension of the meanest capacity.

Note.—As all the rules to the Introductory Problems are *universal*, and not subject to any restrictions or peculiarities, there will be no necessity whatever for “repeating the operation,” or “going over the work again :” provided, *always*, that the operation be performed correctly in the first instance.

We will now proceed to the solution of

PROBLEMS RELATIVE TO THE LATITUDE.

The *Latitude* of any place on the earth is expressed by the distance of such place from the equator, either north or south, and is measured by an arc of the meridian intercepted between the said place and the equator.—Or,

The *Latitude* of any place on the earth is equal to the elevation of the pole of the equator above the horizon of such place ; or (which amounts to the same), it is equal to the distance of the zenith of the place from the equinoctial in the heavens. The *complement* of the latitude is the distance of the zenith of any place from the pole of the equator, and is expressed by what the latitude wants of 90 degrees. The latitude is named north or south, according as the place is situate with respect to the equator.

PROBLEM I.

Given the Sun's Meridian Altitude, to find the Latitude of the Place of Observation.

RULE.

Find the true altitude of the sun's centre, by Problem XXIII., page 374, and call it north or south, according as that object may be situate with respect to the observer at the time of observation; which subtracted from 90° , will give the sun's meridional zenith distance of a contrary denomination to that of its altitude.

Reduce the sun's declination to the meridian of the place of observation, by the rule in page 359, or by Table XV. Then, if the meridional zenith distance and the declination are both north or both south, their sum will be the latitude of the place of observation; but if one be north and the other south, their difference will be the latitude, and always of the same name with the greater term.*

Example 1.

April 10th, 1836, in longitude 75° W., the meridian altitude of the sun's lower limb was $57^\circ 40' 30''$ S., and the height of the eye above the level of the sea 22 feet; required the latitude?

Observed altitude of the sun's lower limb = $57^\circ 40' 30''$ S.

Sun's semidiameter = $15' 58''$
Dip of the horiz. for 22 feet = 4.30 } Diff. = $+ 11.28$

Apparent altitude of the sun's centre = $57^\circ 51' 58''$ S.

Refraction = $0' 35''$
Parallax = 0.5 } Difference = $- 0.30$

True altitude of the sun's centre = $57^\circ 51' 28''$ S.

Sun's meridional zenith distance = $32^\circ 8' 32''$ N. $32^\circ 8' 32''$ N.

Sun's declination at noon, April 10th = $8^\circ 3' 56''$ N.

Correction for longitude 75° W. = $+4.36$

Sun's reduced declination = $8^\circ 8' 32''$ N. $8^\circ 8' 32''$ N.

Latitude, as required = $40^\circ 17' 4''$ N.

* The principles upon which this rule is founded may be seen by referring to "The Young Navigator's Guide to the Sidereal and Planetary Parts of Nautical Astronomy," between pages 98 and 105; reading the word *sun*, instead of *star*.

Note. The meridional zenith distance and the declination are added together, because they are both of the same name: hence, the latitude is $40^{\circ}17'4''$ N

Example 2.

October 24th, 1836, in longitude 90° east, the meridian altitude of the sun's lower limb was $27^{\circ}31'20''$ S., and the height of the eye above the surface of the sea 23 feet; required the latitude?

Observed alt. of the sun's lower limb = $27^{\circ}31'20''$ S.

Sun's semidiameter = $.16'7''$
Dip of the hor. for 23 ft. = 4.36 } Diff. = $+11.31$

Apparent altitude of the sun's centre = $27^{\circ}42'51''$ S.

Refraction = $1'48''$
Parallax = 0.8 } Difference = $. - 1.40$

True altitude of the sun's centre = $27^{\circ}41'11''$ S.

Sun's meridional zenith distance = $62^{\circ}18'48''$ N. — $62^{\circ}18'48''$ N,

Sun's decl. at noon, Oct. 23rd = $11^{\circ}31'48''$ S.

Correction for long. 90° east = $+5.14$

Sun's reduced declination = $11^{\circ}37'2''$ S. $11.87, 2$ S.

Latitude, as required = $50^{\circ}41'46''$ N.

Note.—The difference between the meridional zenith distance and the declination is taken, because they are of contrary names:—Hence, the latitude is $50^{\circ}41'46''$ north.

PROBLEM II.

Given the Moon's Meridional Altitude, to find the Latitude of the Place of Observation.

RULE.

Reduce the moon's passage over the meridian of Greenwich, on the given day, to the meridian of the place of observation, by applying thereto the correction in Table XXXVIII., by addition or subtraction, according as the longitude is west or east; as explained in Examples 1 and 2, pages 101 and 102.

To the time of the moon's passage over the meridian of the place of observation, thus found, let the longitude of that meridian, in time, be added if it be west, or subtracted if east; and the sum, or difference, will be the corresponding time at Greenwich: to which let the moon's declination be reduced by Problem XVI., page 364; and her horizontal parallax, and semidiameter, by Problem XV., page 361 (or by means of Table XVI., as explained in page 27), and let the moon's reduced semidiameter be corrected by the augmentation contained in Table IV.

Find the true altitude of the moon's centre, by Problem XXIV., page 376, and call it north or south, according as it may be situate with respect to the observer at the time of observation; which, subtracted from 90° , will give the moon's meridional zenith distance of a contrary denomination to that of its altitude.

Then, if the meridional zenith distance and the declination are of the same name, their sum will be the latitude of the place of observation; but if they are of contrary names, their difference will be the latitude, of the same name with the greater term.

Example 1.

January 27th, 1836, in longitude 55° west, the meridian altitude of the moon's lower limb was $58^\circ 40' 20''$ S., and the height of the eye above the level of the sea 26 feet; required the latitude?

Time of D 's passage over the meridian of Greenwich = . $7^h 30^m 0^s$
Cor., Table XXXVIII., for long. 55° W., and retard. 47^m . $+ 6.57$

Time of D 's passage over the given meridian = . . . $7^h 36^m 57^s$
Longitude of the given place 55° west, in time = . . . $+ 3.40. 0$

Greenwich mean time = $11^h 16^m 57^s$

D 's semidiameter at noon,	D 's hor. parallax at noon,
Jan. 27th = $14' 46''$	Jan. 27th = $54' 10''$
Cor. of ditto for $11^h 16^m 57^s$ — 1	Cor. of ditto for $11^h 16^m 57^s$ — 3
<hr/>	<hr/>
D 's reduced semidiameter $14' 45''$	D 's reduced hor. parallax $54' 7''$
Augmentation, Table IV. $+ 12$	
<hr/>	
D 's true semidiameter = . $14' 57''$	

D 's declination at 11 hours, Jan. 27th $21^\circ 12' 15''$ N.
Correction for $16^m 57^s$ = $+ 2.20.$

D 's reduced declination = $21^\circ 14' 35''$ N.

Observed altitude of D 's lower limb, = $58^{\circ}40'20''$ South.

D 's true semidiameter = $14'57''$
 Dip of horizon for 26 feet = 4.52 } Difference = . . + 10. 5

Apparent altitude of the moon's centre = $58^{\circ}50'25''$ South.

Correction of ditto, Table XVIII. = + 27.26

True altitude of the moon's centre = $59^{\circ}17'51''$ South.

Moon's meridional zenith distance = $30^{\circ}42' 9''$ North.

Moon's reduced declination = $21.14.35$ North.

Latitude of the place of observation = $51^{\circ}56'44''$ North.

Example 2.

February 3rd, 1836, in longitude 65° east, the meridional altitude of the moon's upper limb was $62^{\circ}45'10''$ north, and the height of the eye above the level of the sea 29 feet; required the latitude?

Time of D 's passage over the meridian of Greenwich = . $13^{\text{h}}19^{\text{m}}54^{\text{s}}$

Cor., Table XXXVIII., for long. 65° east, and retard. 47^{m} — 8.13

Time of D 's passage over the given meridian = . . . $13^{\text{h}}11^{\text{m}}41^{\text{s}}$

Longitude of the place of observation 65° east, in time = $-4.20.00$

Greenwich mean time = $8^{\text{h}}51^{\text{m}}41^{\text{s}}$

D 's semidiameter at noon,

Feb. 3rd = $15'15''$

Correction for $8^{\text{h}}51^{\text{m}}41^{\text{s}}$ = + 3

Augmentation, Table IV. + 14

D 's true semidiameter = $15'32''$

D 's hor. parallax at noon,

Feb. 3rd = $55'58''$

Correction for $8^{\text{h}}51^{\text{m}}41^{\text{s}}$ = + 11

D 's reduced hor. parallax $56' 9''$

D 's declination at 8 hours, Feb. 3rd = $17^{\circ}25'19''$ N.

Correction for $51^{\text{m}}41^{\text{s}}$ = — 9.41

D 's reduced declination = $17^{\circ}15'38''$ N.

Observed altitude of the moon's upper limb = . . 62°45'10" North.

$\left. \begin{array}{l} \text{D's true semidiameter} \quad . \quad . \quad 15'32'' \\ \text{Dip of the horizon for 29 feet} \quad 5.10 \end{array} \right\} \text{Sum} = . \quad . \quad - \quad 20.42$

Apparent altitude of the moon's centre = . . . 62°24'28" North.

Correction of ditto, Table XVIII. = + 25.31

True altitude of the moon's centre = 62°49'59" North.

Moon's meridional zenith distance = 27°10' 1" South.

Moon's reduced declination = 17.15.38 North.

Latitude of the place of observation = 9°54'23" South.

Note.—Since the moon's declination is *now* given in the Nautical Almanac for every *hour*; and since this important element may be readily corrected for the *excess* of the minutes and seconds beyond the *hour* in the Greenwich time, by means of Problem XVI., page 364; therefore, the above method of finding the latitude is not much more troublesome than that for the sun, while it is *equally as correct*.—And thus it is manifest that the latitude of a ship may be deduced from the meridional altitude of the moon, “with nearly as little labour as is required in the case of the sun;” as most justly remarked in page 505 of the Ephemeris for 1836.

PROBLEM III.

Given the Meridional Altitude of a Planet; to find the Latitude of the place of Observation.

Reduce the mean time of observation to the meridian of Greenwich, by Problem III., page 342:—To the Greenwich time, thus found, let the planet's declination be reduced by Problem XVII., page 366. Find the true altitude of the planet's centre, by Problem XXV., page 377; and hence its meridional zenith distance, noting whether it be north or south: then, if the meridional zenith distance and the declination are of the same name, their sum will be the latitude of the place of observation; but if they are of contrary names, their difference will be the latitude, of the same name with the greater term,

Example.

December 30th, 1836, in longitude 80° west, at $15^{\text{h}}20^{\text{m}}5^{\text{s}}$ mean time, the meridional altitude of the planet Mars was $43^{\circ}46'20''$ north, and the height of the eye above the level of the sea 24 feet; required the latitude?

Mean time of obser. = $15^{\text{h}}20^{\text{m}}5^{\text{s}}$	Mars' declination at
Longitude 80° west, in	noon, Dec. 30th. $15^{\circ}53'43''\text{N.}$
time = + 5. 20. 0	Cor. for $20^{\text{h}}40^{\text{m}}5^{\text{s}}$ = + 2. 38
<hr/>	
Greenwich time = . . $20^{\text{h}}40^{\text{m}}5^{\text{s}}$	Mars' red. decl. = $15^{\circ}56'21''\text{N.}$

Observed central altitude of the planet Mars = . . $43^{\circ}46'20''$ North.
Dip of the horizon for 24 feet = — 4. 42

Apparent central altitude of Mars = $43^{\circ}41'38''$ North.
Refraction, Table VIII. = $1'0''$
Parallax, Table VI. = . 0. 8 } Difference = . — $0'52''$

True central altitude of the planet = $43^{\circ}40'46''$ North.

Meridional zenith distance of ditto = $46^{\circ}19'14''$ South.
Reduced declination of ditto = 15. 56. 21 North.

Latitude of the place of observation = $30^{\circ}22'53''$ South.

Note.—The *difference* between the meridional zenith distance and the declination is taken because they are of contrary names: had they been of the *same name*, their *sum* would be the latitude of the place of observation.

PROBLEM IV.

Given the Meridional Altitude of a fixed Star, to find the Latitude of the Place of Observation.

RULE.

Find the true altitude of the star, by Problem XXVI., page 378, and hence its meridional zenith distance, noting whether it be north or south. Take the declination of the star from Table XLIV., and reduce it to the time of observation, or, more correctly, from the

Nautical Almanac. Now, if the star's meridional zenith distance and its declination be of the same name, their sum will be the latitude of the place of observation; but if they are of contrary names, their difference will be the latitude, of the same name with the greater term.

Example 1.

January 1st, 1836, in longitude $85^{\circ}3'$ W., at $12^{\text{h}}49^{\text{m}}4'$ mean time, the meridian altitude of Procyon was $44^{\circ}49'$ S., and the height of the eye above the level of the horizon 16 feet; required the true latitude?

Observed altitude of Procyon = . . . $44^{\circ}49' 0''$ S.

Dip of the horizon for 16 feet = . . . — 3.50

Procyon's apparent altitude = . . . $44^{\circ}45' 10''$ S.

Refraction = . . . — 0.57

Procyon's true altitude = . . . $44^{\circ}44' 13''$ S.

Procyon's meridional zenith distance = . $45^{\circ}15' 47''$ N.

Procyon's declination per N. A. = . . . $5.38.28$ N.

Latitude of the place of observation = . $50^{\circ}54' 15''$ N.

Note.—The principles of finding the latitude by the meridional altitude of a celestial object may be seen by referring to “The Young Navigator's Guide to the Sidereal and Planetary Parts of Nautical Astronomy,” between pages 98 and 105.

PROBLEM V.

Given the Meridional Altitude of a Celestial Object observed below the Pole, to find the Latitude of the Place of Observation.

RULE.

Find the true altitude of the object, as before; to which let its polar distance, or the complement of its corrected declination, be added, and the sum will be the latitude of the place of observation, of the same name with the declination.

Example 1.

June 20th, 1836, in longitude 65° W., the meridian altitude of the

sun's lower limb, observed below the pole, was $9^{\circ}12'$, and the height of the eye 20 feet; required the latitude?

Observed altitude of the sun's lower limb = . . $9^{\circ}12' 0''$

Sun's semidiameter = . . $15'45''$
Dip of the horizon for 20 feet = 4.17 } Diff. =. + 11.28

Apparent altitude of the sun's centre = . . . $9^{\circ}23'28''$

Refraction = $5'34''$
Parallax = . 0.9 } Difference = - 5.25

True meridian altitude below the pole = . . . $9^{\circ}18' 3''$

Sun's corrected polar distance, or co-declination = $66.32.20$ N.

• Latitude of the place of observation = . . . $75^{\circ}50'23''$ N.

Example 2.

June 1st, 1836, in longitude 90° E., at $12^{\text{h}}21^{\text{m}}36^{\text{s}}$ mean time, the observed altitude of Capella, when on the meridian below the pole, was $11^{\circ}48'$, and the height of the eye above the level of the sea 25 feet; required the latitude?

Observed altitude of Capella = $11^{\circ}48' 0''$

Dip of the horizon for 25 feet = - 4.47

Capella's apparent altitude = $11^{\circ}43'13''$

Refraction = - 4.29

Capella's true meridian altitude below the pole = $11^{\circ}38'44''$

Capella's polar distance, per Naut. Almanac = $44.10.28$ N.

Latitude of the place of observation = . . . $55^{\circ}49'12''$

Remarks.—1. When the polar distance or co-declination of a celestial object is less than the latitude of the place of observation (both being of the same name), such celestial object will not set, or go below the horizon of that place: in this case, the celestial object is said to be circumpolar, because it revolves round the pole of the equator, or equinoctial, without disappearing in the horizon.

2. If 12 hours, diminished by half the daily increase of the *mean*

sun's right ascension (viz., 1^h58^m28^s), be added to the mean time of the superior transit of a *fixed star*, it will give the mean time of its inferior transit over the opposite meridian; that is, the mean time of its coming to the meridian below the pole.

3. The least altitude of a circumpolar celestial object indicates its being on the meridian below the pole.

PROBLEM VI.

Given the Altitude of the North Polar Star, taken at any Hour of the Night, to find the Latitude of the Place of Observation.

Although the proposed method of finding the latitude at sea is only applicable to places situate to the northward of the equator, yet, since it can be resorted to at any time of a fine clear night, it deserves the particular attention of the mariner.

Of all the heavenly bodies, the polar star seems best calculated for finding the latitude in the northern hemisphere by nocturnal observation; because a single altitude, taken at any hour of the night by a careful observer, will give the latitude to a sufficient degree of accuracy, provided the mean time of observation be but known within a *few* minutes of the truth: however, an error in the mean time, even as considerable as 20 minutes, will not affect the latitude to the value of half a minute, when the polar star is on the meridian, either above or below the pole; nor will it ever affect the latitude more than about 8 or 9 minutes, even at the star's greatest distance from the meridian. But, as it is highly improbable, in the present improved state of watches, that the mean time at a ship can ever be so far out as five minutes, the latitude resulting from this method will, in general, be as near to the truth as the common purposes of navigation require.

RULE.

To the *mean* sun's right ascension, as given in the Nautical Almanac, or in Table XII. (reduced to the meridian of the place of observation, by Problem V., page 344), add the mean time of observation; and the sum (rejecting 24 hours, if necessary), will be the right ascension of the meridian, or mid-heaven;* with which enter Table X., and take out the corresponding correction. Find the true altitude of the star, by Problem XXVI., page 378; to which let the correction, so found,

* See Article 32, page 309.

be applied by addition or subtraction, according to the directions contained in the Table, and the sum or difference will be the approximate latitude.

Enter Table XI., with the approximate latitude, thus found, at top of the page, and the right ascension of the meridian in one of the side columns; in the angle of meeting will be found a correction, which, being applied by *addition* to the approximate latitude, will give the true latitude of the place of observation.

Remark.—Since the corrections of the polar star's altitude, in Table X., have been computed for the beginning of the year 1836, a reduction therefore becomes necessary, in order to adapt them to subsequent years and parts of a year. The method of finding this reduction is illustrated in the example, pages 17 and 18.

Example 1.

January 2nd, 1836, in longitude 60° west, the mean of several observed altitudes of the north polar star was $52^{\circ}15'5''$, and that of the corresponding times, per watch, $8^h6^m23^s$ mean time; the height of the eye above the level of the horizon was 16 feet; required the latitude?

Mean time of observ. = $8^h6^m23^s$	Mean sun's R. A. at
Long. 60° W., in time = $+4.0.0$	noon = $. . . . 18^h44^m40^s$
Greenwich time = $. . 12^h6^m23^s$	Cor. for $12^h6^m23^s$ = $+ 1.59$
	Mean sun's red. R. A. = $18^h46^m39^s$
Mean time of observation, per watch = $. 8.6.23$	
Right ascension of the meridian, or mid-heaven = $. . . 2^h53^m2^s$	
Observed altitude of the polar star = $. . . . 52^{\circ}15'5''$	
Dip of the horizon for 16 feet = $. . . . - 3.50$	
Apparent altitude of the polar star = $. . . . 52^{\circ}11'15''$	
Refraction of altitude = $. . . . - 0.44$	
True altitude of the polar star = $. . . . 52^{\circ}10'31''$	
Correction from Table X., answering to $2^h53^m2^s$ = $-1.22.58$	
Approximate latitude = $. . . . 50^{\circ}47'33''$ North.	
Correction of ditto, Table XI. = $. . . . + 0.23$	
Latitude of the place of observation = $. . . 50^{\circ}47'56''$ North.	

Note.—The true latitude, computed by spherical trigonometry, is $50^{\circ}48'14''$; the difference between which and that deduced as above being only 19 seconds in a period of 28 years:—hence, it is evident that the latitude may be always determined by means of Tables X. and XI., to every desirable degree of exactness for the ordinary purposes of navigation.

The elementary principles of the above method of finding the latitude are given in “the Young Navigator’s Guide to the Sidereal and Planetary Parts of Nautical Astronomy,” between pages 144 and 156, where a diagram may be seen illustrative of the star’s *apparent motion* round its orbit.

Remark.—The polar star has long been considered such a favourable object for determining the latitude of a ship in the northern hemisphere, that mostly all *the professors of mathematics* who have touched upon nautical astronomy during the last century, have given “Tables for determining the Latitude by Observations of the Polar Star out of the meridian.”—But, as “the learned professors” have been, in general, *mere theoretical writers*, they seem, therefore, to have been unconscious of how *extremely difficult* it is to observe the altitude of the polar star by means of a common Hadley’s quadrant:—for, since *Polaris* only ranks between the second and third magnitudes, its altitude cannot always be taken to the necessary degree of exactness on board of a ship at sea; particularly in *high latitudes*. The splendour of the polar star, like that of all other celestial objects, is materially lessened by *reflection* in the moveable and fixed reflectors of the instrument:—its brilliancy first suffers a diminution in the index-glass, and a still further diminution when it is reflected from that into the horizon-glass:—hence, as the star is *scarcely of the second magnitude*, its image grows *less and less* as it is brought down, by the motion of the index, from the heavens to the grosser parts of the atmosphere; and thus before the reflected image reaches the horizon, it becomes so faint and indistinct as to be often imperceptible to the sight of an observer in places to the *northward of the parallel of London*. From this it is manifest that although the method in question is *strictly correct in theory*; yet, it is not always practicable in *high northern latitudes*:—for, as before noticed, the altitude of the star cannot be taken to the necessary degree of exactness, unless the observer is furnished with a quadrant far better adapted to the purpose, in *reflectors* and *distinct object-glasses*, than is to be found amongst the common instruments used at sea. However, in *low latitudes*, or betwixt the equator and the mean parallel, the above method will always answer the navigator’s expectations; because in those latitudes the altitude of the star can be readily

taken at any time of a clear night, even by a common quadrant, provided the observer abides by the directions given in Article 63, page 327.

PROBLEM VII.

Given the Latitude by Account, the Sun's Declination, and two observed Altitudes of its lower or upper Limb, the elapsed Time, and the Course and Distance run between the Observations ; to find the Latitude of the Ship at the Time of Observing the greatest Altitude.

RULE.

To reduce the least Altitude to what it would be, if taken at the Place where the greatest Altitude was observed :—

Find the angle contained between the ship's course (corrected for leeway, if any), and the sun's bearing at the time of taking the least altitude ; with which, if less than 8, or with what it wants of 16 points if it be more than 8, enter the general Traverse Table, and find the difference of latitude corresponding thereto and the distance made good between the observations, which call the *reduction of altitude*.

Now, if the least altitude be observed in the forenoon, the reduction of altitude is to be applied thereto by *addition* when the above angle is less than 8 points, but by *subtraction* when it is more than 8 points ; the sum, or difference, will show what the less altitude would be if observed at the same place with the greater altitude. Again, if the less altitude be observed in the afternoon, a contrary process is to be observed ; viz., the reduction of altitude is to be *subtracted* therefrom, when the above angle is less than 8 points, but to be *added* thereto when it is greater.

To compute the Latitude :—

Reduce the sun's declination by Problem XIV., page 357, to the time and place where the greatest altitude was observed ; then, to the log. secant of the latitude by account, add the log. secant of the corrected declination ; the sum, rejecting 20 from the index, will be the *logarithmic ratio*.

To the log. ratio, thus found, add the logarithm of the difference of the natural co-verses of the two corrected altitudes, and the logarithm of the half-elapsed time (Table XXX.) ; the sum of these three logarithms will be the logarithmic middle time. Find the time corresponding to this in Table XXXI. ; the difference between which

and the half-elapsed time will be the time from noon when the greatest altitude was observed.* From the log. rising (Table XXXII.), answering to this time, subtract the log. ratio; and the remainder will be the logarithm of a natural number, which, being subtracted from the natural co-versed sine of the greatest altitude, will leave the natural versed sine of the sun's meridional zenith distance; to which let the corrected declination be applied by addition or subtraction, according as it is of the same or of a contrary name: and the sum, or difference, will be the latitude of the ship at the time that the greatest altitude was taken; which may be reduced to noon, by means of the log, if necessary.

If the latitude, thus found, differ considerably from that by account, the *operation must be repeated*, using the computed latitude in place of that by account, until the latitude last found agrees nearly with the latitude used in the computation.

Remarks.—1. Since this method is only an approximation to the truth, it requires to be used under certain restrictions; viz., the observations must be taken between nine o'clock in the forenoon, and three in the afternoon. If both observations be in the forenoon, or both in the afternoon, the elapsed time must not be less than the distance of the observation of the greatest altitude from noon. If one observation be in the forenoon, and the other in the afternoon, the elapsed time must not exceed four hours and a half; and, in all cases, the nearer the greater altitude is to noon, the better.

2. If the sun's meridional zenith distance be less than the latitude, the limitations are still more contracted. If the latitude be double the meridian zenith distance, the observations must be taken between half-past nine in the forenoon and half-past two in the afternoon; and the elapsed time must not exceed three hours and a half. The observations must be taken still nearer to noon, if the latitude exceeds the meridian zenith distance in a greater proportion.

Example 1.

At sea, January 9th, 1836, in latitude $50^{\circ}42'$ N., by account, and longitude $30^{\circ}10'$ W., at $21^{\text{h}}37^{\text{m}}35^{\text{s}}$ mean time, the observed altitude of the sun's lower limb was $10^{\circ}35'30''$ S, and the bearing of its centre, by azimuth compass, S. E. $\frac{1}{4}$ S.; and at $23^{\text{h}}17^{\text{m}}45^{\text{s}}$ the observed alti-

* When the middle time is greater than the half-elapsed time, both observations will be on the same side of the meridian; otherwise, on different sides.

tude was $17^{\circ}0'50''$ S : the height of the eye above the level of the sea was 20 feet ; and the ship's course during the elapsed time S.S.E., at the rate of 10 knots an hour ; required the latitude of the ship at the time of observing the greater altitude?

Solution.—The sun's bearing at time of first, or the *least* observation, was S. E. $\frac{3}{4}$ S., or $3\frac{1}{4}$ points, and the ship's course S.S.E., or 2 points : hence, the *contained angle* is $1\frac{1}{4}$ point.

The elapsed time between the observations is $1^h40^m10^s$; and the rate of sailing 10 knots, or *miles*, an hour:—Then, as 1 hour : 10 miles :: $1^h40^m10^s$ to $16'42''$, or 17 miles *nearly* ; which is the distance run between the observations.

Now, to course $1\frac{1}{4}$ point and distance 17 miles in Table XLII., the difference of latitude is 16.5 miles ; which is the *reduction* of altitude, and which is *additive* to the *least altitude*, because the *contained angle* is *less than* 8 points or 90° , and the observation made in the forenoon.

Time of observ. greatest altitude = . . . $23^h17^m45^s$	☉ declination at noon, January 10th . . . $22^{\circ}3'52''$ S.
Longitude $30^{\circ}10'$ W., in <i>time</i> = . . . +2. 0.40	Correc. for $1^h18^m25^s$: — 0.29
Greenwich time, past noon, January 10th = $1^h18^m25^s$	☉ reduced declination $22^{\circ}3'23''$ S.

<i>First</i> observed altitude of the sun's lower limb = . . . $10^{\circ}35'30''$	
Sun's semidiameter = . . . $16'17''$	} Difference = . . + 12. 0
Dip of horizon for 20 feet = $4'17''$	

Sun's apparent altitude = . . . $10^{\circ}47'30''$	
Refraction = . . . $4'52''$	} Difference = . . — 4. 43
Parallax = . . . 9.	
Reduction of altitude = . . . + 16. 30	

Sun's reduced altitude = . . . $10^{\circ}59'17''$
which is the altitude that the sun's *centre* would have, if observed at the second *place* of observation.

<i>Second</i> observed altitude of the Sun's lower limb = . . $17^{\circ} 0'50''$	
Sun's semidiameter = . . . $16'17''$	} Difference = . . + 12. 00
Dip of horizon for 20 feet = . 4. 17.	
Sun's apparent altitude = . . . $17^{\circ}12'50''$	
Refraction = . . . $3'2''$	} Difference = . . 2. 54
Parallax = . . . 8.	
Sun's true central altitude = . . . $17^{\circ} 9'56''$	

Lat. by account, or *dead reckoning* $50^{\circ}42' 0''$ Log. sec. = 10.198335
 Sun's reduced declination . . . 22. 3.23 Log. sec. = 10.033007

Logarithmic ratio = 0.231342

First time $21^{\circ}37'35''$; alt. $10^{\circ}59'17''$ N.co-v.S. 809396

Second time 23. 17. 45; alt. 17. 9. 56 N.co-v.S. 704866

Elapsed time = $1^{\circ}40'10''$ — Difference = . . . 104530 L.5. 019241

Half E. time = $0^{\circ}50' 5''$; the Log. of which, Table XXX., is 0.663950

Middle time = 1.37. 0. — Log. mid. time, Table XXXI. = 5.914533

Time fr. noon = $0^{\circ}46'55''$, when the *greatest* altitude was taken; the
 Logarithmic rising of which, Table XXXII., is . . . 4.319740

Logarithmic ratio = 0.231342

Natural number = 12257 Logarithm 4.088398

Nat. co-versed sine of greatest alt. = 704866

Nat. versed sine of merid. zenith dist. = 692609 = $72^{\circ}5'53''$ North.

Sun's reduced declination = 22.3.23 South.

Latitude of the ship = $50^{\circ}2'30''$ North.

But, since this differs more than 39 miles from the latitude by account,
 it becomes essentially necessary to *repeat the operation*.

Computed latitude = . . . $50^{\circ}2'30''$ Log. secant = 10.192309

Sun's reduced declination = . 22.3.23 Log. secant = 10.033007

Logarithmic ratio, for second operation = 0.225316

Difference of nat. co-versed sines, as above, 104530, Log. 5.019241

Half elapsed time = $0^{\circ}50' 5''$ Logarithm, as before = . 0.663950

Middle time = . . 1.35.34 Log. middle time = . . 5.908507

Time from noon = $0^{\circ}45'29''$, when the greatest altitude was taken;

The logarithmic rising of which is 4.292880

Logarithmic ratio = 0.225316

Natural number = 11683 Logarithm 4.067564

Nat. co-versed sine of the greatest alt. = 704866

Nat. versed sine of merid. zenith dis. = 693183 = $72^{\circ}7'57''$ North.

Sun's reduced declination = 22.3.23 South.

Latitude of the ship = $50^{\circ}4'34''$ North.

And since this differs only about two miles, from the computed lati-

tude, as above ; it may, therefore, be esteemed as the true latitude of the ship.

Example 2.

At sea, April 14th, 1836, in latitude $43^{\circ}17'$ S., by account, and longitude $60^{\circ}25'$ E., at $23^{\text{h}}20^{\text{m}}52^{\text{s}}$ mean time, the observed altitude of the sun's lower limb was $35^{\circ}30'30''$ N. and at $2^{\text{h}}10^{\text{m}}22^{\text{s}}$ mean time, April 15th, the observed altitude of that limb was $28^{\circ}38'40''$ N., and the bearing of the sun's centre, by azimuth compass, N.W. $\frac{1}{4}$ N.; the height of the eye above the level of the horizon was 24 feet, and the ship's course during the elapsed time S.W., at the rate of 9 knots an hour ; required the latitude of the ship at the time of observation of the greater altitude ?

Solution.—The sun's bearing at the time of observing the least altitude was N.W. $\frac{1}{4}$ N., or $3\frac{1}{4}$ points, and the ship's course S.W.:—hence the *contained angle* is $8\frac{1}{4}$ points. The elapsed time between the observations is $2^{\text{h}}49^{\text{m}}30^{\text{s}}$, and the rate of sailing 9 knots, or *miles* an hour. Then, as 1 hour is to 9 miles, so are $2^{\text{h}}49^{\text{m}}30^{\text{s}}$ to $25^{\circ}26'$; which is the distance run between the observations. Now, to course $7\frac{1}{4}$ points (viz. $16 - 8\frac{1}{4} = 7\frac{1}{4}$), and distance 25 miles, the difference of latitude is 3.7 miles, or $3^{\circ}42''$, which is the reduction of altitude, and which is *additive* to the least *altitude*, because the contained angle is *more* than 8 points, and that altitude observed in the afternoon.

Time of observ. great-		☉'s declination at noon,
est altitude = . . .	$23^{\text{h}}20^{\text{m}}52^{\text{s}}$	April 14th = . . . $9^{\circ}31'20''$ N.
Longitude $60^{\circ}25'$ E.,		Cor. for $19^{\text{h}}19^{\text{m}}12^{\text{s}}$ = $+17.18$
in time = . . .	$-4.1.40$	
	<hr/>	☉'s reduced decl. = $9^{\circ}48'38''$ N.
Greenwich time, past		<hr/>
noon, April 14th =	$19^{\text{h}}19^{\text{m}}12^{\text{s}}$	
	<hr/>	

First observed altitude =	$35^{\circ}30'30''$
Sun's semidiameter = . . . $15'57''$	} Difference = . . . $+11.15$
Dip of hor. for 24 feet = . . . 4.42	
	<hr/>
Sun's apparent altitude =	$35^{\circ}41'45''$
Refraction = $1'19''$	} Difference = . . . -1.12
Parallax = 0.7	
	<hr/>
Sun's true central altitude =	$35^{\circ}40'33''$
	<hr/>

Second observed altitude = 28°38'40"
 Sun's semidiameter = . 15'57" }
 Dip of hor. for 24 feet = . 4.42 } Difference = . . +11.15

Sun's apparent altitude = 28°49'55"
 Refraction = 1'43" }
 Parallax = 0.8 } Difference = . . -1.35
 Reduction of altitude = +3.42

Sun's reduced altitude = 28°52' 2"
 which is the altitude that the sun's centre would have, if observed at
 the first place of observation at 2^h 10^m 22^s past noon, April 15th.

Lat. by account, or *dead reckoning* 43°17' 0" Log. sec. = 10.137885
 Sun's reduced declination = . . 9.48.38 Log. sec. = 10.006398

Logarithmic ratio = 0.144283

First time 23^h 20^m 52^s, alt. 35°40'33"N.co-v.S. 416801
 Second time 2. 10. 22, alt. 28.52. 2 N.co-v.S. 517218

Elapsed time = 2^h 49^m 30^s. — Difference = . . 100417 L. 5.001807

Half E. time = 1^h 24^m 45^s, the Log. of which, Table XXX., is 0.441990

Middle time = 0.44.40. — Log. mid. time, Table XXXI. = 5.588080

Time fr. noon = 0^h 40^m 5^s, when the greatest altitude was taken; the
 Logarithmic rising of which in Table XXXII. is . . . 4.183480
 Logarithmic ratio = 0.144283

Natural number = 10943 Logarithm 4.039147
 Nat. co-versed sine of greatest altitude = 416801

Nat. versed sine of merid. zenith dist. = 405858 = 53°32'55" South.
 Sun's reduced declination = 9.48.38 North.

Latitude of the ship = 43°44'17" South.

But, since this differs more than 27 miles from the latitude by account, it becomes indispensably necessary to *repeat the operation*.

Computed latitude 43°44'17" Log. secant = 10.141157
 Sun's reduced declination . . 9.48.38 Log. secant = 10.006398

Logarithmic ratio for second operation = 0.147555
 Dif. of natural co-versed sines, as above, 100417, Log. = 5.001807
 Half elapsed time 1°24'45", Log. of which, as above = . 0.441990

Middle time . 0.45. 1, Log. middle time = . . . 0.591352

Time from noon = 0°39'44", when the greatest altitude was taken; the
 Logarithmic rising of which is 4.175826
Logarithmic ratio for second operation = 0.147555

Natural number = 10673 Logarithm 4.028271
 Nat. co-versed sine of greatest altitude = 416801

Nat. versed sine of merid. zenith distance 406128 = 53°34' 4" South.
 Sun's reduced declination = 9.48.38 North.

Latitude of the ship = 43°45'26" South.
 And, since this only differs 1'9", or little more than one mile from the computed latitude, as above, it may, therefore, be esteemed as the true latitude of the ship.

Remark.—The method of finding the latitude by double altitudes, as above, being a very tedious and indirect operation, and generally a very inaccurate one, unless the limitations pointed out in the remarks (page 399) are strictly attended to, no notice, therefore, would have been taken of it in this work, were it not for the purpose of giving the most ample illustration of the general use of the Tables. And, notwithstanding what has been said in favour of double altitudes by *theoretical writers*, this method of finding the latitude at sea is evidently far from being one of the most advantageous in practical navigation: for the operation, besides being rather circuitous, requires a considerable portion of time to go through with it correctly; and, after all, it frequently happens, that although every seeming precaution has been taken, the mariner's hopes are disappointed in the

result. We will now proceed to a more direct and universal method of finding the latitude, either at sea or on shore.

PROBLEM VIII.

Given the Altitudes of two known fixed Stars observed at the same instant, at any time of the Night, to find the Latitude of the Place of Observation, independent of the Latitude by Account, the Longitude, or the Mean Time.

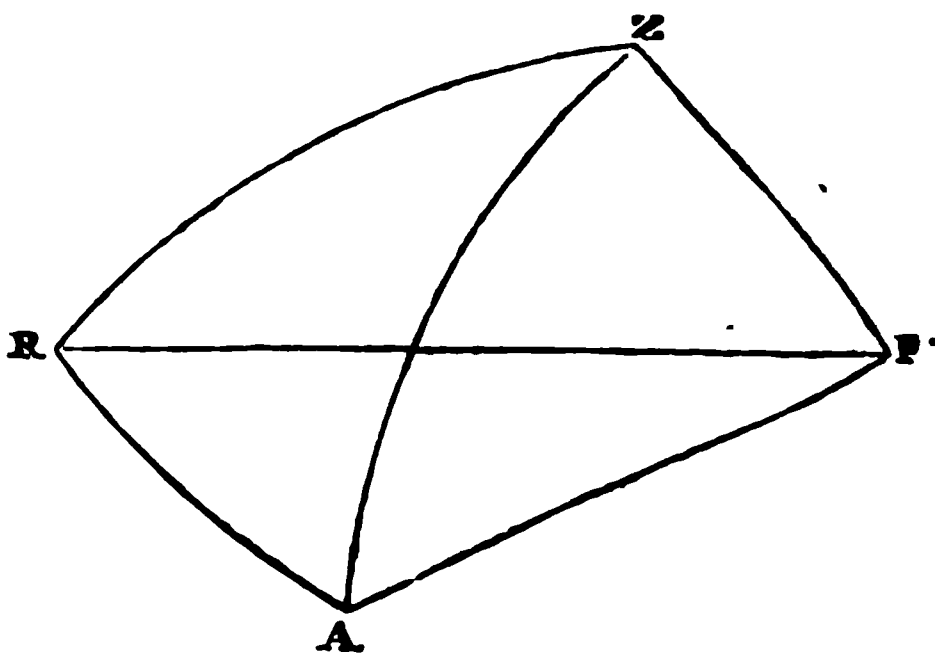
In the preceding problems for finding the latitude (the two last excepted), the meridional altitudes of the celestial objects were the principal elements under consideration; however, since it frequently happens that, in consequence of the interposition of clouds, or other causes, the altitudes of the heavenly bodies cannot always be taken at their respective times of transit, the present problem is, therefore, proposed, which possesses the peculiar advantage of enabling the mariner to determine the position of his ship, with respect to latitude, by the altitudes of two known fixed stars, observed at the same instant, and at any hour of the night, either before or after their passing the meridian, and independent of the latitude by account, the longitude, or the mean time of observation. Nor will the mariner, in this method, be subjected to the necessity of *repeating the operation*, or of puzzling himself with a variety of cases and corrections, in finding an approximate latitude.

In the proposed method of finding the latitude, the principal element which enters into the calculation, viz., *the true spherical distance between the two stars*, will be found in the eighth column of the page, in Table XLIV.; and although that distance is adapted to the *beginning* of the year 1824, yet, it may be very readily reduced to any subsequent, or *future* period: for, the product of the “annual variation in distance,” by the number of years and parts of a year that have elapsed since the 1st of January, 1824, being applied to the corresponding tabular distance, between the two given stars, by addition or subtraction, according to its sign; the result will be the correct spherical distance between the two stars at the given time. And for this particular purpose, viz., *the determination of the latitude*,

the tabular distance, so reduced, will prove to be sufficiently near the truth during all the *remaining years of the nineteenth century*.

As the method in question is most highly deserving of the young navigator's attention, I shall, therefore, give an illustration of the principles upon which it is founded:—as thus,

In the annexed diagram, let the points R and A represent the places of two fixed stars; P, the pole of the equator, and Z the zenith: hence the side PZ is the co-latitude of the place of observation.



Let PR be the polar distance of the star which is most *remote* from the pole; and PA that of the star which is nearest to it; then, the angle RPA expresses the difference of right ascension between the two stars; and the arc RA, their distance asunder in the firmament. The arc ZR is the zenith distance, or co-altitude of the star which is most *remote* from the pole P; and ZA that of the star which is *nearest* to it.

Now, in this diagram, it is evident that there are three oblique angled spherical triangles to work in, viz., PRA, ZRA, and ZRP; to find the side PZ, or the co-latitude of the ship or place.

First.—In the triangle PRA, the three sides and one angle are given; viz., the polar distance PR; the polar distance PA; the tabular distance between the stars RA, and the angle RPA = the difference of right ascension; to find the angle PRA.

Second.—In the triangle ZRA, the three sides are given, viz., the zenith distance, or co-altitude ZR; the co-altitude ZA; and the tabular distance between the stars RA; to find the angle ZRA:—The difference between which and the angle PRA, gives the angle ZRP; which is the angle opposite to the co-latitude of the place of observation.

Third.—In the triangle ZRP, two sides and the included angle are given, viz., the polar distance PR; the zenith distance ZR; and the contained angle ZRP; to find the third side, or co-latitude PZ:—

And thus we see the principles upon which the problem is based; the direct solution of which may be seen in the following

Example.

January 1st, 1836, in north latitude, the *true* altitude of the star Alphard was $22^{\circ}11'50''$, and, at the same instant, that of the star Regulus was $35^{\circ}0'10''$; required the latitude of the place of observation?

Alphard's red. right ascension $9^h19^m31^s$, and declination $7^{\circ}57'0''$ S.
 Regulus's red. ditto . $9.59.39$, and declination $12.45.59$ N.

Difference of ditto . . . $0^h40^m8^s = 10^{\circ}2'0''$, the angle R P A.
 Reduced tabular distance between the two stars is $22^{\circ}59'20'' =$ the side R A.

When the latitude of a place, and the declination of a celestial object are of different names, 90° *plus* the declination; but when of the same name, 90° *minus* the declination, expresses the polar distance of the object; hence the polar distance of Alphard = P R, is $97^{\circ}57'0''$, and that of Regulus = P A, $77^{\circ}14'1''$. As the zenith distances are simply the co-altitudes of the objects, therefore, the zenith distance of Alphard = Z R, is $67^{\circ}48'10''$, and that of Regulus = Z A, $54^{\circ}59'50''$.* Now, with those elements, the latitude is to be determined in the following manner, viz.:—

First.—To find the angle P R A.—See *Spherics*, Problem I., page 198.

As the distance, or side R A = $22^{\circ}59'20''$, Log. co-secant 10.408320
 Is to the angle R P A = . . . 10. 2. 0, Log. sine . . . 9.241101
 So is the polar distance P A = $77.14.1$, Log. sine . . . 9.989129

To the angle P R A = . . . $25^{\circ}47'21''$, Log. sine . . . 9.638550

* As this illustration is intended for the guidance of young gentlemen who are not supposed to be perfect masters of *the doctrine of spherics*, I am, therefore, desirous of rendering it so very familiar as not to subject them to the necessity of *guessing at my meaning*.

Second.—To find the angle Z R A.—See *Spherics*, Problem V. p. 207.

Zenith distance Z A = 54°59'50"
 Zenith distance Z R = 67. 48. 10, Log. co-sec. 10. 033441
 Distance between the stars, or RA = 22. 59. 20, Log. co-sec. 10. 408320

Sum 145°47'20"

Half sum 72°53'40" Log. sine . 9. 980351
 Remainder 17. 53. 50. Log. sine . 9. 487577

Sum = 19. 909689

Half the required angle = . . 25°40'38" L. co-sine = 9. 954844½

Angle Z R A = 51°21'16" — PRA, 25°47'21" = the
 angle Z R P, 25°33'55", opposite to the co-latitude P Z.

Third.—To find the side P Z.—See *Spherics*, Problem III., page 202.

Angle Z R P,

25°33'55", half ditto = 12°46'57½", twice its L. sine = 18. 689778

Polar dis. or side P R = 97. 57. 00. Log. sine = . . 9. 995806

Zenith dis. or side Z R = 67. 48. 10. Log. sine = . . 9. 966559

Constant log. = 6. 301030

Difference of the sides = 30° 8'50" N.V.S. = 135262

Natural number = 89779 L. 4. 953173

Natural versed sine of the side P Z = 225041 = 39°11'55",
 the co-latitude. Hence the latitude of the place of observation is
 50°48'5" North.

Note.—The last operation is agreeably to the *formula* under Remark 2, page 204.

Now, from the spherical principles thus established, and, I trust, clearly illustrated, we obtain the following

GENERAL RULE.

For deducing the Latitude from the Altitudes of two fixed Stars.

RULE.

Let the altitudes of two stars be observed, at the same moment, whose computed spherical distance asunder is given in Table XLIV.; and let those *observed* altitudes be reduced to the *true* by Problem XXVI., page 378. Take the right ascensions and declinations of the two stars, and also their computed spherical distance, from Table XLIV., and let these be reduced, respectively, to the *night* of observation, as shown in page 115. Let the star which is adjacent, or nearest to the elevated pole, be distinguished by the letter A, and that which is remote, or farthest, by the letter R.—Now,

To the log. sine of the reduced tabular distance between the two stars, add the log. secant of the declination of the star A, and the log. half-elapsed time of the difference of right ascension; the sum, rejecting 20 from the index, will be the log. half-elapsed time of *arch the first*.

From the natural co-versed sine of the altitude of the star A, subtract the natural co-versed sine of the sum of the tabular distance between the stars and the altitude of the star R, and find the logarithm of the remainder; to which add the log. co-secant of the reduced tabular distance, and the log. secant of the altitude of the star R;—the sum of these three logarithms, abating 20 in the index, will be the log. rising of *arch the second*; the difference between which and *arch the first*, will be *arch the third*.

To the log. rising of *arch the third*, add the log. co-sines of the declination and altitude of the star R, and the sum, abating 20 in the index, will be the logarithm of a natural number; which, being added to the natural versed sine of the difference between the altitude and declination of the star R, when its declination and the latitude are of the same name, or to that of their sum when of contrary names, the result will be the natural co-versed sine of the latitude.

Remark.—The reader is requested to bear in mind that he is to make choice of two stars whose *spherical distance asunder* is given in the eighth column of the page in Table XLIV.; and here it may be ne-

cessary to observe, that *in many cases* the distance of the same bright star has been computed from two or three stars situated in *different parts of the heavens*, so as to afford an opportunity of selecting the object which may appear to be in the most favourable position for observation. The tabular distances are adapted to the beginning of the year 1824; but they have been calculated with so much care that, when reduced to a *subsequent period*, or to any distant year, they will prove to be sufficiently correct *for all nautical purposes* till the end of the nineteenth century, or till the 1st of January, 1900. For the manner of reducing those distances to any *future time*, see the precepts in the lower half of page 115.

Caution.—In order to guard against falling into an error, by working in an *impossible triangle*, it will be advisable to make choice of two stars whose tabular distance is *not less than 20 degrees*, and difference of right ascension *not less than a quarter of an hour*: and, since the Table contains an extensive variety of *distances* and *differences of right ascension* greater than those values, the practical navigator can never be at a loss in finding out two eligible stars for observation; so far as regards their situation in the heavens, and their position with respect to the horizon.

Example 1.

At sea, January 1st, 1836, in north latitude, the *mean* of several observed altitudes of the star Alphard, *reduced to the true altitude*, was $22^{\circ}11'50''$; and, at the same time, that of the star Regulus was $35^{\circ}0'10''$; required the latitude of the ship, or place of observation?

Let Regulus, the star *nearest to the elevated pole*, be called A; and Alphard, the most remote from it, R:—Then the tabular distance, right ascensions and declinations of these, *reduced to the given day*, are as follow, viz.:—

True spherical distance between the two given stars = $22^{\circ}59'20''$
 A, or Regulus's R. A. = $9^{\text{h}}59^{\text{m}}39^{\text{s}}$; and declination . $12.45.59$ N.
 R, or Alphard's R. A. = $9.19.31$; and declination . $7.57.0$ S.

Difference of R. A. . = $0^{\text{h}}40^{\text{m}}8^{\text{s}}$ L. half-elapsed time = 0.758900
 Dist. between the stars $22^{\circ}59'20''$ Log. sine 9.591680
 Declina. of the star A . $12.45.59$ Log. secant 10.010871

Arch the first . . . = $1^{\text{h}}43^{\text{m}}9^{\text{s}}$ L. half-elapsed time = 0.361451

OF FINDING THE LATITUDE BY THE ALTITUDES OF TWO STARS. 411

Dist. betw. the stars = $22^{\circ}59'20''$ Log. co-secant . . 10.408320
 Altitude of the star R = $22.11.50$ Log. secant . . . 10.033441

Sum of ditto . . = $45^{\circ}11'10''$ Nat. co-v. sine 290600
 Altitude of the star A = $35.0.10$ Nat. co-v. sine 426384

Remainder, or difference of natural co-versed sines 135784 L. 5. 132849

Arch the second . . = $3^{\circ}25'25''$ Log. rising . . = 5.574610

Arch the first . . . = 1.43. 9

Arch the third . . = $1^{\circ}42'16''$ Log. rising . . = 4.990860

Declina. of the star R = $7^{\circ}57'0''$ Log. co-sine . . . 9.995806

Altitude of ditto . . . $22.11.50$ Log. co-sine . . . 9.966559

Sum of declin. and alt. = $30^{\circ}8'50''$ N. V. sine 135262

Natural number 89789 Log. 4.953225

Natural co-versed sine of the latitude = . . 225051 = $50^{\circ}48'2''$ N.;
 which is the latitude of the place of observation, as required.

Note.—The latitude, thus found, differs $3''$, or *the twentieth part of a minute*, from that determined by spherical trigonometry in the preceding page: this trifling difference is owing to the numbers in the Tables of Log. Half-elapsed Time and Log. Rising, being *only extended to five places* of decimals; while the numbers in the Tables used for the spherical computation, are all extended *to six places of figures*.

Example 2.

At sea, January 1st, 1836, in north latitude, the mean of several observed altitudes of the star α Arietis, *reduced to the true altitude*, was $16^{\circ}45'34''$, and, at the same instant, that of the star Aldebaran was $41^{\circ}41'40''$; required the latitude of the ship, or place of observation?

Let α Arietis, the star nearest to *the elevated pole*, be called A; and Aldebaran, the most remote from it, R:—Then, the tabular distance, right ascensions, and declinations of these, *reduced to the given day*, are as follow, viz. :—

True spherical distance between the two given stars = $35^{\circ}32'7''$

A, or α Arietis' R. A. = $1^h57^m56^s$; and declination . $22^{\circ}41'2''$ N.
 R, or Aldebaran's R. A. = $4.26.31$; and declination . $16.10.24$ N.

Difference of R. A. . . . $2^h28^m35^s$ L. half-elapsed time 0.219070
 Dist. between the stars . $35^{\circ}32'7''$ Log. sine 9.764329
 Declina. of the star A = $22.41.2$ Log. secant 10.034965

Arch the first = $4^h53^m50^s$ L. half-elapsed time = 0.018364
 Dist. betw. the stars $35^{\circ}32'7''$ Log. co-secant 10.235671
 Alt. of the star R = $41.41.40$ Log. secant 10.126852

Sum of ditto . . = $77^{\circ}13'47''$ Nat. co-v. sine 024736
 Alt. of the star A = $16.45.34$ Nat. co-v. sine 711648

Remainder, or difference of nat. co-versed sines 686912 L. 5.836901

Arch the second . . . $8^h22^m35^s$ Log. rising 6.199424

Arch the first $4.53.50$

Arch the third $3^h28^m45^s$ Log. rising 5.587620
 Declina. of the star R $16^{\circ}10'24''$ Log. co-sine 9.982463
 Altitude of ditto . . $41.41.40$ Log. co-sine 9.873148

Diff. of dec. and alt. . $25^{\circ}31'16''$ Nat. v. sine 097574
 Natural number 277479 Log. 5.443231

Natural co-versed sine of the latitude . . . $375053 = 38^{\circ}40'42''$ N.
 which is the latitude of the place of observation, as required.

Note.—The *difference* betwixt the declination and altitude of the star R, is taken in this example because the latitude and declination of *that star* are of *the same name*: in the preceding example, the *sum* was taken because the latitude and declination were of *contrary names*. And, as this is *the only case* to which the rule is subject; it is presumed that any further illustration, by way of examples, would be unnecessary.

Hence, it is manifest that in the above problem the mariner is provided with a direct and *most accurate method* of finding the latitude at sea; and, since it prevents the uncertainty and confusion arising from an error in the assumed latitude, or that by account, and, besides, being free from all ambiguity, restriction, and variety of cases whatever,—it may, therefore, be employed with a certainty of success, at any hour

of the night, whenever two known fixed stars are visible. Indeed, if the altitudes of the objects be determined with but common attention, the latitude resulting therefrom will be always true to the nearest second of a degree, without the necessity of *repeating the operation*, or of applying any correction whatever to the result.

Remarks.—Although it is at all times advisable for two observers to take the altitudes of the stars at the same moment of time, yet, should one person be desirous of going through the whole operation himself, he is to proceed as follows:—viz., let the altitude of one star be taken, and the time of observation noted by a watch that shows seconds; then let the altitude of the other star be observed, and the time noted also; and let the altitude of the *first observed star* be again taken, and the time of observation noted.

Now, find the difference between the first and last times of observation, and between the altitudes of the first observed star; and find, also, the difference between the first time of observation of the first star, and the time of observing the second star. Then say, as the interval or difference of time between the two observations of the first star, is to the difference of altitude in that interval; so is the interval, or difference of time, between the observations of the first and second star, to a correction; which, being applied by addition or subtraction, to the *first observed altitude* of the first star, according as it may be increasing or decreasing, the sum or difference will be the altitude of that star reduced to the time that the altitude of the second star was taken. This part of the operation may be readily performed by proportional logarithms;—see Example, page 75. The interval between the observations ought, however, to be as much contracted as possible, on account of guarding against any irregularities in the change of altitude.—With the altitudes, thus found, and the other requisite elements, the latitude is to be computed as above.

PROBLEM IX.

Given the Latitude by Account, the Altitude of the Sun's lower or upper Limb observed near the Meridian, the Mean Time of Observation, and the Longitude; to find the true Latitude.

Since it frequently happens at sea, particularly during the winter months of the year, that the sun's meridional altitude cannot be taken, in consequence of the interposition of clouds, fogs, rain, or other causes; and since the true determination of the latitude becomes an

object of the greatest importance to the mariner when his ship is sailing in any narrow sea trending in an easterly or a westerly direction, such as the British Channel; the present problem is, therefore, given, by means of which the latitude may be very readily and correctly inferred from the sun's altitude taken at a given interval from noon, within the following limits; viz.,—The *number* of minutes and parts of a minute, contained in the interval between the time of observation and noon, must not exceed the *number* of degrees and parts of a degree contained in the object's meridional zenith distance at the place of observation. And, since the meridional zenith distance of a celestial object is expressed by the difference between its declination and the latitude of the place of observation, when they are of the same name, or by their sum, when of contrary names, the extent of the interval from noon *within which the altitude should be observed*, may, therefore, be readily ascertained, by means of the difference between the latitude, and the declination, when they are both north or both south, or by their sum when one is north and the other south: thus, if the latitude be 60 degrees, and the declination 23 degrees, both of the same name, the interval between the time of observation and noon ought not to exceed 37 minutes; but if one be north and the other south, the interval may be extended, if necessary, to 83 minutes before or after noon. The altitude, however, may be taken as near to noon as the mariner may think proper; the only restriction being, that the observation must be made *within* the above-mentioned limits.

The interval between the mean time of observation and noon must be accurately determined: this may be always done, by means of a chronometer or any well-regulated watch showing seconds; proper allowance being made for the difference of time answering to the change of longitude, if any, since the last observation for determining its error.

Now, if the sun's altitude be observed *at any time within the above-mentioned limits*, the latitude of the place of observation may then be determined, to every degree of accuracy desirable in nautical operations, by the following rule; which, being performed by proportional logarithms, renders the operation nearly as simple as that of finding the latitude by the meridional altitude of a celestial object.

See explanation to Tables LI. and LII., between pages 138 and 143.

RULE.

Reduce the sun's declination to the mean time and place of observation, by Problem XIV., page 357, and let the observed altitude of

the sun's lower limb be reduced to the true central altitude, by Problem XXIII., page 374. Then, with the sun's reduced declination, and the latitude by account, enter Table LI. or LII. (according as the latitude and the declination are of the same or of a contrary denomination), and take out the corresponding correction in seconds and thirds, which are to be esteemed as *minutes and seconds*, agreeably to the Rule in page 139. Now,

To the proportional logarithm of this correction, add *twice* the proportional logarithm of the interval between the time of observation and noon, and the constant logarithm 7.2730; the sum of these three logarithms, abating 10 in the index, will be the proportional logarithm of a correction, which being *added* to the true altitude of the sun's centre, the sum will be the meridional altitude of that object: hence the sun's meridional zenith distance will be known; to which let its reduced declination be applied by addition or subtraction, according as it is of the same or of a contrary name, and the sum or difference will be the latitude of the place of observation.

Remark.—In taking out the equation from Table LI. or LII., proportion must be made for the *excess* of the given degrees of latitude and declination above the *next less* tabular correction; agreeably to the formula, or mode of computation, at the bottom of page 141.

Example 1.

At sea, January 1st, 1836, at $22^{\circ}48'3''$ mean time, in latitude $51^{\circ}36'$ north, by account, and longitude $10^{\circ}45'30''$ west; the mean of several observed altitudes of the sun's lower limb *reduced to the true central altitude*, was $13^{\circ}33'58''$ south: required the true latitude of the place of observation?

Mean time of observ. $22^{\circ}48'3''$	Sun's declination at
Longitude $10^{\circ}45'30''$	noon, Jan. 1st = $23^{\circ}4'16''$ S.
west, in time . . . + 0.43.2	Cor. for $23^{\circ}31'5''$ = -4.49
Greenwich time . . . <u>$23^{\circ}31'5''$</u>	Sun's reduced dec. . <u>$22^{\circ}59'27''$ S.</u>

Mean time of observation . $22^{\circ}48'3''$
 Reduced equation of time . -4.3

Apparent time of observation $22^{\circ}44'0''$; which is $1^{\circ}16'0''$ from apparent noon.

The cor. in Table LII., corresp. to lat. 50° N., and dec. $22^{\circ}59'27''$ S.,
 taken as 23° , is $1^{\circ}12'9''$
 Reduction for $1^{\circ}36''$ of latitude — 3.1

Tabular correction = $1^{\circ}9'8''$ Prop. logarithm = . 2.1806
 Apparent time from noon = $1^{\circ}16'0''$ Twice its prop. log. = 0.7490
 Constant logarithm = 7.2730

Correction of the sun's altitude = . $1^{\circ}50'34''$ Prop. log. 0.2116
 True altitude of the sun's centre = . 13.33.58 South.

Sun's meridional altitude = . . . $15^{\circ}24'32''$ South.

Sun's meridional zenith distance = . $74^{\circ}35'28''$ North.
 Sun's reduced declination $22.59.27$ South.

Latitude of the place of observation = $51^{\circ}36'1''$ North.

Note.—In all problems relating to *the sun*, the mean time of observation is to be reduced to apparent time, as above; so that the interval from noon may be expressed in *apparent time*.

Example 2.

At sea, March 21st, 1836, at $0^{\circ}57'39''$ mean time, in latitude $51^{\circ}5'$ north, by account, and longitude $35^{\circ}45'$ west, the mean of several observed altitudes of the sun's lower limb, *reduced to the true central altitude*, was $38^{\circ}13'37''$ south of the observer; required the correct latitude of the place of observation?

Mean time of observ. = $0^{\circ}57'39''$	☉'s declination at noon,
Longitude $35^{\circ}45'$ W.,	March 21st = . $0^{\circ}22'4''$ N.
in time = . . . +2.23. 0	Correc. for $3^{\circ}20'39''$ + 3.18
Greenwich time = . <u>$3^{\circ}20'39''$</u>	☉'s reduced declina. <u>$0^{\circ}25'22''$ N.</u>

Mean time of observ. = $0^{\circ}57'39''$
 Reduced equation of time — 7.14

Apparent time of observ. $0^{\circ}50'25''$; which is the apparent time from noon.

The correction in Table LI., corresponding to latitude 50° north, and Declination 0° is $1^{\circ}38''8$
 Red. for lat. $1^{\circ}5'$ and decl. $25^{\circ}22'' = 3.2$

Tabular correction = $1^{\circ}35''6$, Prop. logarithm 2.0530
 Apparent time from noon = . . $0^{\circ}50'25''$ Twice its P. L. 1.1054
 Constant logarithm = 7.2730

Correction of the sun's altitude = . . $1^{\circ}6'40''$ Prop. log. 0.4314
 True altitude of the sun's centre = . $38.13.37$ South.

Sun's meridional altitude = $39^{\circ}20'17''$ South.

Sun's meridional zenith distance = . $50^{\circ}39'43''$ North.
 Sun's reduced declination = $0.25.22$ North.

Correct latitude of the place of observ. = $51^{\circ}5'5''$ North.

Hence it is evident, that the latitude may be determined by this method to all the accuracy desirable in nautical operations. It possesses a decided advantage over that by *double altitudes*; and, since the calculation is so extremely simple, the mariner will do well to avail himself thereof on every occasion; because the latitude, thus reduced, will be equally as correct as that resulting from the observed meridional altitude, provided the observation be made within the prescribed limits. When, however, the latitude and the declination are of different names, it will not produce any sensible error in the result, if the altitude be observed *a few seconds without* those limits, as may be seen in Example 1, page 415.

But it is to be remembered, that the mean time of observation must be well determined, because a trifling error in the *interval* from *apparent noon* would sensibly affect the resulting latitude.

PROBLEM X.

Given the Latitude by Account, the Altitude of the Moon's lower or upper Limb, observed near the Meridian, the mean Time of Observation, and the Longitude ; to find the true Latitude.

RULE.

To the mean time of observation apply the longitude in time by Problem III., page 342, and the mean time at Greenwich will be obtained: to which let the *mean* sun's right ascension be reduced by Problem V., page 344.—Let the moon's semidiameter and horizontal parallax be reduced to the Greenwich time by Problem XV., page 361; her right ascension and declination by Problem XVI., page 364: and let the observed altitude of her lower limb be reduced to the true central altitude by Problem XXIV., page 376.

To the mean time of observation add the *mean* sun's reduced right ascension; the sum, abating 24 hours if necessary, will be the right ascension of the meridian; the difference between which and the moon's reduced right ascension will be her horary distance from the meridian.

Now, with the moon's reduced declination, and the latitude by account, enter Table LI. or LII., according as they are of the same or of a contrary denomination, and take out the corresponding correction, agreeably to the Rule in page 139: with which and the moon's horary distance from the meridian, compute the correction of altitude; and hence, the latitude of the place of observation, by Problem IX., page 413.

Note.—The limits within which the altitude of the moon should be observed, are to be determined in the same manner, precisely, as if it were the sun that was under consideration; observing, however, to estimate the interval from the moment of transit over the meridian of the place of observation, instead of from noon.

Example.

At sea, January 23rd, 1836, at $3^h 53^m 12^s$ mean time, in latitude $51^{\circ} 15'$ north, by account, and longitude 45° west, the mean of several observed altitudes of the moon's lower limb, *reduced to the true central altitude*, was $39^{\circ} 15' 3''$ south; required the true latitude?

n time of observ. 3 ^h 53 ^m 12 ^s : g. 45° west, in ne +3. 0. 0 <hr/> nwich time = . 6 ^h 53 ^m 12 ^s : R. A. at 6 hours, n. 23rd. = . . 0 ^h 48 ^m 54 ^s : act. for 53 ^m 12 ^s : + 1. 40 <hr/> educed R. A.= 0 ^h 50 ^m 34 ^s : <hr/> declination at 6 urs, Jan. 23rd. . 1°20′56″ N. act. for 53 ^m 12 ^s : = +12. 30 <hr/> educed declin.= 1°33′26″ N. lar correction = . . . 1 ^h 37 ^m 0 r's meridional distance 0 ^h 48 ^m 47 ^s : tant. logarithm 7.2730 <hr/> ection of the moon's altitude . . 1° 3′19″ Prop. log. 0.4537 altitude of the moon's centre . 39. 15. 3 South. <hr/> r's meridional altitude = . . . 40°18′22″ South. <hr/> r's meridional zenith distance . 49°41′38″ North. r's reduced declination . . . 1.33.26 North. <hr/> latitude, as required = . . . 51°15′ 4″ North.	Mean sun's R. A. at noon, Jan. 23. = 20 ^h 7 ^m 27 ^s :31 Cor. for 6 ^h 53 ^m 12 ^s : . +1. 7.88 <hr/> Mean sun's red.R.A. 20 ^h 8 ^m 35 ^s :19 Mean time of obs. . 3.53.12. — <hr/> R. A. of meridian . 0 ^h 1 ^m 47 ^s : — D 's reduced R. A. . 0.50.34 — <hr/> D 's dist. from mer.= 0 ^h 48 ^m 47 ^s : — <hr/> Cor. Table LI. to lat. 50° and dec. 1° . 1 ^h 40 ^m 3 Do. for 1°15′ of lat. and 33′26″ of dec. — 3.3 <hr/> Tabular correction . 1 ^h 37 ^m 0 Prop. logarithm . . 2.0467 Twice its prop. log. 1.1340 <hr/> 0.4537 South. North. North. North.
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PROBLEM XI.

duce the Latitude from the Altitude of a Planet taken near the Meridian.

RULE.

duce the mean time of observation to the meridian of Green- by Problem III., page 342 ; to which, let the mean sun's right

ascension be reduced by Problem V., page 344; and the geocentric right ascension and declination of the planet by Problem XVII., 366.—Find the planet's horary distance from the meridian by Problem XXII., page 373; and let its observed *central* altitude be reduced to the *true* altitude by Problem XXV., page 377.—Now, with the true altitude by account, and the planet's reduced declination, enter Table LI. or LII., according as they are of the same or of contrary denominations, and take out the corresponding correction, agreeably to the index in page 139; with which, and the planet's distance from the meridian, compute the correction of altitude, and, hence, the latitude of the place of observation, by Problem IX., page 413.

Note.—The measure of the interval between the time of observation and the time of transit,—that is, the *number* of minutes and seconds contained in the planet's distance from the meridian, must not be confounded with the number of degrees and minutes contained in that object's meridian zenith distance at the place of observation, as particularly pointed out in page 414.

See explanation to Tables LI. and LII., page 138, and thence to

Example.

At sea, January 4th, 1836, at 10^h 47^m 56^s: mean time, in latitude 45° 28' south, by account, and longitude 60° 12' east, the mean of several observed central altitudes of the planet Jupiter, *reduced to true altitude*, was 19° 52' 38" north of the observer; required the latitude of the place of observation?

Mean time of obs.	10 ^h 47 ^m 56 ^s :	Mean sun's R.A. at noon	18 ^h 53 ^m
Longitude 60° 12' east		Correct. for 5 ^h 59 ^m 56 ^s :	+6 ^m
in time	—4. 48. 0		
Greenwich time =	5 ^h 59 ^m 56 ^s :	Mean sun's red. R. A.	18 ^h 53 ^m

Jupiter's geocentric right ascension, *reduced to the Greenwich time*, is 6^h 45^m 48^s., and his declination 23° 6' 37" north.

Mean time of obs.	10 ^h 47 ^m 56 ^s	Cor. Table LII, answ. to	
Mean sun's red. R. A.	18. 53. 32	lat. 45°, and dec. 23°	1 ^m 22 ^m 7 ^s
		Cor. for 28 ^h Lat. and	
R. A. of meridian	5 ^h 41 ^m 28 ^s	6 ^m 37 ^s declination	-1.0
Jupiter's R. A.	6. 45. 48	Tabular correction =	1 ^m 21 ^m 7 ^s
Ditto meridian distance	1 ^h 4 ^m 20 ^s	Twice its prop. log.	0.8936
Tabular correction	1 ^m 21 ^m 7 ^s	Prop. logarithm	2.1212
Constant logarithm			7.2730
Correction of Jupiter's altitude		1 ^h 32 ^m 47 ^s Prop. log. =	0.2878
True altitude, per observation		19. 52. 38	North.
Jupiter's meridional altitude =		21 ^h 25 ^m 25 ^s	North.
Ditto meridional zenith distance		68 ^h 34 ^m 35 ^s	South.
Ditto reduced declination		23. 6. 37	North.
Latitude of the place of observation =		45 ^h 27 ^m 58 ^s	South.

Note.—The latitude may be deduced in the same manner from the altitude of a fixed star when near the meridian; taking care, however, that the interval between the time of observation and the moment of transit, viz., the star's horary distance from the meridian, does not exceed the limits prescribed in page 414.

PROBLEM XII.

To deduce the Latitude from the Altitude of a Celestial Object observe near the Meridian below the Pole.

RULE.

If the object be the sun, let its horary distance from the meridian be reckoned from *apparent midnight*; but, for any other celestial object, let its horary distance from the meridian be reckoned from the *mean time* of its transit *below the pole*. Now, let the correction answering to the latitude and the declination be *always* taken out of Table LII., in the same manner as if those elements were of different denominations:—then, the resulting correction of altitude being applied by

subtraction to the true altitude of the object, its meridional altitude *below the pole* will be obtained. With the meridional altitude *& the pole*, thus found, and the reduced declination of the object, latitude is to be determined by Problem V., page 392.

Note.—The interval between the time of observation and the moment of transit *viz.*, the limits within which the altitude should be served, is to be determined in the same manner as if the celestial object were near the meridian above the pole.—See the first paragraph to Problem IX., in page 414.

Example.

At sea, June 21st, 1836, at 12^h 57^m 15^s mean time, in latitude 71° 55' north, by account, and longitude 65° 0' west, the mean several observed altitudes of the sun's lower limb, *reduced to the central altitude*, was 5° 51' 30" south of the observer; required the latitude of the place of observation?

Mean time of obs.	12 ^h 57 ^m 15 ^s	☉'s reduced declin.	23° 27' 35"
Long. 65° W., in time + 4. 20. 0		☉'s N. polar dist.	66° 32' 25"
Greenwich time =	17 ^h 17 ^m 15 ^s	Correct. Table LII., to lat. 70° and dec. 23° =	0° 37' 1"
Mean time of obs.	12 ^h 57 ^m 15 ^s	Do., to 1° 55' of lat. and 27° 35' of dec.	— 3. 2
Red. equation of time	— 1. 32	Tabular correction =	0° 33' 9"
Apparent time of obs.	12 ^h 55 ^m 43 ^s		
Appar. time from midn.	0 ^h 55 ^m 43 ^s	Twice its prop. log.	1. 0
Tabular correction	0° 33' 9"	Prop. logarithm	2. 5
Constant logarithm =			7. 2
Correction of altitude		— 0° 28' 52"	Prop. log. = 0. 7
True central altitude of the sun		5. 51. 30	South.
Sun's meridional altitude <i>below the pole</i>		5° 22' 38"	South.
Sun's north polar distance		66. 32. 25	
Latitude of the place of observation =		71° 55' 37"	North.

Note.—If the object be a *fixed star*, let the mean time of its superior transit above *the pole* at the given meridian, be determined by Problem VIII., page 348; to this time let 12 hours diminished by half the

urnal increase of the *mean sun's* right ascension, be added (*viz.*, $12^h - 1^h 58' 28'' = 11^h 58' 1'' 72''$), and the sum, abating 24 hours, if necessary, will be the mean time of the star's inferior transit *below the pole*:—Then, the rest of the operation is to be performed exactly the same as that for the sun in the Example, page 422.

Remark.

The following ingenious problem for determining the latitude, was communicated to the author by that scientific and enterprising officer, Captain William Fitzwilliam Owen, of His Majesty's ship Eden, who is so highly renowned for his extensive knowledge in every department of science connected with nautical subjects.

PROBLEM.

Given the Latitude by Account, the true Altitude of the Sun's Centre, and the apparent Time; to find the true Latitude of the Place of Observation.

RULE.

Find the mean between the estimated meridian altitude, and the altitude deduced from observation, which call the middle altitude; then,

To the log. rising of the *apparent* time from noon, add the log. co-sine of the latitude, the log. co-sine of the corrected declination, the log. secant less radius of the middle altitude, and the constant logarithm 7.536274;* the sum of these five logarithms, abating 30 in the index, will be the logarithm of a natural number, which is to be esteemed as minutes, and which, being added to the sun's true central altitude, will give his correct meridional altitude; and, hence, the true latitude of the place of observation?

Example.

December 22nd, 1825, in latitude $8^{\circ} 0'$ south, by account, at $23^h 41^m 15^s$ *apparent time*, the true altitude of the sun's centre was $74^{\circ} 16'$; required the true latitude?

* This is the log. co-secant of one minute with a modified index.

Apparent time from noon = . 0^h 18^m 45^s : *Log. rising* = 3.524365
Latitude by account = . . . 8° 0' 0". *Log. co-sine* = 9.995758
Sun's corrected declination = 23.27. 0 S. *Log. co-sine* = 9.962562

Estimated meridional altitude = 74° 33' 0" *Constant log.* = 7.536274
True central altitude = . . 74. 16. 0 74° 16' 0"

Middle altitude = . . . 74° 24' 30" *Log. secant* = 0.570604

Correction of altitude = + 39' 0" *Log.* = 1.589558

Sun's correct meridional altitude = . . 74° 55' 0"

Sun's correct declination = . . . 23. 27. 0 South.

True latitude of the place of observation = 8° 22' 0" South ; which exactly agrees with the result by spherical trigonometry.

Note.—By this method of computation, an error of one degree in the latitude by account, in places within the tropics, will produce little or no effect on the latitude resulting from calculation : thus, if the latitude by account be assumed at 7° 0', or at 9° 0', the resulting latitude, or that deduced from computation, will not differ more than one minute from the truth ; and the same result would be obtained, if the altitude were observed at the distance of an hour from noon : provided, always, that the measure of the interval from noon be very correctly known. But I have to observe, that this method will *not* answer in places to the *northward* of the Tropic of Cancer, or to the *southward* of the Tropic of Capricorn.

PROBLEM XIII.

Given the Longitude of a Place, the Sun's Declination and Semidiameter, and the Interval of Time between the Instants of his Limbs being in the Horizon ; to find the Latitude of that Place.

RULE.

Reduce the mean time, per watch, of the rising or setting of the sun's centre to the corresponding time at Greenwich, by Problem III., page 342 ; to which time let the sun's declination be reduced, by Problem XIV., page 357.

To the arithmetical complement of the logarithm of the interval of time, expressed in seconds, between the instants of the sun's limbs

being in the horizon, add the logarithm of the sun's semidiameter, reduced to seconds, and the constant logarithm 9.124939;* the sum of these three logarithms, rejecting 10 in the index, will be the logarithmic co-sine of an arch. Now, to the logarithmic sine of the *sum* of this arch and the sun's reduced declination, add the logarithmic sine of their *difference*; then, half the sum of these two logarithms will be the logarithmic sine of the latitude of the place of observation.

Example.

At sea, July 13th, 1836, in north latitude, and longitude 120° west, the sun's lower limb, at the time of its setting, was observed to touch the horizon at 8^h5^m24^s mean time, and the upper limb at 8^h9^m30^s; required the latitude of the place of observation?

Mean time of sun's centre setting = 8^h5^m24^s + 8^h9^m30^s ÷ 2 = 8^h7^m27^s
 Longitude 120° west, in time 8.0. 0

Mean time at Greenwich = 16^h7^m27^s
 —The sun's declination reduced to this time is 21°42'57" north.

Interval of time between the setting of the sun's lower and
 upper limbs is 4^m6^s, or 246 seconds Log. ar. comp. = . 7.609065
 Sun's semidiameter 15'.45".4 or 945".4 Logarithm . . . 2.975616
 Constant logarithm 9.124939

Arch 59°10'31" Log. co-sine = 9.709620
 Sun's reduced declination . . 21.42.57 North.

Sum = 80°53'23" Log. sine . 9.994488
 Difference 37.27.34 Log. sine . 9.784046

Sum = : . 19.778534

Lat. of the place of observation = 50°47'58" Log. sine . 9.889267

Remark.—In this method of finding the latitude, it is indispensably necessary that the interval of time (per watch) between the instants of the sun's lower and upper limbs touching the horizon be determined to the *nearest second*; otherwise the latitude resulting therefrom may be subject to a considerable error, particularly in places where the limbs of that object rise or set in a vertical position; which is frequently the case in parts within the tropics.

* This is the ar. comp. of the prop. log. of 24 hours, esteemed as minutes.

SOLUTION OF PROBLEMS RELATIVE TO MEAN TIME, &c.

Time, as inferred directly from observations of the heavenly bodies, is denominated either *apparent* or *mean solar time*. *Apparent time* is that which is deduced from an altitude of the sun; and *mean time* from the altitudes of the moon, stars, or planets. *Mean time* arises from supposed uniform motion of the sun: hence, a mean solar day is always of the same determinate length; but the measure of an apparent day is ever variable,—being longer at one time of the year, and shorter at another, than a mean day: the instant of apparent noon will, therefore, sometimes precede, and at other times follow, that of mean noon. The difference of those instants is called the *equation of time*; which *equation* is expressed by the difference between the sun's true right ascension and his mean longitude, corrected by the equation of the equinoxes in right ascension, and converted into time at the rate of 4 minutes to every 15 minutes of motion, &c. &c. The equation of time is always equal to the difference between the times shown by an uniform or equable-going clock, and a true sun-dial.

The sun's motion in the ecliptic is constantly varying, and so is his motion in right ascension: but since the latter is rendered further unequal, on account of the obliquity of the ecliptic to the equator, hence follows that the intervals of the sun's return to the same meridian become unequal, and that he will gradually come to the meridian of the same place too late, or too early, every day, for an uniform motion, such as that shown by an equable-going watch or clock.

It is this retardation or acceleration of the sun's coming to the meridian of the same place, that is called the *equation of time*; which implies a correction that is additive to, or subtractive from, the *apparent time*, in order to reduce it to equable or mean time; and *vice versa*, which is subtractive from, or additive to, the *mean time* in order to reduce it to apparent time. But, as this essentially useful element has been particularly treated of in the Explanatory Articles between pages 309 and 316; the reader is, therefore, requested to refer to those pages where he will find all the information that he can desire relative to the *equation of time*.

The young navigator will please to bear in mind, that the equation of time relating to nautical operations (like the sun's declination), that which is contained in page II. of the month in the Nautical

Almanac :—but, since it is calculated for the meridian of Greenwich, and *for noon*, a *correction*, therefore, becomes necessary in order to reduce it to a given time under any other meridian.—The customary method of finding this *correction* is pointed out in Problem XIV., page 357; but, since the rule given in that problem is only calculated to answer the ordinary purposes of navigation, viz., for showing *the reduction of the equation to the nearest second*, therefore, when a more rigid degree of exactness is called for, which is always the case when the equation of time is required for the important purpose of establishing *the error and the rate of a chronometer*, the following mode of computation is to be adopted, by means of which that element may be readily reduced to any given time under a known meridian, to the *sixtieth part of a second*.

RULE.

Reduce the mean time at ship to the meridian of Greenwich, by Problem III., page 342. Take from page II. of the month in the Ephemeris, the equation of time for the noon preceding the Greenwich time; multiply the decimals by 6; cut off the *right-hand figure* of the product, and they will be converted into *thirds* :—hence, the equation of time at noon will be expressed in minutes, seconds, and thirds.

Find the daily difference of the equation for the noons immediately preceding and following the Greenwich time: multiply the decimals in the difference by 6; cut off the *right-hand figure* of the product, and they will be converted into *thirds* :—hence, the daily difference of the equation will be expressed in *seconds* and *thirds*.

Now, to the proportional logarithm of these seconds and thirds (esteemed as *minutes* and *seconds*), add the prop. logarithm of the Greenwich time (reckoning the hours as *minutes*, and the minutes as *seconds*), and the constant logarithm 9.1249; the sum, abating 10 in the index, will be the prop. logarithm of a correction in *minutes and seconds*, to be considered as seconds and thirds, and which, being applied to the equation at noon by addition or subtraction, according as it may be increasing or decreasing, the result will be the correct equation of time at the given meridian.

Example.

January 1st, 1836, at 3^h5^m10^s mean time, in longitude 54°40' west; required the correct equation of time?

Equation at noon,	Mean time = . . . 3 ^h 5 ^m 1 ^s
January 1st = . . 3 ^h 34 ^m 9 ^s 1	Longitude 54 ^h 40 ^m west
Ditto ditto, Jan. 2nd = 4. 3. 25	in time = . . +3. 38. 4
Daily difference = . . 0 ^m 28 ^m 34 ^s 1	Greenwich time = . 6 ^h 43 ^m 5 ^s
Multiply decimals by . . . 6	
Daily dif. of equa. = 28 ^m 20 ^m 4	Prop. logarithm = . . 0. 80
Greenwich time = 6 ^h 43 ^m 50 ^s	Prop. logarithm = . . 1. 42
Constant logarithm =	9. 11
Correction of equation = + 7 ^m 57 ^s	Prop. log. = 1. 36
Equation of time at noon 3. 34. 91 = 3. 34. 54. 6	

Correct equation of time = . . . 3^h 42^m 51^s 6, as required.

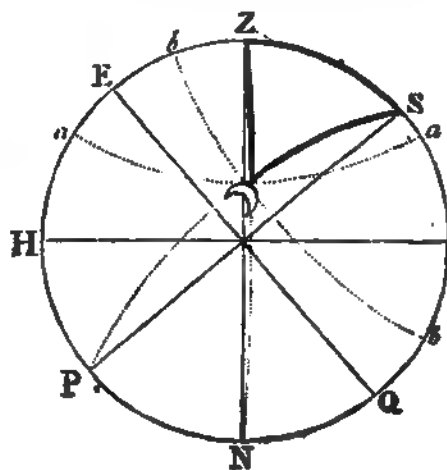
Hence it is manifest that, by the above rule, the equation of time may be readily reduced to any given time under a known meridian, the most rigid degree of astronomical exactness.

PROBLEM I.

Given the Latitude and Longitude of a Ship, or place, and the Observed Altitude of the Sun's Lower Limb; to find the Hour Angle of the object (viz., its Horary Distance from the Meridian), the Mean Time and the Error of the Watch used in noting the time of observation.

Before entering upon the solution of this exceedingly useful problem, it may not be unnecessary to give a concise illustration of the spherical principles upon which it is founded, as thus:—

In the annexed diagram (projected stereographically upon the plane of the meridian), let the primitive circle ZHNO represent the plane of the meridian of the place of observation; S P the earth's axis; E Q the equator; H O the horizon; and Z N the prime vertical, in which Z represents the zenith. Let the small circle *aa* represent the parallel of the object's altitude, and *bb* the parallel of its declination.



The intersection of those small circles in \mathfrak{D} , shows the object's place in the heavens at the time of observation. Draw the oblique circles $S \mathfrak{D} P$, and $Z \mathfrak{D} N$, the first of which is the object's circle of right ascension, and the other its azimuth, or circle of altitude. Now, in the oblique angled spherical triangle $Z \mathfrak{D} S$, the three sides are given to find the hour angle $Z S \mathfrak{D}$, viz., the side $S Z$ = the co-latitude of the place of observation; the side $S \mathfrak{D}$ = the object's polar distance, and the side $\mathfrak{D} Z$ = its co-altitude, or zenith distance; to find the object's distance from the meridian expressed by the angle at S :—which angle is to be found by spherical trigonometry, Problem V., page 207.

Example.

January 1st, 1836, in latitude $40^{\circ}27'$ north, and longitude $54^{\circ}40'$ west, at $3^h 5^m 10^s$ per watch, the mean of several observed altitudes of the sun's lower limb, *reduced to the true central altitude*, was $13^{\circ}43'57''$; required his horary distance from the meridian, the correct mean time, and the error of the watch by which the time of observation was noted:

Mean time per watch = $3^h 5^m 10^s$	Sun's red. declina. = $23^{\circ} 2' 53'' S.$
Longitude $54^{\circ}40'$ W.,	
in time = . . . +3.38.40	Ditto north polar
	distance = . $113^{\circ} 2' 53''$
Greenwich time = . $6^h 43' 50''$	Reduced equation of
	time = $3^m 42' 52^s$

Sun's zenith distance, or side $Z \mathfrak{D}$ = $76^{\circ}16' 3''$

Sun's polar distance, or side $S \mathfrak{D}$ = $113. 2. 53$ Log. co-sec. 0.036128

Co-latitude, or side $Z S$ = . . . $49.33. 0$ Log. co-sec. 0.118631

Sum = $238^{\circ}51'56''$

Half sum = $119^{\circ}25'58''$ Log. sine 9.939984

Remainder = $43. 9. 55$ Log. sine 9.835123

Sum = 19.929866

Half the hour angle $Z S \mathfrak{D}$ = $22^{\circ}42'58''$ Log. co-sine = . 9.964933

Sun's horary distance = . $45^{\circ}25'56''$ = $3^h 1^m 43'44^s$ *apparent time.*

Reduced equation of time = +3.42.52

Correct mean time of observation = . $3^h 5^m 26'36^s$

Time of observation per watch = . . . $3.5.10. 0$

Error of the watch = $0^h 0^m 16'36^s$:—Hence the watch is 16 seconds and 36 thirds, or $16\frac{1}{6}$ slow for mean time.

Thus we see the principles upon which the above astronomical problem is founded, and from those principles a variety of methods may be deduced for computing the horary distances of the heavenly bodies from the meridian, the most practically useful of which shall be now given for the information of the reader, viz. :

METHOD I.

Of computing the Hour Angle, or Horary Distance of a Celestial Object from the Meridian.

RULE.

If the latitude of the ship or place, and the declination of the object be of the same name, 90° *minus* the declination; but if of different names, 90° *plus* the declination will express the object's distance from the elevated pole of the heavens, viz., its *polar distance*.

Now, add together the true altitude of the celestial object, its polar distance, and the latitude of the place of observation; take half the sum, and call the difference between it and the true altitude the *remainder*.

Then, to the log. co-secant of the polar distance, add the log. secant of the latitude, the log. co-sine of the half sum, the log. sine of the *remainder*, and the constant logarithm 6.301030: the sum of these 5 terms, abating 20 in the index, will be the log. rising (Table XXXII.) answering to the given celestial object's distance from the meridian, which, in case of the *sun*, will be the interval between the moment of observation and noon in *apparent time*.

Example.

Let the latitude of a ship, or place, be $40^\circ 27'$ north; the true altitude of a heavenly body $13^\circ 43' 57''$, and its declination $23^\circ 2' 53''$ south; required its horary distance from the meridian?

True altit. of the celestial object = $13^\circ 43' 57''$

Polar distance of ditto = . . . 113. 2. 53 Log. co-sec. 0. 036128*

Latitude of the ship = . . . 40. 27. 0 Log. secant 0. 118631*

Sum = $167^\circ 13' 50''$ Const. log. 6. 301030

Half sum = $83^\circ 36' 55''$ Log. co-sine 9. 046120

Remainder = 69. 52. 58 Log. sine . 9. 972661

Object's meridian distance = . . . 3^h 1^m 44^s Log. rising 5. 47457,0

* The 10 *seconds* are rejected from the indices of the log. co-secant, and log. secant; and, with the view of facilitating the future operations in this work, the same plan will be pursued in all the computations that involve those logarithmical expressions.

Remarks.—As there is no variety in this method, except that which simply relates to finding the polar distance of the object, viz., by *adding* the declination to 90° when it is of a name *contrary* to, or *subtracting* it from 90° when of the *same name* as the latitude; we, therefore, presume that another example would be unnecessary.

The young navigator will find the above one of the most *practicable* methods that can be employed for computing the horary distance of a heavenly body, because the meridian distance is obtained directly by the *plain addition of five terms*;—and thus it is peculiarly adapted to the determination of the longitude at *sea* by means of a chronometer, where the *nearest second* will be always sufficiently exact.

Since the log. rising, in Table XXXII., is only computed to *five* places of decimals, therefore, in taking out the meridian distance of a celestial object answering to a given or calculated log. rising, the sixth or *right-hand figure* of such given log. rising is to be rejected; observing, however, to increase the fifth or preceding figure by unity or 1, when the figure so rejected amounts to 5 or upwards:—In the above example had the 0, or right-hand figure, which is struck off by a dot, been a 5, a 6, a 7, &c., then the log. rising would be 5.47458, and so on.

METHOD II.

Of computing the Hour Angle, or Horary Distance of a Celestial Object from the Meridian.

RULE.

If the latitude of the place of observation and the declination of the given celestial object are of contrary names, let their *sum* be taken,—otherwise, their *difference*,—and the meridional zenith distance of the object will be obtained; the natural versed sine of which, being subtracted from the natural co-versed sine of the object's true altitude, will leave a *remainder*. Now, to the logarithm of this remainder add the log. secants of the latitude and the declination, and the sum will be the log. rising of the object's horary distance from the meridian; which, in case of the *sun*, will be the correct interval between the moment of observation and noon in *apparent time*.

Example.

Let the latitude of the ship be $49^\circ 13'$ south; the true altitude of a heavenly body $22^\circ 38' 17''$, and its declination $17^\circ 15' 5''$ south; required its horary distance from the meridian?

Latitude of the ship = . $49^{\circ}13' 0''$ S. . Log. secant 0.184954
 Declination of the object = $17.15. 5$ S. . Log. secant 0.019991

Object's merid. zenith dist. = $31^{\circ}57'55''$ N. v. sine 151631

Object's true altitude = . $22.38.17$ N. co-v. sine 615091

Remainder = 463460 L. 5. 666012

Object's meridian distance = . . $5^{\circ}0'25''$ Log. rising 5.87095,7

Note.—Because the latitude and the declination are of the *same* denomination, their *difference* is taken; had they been of *contrary* denominations, their *sum* would have been taken, as stated in the first part of the Rule.

METHOD III.

Of computing the Hour Angle, or Horary Distance of a Celestial Object from the Meridian.

RULE.

If the latitude of the place of observation and the declination of the given celestial object are of contrary names, let their *sum* be taken,—otherwise, their *difference*,—and the meridional zenith distance of the object will be obtained; from the natural co-sine of which, subtract the natural sine of the object's true altitude, and to the logarithm of the remainder add the log. secants of the latitude and the declination; and the sum will be the log. rising of the object's horary distance from the meridian :—which, in case of the *sun*, will be the *apparent time* from noon.

Example.

Let the latitude of the ship or place be $39^{\circ}47'$ south; the true altitude of a heavenly body $13^{\circ}9'53''$, and its declination $22^{\circ}54'42''$ north; required its horary distance from the meridian?

Latitude of the ship = . $39^{\circ}47' 0''$ S. . Log. secant 0.114373
 Declination of the object = $22.54.42$ N. . Log. secant 0.035690

Object's merid. zenith dist. = $62^{\circ}41'42''$ N. co-sine = 458727

Object's true altitude = . $13. 9.53$ Nat. sine = 227751

Remainder = 230976 L. 5. 363567

Object's meridian distance = . $3^{\circ}10'35''$ Log. rising = 5.51363,0

Note.—Since the latitude and the declination are of *different* names their *sum* is taken; had they been of *the same name* their *difference* would have been taken; as stated in the first part of the Rule.

METHOD IV.

Of computing the Hour Angle, or Horary Distance of a Celestial Object from the Meridian.

RULE.

If the latitude of the place of observation and the declination of the celestial object be of contrary names, let their *sum* be taken,—otherwise, their *difference*,—and the meridional zenith distance of the object will be obtained; to which apply its observed zenith distance, by addition and subtraction, and let half the sum and half the difference be taken; then,

To the log. secant of the latitude add the log. secant of the declination, the log. sine of the half sum, the log. sine of the half difference, and the constant logarithm 6. 301030; the sum of these five logarithms, abating 20 in the index, will be the log. rising of the object's horary distance from the meridian—which, in case of *the sun*, will be the interval from noon in apparent time.

Example.

Let the latitude of the ship or place be $37^{\circ}20'$ south; the true altitude of a heavenly body $26^{\circ}49'58''$, and its declination $19^{\circ}58'26''$ south; required its horary distance from the meridian?

Lat. of the ship = $37^{\circ}20' 0''$ S.	Log. secant 0.099567
Dec. of the object $19^{\circ}58.26$ S.	Log. secant 0.026942

Obj. mer. zen. dist. $17^{\circ}21'34''$

Observed zen. dist. 63. 10. 2

Const log. 6. 301030

Sum =	$80^{\circ}31'36''$	Half = $40^{\circ}15'48''$	Log. sine	9.810435
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Difference	45. 48. 28	Half = 22. 54. 14	Log. sine	9.590158
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Object's meridian distance $4^{\text{h}}43^{\text{m}}42^{\text{s}}$ Log. rising 5.82813,2

Note.—The *difference* between the latitude and the declination is taken because these elements are of the *same name*; had they been of *contrary* denominations *their sum* would have been taken, as stated in the first part of the Rule.

METHOD V.

Of computing the Hour Angle or Horary Distance of a Celestial Object from the Meridian.

RULE.

If the latitude of the ship and the declination of the object be of the same name, 90° *minus* the declination ; but, if of contrary names, 90° *plus* the declination, will be its *polar distance*.

Now, add together the true altitude of the celestial object ; its polar distance, and the latitude of the place of observation ; take half the sum, and call the difference between it and the true altitude the *remainder*.

Then, to the log. co-secant of the polar distance, add the log. secant of the latitude, the log. co-sine of the half sum, and the log. sine of the *remainder* : half the sum of these four logarithms will be the log. sine of an arch ; which being *doubled* and converted into time, by Problem I., page 341, the result will be the object's horary distance from the meridian to the most rigid degree of exactness.

Example.

Let the latitude of the place be 40°27'30" south ; the true altitude of a heavenly body 31°55'23", and its declination 20°49'3" south ; required its horary distance from the meridian ?

True alt. of the celestial object 31°55'23"			
Polar distance of ditto	. . . 69.10.57	Log. co-secant .	0.029319
Latitude of the place	. . . 40.27.30	Log. secant . .	0.118685
<hr/>			
Sum = 141°33'50"		
<hr/>			
Half sum 70°46'55"	Log. co-sine	. 9.517413
Remainder 38.51.32	Log sine	. . 9.797547
<hr/>			
		Sum =	. . . 19.462964
<hr/>			
Arch = 32.36.24	Log. sine =.	. 9.731482
<hr/>			
Hour angle = 65°12'48" = 4 ^h 20 ^m 51 ^s .12 ^t ; which is the correct horary distance of the object from the meridian.			

Note.—In finding the error of a chronometer, the above is the method of calculation that should be adopted; because it is so arranged that the horary distance of an object from the meridian may be accurately determined to the sixtieth part of a second, or to a more rigid degree of exactness if necessary:—whereas, the preceding modes of computation are only properly adapted for showing the horary distance of a heavenly body to the nearest second of time; those being mainly designed for the purpose of expediting the operation for finding the longitude at sea, by a comparison of the Greenwich time indicated by a chronometer, or deduced from a lunar observation; because, on the ocean the difference of a second in the mean time of observation, cannot sensibly affect the place of a ship.

PROBLEM II.

Given the Latitude and Longitude of a Ship, and the Observed Altitude of the Sun's Lower Limb; to find the Mean Time of Observation, and the Error of the Watch.

RULE.

Reduce the time of observation, per watch, to the meridian of Greenwich, by Problem III., page 342:—to which let the sun's declination be reduced by Problem XIV., page 357; and the equation of time by the Rule in page 427. Reduce the observed altitude of the sun's lower limb to the true central altitude, by Problem XXIII., page 374:—then,

With the latitude of the place of observation, the sun's reduced declination, and his true central altitude, compute the horary distance from the meridian, by any of the Methods given in the last Problem between pages 430 and 434.—Now, if the observation be made in the afternoon, the horary distance, thus found, will directly express the apparent time; but, if it be made in the forenoon, 24 hours diminished by the horary distance will be the apparent time past the preceding noon. To the apparent time apply the reduced equation of time, as directed in Problem XIX., page 369; and the result will be the mean time of observation: the difference between which and the time indicated by the watch, will be the error of the latter; which error will be fast for mean time when the time per watch is the greatest; otherwise, it will be slow for mean time.

For the principles of the above Problem, see page 428.

Remark.—In *practice*, it becomes indispensably necessary to take several altitudes of the sun, or of the heavenly body decided on for observation, and to note the corresponding times by a watch that shows seconds: then, the mean of the altitudes divided by their sum, gives the mean altitude; and the sum of the times, so divided, gives the mean time.

Example.

January 20th, 1836, in latitude $37^{\circ}20'$ south, and longitude $49^{\circ}45'$ east, the following altitudes of the sun's lower limb were observed: the height of the eye above the level of the horizon being 16 feet, and the index error of the sextant $1'10''$ *subtractive*; required the correct mean time of observation, and the error of the watch?

Time of ob. per watch	19 ^h 10 ^m 25 ^s
Ditto	19. 11. 10
Ditto	19. 11. 55
Ditto	19. 12. 40
Ditto	19. 13. 25

Sum = 95^h 59^m 35^s

Mn. time per watch 19^h 11^m 55^s
 Lon. $49^{\circ}45'$ E. in *ti*. 3. 19. 0

Greenwich time = 15^h 52^m 55^s

☉'s dec. at noon . $20^{\circ}16' 3''$ S.
 Cor. for 15^h 52^m 55^s — 8. 35.

☉'s reduced dec. = $20^{\circ} 7' 28''$ S.

Equation of time at
 noon 11^m 10^s 94 = 11^m 10^s 56^s 4
 Cor. for 15^h 52^m 55^s 11. 44 —

Red. equa. of time = 11^m 22^s 40^s —

Alt. ☉'s lower limb =	$23^{\circ}19'10''$
Ditto	23. 28. 0
Ditto	23. 36. 50
Ditto	23. 45. 40
Ditto	23. 54. 30

Sum = 118^h 4^m 10^s

Mean altitude . . $23^{\circ}36'50''$
 Index error . . . — 1. 10

Observed altitude = $23^{\circ}35'40''$
 ☉'s se.-d. $16'16''$ }
 Dip of hor. 3. 50 } Dif. = +12. 26

Apparent altitude . . $23^{\circ}48' 6''$
 Refrac. = $2'9''$ }
 Parallax . 8^s } Dif. = —2. 1

☉'s true central alt. = $23^{\circ}46' 5''$

To find the Mean Time and the Error of the Watch.

Sun's true central altitude . $23^{\circ}46' 5''$
 Sun's south polar distance . $69.52.32$ Log. co-secant = 0.027358
 Latitude of the place . . $37.20. 0$ Log. secant . . 0.099567

Sum = . . . $130^{\circ}58'37''$ Constant Log. . 6.301030

Half sum . . . $65^{\circ}29'18\frac{1}{2}$ Log. co-sine . . 9.617919

Remainder . . . $41.43.13\frac{1}{2}$ Log. sine . . 9.823145

Sun's horary dist. from the meridian = $4^{\circ}59'38''$ Log rising = $5.86901,9$

Apparent time of observation . . $19^{\circ} 0'22''$

Red. equa. of time = $11^{\circ}22'40''$ or $+11.23$

Correct mean time of observation = $19^{\circ}11'45''$

Time of observation per watch . $19.11.55$

Difference, or *error* of the watch = $0^{\circ} 0'10''$ Hence, the watch is 10 seconds *fast* for mean time.

Remark.—When the altitudes of a heavenly body are taken for the purpose of regulating a watch or chronometer, the object made choice of for observation ought *not* to be nearer to the meridian than 2 hours, or 30 degrees :—and, in all cases, the farther it is from the meridian, and the nearer it is to the *east* or the *west* points of the horizon, the more correct will be the result ; because, then, the change of altitude is the *quickest* ; and the error of a mile or two in the latitude will not sensibly affect the hour angle or meridian distance :—provided, *always*, that the celestial object be not less than 4 or 5 degrees above the horizon ; so as to guard against the uncertainty of the atmospherical refraction on small angles of altitude. But, this point shall be more fully defined in a subsequent page.

PROBLEM III.

Given the Latitude and Longitude of a Ship, and the observed Altitude of the Moon's Lower or Upper Limb ; to find the Mean Time of Observation and the Error of the Watch used in noting the time.

RULE.

Reduce the time of observation, *per watch*, to the meridian of Greenwich, by Problem III., page 342 : to which time let the *mean*

sun's right ascension be reduced by Problem V., page 344.* Let the moon's semidiameter and horizontal parallax be reduced to the same time by Problem XV., page 361 ; and her right ascension and declination by Problem XVI., page 364 : and let the observed altitude of her limb be reduced to the true central altitude by Problem XXIV., page 376 :—then,

With the latitude of the place of observation, the moon's reduced declination, and her true central altitude, compute her horary distance from the meridian by any of the Methods given in Problem I., between pages 430 and 434. Now, from the horary distance thus found, the mean time is to be deduced agreeably to the *latter part* of the Rule to Problem XX., page 370 : the difference between which and the time of observation per watch, will be the *error* of the latter ; which will be *fast* for mean time when the time indicated by the watch is the *greatest* ; otherwise, it will be *slow* for mean time.

See *Remark*, page 437.

Example.

January 3rd, 1836, in latitude $50^{\circ}10'$ north, and longitude 60° west, the *mean* of several observed altitudes of the moon's lower limb, east of the meridian, was $35^{\circ}47'21''$, and that of the corresponding times, per watch, $7^h49^m30^s$: the height of the eye above the level of the sea was 17 feet, and the index error of the sextant $0'.45''$ *additive* ; required the correct mean time, and the error of the watch ?

Tl. of obs. per watch	$7^h49^m30^s$	D's red. semidiam.=	$14'15''$
Long. 60° W., in time	+ 4. 0. 0	Augmenta., Tab. IV.=	+ 9
Greenwich time	$11^h49^m30^s$	D's true semidiam.=	$14'24''$
Mn. sun's R.A. at n.	$18^h48^m36^s.16$	D's red. hor. paral.=	$54'26''$
Cor. for $11^h49^m30^s$	+ 1. 56. 55	D's reduced R. A. =	$6^h54^m38^s$
Mn. sun's red. R.A.	$18^h50^m32^s.71$	D's reduced dec. =	$26^{\circ}26'32''$ N.

The observed altitude of the moon's lower limb is $35^{\circ}47'21''$: let this be corrected for index error, and then, properly reduced, and it will give $36^{\circ}41'17''$ for the true altitude.

* See Article 37, page 313, relative to the *mean* sun's right ascension.

To find the mean time and the error of the watch by Method II., page 431.

Lat. of the ship . $50^{\circ}10' 0''$ N. Log. secant 0.193442
 D's reduced dec. $26.26.32$ N. Log. secant 0.047991

D's mer. zen. dist. $23^{\circ}43'28''$ Nat. ver. sine 084509

D's true cent. alt. $36.41.17$ Nat. co-v. sine 402542

Remainder = 318083 Log.=5.502472

D's horary dist. *east* of the mer. . $4^{\text{h}}14^{\text{m}}11^{\text{s}}$ Log. rising =5.74390,5

D's reduced right ascension . . $6.54.38$

Right ascension of the meridian = $2^{\text{h}}40^{\text{m}}27^{\text{s}}$

Mean sun's reduced R. A. . . $18.50.33$, to the *nearest* second.

Correct mean time of observation $7^{\text{h}}49^{\text{m}}54^{\text{s}}$

Time of observation per watch . $7.49.30$

Difference, or error of the watch = $0^{\text{h}} 0^{\text{m}}24^{\text{s}}$: hence, the watch is 24 seconds *slow* for mean time.

Note.—Since the moon's right ascension and declination are *now* given in the Nautical Almanac for every hour, the *mean time* can be deduced with *nearly* as much precision from her true central altitude as from that of the sun's.

PROBLEM IV.

Given the Latitude and Longitude of a Ship, and the Observed Altitude of a Planet's Centre ; to find the Mean Time of Observation, and the Error of the Watch.

RULE.

Reduce the time of observation, per watch, to the meridian of Greenwich, by Problem III., page 342, to which let the *mean* sun's right ascension be reduced by Problem V., page 344 ;* and also the geocen-

* See Article 37, page 313, relating to the mean sun's right ascension.

tric right ascension and declination of the planet, by Problem XVII, page 366. Reduce the observed altitude of the planet to its true central altitude by Problem XXV., page 377.

Then, with the latitude of the ship, or place, the planet's reduced declination, and its true central altitude, compute its horary distance from the meridian by any of the Methods given in Problem I., between pages 430 and 434. Now, from the horary distance, thus found, the mean time is to be deduced in conformity with the *latter part* of the Rule to Problem XX., page 370; the difference between which and the time of observation, per watch, will be the error of the latter, which will be *fast* for mean time when the time shown by the watch is the *greatest*; otherwise, it will be *slow* for mean time.

See *Remark*, page 437.

Example.

July 3rd, 1836, in latitude $34^{\circ}45'$ south, and longitude $80^{\circ}30'$ east, the mean of several altitudes of the *centre* of Venus, west of the meridian, was $17^{\circ}48'36''$, and that of the corresponding times, per watch, $5^h43^m25^s$: the height of the eye above the level of the horizon was 18 feet, and the index error of the sextant $0'30''$ *subtractive*; required the correct mean time of observation, and the error of the watch?

Time of observ. per		Venus's hor. parallax = .	24"
watch =	$5^h43^m25^s$	Ditto red. R. A. =	$8^h52^m44^s$
Long. $80^{\circ}30'$ east, in		Ditto red. declin. =	$16^{\circ}3'8''N.$
time =	5.22. 0	Mean sun's R. A. at	
		noon =	$6^h46^m9^s35$
Greenwich time = .	<u>$0^h21^m25^s$</u>	Correc. for 21^m25^s =	+ 3.52
		Mean sun's red. R. A. =	<u>$6^h46^m12^s87$</u>

The observed central altitude of Venus was $17^{\circ}48'36''$; let this be corrected for index error, and then properly reduced, and it will give $17^{\circ}41'53''$ for the true central altitude of that planet.

To find the mean time, and the error of the watch, by Method III., page 432.

Latitude of the ship = $34^{\circ}45' 0''$ S. . . . Log. secant 0.085315
 Venus's declination = $16. 3. 8$ N. . . . Log. secant 0.017272

Venus's mer. zen. dist. $50^{\circ}48' 8''$ Nat. co-sine 631999

Venus's true central alt. $17. 41. 53$ Nat. sine . 304001

Remainder = 327998 Log. 5.515872

Venus's horary dist., *west* of the merid. $3^h36^m54^s$ Log. rising 5.61845,9

Venus's reduced right ascension = $8. 52. 44$

Right ascension of the meridian = $12^h29^m38^s$

Mean sun's reduced right ascension = $6. 46. 13$, to the *nearest* second.

Correct mean time of observation = $5^h43^m25^s$.—And, since this is exactly the same as the time of observation indicated by the watch, there is, therefore, no error in this machine;—hence it shows the true mean time.

Remark.—As the parallaxes of the planets are *now* given in the Ephemeris, the mean time may be deduced from their altitudes to a very great degree of exactness, provided the longitude of the place of observation be known within a few miles of the truth, or that there be a chronometer on board to indicate the mean time at Greenwich.

Should the altitude of Venus be observed on shore by means of an artificial horizon, it is the *lower limb* of the planet that should be taken. See Articles 64 and 67 in pages 328 and 329, and the third paragraph in page 380.

In fine clear weather the altitude of Venus may frequently be taken in the presence of the sun, viz., in broad day-light; particularly when she is drawing near her greatest elongation, *after her inferior conjunction*.

PROBLEM V.

Given the Latitude and Longitude of a Ship or Place, and the observed Altitude of a fixed Star; to find the Mean Time of Observation and the Error of the Watch used in noting the time.

RULE.

Reduce the time of observation, *per watch*, to the meridian of Greenwich by Problem III., page 342; to which let the mean sun's right

ascension be reduced by Problem V., page 344.* Find the true altitude of the star by Problem XXVI., page 378; and take its right ascension and declination from the Nautical Almanac, between pages 368 and 407.—Then,

With the latitude of the ship or place, the star's true altitude, and its declination, compute its horary distance from the meridian by any of the Methods given in Problem I. between pages 430 and 434.

Now, from the horary distance, thus found, the mean time is to be deduced in conformity with the *latter part* of the Rule to Problem XX., page 370 :—the difference between which and the time of observation per watch, will be the error of the latter; which will be *fast* for mean time, when the time shown by the watch is the greatest; otherwise, it will be *slow* for mean time.

Example.

May 1st, 1836, in latitude $39^{\circ}20'30''$ south, and longitude $75^{\circ}40'$ east, the mean of several altitudes of Spica Virginis was $25^{\circ}35'18''$ west of the meridian, and that of the corresponding times, per watch, $14^h58^m49^s$; the height of the eye above the surface of the sea was 19 feet, and the index error of the sextant $1'15''$ additive; required the correct mean time, and the error of the watch?

See Remark, page 437.

Time of observation per		Mean sun's R. A. at
watch =	$14^h58^m49^s$	noon = $2^h37^m46^s20$
Long. $75^{\circ}40'$ east, in		Cor. for $9^h56^m9^s = + 1.37.93$
time =	$-5. 2.40$	
		Mean sun's red. R.A. = $2^h39^m24^s13$
Greenwich time =	$9^h56^m 9^s$	

Right ascension of Spica Virginis $13^h16^m35^s$ and declination $10^{\circ}18'19''$ south.

The observed altitude of the star is $25^{\circ}35'18''$; this being corrected for index error, and then properly reduced, gives $25^{\circ}30'23''$ for the true altitude.

To find the mean time, and the error of the watch, by Method IV., page 433.

* See Article 37, page 313, relative to the mean sun's right ascension.

Latitude of the ship = $39^{\circ}20'30''$ S. . . Log. secant = 0.111607
 Star's declination = $10.18.19$ S. . . Log. secant = 0.007063

Star's mer. zen. dist. = $29^{\circ}2'11''$
 Observed zen. dist. = $64.29.37$ Constant log. = 6.301030

Sum = . . . $93^{\circ}31'48''$ Half = $46^{\circ}45'54''$ Log. sine 9.862460
 Difference = . . $35.27.26$ Half = $17.43.43$ Log. sine 9.483600

Star's hor. dist. west of the merid. = $4^{\circ}21'27''$ Log. rising = 5.76576,0
 Star's right ascension = . . . $13.16.35$

Right ascension of the meridian = $17^{\circ}38'2''$
 Mean sun's red. right ascension = $2.39.24$ rejecting decimals.

Correct mean time of observation = $14^{\circ}58'38''$
 Time of observation per watch = $14.58.49$

Difference, or error of the watch = $0^{\circ}0'11''$ —Hence, the watch is 11 seconds *fast* for mean time.

Remark.—In finding the error of a watch by sidereal observations, two or more stars should be observed, and the error of the watch deduced from each star separately. And, if an equal number of stars be observed on different sides of the meridian, and nearly equidistant therefrom, it will conduce to still greater accuracy; because, then, the errors of the sextant or quadrant, and the unavoidable errors of observation, will have a mutual tendency to correct each other. The mean of the errors so deduced should be taken for the real error of the watch.

SOLUTION OF PROBLEMS RELATIVE TO FINDING THE ALTITUDES OF THE HEAVENLY BODIES.

It sometimes happens at sea, particularly in taking a *lunar observation*, that the horizon is so ill-defined as to render it impossible to observe the altitudes of the objects to a sufficient degree of exactness; or, perhaps, that one or both of the objects are directly over the land, at the time of measuring the lunar distance, and the ship so contiguous thereto as to render the absolute value of the horizontal dip uncertain:

in such cases, therefore, the altitudes of the objects must be obtained by computation, the principles of which may be familiarly illustrated in the following manner, viz. :—Since the problem for finding the altitude of a heavenly body resolves itself into an oblique angled spherical triangle, in which two sides and the contained angle are given to find the third side; therefore, in the diagram, page 428, given the side SZ = the co-latitude, the side $S\mathcal{D}$ = the polar distance, and the included angle at S = the horary distance from the meridian; to find the side $\mathcal{D}Z$ = the co-altitude, or zenith distance of the object; which side is to be found by spherical trigonometry, Problem III., page 202, as thus :—

Let the latitude be $40^{\circ}27'$ north, the declination of a celestial object $23^{\circ}2'53''$ south, and its horary distance from the meridian $3^h1^m44^s$; to find the true central altitude of that object.

See Example, page 429.

Object's dist. from merid. $3^h1^m44^s = 45^{\circ}26' =$ the hour angle $ZS\mathcal{D}$

Half the hour angle $ZS\mathcal{D} =$	$22^{\circ}43'$	$\left\{ \begin{array}{l} \text{Twice its} \\ \text{Log. sine} \end{array} \right\} =$	19.173568
Co-latitude, or side $SZ =$	$49.33.0$	Log. sine =	9.881369
Polar distance, or side $S\mathcal{D} =$	$113.2.53$	Log. sine =	9.963872
<hr/> Sum =			<hr/> 39.018809 <hr/>
Difference of the sides =	$63^{\circ}29'53''$	Half =	$+19.509404\frac{1}{2}$
Half difference =	$31^{\circ}44'56\frac{1}{2}''$	Log. sine =	$9.721149\frac{1}{2}$
Arch =	$31.33.18$	Log. tangt. =	9.788255
Log. sine of the arch =			-9.718764
Half the required side =	$38^{\circ}8'3''$	Log. sine =	$9.790640\frac{1}{2}$
Side $\mathcal{D}Z$, or zenith distance =	$76^{\circ}16'6''$	—Hence, the true altitude of the celestial object is $13^{\circ}43'54''$	

See the formula under Remark I., page 203.

Now, from the principles thus established, the Rules in the following problems have been deduced for determining the altitudes of the heavenly bodies.

PROBLEM I.

Given the Latitude and Longitude of a Ship, or Place, and the Mean Time; to find the True, and the Apparent Altitude of the Sun's Centre.

RULE.

Reduce the given mean time to the meridian of Greenwich, by Problem III., page 342; to which let the sun's declination, and the equation of time, be reduced by Problem XIV., page 357.—Convert the given mean time into *apparent time* by Problem XXI., page 372, and it will express the sun's horary distance from the meridian, if the given time be in the *afternoon*; but, if it be in the *forenoon*, 24 hours *minus* the apparent time, will be the sun's distance from the meridian, viz., from apparent noon.

If the latitude of the place and the sun's declination are of *contrary* names, let their *sum* be taken; otherwise, their *difference*: and the meridional zenith distance of that object will be obtained. Then,

To the logarithmic rising answering to the sun's horary distance from the meridian, add the logarithmic co-sines of the latitude of the place, and of the sun's reduced declination: the sum, rejecting 20 from the index, will be the logarithm of a natural number; which, being added to the natural versed sine of the sun's meridian zenith distance, will give the natural co-versed sine of its true central altitude.

To the sun's true altitude, thus found, let the correction corresponding thereto in Table XIX., be added; and the sum will be the *apparent* altitude of the sun's centre.

Example.

Required the true, and the apparent altitude of the sun's centre, January 1st, 1836, at 3^h 5^m 27^s: mean time, in latitude 40° 27' north, and longitude 54° 40' west of the meridian of Greenwich?

Given mean time = . 3 ^h 5 ^m 27 ^s :	☉'s declination at
Long. 54° 40' west, in	noon = . 23° 4' 16" South.
time = . . . + 3. 38. 40	Cor. for 6 ^h 44 ^m 7 ^s : — 1. 23
<hr/>	<hr/>
Greenwich time = . 6 ^h 44 ^m 7 ^s :	☉'s red. decl. = 23° 2' 53" South.
<hr/>	Equation of time
Given mean time = . 3 ^h 5 ^m 27 ^s :	at noon = . 3 ^m 35 ^s : Sub.
Equation of time = . — 3. 43	Cor. for 6 ^h 44 ^m 7 ^s : + 8
<hr/>	<hr/>
Apparent time = . 3 ^h 1 ^m 44 ^s :	Red. equa. of time = 3 ^m 43 ^s : Sub.

To find the true Central Altitude, &c.

Sun's hor. mer. dist. = $3^{\circ} 1'44''$. . . Log. rising 5.47

Sun's red. declin. = $23^{\circ} 2'53''$ S. . . . Log. co-sine 9.99

Latitude of the ship = $40.27.0$ N. . . . Log. co-sine 9.88

Sun's mer. zen. dist. = $63^{\circ}29'53''$ Nat. v. sine 553772

Natural number = 208814 Log. 5.32

True alt. of sun's centre = $13^{\circ}44' 2''$ N. co-v. sine 762586

Correc., Table XIX. = 3.40

Sun's apparent alt. = $13^{\circ}47'42''$, as required.

Remarks.—In computing the sun's altitude for a particular point on shore, where an extreme degree of exactness may be necessary, the equation of time ought to be reduced agreeably to the Rule in Art. 427 :—but, for the ordinary purposes of navigation, in which a rigid degree of accuracy is *never* called for, it will be quite near enough to the truth to reduce the equation of time to the *nearest* second, as in the above Example.

The young navigator must bear in mind that the sun's horary distance from the meridian is always to be expressed in *apparent time*. And he should remember that, in determining the sun's altitude at any given time before mid-day, it is the *difference* betwixt the *apparent time* and 24 hours (the time being *reckoned, astronomically, from preceding noon*) that will express the horary distance from the meridian.

Note.—There is a cipher annexed to the log. rising, so as to' make the *number* of its decimal places correspond with *that* of the log. sines.

PROBLEM II.

Given the Latitude and Longitude of a Ship, or Place, and the Time; to find the True, and the Apparent Altitude of the Moon's Centre.

RULE.

Reduce the given mean time to the meridian of Greenwich by Problem III., page 342; to which let the *mean* sun's right ascension be reduced by Problem V., page 344 :—and let the moon's horary parallax be reduced by Problem XV., page 361; and her right ascension and declination by Problem XVI., page 364.

To the *mean* sun's reduced right ascension let the given mean time be added; and the sum, abating 24 hours, if necessary, will be the right ascension of the meridian (Problem VI., page 345); the difference between which and the moon's reduced right ascension will be her horary distance from the meridian.

If the latitude of the ship and the moon's declination are of *contrary* names, let their *sum* be taken; but, if of the *same name*, their *difference*; the result will be her meridional zenith distance.—Then,

To the log. rising answering to the moon's horary distance from the meridian, add the log. co-sines of her declination, and the latitude of the ship; the sum, abating 20 in the index, will be the logarithm of a natural number; which, being added to the natural versed sine of the moon's meridional zenith distance, will give the natural co-versed sine of her true central altitude.—From the true altitude of the moon, thus found, subtract the correction corresponding thereto, and her reduced horizontal parallax, in Table XIX., and the remainder will be her *apparent* central altitude.

Example.

Required the true, and the apparent altitude of the moon's centre, January 7th, 1836, at 11^h 24^m 25^s mean time, in latitude 50° 10' north, and longitude 60° west of the meridian of Greenwich?

Given mean time = .	11 ^h 24 ^m 25 ^s	Mean sun's R. A. at	
Long. 60° W., in time =	4. 0. 0	noon = . . .	19 ^h 4 ^m 22 ^s 41
		Cor. for 15 ^h 24 ^m 25 ^s =	+ 2. 31. 86
Greenwich time =	15 ^h 24 ^m 25 ^s		
Given mean time =	11 ^h 24 ^m 25 ^s	M ⁿ . sun's red. R. A. =	19 ^h 6 ^m 54 ^s 27
Mean sun's red. R. A. =	19. 6. 54	☾'s red. hor. par. =	56'. 8"—
Right ascen. of merid. =	6 ^h 31 ^m 19 ^s	☾'s red. R. A. =	10 ^h 29 ^m 58 ^s —
☾'s reduced R. A. =	10. 29. 58	☾'s red. declin. =	14 ^h 54 ^m 53 ^s N.
☾'s hor. dist. fr. mer. =	3 ^h 58 ^m 39 ^s		
☾'s reduced declin. =	14. 54. 53 N.	Log. rising	5. 774460
Latitude of the ship =	50. 10. 0 N.	L. co-sine	9. 985116
		L. co-sine	9. 806558
☾'s merid. zen. dist. =	35° 15' 7"	Nat. ver. sine	183378
	Natural number =		368243 Log. 5. 566134
True alt. of ☾'s centre =	26° 38' 23" N.	co-v. sine	551621
Red. of do., Table XIX. =	— 48. 34		
App. alt. of ☾'s centre =	25° 49' 49"		as required.

Note.—There is a cipher annexed to the log. rising, so as to make the *number* of its decimal places correspond with *that* of the log. co-sines.

PROBLEM III.

Given the Latitude and Longitude of a Ship, or Place, and the Mean Time; to find the True, and the Apparent Altitude of a Planet's Centre.

RULE.

Reduce the given mean time to the meridian of Greenwich by Problem III., page 342:—to which let the *mean* sun's right ascension be reduced by Problem V., page 344: and the planet's geocentric right ascension and declination by Problem XVII., page 366.

To the *mean* sun's reduced right ascension let the given mean time be added; and the sum, abating 24 hours if necessary, will be the right ascension of the meridian (Problem VI., page 345); the difference between which and the planet's reduced right ascension will be its horary distance from the meridian.

If the latitude of the ship and the planet's declination are of *contrary* names, let their *sum* be taken; otherwise, their *difference*; and the planet's meridional zenith distance will be obtained.—Then,

To the log. rising answering to the planet's horary distance from the meridian, add the log. co-sines of its declination, and the latitude of the ship; the sum, abating 20 in the index, will be the logarithm of a natural number; which, being added to the natural versed sine of the planet's meridional zenith distance, will give the natural co-versed sine of its true central altitude.—Now, with the planet's true altitude, thus found, enter Table XIX., and take out the equation, or correction corresponding to the *reduction of a star's true altitude*; the difference between which and the planet's parallax in altitude, Table VI., will leave a correction; which, being added to the true altitude, will give the *apparent* altitude of the planet.

Example.

Required the true, and the apparent altitude of Venus, January 3rd, 1836, at 7^h 50^m 45^s mean time, in latitude 45° 34' south, and longitude 80° 30' east of the meridian of Greenwich?

Given mean time = . 7 ^h 50 ^m 45 ^s :	<i>Mean</i> sun's R. A. at
Long. 80°30' east, in	noon = . . . 18 ^h 48 ^m 36 ^s : 16
time = . . . 5. 22. 00	Cor. for 2 ^h 28 ^m 45 ^s : + 0. 24. 43
Greenwich time = . 2 ^h 28 ^m 45 ^s :	<i>Mn.</i> sun's red. R. A. = 18 ^h 49 ^m 00 ^s : 59
Given mean time = . 7 ^h 50 ^m 45 ^s :	Venus's hor. par. = . . . 6" —
<i>Mean</i> sun's red. R. A. = 18. 49. 1	Venus's red. geo.
	R. A. = . . . 20 ^h 28 ^m 42 ^s : —
Right ascen. of merid. = 2 ^h 39 ^m 46 ^s :	Ditto, declination = 20°42'16" S.
Venus's reduced R. A. = 20. 28. 42	

Venus's horary dist. = 6 ^h 11 ^m 4 ^s	Log. rising 6. 020470
Ditto, red. decl. = . 20°42'16" S.	Log. co-sine 9. 971005
Latitude of the ship = 45. 34. 0 S.	Log. co-sine 9. 845147

Venus's mer. zen. dis. = 24°51'44" Nat. ver. sine 092678	
Natural number = 686470	Log. 5. 836622

Venus's true cen. alt. = 12°45'33" Nat. co-v. sine 779148

Red., Ta. XIX. 4'6" } Diff. + 4'00"
 Par., Table VI. 0. 6 }

Venus's apparent alt. = 12°49'33", as required.

Note.—There is a cipher annexed to the log. rising, so as to make the *number* of its decimal places correspond with *that* of the log. co-sines.

PROBLEM IV.

Given the Latitude and Longitude of a Ship, or Place, and the Mean Time ; to find the True, and the Apparent Altitude of a fixed Star.

RULE.

Reduce the given mean time to the meridian of Greenwich by Problem III., page 342 ; to which let the *mean* sun's right ascension be reduced by Problem V., page 344.—Take the star's right ascension and declination from the Nautical Almanac, between pages 368 and 407

To the *mean* sun's reduced right ascension let the given mean time be added ; and the sum, abating 24 hours, if necessary, will be the right ascension of the meridian (Problem VI., page 345) ; the difference

between which and the star's right ascension will be its horary distance from the meridian.

If the latitude of the ship and the star's declination are of *contrary* names, let their *sum* be taken ; but if of the *same name*, their *difference* : the result will be the star's meridional zenith distance.—Then,

To the log. rising answering to the star's horary distance from the meridian, add the log. co-sines of its declination, and the latitude of the ship : the sum, abating 20 in the index, will be the logarithm of a natural number ; which, being added to the natural versed sine of the star's meridional zenith distance, will give the natural co-versed sine of its true altitude.—Now, to the star's true altitude, thus found, let the correction corresponding thereto in Table XIX. be added ; and the sum will be its *apparent altitude*.

Example.

Required the true, and the apparent altitude of the star Procyon, January 10th, 1836, at 9^h31^m39^s mean time, in latitude 39°20'30" south, and longitude 75°40' east of Greenwich ?

Given mean time = . 9 ^h 31 ^m 39 ^s	Mean sun's R. A. at
Long. 75°40' east, in	noon = . . . 19 ^h 16 ^m 12 ^s 07
time = . . . 5. 2. 40	Cor. for 4 ^h 28 ^m 59 ^s = + 0. 44. 19
Greenwich time = . 4 ^h 28 ^m 59 ^s	Mn. sun's red. R. A. = 19 ^h 16 ^m 56 ^s 26
Given mean time = 9 ^h 31 ^m 39 ^s	Procyon's right as-
Mean sun's red. R. A. = 19. 16. 56	cension = . . . 7 ^h 30 ^m 44 ^s
Right ascen. of merid. = 4 ^h 48 ^m 35 ^s	Ditto, declination = 5°38'27"N.
Procyon's right ascen. = 7. 30. 44	
Procyon's horary dist. = 2 ^h 42 ^m 9 ^s	Log. rising 5. 380240
Ditto, declination = . 5°38'27"N.	Log. co-sine 9. 997891
Latitude of the ship = 39. 20. 30 S.	Log. co-sine 9. 888393
Star's mer. zen. dist. = 44°58'57" Nat. ver. sine 292678	
Natural number = 184724	Log. 5. 266524
Star's true altitude = 31°30'24" Nat. co-v. sine 477402	
Red. Table XIX. = + 1. 33	
Star's apparent alt. = 31°31'57", as required.	

Note.—In the above Example, as in the three preceding ones, a *cipher* is annexed to the log. rising, so as to make the number of its decimal places correspond with the number of decimals in the log. co-sines.

The reader will please to observe that the *natural sines* may be used in the solution of the four last problems, instead of the *versed sines*: in this case, if the natural number be *subtracted* from the *natural co-sine* of the object's meridional zenith distance, the *natural sine* of its true altitude will be obtained.—Thus, in the above Example, the star's meridional zenith distance is $44^{\circ}58'57''$:—Now, the natural co-sine of this is 707322; from which let the natural number 184724 be subtracted, and the remainder 522598, is the natural sine of the star's true altitude; the arch corresponding to which is $31^{\circ}30'24''$.—The natural sines, and natural co-sines are comprehended in Table XXVII., between pages 93 and 137 of the second volume.—See the first paragraph in page 57, and the *Remark* in page 60, of the present volume.

The four preceding problems are evidently the converse of those for finding the mean time, as given in pages 435, 437, 439, and 441.

SOLUTION OF PROBLEMS RELATIVE TO THE LONGITUDE.

The *Longitude* of a given place on the earth, is that arc or portion of the equator which is intercepted between the first or principal meridian and the meridian of the given place; and is denominated east or west, according as it may be situate with respect to the first meridian.

The *first or principal meridian* is an imaginary great circle passing through any remarkable place and the poles of the world: hence it is entirely arbitrary; and, therefore, the British reckon their first meridian to be that which passes through the Royal Observatory at Greenwich; the French esteem their first meridian to be that which passes through the Royal Observatory at Paris; the Spaniards, that which passes through Cadiz, &c. &c. &c. Every part of the terrestrial sphere may be conceived to have a meridian line passing through it, cutting the equator at right angles: hence there may be as many different meridians as there are points in the equator.

Every meridian line, with respect to the place through which it passes, may be said to divide the surface of the earth into two equal parts, called the eastern and western hemispheres. Thus, when the face of an observer is turned towards the north pole of the world, the hemisphere which lies on his right-hand is called east, and that on his

left-hand west ; and, *vice versa*, when the face is directed towards the south pole of the world, the hemisphere which lies on the left-hand is called east, and that on the right-hand west.

The longitude is reckoned both ways from the first meridian, east and west, till it meets with the same meridian on the opposite part of the equator : hence the longitude of any place on the earth can never exceed 180 degrees. The difference of longitude between two places on the earth is an arc of the equator contained between the meridians of those places, showing how far one of them is to the eastward or westward of the other, and can never exceed 180 degrees, or half the earth's circumference.

All places that are situated under the same meridian have the same longitude ; but places which lie under different meridians have different longitudes : hence, in sailing due north or due south, since a ship does not change her meridian, she keeps in the same parallel of longitude ; but, in sailing due east or due west, she constantly changes her meridian, and therefore passes through a variety of longitudes.

When the meridian of any place is brought, by the diurnal revolution of the earth round its axis, to point directly to the sun, it is then noon or mid-day at that place.

The motion of the earth on its axis is, at all times, equable and uniform ; and, since it turns round its axis *eastward* once in every 24 hours, all parts of the equator, or great circle of 360 degrees, will pass by the sun, or star, in equal portions of time : therefore the twenty-fourth part of the equator, viz., 15 degrees, will pass by the sun in one hour of time : for, $24^h \times 15^\circ$ or 1 hour, = 360 degrees ; and, conversely, $360 \text{ degrees} \div 24 \text{ hours} = 15 \text{ degrees or 1 hour}$.

Every place on the earth, whose meridian is 15 degrees east of the Royal Observatory at Greenwich, will have noon and every other hour *one hour sooner* than at the meridian of that Observatory ; if the meridian be 30 degrees east of Greenwich, it will have noon and every other hour *two hours sooner* than at the meridian of that place, and so on ; the time always differing at the rate of 1 hour for every 15 degrees of longitude, 1 minute of time for every 15 minutes of longitude, and 1 second of time for every 15 seconds of longitude. Again, every place whose meridian is 15 degrees west of the Royal Observatory at Greenwich will have noon and every other hour *one hour later* than at the meridian of that Observatory ; if the meridian be 30 degrees to the westward of Greenwich, it will have noon and every hour *two hours later* than at the meridian of that place, and so on. Hence it is evident, that if the time at the meridian of a ship or place be *greater* than the time, at the same instant, at the meridian of Greenwich, such ship or

place will be to the *eastward* of Greenwich ; but if the time at a ship or place be *less* than the time, at the same instant, at Greenwich, such ship or place will be to the *westward* of Greenwich.

Since the longitude of any place on the earth is expressed by the difference of time between that place and the Royal Observatory at Greenwich ; therefore, to determine the longitude of a given place, we have only to find the time of the day at that place, and also at Greenwich, at the same instant ; then, the difference of these times being converted into motion, by Problem II., page 342, or, more readily, by Table I., Vol. II. in this work, the longitude of such given place will be obtained.

The readiest, and, indeed, the most simple method of finding the longitude at sea, *in theory*, is by a chronometer, or other machine, that will measure time so exactly true as to go uniformly correct in all places, seasons, and climates : for, such a machine being once regulated to the meridian of the Royal Observatory at Greenwich, would always show the true time under that meridian, though removed in a ship to the most distant parts of the globe,—even to the utmost extent of longitude.

Although such a perfect piece of mechanism can scarcely be hoped for or expected to result from the ablest and best applied course of human industry,—yet, in short voyages, a chronometer may be safely employed in the determination of the longitude.

OF FINDING THE ERROR AND THE RATE OF A CHRONOMETER.

Before a chronometer can be applied to the important purpose of navigating a ship over the boundless ocean, it becomes indispensably necessary to determine its *error* for mean time at Greenwich, and to establish its daily *rate* of going to the most rigid degree of mathematical exactness.

The *error* of a chronometer signifies how much it is *fast* or *slow* for mean time at the Royal Observatory at Greenwich ; the *rate*, how much it *gains* or *loses* on mean time in 24 hours.

In the Royal Navy it is customary—it is the *general custom*—to send chronometers on board the ships with the *error* and the *rate* as established at the Naval College in the Dock-yard at Portsmouth.—This, to say the least of it, is a most *mistaken custom* :—indeed, it is fraught with danger :—for, were it possible to waft a chronometer on board a man-of-war on the wings of an angel, or to transport it by the

magnetic touch of a neighbouring ship's wind, yet, the change of atmosphere, the difference of temperature, and the *local attraction* arising from the presence of iron with which one of His Majesty's ships abounds, would most assuredly cause its rate, as settled at the College, to suffer a certain degree of variation.—It is a fact well known to *acute* observers, that the rate of a chronometer will be *altered* by simply placing it within the magnetic influence of a ship's compass; and hence it invariably happens that, after an interval of about three weeks, or in the *short run betwixt Portsmouth and Malta*, the master finds it necessary to give the timekeeper a *new rate*.

In the year 1823, on my return from the East Indies, I had the pleasure of a friendly interview, at the Cape of Good Hope, with the Reverend Mr. Fallows, then His Majesty's Astronomer at the Cape: when our conversation turned upon chronometers, the learned divine communicated a fact of the greatest importance to the nautical world; which fact I shall now narrate for the information of the captains and masters of the Royal Navy, and for the guidance of all persons who trust to a timekeeper in the navigating of a ship.

The Reverend Mr. Fallows, a most accomplished mathematician, and an excellent astronomer, was furnished, by order of Government, with three chronometers, for the use of the Observatory at the Cape of Good Hope:—those were of the very best description, and each equally good; for they had severally withstood the test of astronomical scrutiny, for many months, in the Royal Observatory at Greenwich.—One of those superior machines he placed *abast the foremost bulk-head* of the captain's cabin, and the other two at the extremities thereof;—one close to an *iron-knee* on the starboard side, and the other equally close to an *iron-knee* on the larboard side of the ship.—After the lapse of a few days, the reverend gentleman found that the starboard and larboard chronometers (names by which the two machines at the ship's sides were distinguished) did *not keep uniform pace* with that which was placed at the midship part of the bulk-head.—On reaching Madeira, he ascertained, by celestial observation, that the last-mentioned machine had preserved the *rate* which he gave it on *board* previous to sailing from England, and that the *rates* of the other two had undergone a *sensible alteration*.

Being rather surprised at witnessing such an unexpected difference in timekeepers of equally approved excellence, he therefore changed the position of the side chronometers,—the starboard one was placed on the larboard side, and *vice versa*, the larboard one on the starboard side of the ship. At the end of the week he observed that the side machines had again changed their *rates* with respect to that which

stood midway betwixt them. Those machines were then restored to their original *berths*; and after a few days they were observed to *return to the respective rates which they exhibited previous to their removal*. The reverend gentleman tried the same experiment several times betwixt the equator and the Cape of Good Hope; and in every trial the side chronometers were found to depart from their daily *rates*; to which *rates* they again returned in the course of a few days, after being replaced in their original positions; and kept to them so long as they were *suffered to remain undisturbed*. Here, then, we have an authenticated proof of *the extreme sensitiveness* of chronometers; and of how very susceptible they are of being *affected* by a transition of place, a change of temperature, or a difference in the sphere of local attraction to which they are exposed. And from this we can easily perceive *the great impropriety,—nay, the evident danger,—*of putting confidence in a timekeeper whose *error and rate* have been established on shore. And, hence it becomes clearly manifest, that a chronometer ought to be fixed in its *resting-place* on board a man-of-war, 8 or 10 days *before its rate is determined*; so that it may have time to become reconciled to the change of atmosphere,—to the difference of temperature,—and to the local magnetic attraction of the ship; from which *resting-place* it should never be removed until it is about to be returned to one of His Majesty's stores.

Now, since it is clearly evident that a chronometer should *never be disturbed after its rate has been duly established*,—the captain, therefore, in whose charge it is, ought to be provided with an excellent pocket-watch, or *working chronometer*, that will go *perfectly uniform* for a short interval, or during the *space of a few hours*; which watch he should *lend* to the master for the purpose of noting the times of observation. By means of a watch of that description, the *error* and the *rate* of a chronometer can be correctly determined on board a ship, according to the following Problems:—

PROBLEM I.

To find the Error and the Rate of a Chronometer by Equal Altitudes of the Sun.

RULE.

Immediately before leaving the ship, let the master or some other competent person, carefully compare a *good pocket-watch* (one of the above description and *nearly regulated to mean time* at the given meri-

dian) with the chronometer ; and note down the respective times indicated by both machines.—Then, taking with him a sextant, an artificial horizon, and an assistant to write down the times and the corresponding altitudes, he should go on shore at a place *whose longitude is very correctly known*, and proceed in the following manner, viz. :—

In the morning, when the sun is nearly in the prime vertical, or, at least, when he is *rather more than two hours* distant from the meridian, let an observer bring down his lower limb, by moving the index of the sextant, till it is in contact with the upper limb as seen in the artificial horizon :—then move the index forward ;—set it to the next tenth or twentieth minute, according to the quickness of the sun's ascension, and wait, with the sight directed through the telescope to the sun's image in the artificial horizon, till the contact of the limbs takes place. At the precise moment of contact, the observer is to cry out *stop* ; on which the assistant is to note the exact time per watch ; which being increased by 12 hours, is to be written down abreast of the sun's altitude. Move the index forward to the next tenth, or twentieth minute, and proceed as before, until five or *more* observations are taken. In the afternoon, when the sun's distance from the meridian is within *a few minutes* of what it was at *the last of the forenoon* observations, let the observer bring the sun's limbs again in contact in his artificial horizon :—then, let him set the index of his sextant to the *last* of the morning altitudes, and wait, with the sight directed through the telescope to the sun in the artificial horizon, till there is a perfect contact of the limbs : at the *precise moment of contact* he is to cry out *stop*, and the assistant is to note the exact time ; which being increased by 24 hours, is to be written down abreast of the corresponding time in the morning.—Move the index *back* to the *next less* of the morning altitudes, and proceed as before ; then, to the *next less* ; and so on, till the times corresponding to all the altitudes are noted down.

On the observer's return to the ship, which ought to be without a moment's delay, let him again compare the watch with the chronometer, and note down the respective times indicated by both machines :—hence, he can be satisfied relative to the *uniform* going of the watch during the interval ; this is known by its showing the *same difference* of time at both comparisons.

The sum of the morning times divided by their number, will give the forenoon *mean* :—the same being done with the times past noon, the result will be the afternoon *mean*.—Add the two *means* together ; take half the sum, and it will be the mean time of noon *per watch*, *incorrect*.—Reduce this to the meridian of Greenwich by Problem III.,

page 342; to which let the sun's declination be reduced by Problem XIV. page 357; and the equation of time by the Rule in page 427.

When the equation of time is *additive* in page I. (not II.) of the month in the Nautical Almanac, 24 hours + the reduced equation; but when *subtractive*, 24 hours — the reduced equation will be the true time of mean noon at the place of observation.

The difference between the *means* will be the *interval* betwixt the observations; with which *interval* and the latitude of the place enter Table XIII.; and, with the *interval* and the reduced declination, enter Table XIV.:—take out the corresponding equations, and note whether they are *affirmative* or *negative*, agreeably to the Rule in page 22:—then, with the sum or difference of those two corrections, according as they are of the same or of contrary signs, and half the sum of the variation of the sun's declination for the days immediately preceding and following the given mean noon, compute the equation of equal altitudes by the formula in page 23.—Now, to the mean time of noon per watch *incorrect*, apply the equation of equal altitudes, by addition or subtraction, according as its sign may be *affirmative* or *negative*; and the sum or difference will be the correct mean time per watch when the sun's centre was on the meridian of the place of observation: the difference between which and the *true time* of mean noon, found as above, will be the *error* of the watch for mean time at the given place.

To the time shown by the watch on its *comparison* with the chronometer, add the *error* if it be *slow*, or subtract it if *fast*; and the sum or difference will be the correct mean time at *the moment of comparison*. To this apply the *established* longitude of the place by addition if *west*, or subtraction if *east*; and the result will be the correct mean time at Greenwich. Now, the difference between this and the time indicated by the chronometer at *the moment of comparison*, will be the *error* of this machine for mean time at Greenwich; which will be *fast* when the chronometer time is the *greatest*; otherwise it will be *slow*.

Repeat this operation for a succession of days, or as often as possible; note down the errors for each day, and thus any *irregularity* in the going of the chronometer will be detected:—then, the *sum* of the errors divided by their number, will be the *mean error*; and this being divided by the number of days (an *uneven* number, such as 9, 11, 13, &c., should be preferred) contained between the first and the last times of observation, the result will be the *mean rate*.—The *error* and the

rate thus established will be for *the day which stands evenly between the first and the last days* of observing the equal altitudes.

Should the duties of the ship render it inconvenient to repeat the operation for a succession of days, the observer is, in this case, to determine the *error* of the chronometer *again*, after a lapse of 10 or 12 days, or a longer interval if possible ;—let him find the *difference* between the two *errors* ; then, this being divided by the number of days contained in the interval between the observations, the result will be the *daily rate* of the chronometer.—Reduce the *error* on the last day of observation to *noon* ; and it will be a constant quantity to be applied to the *future times* shown by the chronometer.

Now, to know *whether the daily rate is additive or subtractive*. If the chronometer be *fast* at the time of the first observation, and its *error increasing*, the machine will evidently be *gaining* on mean time ; but if *decreasing*, it will be *losing* for mean time :—in the first case, the *daily rate* is to be applied by *subtraction* ; and in the second by *addition*. Again,—if the chronometer be *slow* at the first observation, and its *error increasing*, the machine will be manifestly *losing* for mean time ; but, if *decreasing*, it will be *gaining* on mean time :—in the first part of this instance the *daily rate* is to be applied by *addition* ; and in the other, by *subtraction* to the times indicated by the chronometer.

Example.

May 1st, 1836 (*civil or nautical time*) the following equal altitudes of the sun were observed at a point about half a mile *due south* of the Observatory at Naples ; the latitude of which point is $40^{\circ}21'47''$ north, and its longitude $14^{\circ}15'4\frac{1}{2}''$ east, or $0^{\text{h}}57^{\text{m}}0^{\text{s}}18'$.—previous to observing in the forenoon the watch was compared with the chronometer ; the time by the former was $7^{\text{h}}42^{\text{m}}32'$, and by the latter $6^{\text{h}}45^{\text{m}}52'$; and in the afternoon, when the watch showed $4^{\text{h}}45^{\text{m}}38'$, the chronometer indicated $3^{\text{h}}48^{\text{m}}58'$.—and since the difference, viz., $0^{\text{h}}56^{\text{m}}40'$ is *the same* at both comparisons, it is a proof that the watch went uniform during the *interval* :—now, from those elements, the *error* of the chronometer for mean time at Greenwich is required.

Altitude of the Sun's Lower Limb.	Forenoon Mean Times of Obs., per Watch.	Afternoon Mean Times of Ob- servation, per Watch.
These are the double angles, as seen in an artificial hori- zon.	30° 0' . . 20 ^h 2 ^m 45 ^s	28 ^h 24 ^m 14 ^s
	30. 10 . . 20. 3. 14	28. 23. 45
	30. 20 . . 20. 3. 43	28. 23. 16
	30. 30 . . 20. 4. 12	28. 22. 47
	30. 40 . . 20. 4. 41	28. 22. 18

Mean = . . . 20^h 3^m 43^s Mean . . . 28^h 23^m 16^s

Afternoon mean = 28. 23. 16 Forenoon mn. = 20. 3. 43

Interval betw. the obs. = 8^h 19^m 33^s Sum = . . 48^h 26^m 59^s

Mean time of noon per watch *incorrect* = . . . 24^h 13^m 29^s 30^t

Established longitude of the given place, *east* = . . 0. 57. 0. 18

Greenwich time past noon, February 29th = . . . 23^h 16^m 29^s 12^t

The sun's declination *reduced to Greenwich time*, is 7° 27' 53" south; and the equation of time, 12^m 33^s 33^t :—and since this element is additive to *apparent* time; therefore the true time of mean noon is 24^h 12^m 33^s 33^t.

To find the Equation corresponding to the Equal Altitudes.

Equation, Table XIII., answ. to lat.

40° 21' 47" and interv. 8^h 19^m 33^s : = 11^m 59^s Negative.

Equation, Table XIV., answ. to dec.

7° 27' 53" and interv. 8^h 19^m 33^s : = 0. 52 Negative.

Sum of the equations = . . . 12^m 51^s Prop. Log. . 1. 1464

Variation of sun's declination = . 22^m 49^s 15 Prop. Log. . 0. 8970

Equation of equal altitudes = . . . —16^m 17^s Prop. L. 1. 0434

Mn. time of noon per watch *incorrect* = 24^h 13^m 29^s 30^t

Correct mean time of noon per watch = 24^h 13^m 13^s 13^t

True time of mean noon, as *above*. . 24. 12. 33. 33

Error of watch for true mean time = —0^m 39^s 40^t; which is *fast*.

Time per watch at last comp. with chron. 4. 45. 38. 0

Correct mean time at last comparison 4^h 44^m 58^s 20^t

Established longitude of the place, *east* = 0. 57. 0. 18

Correct mean time at Greenwich . . 8^h 47^m 58^s 2^t

Mean time by chron. at last comparison 3. 48. 58. 0

Error of the chron. for Greenw. time = 0^m 59^s 58^t; which is *fast*.

Notes.—1. Both parts of the equation of equal altitudes are *additive*: the first, because the sun is advancing towards the elevated pole of the heavens; and the second, because the sun's declination is increasing.—See *Remark*, page 23.

2. Since the morning observations belong *astronomically* to February 29th, therefore half the sum of the variation of the sun's declination for the days preceding and following the given one, viz., $22^{\circ}52'3'' + 22^{\circ}52'3'' + 2 = 22^{\circ}49'15''$, is to be taken as the variation of the declination.

3. Unity or 1, is rejected from the *index* of the sum of the proportional logarithms, agreeably to the Rule in page 22.

Again.

March 15th, 1836 (*civil or nautical time*), at the same place, in forenoon, when the mean of several altitudes of the sun's lower limb, viz., the double angle as seen in an artificial horizon, was $30^{\circ}20'$; mean of an equal number of noted times per watch was $7^{\circ}37'47''$; in the afternoon the mean of the times corresponding to the same altitudes was $4^{\circ}39'53''$.—After returning on board, the watch, on being compared with the chronometer, showed $4^{\circ}56'24''$, and this made $4^{\circ}0'49''$, the difference, viz., $0^{\circ}55'35''$, being the same that was found at the forenoon comparison:—required the error of chronometer mean time at Greenwich, and also its daily rate of going?

Mn. of foren. times =	$19^{\circ}37'47''$	Mn. of aft. times =	$28^{\circ}39'53''$
Afternoon mean . .	$28.39.53$	Forenoon mean . .	$19.37.47$
<hr/>		<hr/>	
Interval =	$9^{\circ}2^{\circ}6'$	Sum =	$48^{\circ}17'40''$
<hr/>		<hr/>	
Mean time of noon per watch <i>incorrect</i> =	$24^{\circ}8'50''$		
Established longitude of the place, <i>east</i> =	$0.57.0$		
<hr/>		<hr/>	
Greenwich time, past noon, March the 14th =	$23^{\circ}11'49''$		

The sun's declination reduced to Greenwich time is $2^{\circ}1'0''$ south, and the equation of time, $9^{\circ}3'5''$; and since this element is additive to *apparent* time; therefore the true time of mean noon is $24^{\circ}9'3''$.

To find the Equation corresponding to the Equal Altitudes.

Equation, Table XIII., answ. to lat.

$$40^{\circ}21'47'' \text{ and interv. } 9^{\text{h}}2^{\text{m}}6^{\text{s}} = 12^{\text{m}}13^{\text{s}} \text{ Negative.}^*$$

Equation, Table XIV., answ. to dec.

$$2^{\circ}1'0'' \text{ and interval } 9^{\text{h}}2^{\text{m}}6^{\text{s}} = 0^{\text{m}}13^{\text{s}} \text{ Negative.}^*$$

$$\text{Sum of the equations} = 12^{\text{m}}26^{\text{s}} \text{ Prop. Log.} = 1.1607$$

$$\text{Variation of the sun's declination } 23^{\circ}41'5''^* \text{ Prop. Log.} = 0.8807$$

$$\text{Equation of equal altitudes} = -16^{\circ}22' \text{ P.L.} = 1.0414^*$$

$$\text{Mn. time of noon per watch, incorrect} = 24^{\text{h}}8^{\text{m}}50^{\text{s}}0^{\text{s}}$$

$$\text{Correct mean time of noon per watch} = 24^{\text{h}}8^{\text{m}}33^{\text{s}}38^{\text{s}}$$

$$\text{True time of mean noon, as above} = 24.9.3.5$$

$$\text{Error of watch for true mean time} . . . +0^{\text{m}}29^{\text{s}}27^{\text{s}}; \text{ which is slow.}$$

$$\text{Time per watch at last comp. with chron. } 4^{\text{h}}56^{\text{m}}24^{\text{s}}0^{\text{s}}$$

$$\text{Correct mean time at comparison} = . . . 4^{\text{h}}56^{\text{m}}53^{\text{s}}27^{\text{s}}$$

$$\text{Established long. of the place, east} = 0.57.0.18$$

$$\text{Correct mean time at Greenwich} = . . . 3^{\text{h}}59^{\text{m}}53^{\text{s}}9^{\text{s}}$$

$$\text{Mn. time by chron. at last comparison } 4.0.49.0$$

$$\text{Error of the chron. for Greenwich time} = 0^{\text{m}}55^{\text{s}}51^{\text{s}}; \text{ which is fast.}$$

To find the Daily Rate of the Chronometer.

March 1st, 1836, chron. *fast* for mn. time at Greenw. $0^{\text{m}}59^{\text{s}}58^{\text{s}}$

March 15th, ditto, ditto $0.55.51$

$$\text{Difference in the interval betw. the observations} = 4^{\text{d}}7^{\text{d}} = 4^{\text{d}}117^{\text{d}}$$

Now, this being divided by 14 (the number of days in the *interval*), the quotient, or $0^{\text{m}}29^{\text{s}}4$, is the actual daily rate (*losing*) of the chronometer, on the supposition of *an uniform motion*; which rate is *additive*, for the reason shown in the last paragraph of the Rule in page 458.

Hence, the *error* of the chronometer for mean time at Greenwich,

* See the Notes 1, 2, and 3 at the end of the last Example, page 460.

at the moment of its last comparison with the watch on March 15th, is $0^{\circ}55'51''$; which reduced to noon is $0^{\circ}55'54''$, or $55^{\circ}9'$ and therefore *subtractive*;—and its daily rate is $0^{\circ}294'$; which *losing rate*, and therefore *additive*.

Remark.—In the above Problem it is *not* indispensably necessary that the latitude of the place and the value of the sun's declination should be very rigidly determined; because, as those *elements* are employed in taking out the equations from Tables XIII. and XI' trifling error therein will not sensibly affect the value of the resulting equation of the equal altitudes:—neither is the adjustment of a sextant of any great importance; provided, always, that it is correct, and shows the same angle at both observations.—And it is manifest, that the above is *the very best method* that can be adopted by persons *not possessing a transit instrument*, for finding the *error* and the *rate* of a chronometer.

PROBLEM II.

To find the Error and the Rate of a Chronometer by single Altitude of the Sun.

RULE.

In the forenoon, or afternoon, when the sun is nearly in the perpendicular, or at a proper distance from the meridian, let several altitudes of his lower limb be observed *on shore* by means of a sextant and artificial horizon, and the corresponding times per watch noted do this machine being *nearly regulated* to mean time at the given place. The sum of the altitudes divided by their number will give the mean altitude; and the sum of the times so divided, will give the mean time per watch. Reduce this to the meridian of Greenwich by Problem III., page 342; to which let the sun's declination be reduced by Problem XIV., page 357; and the equation of time by the Rule in page 427.—Reduce the *double* altitude of the sun to the true celestial altitude by Problem XXVII., page 379.—Then,

With the sun's true altitude, his reduced declination, or *polar distance*, and the latitude of the place; compute his horary distance from the meridian by Method V., page 434.

Now, if the observation be made in the afternoon, the sun's horary distance will be the *apparent time*; but if in the forenoon, its differ-

to 24 hours will be *the apparent time* : to this apply the reduced equation of time, as directed in page I. of the month in the Nautical Almanac, and the result will be the correct mean time of observation : the difference between which and the mean of the times per watch will be the *error* of this machine. To the time indicated by the watch at its *last* comparison with the chronometer, apply the *error* by *addition* if it be *slow*, or by *subtraction* if *fast*; the sum or difference will be the true mean time at that comparison. To this let the established longitude of the place, in time, be added if west, or subtracted if east; and the result will be the correct mean time at Greenwich :—then, the difference between this and the time shown by the chronometer at the *last* comparison, will be its *error* for mean time at Greenwich : which will be *fast* when the chronometer time is the *greatest*; otherwise, it will be *slow*.—Repeat this operation on some *future* day, and as *near to the same hour as possible* :—then, the difference between the *two errors*, so determined, being divided by the number of days contained in the interval between the observations, will be *the daily rate* of the chronometer on the *supposition of an uniform motion* :—and to know whether it is a *losing rate* or a *gaining rate*, see the *last* paragraph in the preceding Rule, page 458. Hence, the *error* and the *rate* of the chronometer will be correctly established. Reduce the *error* to noon, and it will be a constant quantity to be applied to all the future times shown by the *same machine*.

Remark.—The watch should be carefully compared with the chronometer *immediately* before and after the observation; and the times indicated by both machines written down: if the same difference be seen at each time, it will be a proof that the watch has gone equally in the interval between the moments of observation and comparison.

The index error of the sextant must be truly determined; and the position of the place of observation correctly established;—its longitude ought to be known to *the sixtieth part of a second*.

Example

May 1st, 1836, at a point in the vicinity of Cadiz, and under the meridian of the observatory of *St. Fernandó*, the latitude of which point is $36^{\circ}27'15''$ north, and its longitude $6^{\circ}12'16\frac{1}{2}''$ west, or $0^{\text{h}}24^{\text{m}}49^{\text{s}}6'$; the following altitudes of the sun's lower limb were observed in an artificial horizon: the index error of the sextant was $1'20''$ *additive*.—Previous to the observation the watch was compared with the chronometer; the time shown by the former was $4^{\text{h}}10^{\text{m}}15'$,

and by the index $4^{\circ}34'44''$:—and, after the observation, when the watch showed $4^{\circ}35'11''$, the chronometer indicated $5^{\circ}17'40''$; the difference was $0^{\circ}24'26''$, being the same at both comparisons, it proved that the watch went uniform during the interval.—Now, from these elements the error of the chronometer is required.

Time of obs. per watch $4^{\circ}25'25''$

Ditto $4^{\circ}25'35''$

Ditto $4^{\circ}26'18''$

Ditto $4^{\circ}26'35''$

Ditto $4^{\circ}27'18''$

Sum = $20^{\circ}12'20''$

Me. time per watch = $4^{\circ}26'28''$

Long. of the place W.

In time = $0^{\circ}24'49.6''$

Greenwich time = $4^{\circ}51'17.6''$

Sun's dec. at noon.

May 1st. $15^{\circ}10'21''$ N.

Correction for

$4^{\circ}51'17.6''$ = $-3.38''$

Sun's red. dec. $15^{\circ}13.59''$

Sun's N. pol. dist. $74^{\circ}46'1''$

Alt. of sun's limb = $53^{\circ}46.5'$

Ditto $53.34.5$

Ditto $53.22.5$

Ditto $53.10.5$

Ditto $52.58.5$

Sum = $266^{\circ}54.1'$

Mean altitude = $53^{\circ}22'50''$

Index error $+1'29''$

Obs. double angle = $53^{\circ}24'10''$

Sun's observed alt. $26^{\circ}42'10''$

Sun's semidiameter $+15.5''$

Sun's apparent alt. $26^{\circ}57'50''$

Refrac. = $1'51''$ } Diff. $1.4''$

Parallax $0.8''$ }

Sun's true altitude = $26^{\circ}56'11''$

Equa. of time at

noon, May 1 = $3^{\circ}6'8''$ S.

Correction for

$4^{\circ}51'17.6''$ = $+1.29''$

Red. equa. of time $3^{\circ}7'37''$ S.

To find the Hour Angle and the Error of the Chronometer.

Sun's true altitude =	. . . 26° 56' 15"		
Sun's north polar distance	74. 46. 1	Log. co-secant =	. . . 0.015533
Latitude of the place	. 36. 27. 15	Log. secant =	. . . 0.094564
			<hr/>
Sum =	. . . 138° 9' 31"		
			<hr/>
Half sum	. . . 69° 4' 45½"	Log. co-sine =	. . . 9.552760
Remainder	. . . 42. 8. 30½	Log. sine =	. . . 9.826702
			<hr/>
		Sum =	. . . 19.489559
			<hr/>
Arch =	. . . 33° 45' 13"	Log sine =	. . . 9.744779½
			<hr/>
Hour angle =	. . . 67° 30' 26" = 4 ^h 30 ^m 1' 44"	<i>Apparent time.</i>	
Reduced equation of time, <i>subtractive</i>	= - 3. 7. 29		
			<hr/>
Correct mean time of observation	. . . 4 ^h 26 ^m 54' 15"		
Time of observation <i>per</i> watch =	. . . 4. 26. 28. 0		
			<hr/>
Error of the watch for mean time =	. . . + 0 ^m 26' 15"	<i>Slow.</i>	
Time per watch at <i>last</i> comp. with chron.	4. 53. 11. 0		
			<hr/>
Correct mean time at comparison	. . . 4 ^h 53 ^m 37' 15"		
Estab. long. of the place, <i>west</i> , in time =	+ 0. 24. 49. 6		
			<hr/>
Correct mean time at Greenwich =	. . . 5 ^h 18 ^m 26' 21"		
Mean time by chron. at <i>last</i> comparison	5. 17. 40. 0		
			<hr/>
Error of the chron. for Greenwich time =	0 ^m 46' 21"	<i>Slow.</i>	

Again.

May 22nd, 1836, at the same place, the mean of several altitudes of the sun's lower limb, viz., the double angle, as seen in an artificial horizon, was 58° 41' 8", and the mean of an equal number of noted times *per* watch 4^h 23^m 49"; and the index error of the sextant 1' 20" *additive*.—After returning on board, the watch was compared with the chronometer, the time shown by the former was 4^h 53^m 15", and by the latter 5^h 16^m 54" (the difference, viz., 0^h 23^m 39"; being the same that was found at the comparison previous to observation): required the

H H

error of the chronometer for mean time at Greenwich, and also its daily rate of going ?

Time of observ. per watch = . . . 4 ^h 23 ^m 49 ^s ·0	Sun's red. declin.= 20°29'27"N.
Long. of the place, west = . . . 0.24.49.6	Sun's north polar distance = . . 69°30'33"
Greenwich time = 4 ^h 48 ^m 38 ^s ·6	Reduced equation of time = . . 3 ^m 39 ^s ·55 ^{sub} .

The double obs. angle 58°41'8" being cor. for index error, and then red., shows the sun's true alt. to be 29°35'31".

Sun's north polar distance = . . 69.30.33	Log. co-sec.0.028387
Latitude of the given place = . . 36.27.15	Log. secant 0.094564

$$\text{Sum} = 135:33:19"$$

Half sum = . . . 67°46'39½"	Log.co-sine 9.577723
Remainder = . . . 38.11.8½	Log. sine 9.791137

$$\text{Sum} = . 19.491811$$

$$\text{Arch} = 33:51:11" \quad \text{Log. sine} = 9.745905\frac{1}{2}$$

$$\text{Hour angle} = 67:42:22" = 4:27: 9:32' \text{ Ap. time.}$$

$$\text{Reduced equation of time, subtractive} = . . . 3.39.55$$

$$\text{Correct mean time of observation} = . . . 4:23:29:37'$$

$$\text{Time of observation per watch} = . . . 4.23.49 -$$

$$\text{Error of the watch for mean time} = . . . -0:19:23' \text{ Fast.}$$

$$\text{Time per watch at last comparison with chron.} = 4.53.15. 0$$

$$\text{Correct mean time at comparison} = . . . 4:52:55:37'$$

$$\text{Established long. of the place, west, in time} = +0.24.49. 6$$

$$\text{Correct mean time at Greenwich} = . . . 5:17:44:43'$$

$$\text{Mean time by chronometer at last comparison} = 5.16.54. 0$$

$$\text{Error of the chronometer for Greenwich time} = 0:50:43' \text{ Slow.}$$

To find the daily Rate of the Chronometer.

May 1st, 1836, chron. slow for mn. time at Greenwich 0^m 46' 21"
 May 22nd, ditto ditto. 0. 50. 43

Difference in the interval between the observations = 4:22' or 4:367.
 Now, this being divided by 21 (the number of days in the interval), the quotient, or 0. 208, is the actual daily *rate*, *losing*, on the supposition of an uniform motion; which *rate* is *additive*, for the reason shown in the second part of the *last* paragraph of the Rule in page 456. Hence, the *error* of the chronometer for mean time at Greenwich, at the moment of its comparison with the watch on May 22nd, is 50:43: *slow*; which, reduced to noon of that day, is 50:40: *slow*; and therefore *additive* to the *future* times shown by the chronometer:—And the *daily rate* of the machine is 0:208, which is *additive*, because it is a *losing rate*.

The uniformity of the *rate* may be satisfactorily proved by repeating the above operation, agreeably to the directions contained in the *third* paragraph, *reckoning from the bottom* of the preceding Rule, page 463.

With a chronometer whose *error* and *rate* have been thus duly established *on board the ship to which it belongs*, and whose daily *gain*, or *loss*, is perfectly uniform, the navigator may proceed to sea with the most unbounded confidence; for, by means thereof, the longitude of his ship may be correctly determined in any part of the ocean, so long as its *rate* remains unaltered. Whereas, by carrying a chronometer on board a ship after its *error* and *rate* have been settled at any observatory on shore, and then trusting to its correctness, the longitude deduced therefrom may be *out* 60 or 80 miles before the ship is thirty days at sea; for, as stated in page 454, *the removal of a chronometer always alters its daily rate*.

Remarks.—The position of a celestial object most favourable for determining the mean time with the greatest accuracy, is, when it is in the prime vertical; that is, when it bears either due east or due west at the place of observation, or, if it be circumpolar, when it is in that part of its diurnal path which is in contact with an azimuth circle; viz., when the log. sine of its altitude = log. sine of the latitude + radius — log. sine of its declination; because, then, the change of altitude is quickest, and the extreme accuracy of the latitude not very essentially requisite. The nearer a celestial object is to either of these positions, the nearer will the time, deduced from its altitude, be to the truth; as, then, the unavoidable small errors which generally creep

into the observations, or a few miles difference in the latitude, will have little or no effect on the resulting time.

Table XLVII. contains the time or distance of a celestial object from the meridian at which its altitude should be observed, in order to determine the *apparent* time with the greatest accuracy; and Table XLVIII. contains the corresponding altitude most advantageous for observation. But, since those Tables are adapted to the declination of a celestial object when it is of the same name with the latitude of the place of observation, they will not, therefore, indicate either the proper time or the altitude when those elements are of contrary denominations: in this case, since the sun or other celestial object comes to the prime vertical before it rises, and therefore does not bear due east or west while above the horizon, the observation for determining the *apparent* or mean time from its altitude must be made while the object is near to the horizon; taking care, however, not to take an altitude for that purpose under 3 or 4 degrees, on account of the uncertain manner in which the atmospherical refraction acts upon very small angles of altitude observed adjacent to the horizon.—See explanation to the above-mentioned Tables, pages 119 and 120.

PROBLEM III.

Given the Latitude of a Ship, or Place, and the Observed Altitude of the Sun's Lower Limb; to find the Longitude by means of a Chronometer.

RULE.

Let several altitudes of the sun's lower limb be observed, at a proper distance from the meridian, and the corresponding times per watch that shows seconds, noted down; of these take the means respectively.

As the chronometer *must not*, on any account, be disturbed or taken on deck for the purpose of noting the time indicated thereby; therefore, let a good pocket-watch be carefully compared therewith *immediately* before the observation, and the times shown by each written down: and it will be prudent to make another comparison after the observation, so as to be satisfied that there has been no interruption, or *stoppage*, in the movement of the watch.

Find the *interval* shown by the *watch* between the moment of *comparison* and the mean of the times of taking the altitudes; which *add* to the time indicated by the chronometer, if the comparison be made *before*, but *subtract* therefrom if *after*, the observation; the sum, or

difference, will be the time *per chronometer* of observing the mean altitude. To this apply its original *error*, by addition if *slow*, or subtraction if *fast*; and also its *accumulated rate*, affirmatively if *losing*, or negatively if *gaining*; and the result will be *the correct mean time of observation at Greenwich*, according to the chronometer. To the Greenwich time, thus known, reduce the sun's declination, and the equation of time, by Problem XIV., page 357. And let the mean altitude of the sun's limb be reduced to the true central altitude by Problem XXIII., page 374.—Then,

With the sun's true central altitude, its polar distance, and the latitude of the ship, or place, compute the mean time of observation by Problem II., page 435.. Now, the difference between this and the *mean time of observation at Greenwich*, per chronometer, will be the longitude of the place of observation, *in time*:—*East*, if the time at that place be the *greatest*; otherwise, *west*: which convert into motion or degrees by Problem II., page 342.

Note 1.—It will be advisable to adopt the general practice of comparing the watch with the chronometer *previous* to the observation; because this will conduce to the uniformity of the calculation, and it will, moreover, be the means of preventing the computer from falling into an error in going through with the work.

2. The *accumulated rate* is found by multiplying the *daily rate* of the chronometer by the number of days, *and parts of a day*, that may have elapsed between the *noon* of the day on which the *error* and *rate* of the machine were established, and the mean time of observation on the given day.

Example.

At sea, June 10th, 1836, in latitude $30^{\circ}15'0''$ south, the mean of several altitudes of the sun's lower limb was $12^{\circ}10'4''$, and the mean of the corresponding times per watch $20^h2^m16^s$; the height of the eye above the level of the sea was 23 feet, and the index error of the sextant $0'45''$ *subtractive*.—Immediately before the observation, the watch was compared with a chronometer, the *error* and *rate* of which were established the 15th March, 1836, on which day, at noon, it was $55'9$ *fast* for mean time at Greenwich, and *losing* daily at the rate of $0'294$. At the moment of comparison, the time indicated by the watch was $19^h58^m54^s$, and by the chronometer $13^h58^m26^s$ (the times shown by the machines are, of course, increased by 12 hours); required the longitude of the place of observation?

Mean of the times of observation per watch = 20^h 2^m 16^s
 Time shown by watch at comparison with chronometer = 19.58.54

Interval between comparison and observation = +0^h 3^m 22^s
 Time indicated by chronometer at comparison = 13.58.26

Time per chronometer of observing the mean altitude = 14^h 1^m 48^s
 Original *error* of chronometer = . . . 55^m 9^s *fast*, or . . . -56.
 Accumulated rate = $0.294 \times 87\frac{1}{2}$ days = 25^m 7^s *loss*, or . . . +26

Correct mean time of observation at Greenwich = . . . 14^h 1^m 18^s

The sun's declination reduced to the mean time at Greenwich is 23° 5' 32" north, and the equation of time 0^m 50^s, taken to the *nearest second*; which, for the present purpose, is sufficiently near to the truth.—And, the observed altitude of the sun's lower limb, reduced to the true central altitude, is 12° 16' 22".

To compute the Mean Time at Ship, and the Longitude.

Sun's true central altitude = . . . 12° 16' 22"
 Sun's south polar distance = . . . 113. 5.32 Log. co-secant 0.036271
 Latitude of the ship = . . . 30. 15. 0 Log. secant 0.063569

Sum = 155° 36' 54" Constant log. 6.301030

Half sum = 77° 48' 27" Log. co-sine 9.324687
 Remainder = 65.32. 5 Log. sine 9.959143

Sun's hor. distance from the meridian = 3^h 55^m 42^s Log. rising 5.684700

Apparent time of observation = . . . 20^h 4^m 18^s
 Reduced equation of time, *subtractive* = - 0.50

Mean time of observation = . . . 20^h 3^m 28^s
 Mean time of observ. at Greenwich = 14. 1. 18

Longitude of the ship, *in time* = . . . 6^h 2^m 10^s = 90° 32' 30" *East*.

PROBLEM IV.

Given the Latitude of a Ship, or Place, and the Observed Altitude of the Moon's Upper or Lower Limb; to find the Longitude by a Chronometer.

RULE.

Let several altitudes of the moon's limb be observed, when she is at a proper distance from the meridian, and the corresponding times, per watch that shows seconds, noted down; of these take the means respectively.

Then proceed, as directed in the second and third paragraphs of the preceding Rule, until the *correct mean time of observation at Greenwich*, per chronometer, is found:—To this time reduce the *mean sun's right ascension* by Problem V., page 344.* To the same time let the moon's *semidiameter* and *horizontal parallax* be reduced by Problem XV., page 361, and her *right ascension* and *declination* by Problem XVI., page 364. And, let the observed altitude of her limb be reduced to the *true central altitude* by Problem XXIV., page 376.—Then,

With the moon's *true central altitude*, her *reduced declination*, or *polar distance*, and the *latitude* of the ship; compute the *mean time of observation* by Problem III., page 437.—Now, the difference between this *and the mean time of observation at Greenwich*, per chronometer, will be the *longitude* of the place of observation, in time:—*East*, if the time at that place be the *greatest*; otherwise, *west*: which convert into motion, or degrees, by Problem II., page 342.

See Notes 1 and 2, at bottom of the preceding Rule, page 469.

Example.

August 1st, 1836, in latitude $50^{\circ}48'$ north, the mean of several altitudes of the moon's upper limb, east of the meridian, was $27^{\circ}31'32''$ and the mean of the corresponding times, per watch, $13^{\text{h}}5^{\text{m}}10^{\text{s}}$; the height of the eye above the level of the horizon was 17 feet; and the index error of the sextant $2'25''$ additive. Immediately before the observation the watch was compared with a chronometer, the error and rate of which were established on the 22nd May 1836, on which day, at noon, it was $50'7''$ slow for mean time at Greenwich, and losing daily at the rate of $0'208''$: at the moment of comparison when the

* See Article 37, page 313, relative to the mean sun's right ascension.

watch showed $13^{\text{h}}2^{\text{m}}4^{\text{s}}$, the chronometer indicated $17^{\text{h}}37^{\text{m}}15^{\text{s}}$; required the longitude of the place of observation?

Mean of the times of observation, per watch = . . . $13^{\text{h}}5^{\text{m}}10^{\text{s}}$

Time shown by watch at comparison with chronometer = 13. 2. 4

Interval between comparison and observation = . . . $+0^{\text{h}}3^{\text{m}}6^{\text{s}}$

Time indicated by chronometer at comparison = . . . 17.37.15

Time, per chronometer, of observing the mean altitude = $17^{\text{h}}40^{\text{m}}21^{\text{s}}$

Original *error* of chronometer = . $50^{\text{m}}7^{\text{s}}$ *slow*, or . . . + 51

Accumulated rate = $0^{\text{m}}20^{\text{s}}8 \times 71\frac{1}{4}$ days = $14^{\text{m}}9^{\text{s}}$ *loss*, or . . . + 15

Correct mean time of observation at Greenwich = . . . $17^{\text{h}}41^{\text{m}}27^{\text{s}}$

The *mean* sun's right ascension reduced to the mean time at Greenwich is $8^{\text{h}}43^{\text{m}}24^{\text{s}}$ to the *nearest second*. The moon's reduced semi-diameter, corrected for *augmentation*, is $15'30''$, and horizontal parallax $56'53''$; her reduced right ascension, taken to the *nearest second*, is $1^{\text{h}}1^{\text{m}}36^{\text{s}}$, and declination $3^{\circ}53'46''$ north. And the observed altitude of her upper limb reduced to the true central altitude is $28^{\circ}3'8''$

To compute the Mean Time at Ship, and the Longitude.

Moon's true central altitude = . $28^{\circ}3'8''$

Moon's north polar distance = . 86. 6. 14 Log. co-sec. 0.001005

Latitude of the ship, or place = . 50. 48. 0 Log. secant 0.199263

Sum = $164^{\circ}57'22''$ Const. log. 6.301030

Half sum = $82^{\circ}28'41''$ Log. co-sine 9.116959

Remainder = 54.25.33 Log. sine 9.910283

Moon's hor. dist., *east* of the merid. = $3^{\text{h}}14^{\text{m}}6^{\text{s}}$ Log. rising 5.528540

Moon's reduced right ascension = 1. 1.36

Right ascension of the meridian = $21^{\text{h}}47^{\text{m}}30^{\text{s}}$

Mean sun's reduced right ascension = 8.43.24

Mean time of observation = . . . $13^{\text{h}}4^{\text{m}}6^{\text{s}}$

Mean time of observ. at Greenwich = 17.41.27

Longitude of the ship, in time = . $4^{\text{h}}37^{\text{m}}21^{\text{s}}$ = $69^{\circ}20'15''$ west.

Note.—The *horary distance* is determined by Method I., page 430, this being the most concise and uniform mode of computation.

PROBLEM V.

Given the Latitude of a Ship, or Place, and the Observed Altitude of a Planet's Centre ; to find the Longitude by a Chronometer, or Time-keeper.

RULE.

Let several altitudes of the planet's centre be observed, when it is at a proper distance from the meridian, and the corresponding times, per watch that shows seconds, noted down ; of these take the means respectively.

Then proceed, as directed in the second and third paragraphs of the *Rule*, for the sun, Problem III., page 468, until *the correct mean time of observation* at Greenwich, per chronometer, is found :—To which time let the *mean sun's* right ascension be reduced by Problem V., page 344 ;* and also the geocentric right ascension and declination of the planet, by Problem XVII., page 366 ; and let the observed altitude be reduced to the true altitude by Problem XXV., page 377. —Then,

With the planet's true altitude, its declination, or polar distance, and the latitude of the ship, compute the mean time of observation by Problem IV., page 439.—Now, the difference between this and the *mean time of observation* at Greenwich, per chronometer, will be the longitude of the place of observation, in *time* :—*East*, if the time at such place be the greatest ; otherwise, *west* : which convert into motion, or degrees, by Problem II., page 342.

See *Notes* 1 and 2, at bottom of the Rule to Problem III., page 469.

Example.

At sea, March 1st, 1836, in latitude $26^{\circ}40'$ south, the mean of several altitudes of the centre of Jupiter, *west* of the meridian, was $23^{\circ}14'43''$, and the mean of the corresponding times, per watch, $10^h51^m13^s$; the height of the eye above the level of the horizon was 18 feet ; and the index *error* of the sextant $2'20''$ *subtractive*.—Imme-

* See Article 37, page 313, relative to the *mean sun's* right ascension.

diately before the observation, the watch was compared with a chronometer, the *error* and *rate* of which were established the 1st January, 1836; on which day, at noon, it was 3^m51^s *slow* for mean time at Greenwich, and *gaining daily* at the rate of 0^m41.—At the moment of comparison, when the watch showed 10^h47^m54^s, the chronometer indicated 6^h34^m10^s; required the longitude of the place of observation?

Mean of the times of observation per watch = 10^h51^m13^s
 Time shown by watch at comparison with chronometer = 10. 47. 54

Interval between comparison and observation = 0^h 3^m19^s
 Time indicated by chronometer at comparison = 6. 34. 10

Time, per chronometer, of observing the mean altitude = 6^h37^m29^s
 Original error of the chronometer . 3^m51^s *slow* = . . . + 3. 51
 Accumulated rate = 0^m41 × 60 $\frac{1}{2}$ days = 24^m7^s *gain*, or . . . — 0. 25

Correct mean time of observation at Greenwich = . . . 6^h40^m55^s

The *mean sun's* right ascension reduced to Greenwich time is 22^h38^m22^s to the *nearest second*; the *geocentric right ascension* of Jupiter is 6^h25^m33^s, and his *declination* 23[°]30[']39["] north, each taken to the nearest second. The observed altitude of the planet reduced to the true central altitude is 23[°]6[']8["]

To compute the Mean Time at Ship, and the Longitude.

Jupiter's true central altitude = . 23[°] 6' 8"
 Jupiter's south polar distance = . 113. 30. 39 Log.co-sec 0. 037638
 Latitude of the ship = 26. 40. 0 Log.secant 0. 048841

Sum = 163[°]16[']47["] Const. log. 6. 301080

Half sum = 81[°]38[']23 $\frac{1}{2}$ ["] L.co-sine 9. 162548

Remainder = 58. 32. 15 $\frac{1}{2}$ Log. sine 9. 930940

Jupiter's hor. dist., *west* of the merid. = 3^h 3^m 9^s L. rising 5. 480997

Jupiter's geocentric right ascension = 6. 25. 33

Right ascension of the meridian = . 9^h28^m42^s

Mean sun's reduced right ascension = 22. 38. 22

Mean time of observation = . . . 10^h50^m20^s

Mean time of observ. at Greenwich = 6. 40. 55

Longitude of the ship, in *time* = . 3^h 9^m25^s47["]21["]15["] East.

Note.—The *horary distance* is determined by Method I., page 430, because of its *uniformity and conciseness*.

PROBLEM VI.

Given the Latitude of a Ship, or Place, and the Observed Altitude of a Fixed Star; to find the Longitude by a Chronometer, or Timekeeper.

RULE.

Let several altitudes of the star be observed, when it is at a proper distance from the meridian, and the corresponding times, per watch that shows seconds, noted down; of these take the means respectively.

Then proceed, as directed in the second and third paragraphs of the *Rule* to Problem III., page 468, till the *correct mean time of observation* at Greenwich, per chronometer, is found; to which time let the *mean sun's* right ascension be reduced by Problem V., page 344.* And let the observed altitude of the star be reduced to the true altitude by Problem XXVI., page 378. Take the right ascension and declination of the star from the Nautical Almanac, between pages 368 and 407. Then, with the star's true altitude, its declination, or polar distance, and the latitude of the ship, or place, compute the mean time of observation by Problem V., page 441.—Now, the difference between this and the *mean time of observation at Greenwich*, per chronometer, will be the longitude of the place of observation, in *time*:—*East*, if the time at such place be the *greatest*; otherwise, *west*: which convert into motion, or degrees, by Problem II., page 342.

See *Notes* 1 and 2, at bottom of the *Rule* to Problem III., page 469.

Example.

At sea, May 1st, 1836, in latitude $40^{\circ}30'$ north, the mean of several altitudes of the star Regules, *west* of the meridian, was $20^{\circ}43'38''$, and the mean of the corresponding times, per watch, $12^h14^m12^s$; the height of the eye above the level of the horizon was 22 feet; and the index error of the sextant $1'10''$ *additive*. Immediately before the observation, the watch was compared with a chronometer, the *error* and *rate* of which were established the 1st January, 1836; on which day, at noon, it was 2^m58^s *fast* for mean time at Greenwich, and *losing* daily at the rate of $0'125$. At the moment of comparison, the watch showed $12^h10^m46^s$, and the chronometer $16^h45^m35^s$; required the longitude of the place of observation?

* See Article 37, page 313, relative to the *mean sun's* right ascension.

Mean of the times of observation, per watch =	12 ^h 13 ^m 45 ^s
Time shown by watch at comparison with chronometer =	12. 14
<hr/>	
Interval between comparison and observation =	0 ^s
Time indicated by chronometer at comparison =	16. 4
<hr/>	
Time, per chronometer, of observing the mean altitude =	16 ^h 4 ^m
Original error of the chronometer	2 ^m 58 ^s fast = —
Accumulated rate = 0 ^s 125 × 121.7 days = 15 ^m 2 ^s less, or	+
<hr/>	
Correct mean time of observation at Greenwich =	16 ^h 4 ^m

The mean sun's right ascension reduced to Greenwich is 2^h 47^m 29^s to the nearest second.—The right ascension of Regulus is 9^h 59^m 39^s and his declination 12^h 46^m 0^s north, each taken to the nearest second.—The observed altitude of Regulus, viz., 20^h 4^m reduced to the true altitude, is 20^h 37^m 49^s.

To compute the Mean Time at the Ship, and the Longitude

Star's true altitude =	20 ^h 37 ^m 49 ^s
Star's north polar distance =	77. 14. 0 Log. co-sec. 0. 0
Latitude of the ship =	40. 30. 0 Log. secant 0. 1
<hr/>	
Sum =	138 ^h 21 ^m 49 ^s Const. log. 6. 3
<hr/>	
Half sum =	69 ^h 10 ^m 54 ^s $\frac{1}{2}$ Log. co-si. 9. 5
Remainder =	48. 33. 5 $\frac{1}{2}$ Log. sine 9. 8
<hr/>	
Star's hor. dist., west of the merid. =	4 ^h 54 ^m 35 ^s Log. rising 5. 8
Star's right ascension =	9. 59. 39
<hr/>	
Right ascension of the meridian =	14 ^h 54 ^m 14 ^s
Mean sun's reduced right ascension =	2. 40. 29
<hr/>	
Mean time of observation =	12 ^h 13 ^m 45 ^s
Mean time of observ. at Greenwich =	16. 49. 18
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Longitude of the ship, in time =	4 ^h 35 ^m 33 ^s = 68 ^h 53 ^m 15 ^s We

Note.—The horary distance is determined by Method I., page this being the most uniform and concise mode of computation,

Remark.—It frequently happens at sea, that, owing to clouds, rains, & other causes, ships are whole days without profiting by the presence of the sun, or obtaining an altitude of that object for the purpose of ascertaining either latitude or longitude; but it must be remembered, that there are few nights, if any, in which some fixed star, a planet, or the moon, does not present itself for observation, as if intended by Providence to relieve the mariner from the great anxiety which the doubtful position of his ship must naturally excite in him, particularly when returning from a long voyage, and about to enter any narrow strait, such as the English Channel. Under such circumstances, the *three* preceding Problems will be found exceedingly useful; because they exhibit safe and certain means of finding the true place of a ship, so far as the going of the chronometer used in the observation can be depended upon. In this case, since a knowledge of the heavenly bodies becomes indispensably necessary, the reader is referred to “*The Young Navigator’s Guide to the Sidereal and Planetary Parts of Nautical Astronomy*,” where a familiar code of practical directions is given for finding out and knowing all the principal fixed stars and planets in the firmament.

PROBLEM VII.

To find the Longitude of a Ship or Place by celestial Observation, commonly called a Lunar Observation.

The direct progressive motion of a ship at sea is so liable to be disturbed by various unavoidable and often imperceptible causes,—such as a frequent aberration from the true course, by the ship’s continually varying a little, in contrary directions, round her centre of gravity; high seas with heavy swells, sometimes with and at other times against, or in directions oblique to the true course; storms, sudden shifts of wind, unknown currents, *local magnetic attraction*, unequal attention in the helm’s-men, with many other casualties which cannot possibly be properly provided for,—that the place indicated by *the dead reckoning* is frequently so erroneous as to be whole degrees to the eastward or westward of the actual position of the ship. Of this every person must be fully aware, who has navigated the short run between England and the nearest of the West Indian Islands.

As the best account by *dead reckoning* is evidently but a very imperfect kind of guess-work, it should be employed only as an auxiliary to the elementary parts of navigation, and never confided in but

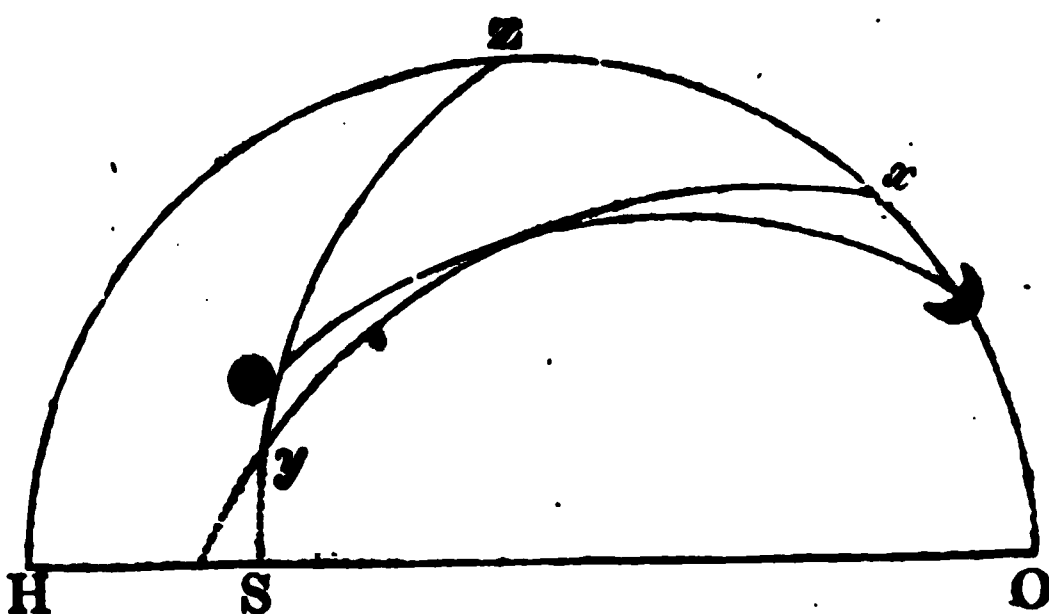
with the utmost caution. Hence it is that celestial observation should be constantly resorted to, because it is the only certain way of detecting the errors of dead reckoning, and of ascertaining, with any degree of precision, the actual position of the ship.

If a chronometer or timekeeper could be so constructed as to go uniformly correct in all seasons, places, and climates, it would immediately obviate all the difficulties attendant on a ship's reckoning, and thus render the longitude as simple a problem as the latitude; for, such a machine being once regulated to the meridian of Greenwich, would always show the absolute time at that meridian; and, hence, the longitude of the place of observation, as has been illustrated in the four preceding Problems; but those pieces of mechanism are so exceedingly complicated, and so extremely delicate, that they are liable to be affected by the common vicissitudes of seasons and climates,—by any sudden exposure to a higher or lower degree of atmospheric temperature, than that to which they have been accustomed; and also by electricity in a thunder-storm:—the celestial bodies ought, therefore, to be consulted, at all times, in preference to machines so subject to mutability, and should ever be confided in by the mariner, as the immutable and unerring timekeepers of nature.

Of all the apparent motions of the heavenly bodies, in the zodiac, with which we are acquainted, that of the moon is by far the most rapid; it being, at a mean rate, about $13^{\circ}10'$ in 24 hours, or nearly half a minute of a degree in one minute of time. Hence, the quickness of the moon's motion seems to adapt her peculiarly to the measurement of small portions of corresponding time; and, therefore, careful observations of the angular distance of that object from the sun, a planet, or a fixed star lying in or near the zodiac, afford the most eligible and practicable means of determining the longitude of a ship at sea: for the true distance deduced from observation, being compared with the computed distances in the Nautical Almanac, will show the corresponding mean time at Greenwich; the difference between which and the mean time at the place of observation will be the longitude of that place in time; which will be east if the time at the place of observation be greater than the Greenwich time, but west if it be less.

The principles of the operation for calculating a *lunar observation*, viz., of reducing the apparent distance between the moon and sun, or a fixed star to the true central distance, may be familiarly illustrated in the following manner, viz. :—

In the annexed diagram, let HO represent the horizon; and Z the zenith of an observer on the earth or sea.—Let the arc OD be the apparent altitude of the moon, and Ox her true central altitude; this



being always *greater* than the apparent altitude by the *excess* of the parallax above the refraction; and let the arc $S\odot$ represent the apparent altitude of the sun, or star, and Sy its true altitude; this being always *less* than the apparent altitude by the *excess* of the refraction above the parallax:—then the arc of the oblique circle $D\odot$, represents the apparent distance; and that of the oblique circle xy , the true central distance between the two objects.

Now, from this diagram, it is manifest that a *lunar observation* consists of two oblique angled spherical triangles; in one of which the three sides are given, to find the angle at the zenith; which angle is the common *vertex* to both triangles;—and, in the other, two sides and the contained angle, viz., *the common vertex at the zenith*, are given, to find the third side.—Thus, in the triangle $ZD\odot$; given the moon's apparent zenith distance ZD ; the sun's or star's apparent zenith distance $Z\odot$, and the apparent central distance $D\odot$; to find the common angle at the zenith:—which angle is to be found by oblique angled spherical trigonometry, Problem V., page 207.—Now, since it is clearly evident that the little variation which is produced in the measures of the sides ZD , and $Z\odot$, by the effects of parallax and refraction (expressed by the small arcs Dx and $\odot y$), will not sensibly affect the value of the contained angle at the zenith:—therefore, in the triangle Zxy ; given the moon's true zenith distance Zx ; the sun's or star's true zenith distance Zy , and the included angle Z ; to find the third side xy = the true central distance between the two objects:—which side is to be found by oblique angled spherical trigonometry, Problem III., page 202.

Example.

Let the apparent central distance between the moon and sun be $17^{\circ}42'28''$, the sun's apparent altitude $10^{\circ}19'19''$, the moon's apparent altitude $42^{\circ}55'1''$, and her horizontal parallax $60'2''$; required the true central distance between the two objects?

The sun's apparent altitude being $10^{\circ}19'19''$, the correction corresponding thereto is $4'56''$ *subtractive*; which makes its true central altitude $10^{\circ}14'23''$:—hence, the sun's apparent zenith distance, or *co-altitude*, is $79^{\circ}40'41''$, and its true zenith distance $79^{\circ}45'37''$.—The moon's apparent altitude being $42^{\circ}55'1''$, the correction corresponding thereto is $42'57''$ *additive*; which makes her true central altitude $43^{\circ}37'58''$:—hence, the moon's apparent zenith distance, or *co-altitude*, is $47^{\circ}4'59''$, and her true zenith distance $46^{\circ}22'2''$.

Computation.—First.

In the triangle $Z \triangleright \odot$, given; the side $Z \odot = 79^{\circ}40'41''$; the sun's apparent zenith distance; the side $Z \triangleright = 47^{\circ}4'59''$, the moon's apparent zenith distance, and the side $\triangleright \odot = 117^{\circ}42'28''$, the apparent distance; to find the angle at the zenith.—Hence, by spherical trigonometry, as above mentioned,—

Appar. cent. dist. or side $\triangleright \odot = 117^{\circ}42'28''$

Sun's app. zen. dist. or side $Z \odot = 79.40.41$ Log. co-sec. 0.007086

Moon's app. zen. dist. or side $Z \triangleright = 47.4.59$ Log. co-sec. 0.135286

Sum = $244^{\circ}28'8''$

Half sum = $122^{\circ}14'4''$ Log. sine . 9.927305

Remainder = $4.31.36$ Log. sine . 8.897204

Sum = . 18.966881

Arch = $72^{\circ}16'41''$ Log. co-sine 9.483440½

Angle at the zenith = $144^{\circ}33'22''$

Now, having found the angle at the zenith, which is common to both triangles; therefore, in the triangle Zxy given; the side $Zy = 79^{\circ}45'37''$, the sun's true zenith distance; the side $Zx = 46^{\circ}22'2''$, the moon's true zenith distance, and the included angle $Z = 144^{\circ}33'22''$; to find the side $xy =$ the true central distance.

Hence, by spherical trigonometry,—Formula, *Remark 2*, p. 204,—

Half the angle at the zenith = $72^{\circ}16'41''$ Twice its log. sine 19.957772

Sun's true zen. dist. or side $Zy = 79.45.37$ Log. sine . . . 9.993027

\triangleright 's true zen. dist. or side $Zx = 46.22.2$ Log. sine . . . 9.859605

Difference of the sides . . $33^{\circ}23'35''$ Constant log. . 6.301030

The nat. vers. sine of which is 165085

Natural number 1292510 Logarithm = . 6.111434

Nat. ver. sine of the side $xy = 1.457595 = 117^{\circ}13'55\frac{1}{2}''$; which is the true central distance between the two objects, as required.

Note.—The above diagram and consequent operation exhibit the pure spherical principles upon which the computation of a *lunar observation* is founded; from which, together with a competent knowledge of *the trigonometrical canon*, and a clear conception of the peculiar properties of the *natural versed sines*, and logarithmic sines, as explained between pages 53 and 56, a variety of methods may be deduced for reducing the apparent distance between the moon and sun, or a star, to the true central distance.—The following direct modes of calculation are all derived from spherical trigonometry, as above; and therefore they will prove to be *general* and *rigidly correct* in all cases where the zenith distances and the apparent central distance of the two objects constitute a perfect or *possible* spherical triangle in the heavens:—*the two first* of these are *universal*; they are *not* subject to any restriction whatever; for they are as applicable to an *impossible* as to the most *perfect triangle* that can be formed by the two objects in the starry firmament:—and so long as conciseness, dispatch, uniformity, and facility of calculation, are points of importance to the practical navigator, they will be found to be *the readiest methods* that can be employed for clearing the apparent distance of the effects of parallax* and refraction; that is, of reducing the apparent distance between the moon and sun &c., to what it would appear to an observer placed at the centre of the earth; supposing this to be transparent.

METHOD I.

Of reducing the apparent Distance between the Moon and Sun, a fixed Star, or a Planet, to the true Central Distance.

RULE.

Take the auxiliary angle from Table XX., and let it be corrected for the sun's, star's, or planet's apparent altitude, as directed in pages 44 and 45.

Find the difference of the apparent altitudes of the objects, and, also, the difference of their true altitudes.

Then, to the natural versed sines supplement of the sum and the difference of the auxiliary angle and the difference of the apparent altitudes, add the natural versed sines of the sum and the difference of

* Respecting the nature of parallax, see Illustration, page 29.

the auxiliary angle and the apparent distance, and the natural versed sine of the difference of the true altitudes: the sum of these five numbers, abating 4 in the radii or left-hand place, will be the natural versed sine of the true central distance.

General Remarks.

1. The *correction of the moon's apparent altitude* is contained in Table XVIII., and is to be taken out therefrom agreeably to the directions given in page 39.

2. The *correction of the sun's apparent altitude* is the difference between the refraction and the parallax corresponding to that altitude in Tables VIII. and VII. See note, page 375.

3. The *correction of a planet's apparent altitude* is the difference between the refraction and the parallax answering to that altitude in Tables VIII. and VI. See note, page 378.—And,

4. The *correction of a star's apparent altitude* is the refraction corresponding thereto in Table VIII. The fixed stars have not any sensible parallax.—See note, page 379.

Example.

Let the apparent central distance between the moon and sun be $117^{\circ}42'28''$, the sun's apparent altitude $10^{\circ}19'19''$, the moon's apparent altitude $42^{\circ}55'1''$, and her horizontal parallax $60'2''$, required the true central distance?

Sun's apparent altitude $10^{\circ}19'19''$ — Corr. $4'56''$ = true alt. $10^{\circ}14'23''$
 Moon's apparent alt. . $42.55.1$ + Corr. $42'57''$ = true alt. $43.37.58$

Diff. of apparent alts. . $32^{\circ}35'42''$ — Difference of true alts. $33^{\circ}23'35''$
 Auxiliary angle . . $60.22.39$
 Appar. central dist. $117.42.28$

Sum of aux. ang. and
 diff. apparent alts. $92^{\circ}58'21''$ Nat. versed sine supt. . 0.948143
 Difference of ditto . $27.46.57$ Nat. versed sine supt. . 1.884724
 Sum aux. ang. & ap. dis. $178.5.7$ Natural versed sine . . 1.999442
 Difference of ditto . $57.19.49$ Natural versed sine . . 0.460205
 Difference of true alts. $33.23.35$ Natural versed sine . . 0.165085

True central dist. = $117^{\circ}13'56''$ Natural versed sine . . 1.457599

See *Example*, page 480; in which the true central distance is $117^{\circ}13'55\frac{1}{2}''$.

Remarks.

1. When the sum of the auxiliary angle and the apparent central distance exceeds a semicircle, or 180 degrees, the natural versed sine supplement of its excess above that quantity is to be taken; or, which amounts to the same thing, the *natural versed sine of its supplement to 360 degrees*.

2. The above is one of the *shortest* and *most uniform* methods that have been deduced *directly* from spherical trigonometry; for it determines the true central distance by *the simple addition* of five natural versed sines, without the introduction of logarithms, or the application of *corrections*.—Besides which, it is *universal*; and *not* subject to any restrictions whatever:—its limits are *unbounded*; for it answers as well in an impossible triangle as in the most perfect or possible triangle that can be formed by the co-altitudes of the objects and their apparent central distance.

3. Here it may be necessary to observe, that a spherical triangle is said to be *impossible*, or imperfect, when the *sum of any two sides* is *less* than the value of the *third side*:—Hence, to render a triangle *possible* or perfect, the *sum of the two shortest sides* must be always *greater* than the measure of the *longest side*.

METHOD II.

Of reducing the Apparent to the True Central Distance.

RULE.

Take the auxiliary angle from Table XX., and let it be corrected for the sun's, star's, or planet's apparent altitude, as directed in pages 44 and 45.

Find the sum of the apparent altitudes of the objects, and, also, the sum of their true altitudes; then,

To the natural versed sines of the sum and the difference of the auxiliary angle and the sum of the apparent altitudes, add the natural versed sines of the sum and the difference of the auxiliary angle and the apparent distance, and the natural versed sine supplement of the sum of the true altitudes; the sum of these five terms, abating 4 in

the radii or left-hand place, will be the natural versed sine of the true central distance.

Note.—See General Remarks, page 482.

Example 1.

Let the apparent central distance between the moon and Venus be $53^{\circ}49'54''$, the apparent altitude of Venus $19^{\circ}10'40''$, and her horizontal parallax $23'$: the moon's apparent altitude $37^{\circ}40'20''$, and her horizontal parallax $59'47''$; required the true central distance?

Venus's apparent alt. = $19^{\circ}10'40''$ — Corr. $2'21''$ = true alt. = $19^{\circ}8'19''$
 Moon's apparent alt. = $37^{\circ}40'20''$ + Corr. 46.5 = true alt. = $38^{\circ}26'25''$

Sum of the app. alts. = $56^{\circ}51'0''$	Sum of the true alts. = $57^{\circ}34'44''$
Auxiliary angle = $60.20.14$	
Apparent cent. dist. = $53.49.54$	

Sum of auxiliary angle

& sum of ap. alts. = $117^{\circ}11'14''$	Nat. versed sine = . . 1.456899
Difference of ditto = $3.29.14$	Nat. versed sine = . . 0.001851
Sum aux. ang. & ap. dis. $114.10.8$	Nat. versed sine = . . 1.409428
Difference of ditto = $6.30.20$	Nat. versed sine = . . 0.006439
Sum of the true alts. = $57.34.44$	Nat. versed sine sup. = 1.536138

True central dist. = $53^{\circ}53'48''$ Nat. versed sine = . . 0.410755

Remark.—When the sum of the auxiliary angle and the apparent central distance exceeds a semi-circle, or 180° , the natural versed sine supplement of its excess above that quantity is to be taken, or, which amounts to the same, the natural versed sine of its supplement to 360° . The same is to be observed in the event of the aggregate of the auxiliary angle and the sum of the apparent altitudes exceeding 180 degrees: this, however, will but very rarely happen.

The above method is *universal*; and therefore *Remark 2*, at the end of the preceding Method, is applicable to it in every respect.

METHOD III.

Of reducing the Apparent to the True Central Distance.

RULE.

Take of the logarithmic difference from Table XXIV., and let it be corrected for the sun's, star's, or planet's apparent altitude, as directed in pages 49, 51, and 52.

Find the difference of the apparent altitudes of the objects, and, also, the difference of their true altitudes.

Then, from the natural versed sine of the apparent distance, subtract the natural versed sine of the difference of the apparent altitudes; to the logarithm of the remainder let the logarithmic difference be added, and the sum (abating 10 in the index) will be the logarithm of a natural number; which, being added to the natural versed sine of the difference of the true altitudes, will give the natural versed sine of the true central distance.

Note.—See General Remarks, page 482.

Example.

Let the apparent central distance between the moon and Mars be $83^{\circ}10'23''$, the apparent altitude of Mars $17^{\circ}10'20''$, and his horizontal parallax $15''$; the moon's apparent altitude $31^{\circ}20'30''$, and her horizontal parallax $58'53''$; required the true central distance?

Mars' apparent alt. = $17^{\circ}10'20''$ — Correc. $2'49''$ = true alt. = $17^{\circ}7'31''$
 Moon's appar. alt. = $31.20.30$ + Correc. 48.44 = true alt. = $32.9.14$

Diff. of appar. alts. = $14^{\circ}10'10''$ Diff. of true altitudes = $15^{\circ}1'43''$

Apparent distance = $83^{\circ}10'23''$ Nat. V. S. = 881129
 Diff. of appar. alts. = $14.10.10$ Nat. V. S. = 030436 Log. diff. = 9.996299

Remainder = 850693 Log. = 5.929773

Natural number = 843476 Log. = 5.926072
 Diff. of true alts. = $15^{\circ}1'43''$ Nat. V. S. = 034204

True central dist. = $82^{\circ}58'26''$ Nat. V. S. = 877680

Remark.—In an *impossible triangle** it may happen that the *difference* of the apparent altitudes will be *greater* than the apparent distance:—in this case, the natural versed sine of the latter term is to be subtracted from that of the former term:—then, the natural number corresponding to the sum of the two logarithms, being *subtracted* from the natural versed sine of the difference of the true altitudes; the result will be the natural versed sine of the true central distance.

METHOD IV.

Of reducing the Apparent to the True Central Distance.

RULE.

Take of the logarithmic difference from Table XXIV., and let it be corrected for the sun's, star's, or planet's apparent altitude, as directed in pages 49, 51, and 52.

Find the sum of the apparent altitudes of the objects, and, also, the sum of their true altitudes; then,

From the natural versed sine supplement of the sum of the apparent altitudes, subtract the natural versed sine of the apparent distance; to the logarithm of the remainder let the logarithmic difference be added, and the sum (abating 10 in the index) will be the logarithm of a natural number; which, being subtracted from the natural versed sine supplement of the sum of the true altitudes, will leave the natural versed sine of the true central distance.

Note.—See General Remarks, page 482.

Example.

Let the apparent central distance between the moon and a fixed star be $41^{\circ}11'7''$, the star's apparent altitude $43^{\circ}10'20''$, the moon's apparent altitude $56^{\circ}48'16''$, and her horizontal parallax $59'25''$; required the true central distance?

* See Remark 3, page 483.

Star's apparent alt. = $43^{\circ} 10' 20''$ — Correc. $1' 1''$ = true alt. = $43^{\circ} 9' 19''$
 Moon's appar. alt. = $56.48.16$ + Correc. 31.56 = true alt. = $57.20.12$

Sum of the ap. alts. = $99^{\circ} 58' 36''$ Sum of the true alts. = $100^{\circ} 29' 31''$

Sum of the ap. alts. = $99^{\circ} 58' 36''$ N.V.S. sup. = 826753

App. central dist. = $41.11.7$ N. ver. S. = 247416 Log. dif. = 9.993895

Remainder = 579337 Log. = 5.762931

Natural number = 571250 Log. = 5.756826

Sum of true alts. = $100^{\circ} 29' 31''$ N.V.S. sup. = 817902

True cent. dist. = $41^{\circ} 7' 8''$ N. ver. sine = 246652

Remark.—In an *impossible* triangle* it may happen that the natural versed sine *supplement* of the sum of the apparent altitudes will be *less* than the natural versed sine of the apparent distance; in this case, the former term is to be subtracted from the latter term: then, the natural number answering to the sum of the two logarithms being *added* to the natural versed sine *supplement* of the true altitudes, the result will be the true central distance.

METHOD V.

Of reducing the Apparent to the True Central Distance.

RULE.

To the logarithmic sines of the sum and the difference of half the apparent distance and half the difference of the apparent altitudes, add the logarithmic difference, Table XXIV., and the constant logarithm 6.301030: the sum of these four logarithms (rejecting 30 in the index) will be the logarithm of a natural number; which, being added to the natural versed sine of the difference of the true altitudes, will give the natural versed sine of the true central distance.

Note.—See General Remarks, page 482.

Example.

Let the apparent distance between the moon and a fixed star be $37^{\circ} 56' 43''$, the star's apparent altitude $19^{\circ} 32'$, the moon's apparent

* See Remark 3, page 483.

altitude $57^{\circ} 33'$, and her instrumental parallel $61^{\circ} 16'$; required the true central distance.

Sun's appar. alt. = $19^{\circ} 32' 0''$ — Correc. $2^{\circ} 40' =$ true alt. = $19^{\circ} 29' 20''$
 Moon's appar. alt. = $57^{\circ} 33' 0''$ — Correc. $33. 9 =$ true alt. = $57. 6. 9$

Diff. of the ap. alts. = $37^{\circ} 1' 0''$ Diff. of the true altitudes = $37^{\circ} 36' 49''$

Half sum of ap. alts. = $18^{\circ} 30' 30''$

Half the app. dis. = $18. 56. 21\frac{1}{2}$

		Log. diff. =	9.998713
Sum = $37^{\circ} 28' 51\frac{1}{2}''$	Log. sine =	9.784259
Difference = $0. 27. 51\frac{1}{2}''$	Log. sine =	7.908677
		Constant log. =	6.301030

Natural number = 9720 Log. = 3.987679

Dif. of true alts. = $37^{\circ} 36' 49''$ Nat. ver. S. = 207855

True central dist. = $38^{\circ} 31' 1''$ Nat. ver. S. = 217575

Remark.—In an *impossible* triangle,* or when *half the sum* of the apparent altitudes *exceeds* half the apparent distance, the natural number corresponding to the sum of the four logarithms, is to be *subtracted* from the natural versed sine of the difference of the true altitudes;—the result will be the natural versed sine of the true distance.

METHOD VI.

Of reducing the Apparent to the True Central Distance.

RULE.

To the logarithmic co-sines of the sum and the difference of half the apparent distance and half the sum of the apparent altitudes, add the logarithmic difference, Table XXIV., and the constant logarithm 6.301030: the sum of these four logarithms (rejecting 30 in the index) will be the logarithm of a natural number; which, being subtracted from the natural versed sine supplement of the sum of the true altitudes, will leave the natural versed sine of the true distance.

Note.—See General Remarks, page 482.

* See Remark 3, page 483.

Example.

Let the apparent central distance between the moon and a fixed star be $69^{\circ}21'25''$, the star's apparent altitude $27^{\circ}32'37''$, the moon's apparent altitude $22^{\circ}28'56''$, and her horizontal parallax $56'17''$; required the true central distance?

Star's apparent alt. = $27^{\circ}32'37''$ — Correc. $1'49''$ = true alt. = $27^{\circ}30'48''$

Moon's appar. alt. = $22.28.56$ + Correc. 49.43 = true alt. = $23.18.39$

Sum of the ap. alts. = $50^{\circ}1'33''$ Sum of the true altitudes = $50^{\circ}49'27''$

Halfsum of ap. alts. = $25^{\circ}0'46\frac{1}{2}''$

Half ap. cent. dist. = $34.40.42\frac{1}{2}$

Sum = $59^{\circ}41'29''$ Log. diff. = 9.997468

Difference = $9.39.56$ Log. co-sine = 9.702997

Constant log. = 6.301030

Natural number = 989204 Log. = 5.995286

Sum of true alts. = $50^{\circ}49'27''$ N. V. S. sup. = 1.631703

True cent. dist. = $69^{\circ}3'11\frac{1}{2}''$ N. ver. sine = .642499

Remark.—The above Method is only adapted to the solution of a perfect or possible spherical triangle.—Hence, when the *sum* of the apparent zenith distances, or co-altitudes, is *less* than the apparent central distance; or when the *sum* of the apparent central distance and one of the *zenith* distances is *less* than the other zenith distance; the true central distance must be determined by one of *the universal methods*, page 481 or 483.

METHOD VII.

Of reducing the Apparent to the True Central Distance.

RULE.

To the logarithmic sines of the sum and the difference of half the apparent distance and half the difference of the apparent altitudes, add the logarithmic difference: half the sum of these three logarithms (10 being previously rejected from the index) will be the logarithmic sine

of an arch. Now, half the sum of the logarithmic co-sines of the sum and the difference of this arch and half the difference of the true altitudes, will be the logarithmic co-sine of half the true central distance.

Note.—See General Remarks, page 482.

Example.

Let the apparent central distance between the moon and Saturn be $110^{\circ}14'34''$, Saturn's apparent altitude $9^{\circ}40'48''$, and his horizontal parallax $1''$, the moon's apparent altitude $15^{\circ}40'6''$, and her horizontal parallax $58'43''$; required the true central distance?

Saturn's appar. alt. $= 9^{\circ}40'48'' - \text{Cor. } 5'24'' = \text{true alt.} = 9^{\circ}35'24''$
 Moon's appar. alt. $= 15.40.6 + \text{Cor. } 53.11 = \text{true alt.} = 16.33.17$

Diff. of the app. alts. $= 5^{\circ}59'18''$ Diff. of the true alts. $= 6^{\circ}57'53''$

Half diff. of ap. alts. $= 2^{\circ}59'39''$ Half diff. of the true alts. $= 3^{\circ}28'56\frac{1}{2}''$
 Half ap. cent. dist. $= 55.7.17$

		Log. diff. =	9.998176
Sum = $58^{\circ}6'56''$	Log. sine =	9.928966
Difference =	. . $52.7.38$	Log. sine =	9.897284

19.824426

Arch = $54^{\circ}47'2''$ Log. sine = 9.912219
 Half dif. of true alts. $= 3.28.56\frac{1}{2}$

Sum = $58^{\circ}15'58\frac{1}{2}''$	Log. co-sine =	9.720963
Difference =	. . $51.18.5\frac{1}{2}$	Log. co-sine =	9.796035

Sum = 19.516998

Half the true dist. $= 55^{\circ}0'80\frac{1}{2}''$ Log. co-sine = 9.758499

True central dist. $= 110^{\circ}1'1''$

Note.—This Method is only adapted to the solution of a *perfect* or *possible* spherical triangle:—see Remark at the end of the preceding Method, page 489.

METHOD VIII.

Of reducing the Apparent to the True Central Distance.

RULE.

o the logarithmic co-sines of the sum and the difference of half the
arent distance and half the sum of the apparent altitudes, add the
rithmic difference: half the sum of these three logarithms (10
g previously rejected from the index) will be the logarithmic co-
of an arch. Now, half the sum of the logarithmic sines of the
and difference of this arch and half the sum of the true altitudes,
be the logarithmic sine of half the true central distance.

Note.—See General Remarks, page 482.

Example.

et the apparent central distance between the moon and a fixed star
1°29'58", the star's apparent altitude 11°31'2", the moon's appa-
altitude 8°44'35", and her horizontal parallax 57'24"; required
true central distance?

's appar. alt. = 11°31' 2" — Cor. 4'34" = true alt. = 11°26'28"

m's app. alt. = 8.44.35 + Cor.50.46 = true alt. = 9.35.21

of the ap. alts. = 20°15'37" Sum of the true alts. = 21° 1'49"

f sum ap. alts. = 10° 7'48½" Half sum of true alts. = 10°30'54½"

f ap. cent. dist. = 20.44.59

h = 30°52'47½"

erence = . . 10.37.10½

Log. diff. = 9.999083

Log. co-sine = 9.933612

Log. co-sine = 9.992497

19.925192

h = 23°26'23"

f sum tr. alts. = 10.30.54½

Log. co-sine = 9.982598

h = 33°57'17½"

erence = . . 12.55.28½

Log. sine = 9.747053

Log. sine = 9.849604

Sum = 19.096657

f the true dist. = 20°41'54½" Log. sine = 9.548328}

e central dist. = 41°23'49"

Note.—This method is only adapted to the solution of a perfect possible spherical triangle.—See *Remark* at end of Method VI., page 489.

METHOD IX.

Of reducing the Apparent to the True Central Distance.

RULE.

To the logarithmic sines of the sum and the difference of half the apparent distance, and half the difference of the apparent altitudes, add the logarithmic difference, its index being increased by 10: from the sum of these three logarithms subtract the logarithmic sine of half the difference of the true altitudes, and the remainder will be the logarithmic tangent of an arch; the logarithmic sine of which, being subtracted from the half sum of the three logarithms, will leave the logarithmic sine of half the true central distance.

Note.—See General Remarks, page 482.

Example.

Let the apparent central distance between the moon and sun be $91^{\circ}26'8''$, the sun's apparent altitude $14^{\circ}45'41''$, the moon's apparent altitude $53^{\circ}41'1''$, and her horizontal parallax $58'29''$; required the true central distance?

Sun's apparent alt. = $14^{\circ}45'41''$ — Cor. $3'26''$ = true alt. = $14^{\circ}42'15''$

Moon's appar. alt. = $53.41.1$ + Cor. 33.56 = true alt. = $54.14.57$

Diff. of the ap. alts. = $38^{\circ}55'20''$ Diff. of the true alts. = $39^{\circ}32'42''$

Half diff. of ap. alts. = $19^{\circ}27'40''$ Half diff. of true alts. = $19^{\circ}46'21''$

Half ap. cent. dist. = $45.43.4$

—————Log.diff. = 19.994220

Sum = . . . $65^{\circ}10'44''$ Log.sine = 9.957905

Difference = . . $26.15.24$ Log.sine = 9.645809

Sum = 39.597934

Half sum = 19.798967 . . 19.798967

Half diff. of true alts. $19^{\circ}46'21''$ Log.sine = 9.529285

Arch = . . . $61^{\circ}44'43''$ Log.tan. = 10.269682 Log.si. 9.94490

Half the true dist. = $45^{\circ}36'38\frac{1}{2}''$ Log. sine = . . . 9.85406

True central dist. = $91^{\circ}13'17''$

Note.—This Method is only adapted to the solution of a perfect or possible triangle.—See *Remark* at the end of Method VI., page 489.

METHOD X.

Of reducing the Apparent to the True Central Distance.

RULE.

To the logarithmic difference (its index being increased by 10), add the logarithmic co-sines of the sum and the difference of half the apparent distance and half the sum of the apparent altitudes; from half the sum of these three logarithms subtract the logarithmic co-sine of half the sum of the true altitudes, and the remainder will be the logarithmic sine of an arch; the logarithmic tangent of which, being subtracted from the half sum of the three logarithms, will leave the logarithmic sine of half the true central distance.

Note.—See General Remarks, page 482.

Example.

Let the apparent central distance between the moon and a fixed star be $68^{\circ}52'40''$, the star's apparent altitude $10^{\circ}52'17''$, the moon's apparent altitude $6^{\circ}39'28''$, and her horizontal parallax $58'31''$; required the true central distance?

Star's apparent alt. = $10^{\circ}52'17''$ — Correc. $4'50''$ = true alt. $10^{\circ}47'27''$

Moon's appar. alt. = $6.39.28$ + Correc. 50.26 = true alt. $7.29.54$

Sum of the app. alts. = $17^{\circ}31'45''$ Sum of the true altitudes $18^{\circ}17'21''$

Half sum of app. alt. = $8^{\circ}45'52\frac{1}{2}''$ Half sum of the true alt. $9^{\circ}8'40\frac{1}{2}''$

Half app. cent. dist. = $34.26.20$

Log. diff. 19.999326

Sum = . . . $43^{\circ}12'12\frac{1}{2}''$ Log. co-si. 9.862684

Difference = . . $25.40.27\frac{1}{2}$ Log. co-si. 9.954856

Sum = 39.816866

Half sum = 19.908433 . 19.908433

Half sum of true alt. = $9^{\circ}8'40\frac{1}{2}''$ Log. co-si. 9.994445

Arch = $55^{\circ}7'4''$ Log. sine = 9.913988 L. T. 10.156675

Half the true distance = $34^{\circ}22'34''$ Log. sine = 9.751758

True central distance = $68^{\circ}45'8''$

Note.—This Method is only adapted to the solution of a perfect spherical triangle.—See *Remark* at the end of Method VI., page 489.

METHOD XI.

Of reducing the Apparent to the True Central Distance.

RULE.

From the natural versed sine supplement of the sum of the apparent altitudes, subtract the natural versed sine of their difference, and call the remainder *arch first*. Proceed in a similar manner with the true altitudes, and call the remainder *arch second*; and from the natural versed sine supplement of the sum of the apparent altitudes, subtract the natural versed sine of the apparent distance, and call the remainder *arch third*.

Now, to the arithmetical complement of the logarithm of *arch first*, add the logarithms of *arches second and third*, and the sum (rejecting 10 from the index) will be the logarithm of a natural number; which, being subtracted from the natural versed sine supplement of the sum of the true altitudes, will leave the natural versed sine of the true central distance.

Note.—See General Remarks. page 482.

Example.

Let the apparent central distance between the moon and a fixed star be $83^{\circ}15'19''$, the star's apparent altitude $7^{\circ}39'4''$, the moon's apparent altitude $10^{\circ}57'36''$, and her horizontal parallax $58'55''$; required the true central distance?

$$* \text{'s ap. alt.} = 7^{\circ}39'4'' - \text{Cor. } 6'45'' = \text{T. alt. } 7^{\circ}32'19''$$

$$\circ \text{'s ap. alt.} = 10^{\circ}57'36'' - \text{Cor. } 53'3'' = \text{T. alt. } 11^{\circ}50'39''$$

$$\text{Sum} = \quad . \quad 18^{\circ}36'40'' \text{ N.V.S. } \left\{ \begin{array}{l} \text{sup.} \\ \text{sup.} \end{array} \right\} 1.947707 \quad \text{Sum } 19^{\circ}22'58'' \text{ N.V.S. } \left\{ \begin{array}{l} \text{sup.} \\ \text{sup.} \end{array} \right\} 1.943322$$

$$\text{Diff.} = \quad . \quad 3.18.32 \text{ N.V.S. } .001668 \quad \text{Diff. } 4.18.20 \text{ N.V.S. } .002822$$

$$\text{Arch first} = 1.946039$$

$$\text{Arch second} = 1.940500$$

$$\text{Sum of ap. alts.} = 18^{\circ}36'40'' \text{ N.V.S. sup.} = 1.947707$$

$$\text{App. cent. dist.} = 83.15.19 \text{ N.V.S.} = .882554$$

$$\text{Arch third} = \quad . \quad . \quad . \quad . \quad . \quad . \quad 1.065153 \text{ Log.} = 6.027432$$

$$\text{Arch second} = \quad . \quad . \quad . \quad . \quad . \quad . \quad 1.940500 \text{ Log.} = 6.287914$$

$$\text{Arch first} = \quad . \quad . \quad . \quad . \quad . \quad . \quad 1.946039 \text{ Log. ar.co. } 3.710848$$

$$\text{Natural number} = \quad . \quad . \quad . \quad . \quad . \quad . \quad 1.062169 \text{ Log.} = 6.026194$$

$$\text{Sum of true alt.} = 19^{\circ}22'58'' \text{ N.V.S. sup.} = 1.943322$$

$$\text{True cent. dist.} = 83^{\circ}10'28'' \text{ N.V.S.} = .881153$$

Note.—This Method is also adapted to the solution of a perfect or possible spherical triangle.—See *Remark* at the end of Method VI., page 489.

METHOD XII.

To the apparent distance add the apparent altitudes of the objects; take half the sum, and call the difference between it and the apparent distance the *remainder*.—Then,

To the logarithmic difference (its index being augmented by 10), add the logarithmic co-sines of the half sum and the remainder; from half the sum of these three logarithms subtract the logarithmic co-sine of half the sum of the true altitudes, and the remainder will be the logarithmic sine of an arch. Now, the logarithmic co-sine of this arch, being added to the logarithmic co-sine of half the sum of the true altitudes (rejecting 10 from the index), will give the logarithmic sine of half the true central distance.

Note.—See General Remarks, page 482.

Example.

Let the apparent central distance between the moon and Spica Virginis be $37^{\circ}12'40''$, the star's apparent altitude $11^{\circ}27'50''$, the moon's apparent altitude $40^{\circ}55'15''$, and her horizontal parallax $54'10''$; required the true central distance?

Star's apparent alt. = $11^{\circ}27'50''$ — Correc. $4'35''$ True alt. = $11^{\circ}23'15''$

Moon's apparent alt. = $40.55.15$ + Correc. 39.51 True alt. = $41.35.6$

Apparent cent. dist. = $37.12.40$

Sum = $89^{\circ}35'45''$
 ————— L. diff. = 19.995703

Half sum = $44^{\circ}47'52\frac{1}{2}''$ L. co-si. 9.851012

Remainder = $7.35.12\frac{1}{2}$ L. co-si. 9.996181

Sum = 39.842896

Half sum = 19.921448

Half sum of true alt. = $26^{\circ}29'10\frac{1}{2}''$ L. co-si. 9.951844 . . . 9.951844

Arch = $68^{\circ}48'45''$ L. sine = 9.969604 L. co-si. 9.558014

Half the true distance = $18^{\circ}52'26\frac{1}{2}''$ Log. sine = 9.509358

True central distance = $37^{\circ}44'53''$

Note.—This Method is only adapted to the solution of a perfect or possible spherical triangle.—See *Remark* at the end of Method VI., page 489.

General Remarks upon the Lunar Observations.

1. When the objects are in the same azimuth, or vertical circle, and the moon to be the higher body, the true central distance will be equal to the apparent distance augmented by the sum of the corrections of the respective altitudes of the objects; but when the moon is the lower body, the apparent distance diminished by the sum of the corrections will be equal to the true central distance: thus, if the moon were the higher body, and the correction of its apparent altitude to be $43'.50''$, and that of the sun's or star's $3'.10''$, the true central distance would be 47 minutes more than the apparent distance; but if the moon were the lower body, the true central distance would be 47 minutes less than the apparent.

2. When the objects are in the same almucantar, that is, when both have the same altitude, the true central distance will be equal to the apparent distance, diminished by the difference between the principal effects of parallax and refraction on that distance, which difference may be very nearly estimated thus:—

To the log. tangent of half the apparent distance, add the constant logarithm 6.742480,* and the sum, rejecting 10 from the index, will be the log. sine of the effects of refraction on the distance, which is always *additive*.—Then,

To the log. sine of the sum of the horizontal parallax of the objects add the log. sine of the common altitude, and the log. tangent of half the apparent distance; the sum, rejecting 20 from the index, will be the log. sine of the principal effects of parallax on the distance, which is always *subtractive*.

Example.

Let the apparent distance between the moon and sun be $48^\circ 40'$, the common altitude 51 degrees, and the moon's horizontal parallax $58'.10''$, to find the correction of the apparent distance.

* This is the log. sine of $1'.54''$, the effects of refraction on a lunar distance of 50 degrees.

First, to find the principal effects of refraction.

Half the apparent distance =	24° 20'	Log. tangent	9. 655348
Constant logarithm =			6. 742480
Principal effects of refraction =	+ 0' 52"	Log. sine	<u>6. 397828</u>

Second, to find the principal effects of parallax.

☽'s hor. parallax =	0° 58' 10"	+ sun's par. 9"	= 58' 19"	L. sine	8. 229498
Common altitude =	51. 0. 0			L. sine	9. 890503
Half the app. dis. =	24. 20. 0			L. tan.	9. 655348
Principal effects of parallax =		- 20' 30"	Log. sine	7. 775349	
Principal effects of refraction =		+ 0. 52			

Correction of the apparent distance = - 19' 38"

Apparent distance = 48. 40. 0

True central distance = 48° 20' 22", which is but 6 seconds less than the result by the direct method of calculation.

3. When the angle at the moon is a right angle, or 90°: that is, when the log. sine of the moon's altitude is equal to the remainder found by subtracting the log. co-sine of the apparent distance from the log. sine of the apparent altitude of the sun or star, the correction of the apparent distance is very little; thus:—

Suppose it were required to find what altitude the moon should have when the sun's or star's altitude is 24°, so that the apparent distance of 48° 40', may be as little subject to the effects arising from parallax and refraction as the triangle formed by the altitudes and distance will admit of.

Sun's apparent altitude =	24° 0' 0"	Log. sine	9. 609313
Apparent distance =	48. 40. 0	Log. co-sine	9. 819832
Altitude the moon should have =	38° 0' 51"	Log. sine =	<u>9. 789481</u>

Now, the distance and altitude of the sun remaining the same, the true central distance will be greater or less than the apparent, according as the moon's apparent altitude is greater or less than that found by the above formula.*

* If the moon's horizontal parallax be assumed 56' 30", and the apparent distance and apparent altitudes to remain as above, the true central distance will be found by calculation to be 1' 18" more than the apparent distance.

4. When the log. sine of the sun's or star's apparent altitude is equal to the sum of the log. co-sine of the apparent distance, and the log. sine of the moon's apparent altitude, the correction of the apparent distance is also very little; the angle at the moon being a right angle. Thus :—

Suppose it were required to find what altitude the sun or star should have, when the moon's apparent altitude is $58^{\circ}30'$, so that the apparent distance of $48^{\circ}40'$ may be as little affected by parallax and refraction as the triangle formed by the altitudes and distance will admit of.

Apparent distance =	$48^{\circ}40' 0''$	Log. co-sine	9.819832
Moon's apparent altitude =	$58.30. 0$	Log. sine	9.930766
				<hr/>
App. altitude the star should have =		$34^{\circ}16'18''$	Log. sine =	<u>9.750598</u>

Now, the apparent distance and altitude of the moon remaining the same, the true central distance will be greater or less than the apparent, according as the sun's or star's altitude is greater or less than that deduced from the above formula.*

Those remarks will be rendered very familiar by projecting the figure of the lunar observation upon stereographic principles.

5. When the moon is in the nonagesimal; that is, when a line joining the cusps or horns of the moon is perpendicular to the plane of the horizon, the correction of the apparent distance is also very little, the objects being nearly at equal distances from the ecliptic, and on the same side.

Now, if the star be nearer the elevated pole of the ecliptic than the moon, or, which is the same thing, if the star be the highest object, the true distance will be less than the apparent; but if the moon be the nearest object to the elevated pole of the ecliptic, viz., if its altitude be greater than that of the object with which it is compared, the true central distance will be greater than the apparent distance.

6. When the apparent central distance is more than the sun's or star's apparent distance from the zenith, the true central distance will be *less* than the apparent: an exception to this sometimes takes place when the sun's or star's zenith distance is more than 80 degrees.

* If the moon's horizontal parallax be assumed at $56'30''$, and the apparent distance and apparent altitudes to remain as above, the true central distance will be found by calculation to be $1'29''$ more than the apparent distance.

7. When the apparent distance is more than 90 degrees, it is, in *general*, greater than the true central distance.

8. If it be required to know what effect an error of a minute in the lunar distance will have upon the longitude in places under the equator, and thence in any given latitude, the following rules are to be attended to, which will give the required error sufficiently near the truth for every practical purpose at sea.

First, to find the error in longitude arising from an error of a minute in the distance in places under the equator ; say,—

The mean diurnal motion of the moon in her orbit is $13^{\circ}10'35''$, and that of the sun $59'8''$, each taken to the nearest second.—See Articles 11 and 13, pages 305 and 306.—Now, since both objects are moving in the same direction, viz., from west to east, therefore, the difference of their motions, viz., $12^{\circ}11'27''$, is the actual relative motion of the moon in the heavens :—Hence, as the moon's relative mean motion is to the great circle of 360 degrees, so is an error of one minute in distance to the effects of that error on the longitude at the equator ; as thus, by logarithms :—

Moon's relative motion =	$12^{\circ}11'27'' = 43887''$	Log. ar. comp.	5.357664
Great circle of the equa. =	360. 0. 0. = 21600'	Logarithm	4.334454
Error in distance =	0. 1. 0. or 60''	Logarithm	1.778151

Corresponding error in the longitude = $29'53''$ Logarithm = 1.470269

Now, from this the error of longitude in any given latitude may be readily found in the following manner, viz. :—

To the log. co-sine of the given latitude, add the logarithm of the error at the equator, viz. 1.470269 ; and the sum, abating 10 in the index, will be the logarithm of the error in longitude in the given parallel of latitude, corresponding to an error of 1' in the lunar distance.

Example.

Required the effect that an error of one minute in the distance has upon the longitude in a place which is $50^{\circ}48'$ north, or south of the equator ?

Given parallel of latitude =	50°48'	Log. co-sine	9.800737
Constant logarithm =			1.470269

Error of longitude = 18.66 Logarithm = 1.271006

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Hence it is manifest that an error of *one minute* in a lunar distance would produce an error of $29\frac{1}{2}$ miles in the longitude at the equator, and nearly $18\frac{3}{4}$ miles in the parallel of $50^{\circ}48'$ north, or south.

9. Respecting *impossible* spherical triangles, as hinted at in the preceding Methods of reducing the apparent to the true central distance.

When the angular distance between the moon and star &c. is less than 90° , it frequently happens, *nay*, it occurs at a certain hour every night, particularly in *low latitudes*, that the apparent zenith distances of the objects and their apparent central distance constitute an imperfect or an *impossible* spherical triangle. And thus it is that instances often occur in which an observer, after taking the utmost pains to measure a correct lunar distance, finds his hopes disappointed; because the method of computation by which he is accustomed to work may not be adapted to all species of triangles, whether *possible* or *impossible*.

When the moon and star &c. are on the same side of the equinoctial, and nearly of the same declination, and the value of the declination not to differ much from that of the latitude of the place of observation, both being of the same name, the apparent zenith distances of the two objects and their apparent central distance will form an *impossible* triangle while rising in the eastern hemisphere; and, also, while falling in the western hemisphere. If the moon's right ascension be the *least*, she will be vertical to, or *above* the star on the east side of the meridian, and *below* it on the west side:—The converse of this takes place when the star's right ascension is the *least*; because, then, the star will be *above* the moon in ascending above the eastern horizon, and *below* her in descending towards the western horizon.—As this phenomenon is peculiar to all places between the tropics of Cancer and Capricorn, during certain stages of the moon, therefore the instances of *imperfect* or *impossible spherical triangles* are very numerous:—however, the solution of one of those is full as simple as that of the most *perfect* triangle, provided the operation be performed by either the *first* or the *second* Method of computation, page 481 or page 483.

In certain cases the *imperfect* possesses many advantages over the perfect spherical triangle: for, when the moon is the *highest* object, the distance between her enlightened limb and a fixed star, or planet, may be very easily measured; because, in this instance, as the sextant is to be held in its natural, or vertical position, the distance may be observed to the most rigid degree of exactness; and that, too, in the most *boisterous weather*, when, perhaps, it would be almost impossible to measure a strictly correct distance under other circumstances; such

as the two objects being on different sides of the meridian, and the moon to be to the left-hand of, and considerably *lower* than, the star or planet.

Having made those remarks, we will now proceed to the practical part of deducing the longitude from *the lunar observations*.

PROBLEM IX.

Given the Latitude of a Place and its Longitude by Account, the observed Distance between the Moon and Sun, a fixed Star, or a Planet, and the observed Altitudes of those Objects ; to find the true Longitude of the Place of Observation.

RULE.

Reduce the mean time of observation to the meridian of Greenwich, by Problem III., page 342 ; to this time let the moon's horizontal parallax and semidiameter be reduced, by Problem XV., page 361, and let the moon's reduced semidiameter be increased by the augmentation (Table IV.) answering to her observed altitude.

Find the apparent and the true altitude of each object's centre, by the respective problems, for that purpose, contained between pages 374 and 379.

To the observed distance between the nearest limbs of the moon and sun, corrected for index error, if any, add their respective semidiameters, and the sum will be the apparent central distance. But, if the distance be observed between the moon and a fixed star or planet, then the moon's true semidiameter is to be applied to that distance by addition when it is measured from the nearest limb, but by subtraction when it is measured from the remote limb : in either case, the result will be the apparent central distance. With the apparent and the true altitudes of the objects, and their apparent central distance, compute the true central distance, by any of the Methods given in Problem VII., between pages 481 and 495.

Now, from the true central distance, thus found, and the given mean of observation, the longitude is to be determined by Problem XXX., page 383.

Remarks.—If the watch be not well regulated to the time of observation, the mean time may be deduced from the true altitude of the sun, moon, star, or planet, used in the computation, provided the object made choice of for this purpose be sufficiently far from the meridian

at the time of measuring the lunar distances ; if not, the error of the watch must be inferred from the true altitude of one of those objects, when in a more favourable position with respect to the meridian: then the error of the watch, thus found, being applied to the mean of the times of measuring the lunar distances, by addition or subtraction, according as it is slow or fast, the sum or difference will be the mean time of taking the lunar observation, agreeably to the meridian under which the error of the watch was obtained. The error of the watch is to be found by Problems II., III., IV., or V., between pages 435 and 443, according as the object may be the sun, a fixed star, a planet, or the moon.

In taking a lunar observation, it is necessary that several distances be measured,—that the corresponding times, per watch, be carefully noted down,—and that the altitudes of the objects be observed at the same instants with the distances : then, the respective sums of the times, of the observed distances, and of the altitudes, being divided by their common number, will give the mean time of observation, the mean observed distance, and the mean observed altitude of each object, as particularly explained in *the Method of taking a complete set of lunar observations*, page 335.

Example 1.

January 25th, 1836, in latitude 19°30' north, and longitude by account 10°35' east, the following observations were taken : the index error of the sextant with which the distances were measured was 1'10' subtractive, and the height of the eye above the level of the sea 20 feet; required the true longitude of the place of observation ?

Mean Times of Observation.	Observed Distance between Moon and Sun.	Altitude of Sun's Lower Limb.	Altitude of Moon's Lower Limb.
3 ^h 8 ^m 30 ^s	89°57' 30"	44° 0' 40"	46°15' 10"
3. 9. 35	89.58. 0	43.46.40	47.25.30
3. 10. 40	89.58.30	43.32.40	48.35.50
3. 11. 45	89.59. 0	43.18.30	49.46.10
3. 12. 50	89.59.30	43. 4.30	50.56.30
Mean = . . 3 ^h 10 ^m 40 ^s	Mean = . . 89°58' 30"	Mean = 43°32' 36"	Mean = 48°35' 50"
Longitude } —0. 42. 20	Index error = — 1.10		
in time }	»'s semidiam. + 15. 7		
	○'s semidiam. + 16.16		
Greenw. ti. = 2 ^h 28 ^m 20 ^s	Appar. dist. = 90°28' 43"	»'s augmented semidiameter = 15' 7"	»'s reduced horiz. parallax = 54.47
		Auxiliary angle = 60°22' 49"	
Mean of the sun's altitudes = . . . 43°32' 36"		Mean of the moon's alts. = 48.35.50	
Sun's semidiameter = + 16.16		Moon's semidiameter = . . . + 15. 7	
Dip of the horizon for 20 feet = . . . — 4.17		Dip of the hor. for 20 feet = — 4.17	
Sun's apparent altitude = 43°44' 35"		Moon's apparent altitude = 48°46' 40"	
Correction of ditto = — 0.54		Correction of ditto = . . . + 35.16	
Sun's true altitude = 43°43' 41"		Moon's true altitude = . . . 49°21' 56"	

To find the True Central Distance, and the Longitude.

Difference of app. altitudes = $5^{\circ} 2' 5''$	} See Method I., page 481.
Auxiliary angle = . . . $60.22.49$	
Apparent central distance = $90.28.43$	

Sum of the two first terms = $65^{\circ}24'54''$ Nat. ver. sine sup. = 1.416042
 Difference of ditto = . . . $55.20.44$ Nat. ver. sine sup. = 1.568626
 Sum of the two last terms = $150.51.32$ Natural ver. sine = 1.873423
 Difference of ditto = . . . $30.5.54$ Natural ver. sine = 0.134834
 Difference of the true alts. = $5.38.15$ Natural ver. sine = 0.004836

True central distance = . $89^{\circ}52'18''$ Natural ver. sine = 0.997761
 Nearest preced. dist. at 0^h = $88.44.4$ P. log. 3372.—Diff. 9, *increasing*; see R., p. 383.

Difference of ditto = . . . $1^{\circ}8'14''$ P. log. 4213

Portion of time = . . . $2^h28^m18^s$ P. log. 0841
 Time at nearest preced. dist. = 0. 0. 0

Approx. time at Greenwich = $2^h28^m18^s$
 Correction, Table A = . . . — 2 :—See Art. 80 & 81, p. 335 & 337.

Cor. mean time at Greenwich = $2^h28^m16^s$
 Mean time at place of obs. = $3.10.40$

Longitude of ditto, in time = $0^h42^m24^s = 10^{\circ}36'0''$ East.

Note.—It is essentially necessary to attend to the correction in Table A ; because, in some cases, its omission might affect the longitude to the value of 12 or 13 miles : an instance of this will be shown in the *seventh Example of this series*.

Example 2.

February 1st, 1836, in latitude $45^{\circ}40'$ north, and longitude by account $60^{\circ}15'$ west ; the mean of several distances between the moon's *nearest* limb and the star α , Arietis (taken after the manner set forth in the preceding Example) was $86^{\circ}42'20''$; and that of the corresponding times, per watch, properly *regulated to mean time* at the place of observation, $6^h20^m40^s$; at the same instant the mean of an equal number of altitudes of the moon's lower limb was $23^{\circ}12'12''$, and that of the star $63^{\circ}19'46''$:—the index error of the sextant used in measuring the distances was $1'30''$ *additive* ; and the height of the eye above the level of the horizon 22 feet ; required the true longitude of the place of observation ?

Mean time of observ. = 6 ^h 20 ^m 40 ^s	☾'s red. semidiam. = 15' 3"
Longitude 60° 15' west	Augmentation = . . . + 5
in time = . . . 4. 1. 0	
Greenwich time = 10 ^h 21 ^m 40 ^s	☾'s cor. semidiam. = 15' 8"
	☾'s cor. horiz. par. = 55.14
Observed distance = 86° 42' 20"	Auxiliary angle = 60° 11' 45"
Index error = . . . - 1.30	☾'s sub.alt., lower limb = 23° 12' 12"
☾'s correct semidiam. = - 15. 8	☾'s semidiameter = + 15. 8
	Dip for 22 feet = . . . - 4.30
App. cent. distance = 86° 58' 58"	
Star's observed alt. = 63° 19' 46"	☾'s apparent altitude = 23° 22' 50"
Dip of hor. for 22 feet = - 4.30	Correction of ditto = + 48.31
Star's appar. altitude = 63° 15' 16"	
Correction of ditto = - 0.29	
Star's true altitude = 63° 14' 47"	Moon's true altitude = 24° 11' 21"

To find the True Central Distance, and the Longitude.

Sum of the apparent alts. = 86° 38' 6"	} See Method II., page 483.
Auxiliary angle = . . . 60. 11. 45	
Apparent central distance = 86. 58. 58	

Sum of the two first terms = 146° 49' 51"	Natural ver. sine = 1.837059
Difference of ditto = . . . 26. 26. 21	Natural ver. sine = 0.104592
Sum of the two last terms = 147. 10. 43	Natural ver. sine = 1.840364
Difference of ditto = . . . 26. 47. 13	Natural ver. sine = 0.107311
Sum of the true altitudes = 87. 26. 8	Nat. ver. sine sup. = 1.044743

True central distance = . . . 86° 13' 11"	Natural ver. sine = 0.934069
Nearest preceding dist. at 9 ^h = 85. 31. 45	P.L. 2949. — Diff. 10 decreasing;
	see Rule, p. 383.

Difference of ditto = . . . 0° 41' 26" P.L. 6379

Portion of time = . . . 1^h 21^m 43^s P.L. 3430

Time at nearest preced. dist. = 9. 0. 0

Approx. time at Greenwich = 10^h 21^m 43^s

Correction, Table A = . . . + 3 — See Art. 80 & 81, p. 335 & 337.

Cor. mn. time at Greenwich = 10^h 21^m 46^s

Mean time at place of obs. = 6. 20. 40

Longitude, in time = . . . 4^h 1^m 6^s = 60° 16' 30" West.

Note.—This Method is also adapted to the solution of a perfect or possible spherical triangle.—See *Remark* at the end of Method VI., page 489.

METHOD XII.

To the apparent distance add the apparent altitudes of the objects ; take half the sum, and call the difference between it and the apparent distance the *remainder*.—Then,

To the logarithmic difference (its index being augmented by 10), add the logarithmic co-sines of the half sum and the remainder ; from half the sum of these three logarithms subtract the logarithmic co-sine of half the sum of the true altitudes, and the remainder will be the logarithmic sine of an arch. Now, the logarithmic co-sine of this arch, being added to the logarithmic co-sine of half the sum of the true altitudes (rejecting 10 from the index), will give the logarithmic sine of half the true central distance.

Note.—See General Remarks, page 482.

Example.

Let the apparent central distance between the moon and Spica Virginis be $37^{\circ}12'40''$, the star's apparent altitude $11^{\circ}27'50''$, the moon's apparent altitude $40^{\circ}55'15''$, and her horizontal parallax $54'10''$; required the true central distance ?

Star's apparent alt. = $11^{\circ}27'50''$ — Correc. $4'35''$ True alt. = $11^{\circ}23'15''$

Moon's apparent alt. = $40.55.15$ + Correc. 39.51 True alt. = $41.35.6$

Apparent cent. dist. = $37.12.40$

Sum = $89^{\circ}35'45''$

————— L. diff. = 19.995703

Half sum = . . . $44^{\circ}47'52\frac{1}{2}''$ L. co-si. 9.851012

Remainder = . . . $7.35.12\frac{1}{2}$ L. co-si. 9.996181

Sum = 39.842896

Half sum = . . . 19.921448

Half sum of true alt. = $26^{\circ}29'10\frac{1}{2}''$ L. co-si. 9.951844 . . . 9.951844

Arch = $68^{\circ}48'45''$ L. sine = 9.969604 L. co-si. 9.558014

Half the true distance = . . . $18^{\circ}52'26\frac{1}{2}''$ Log. sine = 9.509358

True central distance = . . . $37^{\circ}44'53''$

To compute the true Distance by Method III., page 485.

Apparent central dist. $92^{\circ}57'45''$ N.V.S. 1.051682

Diff. of apparent alts. 19.28.40 N.V.S. .057229 Log.diff. 9.994463

Remainder = .994453 Log. 5.997584

Natural number 981854 Log. = 5.992047

Difference of true alts. $20^{\circ}7'39''$ N.V.S. = 061070

True central distance $92^{\circ}27'36''$ N.V.S. 1.042924

Near. pr. dist. at 15° = 93.29.52 Prop. log. 2627.—Diff. of Logs. = 0.
—See R., p. 383.

Difference of ditto = $1^{\circ}2'16''$ Prop. log. 4610

Portion of time = . . $1^{\circ}54'1''$ Prop. log. 1983

Ti. at near. prec. dist. = 15. 0. 0

Cor. mn. time at Gr. = $16^{\circ}54'1''$:—The sun's declination reduced to this time is $4^{\circ}5'15''$ south; and the equation of time $10^{\circ}30'$ *additive to apparent time*.

To find the Mean Time at ship or place, and the Longitude.

Sun's true central altitude = $30^{\circ}11'44''$

Sun's south polar distance = 85.54.45 Log. co-secant = 0.001106

Latitude of the ship or place = 43.17.0 Log. secant = . 0.137855

Sum = $159^{\circ}23'29''$ Constant log. . 6.301030

Half sum = . . $79^{\circ}41'44\frac{1}{2}''$ Log. co-sine . . 9.252553

Remainder = . . 49.30.00 $\frac{1}{2}$ Log. sine . . . 9.881047

Sun's horary distance from meridian $3^{\circ}25'10'$ Log. rising = 5.573591

Apparent time of observation = . $20^{\circ}34'50'$

Reduced equation of time = . . +10.30

Mean time at place of observation = $20^{\circ}45'20'$

Mean time at Greenwich = . . . 16.54.1

Longitude of ship or place, in time $3^{\circ}51'19'$ = $57^{\circ}49'45''$ East.

Example 4.

April 1st, 1836, in latitude $49^{\circ}30'$ south, and longitude by account $61^{\circ}30'$ east; the mean of several distances between the moon's nearest limb and the star Antares was $56^{\circ}23'52''$, and that of the corresponding times per watch, *not regulated*, $13^h37^m10^s$; at the same instant the mean of an equal number of altitudes of the star was $56^{\circ}49'16''$, and that of the moon's upper limb $38^{\circ}13'12''$: the index error of the sextant used in measuring the distance was $1'20''$ additive; and the height of the eye above the level of the horizon 23 feet; required the longitude of the place of observation?

Time of obs. per watch $13^h37^m10^s$	Δ 's augm. semidiam. = $16'13''$
Long. $61^{\circ}30'E$, in time — 4. 6. 0	Δ 's red. horiz. parallax = $58'52''$

Greenwich time = $9^h31^m10^s$

The star's obs. alt. reduced to app. alt. is $56^{\circ}44'40''$
 Correction of ditto . -0.37

Alt. of moon's up. limb
 red. to the app. alt. is $37^{\circ}52'23''$
 Correction of ditto . 45.15

Star's true altitude = $56^{\circ}44'3''$

Moon's true alt. = $38^{\circ}37'38''$

Obs. dist. of Δ 's near. li. = $56^{\circ}23'52''$

Index error = . . . $+1.20$

Moon's semidiameter . $+16.13$

Log difference = 9.995616
 See Method IV., page 486.

Apparent central dist. = $56^{\circ}41'25''$ N. V. S. 450835

Sum of the app. alts. . $94.37.3$ N.V.S. sup. 919496

Remainder = 468661 Log. = 5.670858

Logarithmic difference = 9.995616

Natural number 463953 Log. = 5.666474

Sum of the true alts. = $95^{\circ}21'41''$ N.V.S. sup. 906562

True central distance $56^{\circ}7'28''$ N. V. S. 442609

Near. prec. dist. at 9^h = $56.24.58$ Prop. log. 2317 Diff. 11 *decr.* —
 See Rule, p. 383.

Difference of ditto . $0^{\circ}17'30''$ Prop. log. 1.0122

Portion of time . . $0^h29^m50^s$ Prop. log. .7805

Time at near. prec. dist. 9. 0. 0

Approx. time at Greenw. $9^h29^m50^s$

Correction, Table A = $+2$:—See Articles 80 and 81, p. 335 & 337.

Cor. mn. ti. at Greenw. = $9^h29^m52^s$:—The *mean* sun's right ascension reduced to this time is $0^h41^m3^s$, taken to the nearest second.

To find the Mean Time at ship or place, and the Longitude

True altitude of Antares = $56^{\circ}44'3''$
 Polar distance of ditto . . $63.56.15$ Log. co-secant=
 Latitude of ship or place . $49.30.0$ Log. secant =

Sum = . . . $170^{\circ}10'18''$ Constant log. =

Half sum = . $85^{\circ}5'9''$ Log. co-sine =

Remainder = . $28.21.6$ Log. sine = . .

Star's horary dist. east of merid. = $2^{\circ}2'29'$ Log. rising =

Star's right ascension $16.19.22$ to the nearest

Right ascension of the meridian = $14^{\circ}16'53'$

Mean sun's red. right ascension = $0.41.3$

Mean time at place of observation = $13^{\circ}35'50'$

Cor. mean time at Greenwich . . $9.29.52$

Longitude of ship or place, in time = $4^{\circ}5'58' = 61^{\circ}29'30'$

Example 5.

April 1st, 1836, in latitude $40^{\circ}10'$ north, and longitude by $40^{\circ}20'$ west; the mean of several distances between the moon's limb and the star Pollux was $82^{\circ}15'7''$, and that of the corrected time per watch *not regulated* $7^{\circ}39'36'$: at the same instant, of an equal number of altitudes of the star was $74^{\circ}55'15''$, of the moon's upper limb $15^{\circ}24'6''$: the index error of the sextant used in measuring the distance was $1'20''$ *subtractive*; and the height of the eye above the level of the horizon 21 feet; required the longitude and latitude of the place of observation?

Time of obs. per watch $7^{\circ}39'36'$

Long. $40^{\circ}20'$ W., in ti. + $2.41.20$

Greenwich time = . $10^{\circ}20'56'$

The star's obs. alt. re-

duced to app. alt. is $74^{\circ}50'51''$

Correction of ditto . . . -0.15

Star's true altitude = $74^{\circ}50'36''$

☾'s augm. semidiam. =

☾'s red. horiz. parallax =

Alt. of moon's up. limb

red. to the app. alt. is 15°

Correction of ditto : +

Moon's true altitude = 15°

Log. difference = 9.998250

os. dist. γ 's rem. limb = $82^{\circ}15'7''$
 dex error -1.20
 moon's semidiameter . -16.7 } See Method V. page 487.

apparent central dist. = $81^{\circ}57'40''$ Half = $40^{\circ}58'50''$ Log. diff. 9.998250
 difference of app. alts. $59.47.16$ Half = $29.53.38$ Const. L. 6.301030

Sum = $70^{\circ}52'28''$ Log. sine 9.975342
 Difference = $11.5.12$ Log. sine 9.283964

natural number 361898 Log. = 5.558586
 ff. of the true alts. = $58^{\circ}53'39''$ N.V.S. 483380

true central distance $81^{\circ}5'58''$ N.V.S. 845278
 near. prec. dist. at $9^h = 80.17.48$ Prop. log. 2310 Diff. 10 *decr.*—See Rule, page 383.

difference of ditto . $0^{\circ}48'10''$ Prop. log. 5725

portion of time = . . $1^h22^m0^s$ Prop. log. 3415

time at near. prec. dist. 9. 0. 0

approx. mn. ti. at Gr. $10^h22^m0^s$

correction, Table A = +3 :—See Art. 80 and 81, p. 335 & 337.

r. mn. time at Gr. = $10^h22^m3^s$:—The *mean* sun's right ascension reduced to this time is $0^h41^m12^s$ to the *nearest second*; the moon's right ascension is $12^h50^m27^s$, and her declination $1^{\circ}56'17''$ south: hence her north polar distance is $91^{\circ}56'17''$

To find the Mean Time at ship or place, and the Longitude.

true alt. of the moon's centre = $15^{\circ}56'57''$

moon's north polar distance . $91.56.17$ Log. co-secant = 0.000249

latitude of the ship or place . $40.10.0$ Log. secant = 0.116809

Sum = $148^{\circ}3'14''$ Constant log. = 6.301030

Half sum = $74^{\circ}1'37''$ Log. co-sine = 9.439625

Remainder = $58.4.40$ Log. sine = 9.928788

moon's hor. dist. *east* of meridian $4^h28^m35^s$ Log. rising = 5.786501

moon's reduced right ascension . $12.50.27$

right ascension of the meridian . $8^h21^m52^s$

mean sun's red. right ascension . $0.41.12$

mean time at place of observation $7^h40^m40^s$

mean time at Greenwich . . . $10.22.3$

longitude of ship or place, in time $2^h41^m23^s = 40^{\circ}20'45''$ West.

Example 6.

May 22nd, 1836, in latitude $34^{\circ}45'$ south, and longitude by account $44^{\circ}30'$ east, at $10^h28^m0^s$ per watch *not regulated*, the mean of several distances between the moon's remote limb and the star Spica Virginis was $61^{\circ}41'58''$: at the same time the mean of an equal number of altitudes of the star was $39^{\circ}55'0''$, and that of the moon's lower limb $27^{\circ}51'10''$ (the altitudes were but *imperfectly observed*, owing to an obstructed horizon); the index error of the sextant used in measuring the distance was $0'10''$ *additive*, and the height of the eye above the level of the horizon 19 feet; required the true longitude of the place of observation?

Time of obs. per watch $10^h28^m0^s$	☾'s aug. semidiameter = $15'46''$
Long. $44^{\circ}30'$ E. in ti. — $2.58.0$	☾'s red. horiz. parallax = $57'27''$

Greenwich time = $7^h30^m0^s$

The star's obs. alt. reduced to app. alt. is $39^{\circ}50'49''$
 Correction of ditto . . . — 1.9

Alt. of moon's low. limb red. to the app. alt is $28^{\circ}2'45''$
 Correction of ditto . . . + 48.56

Star's true alt. = $39^{\circ}49'40''$

Moon's true altitude = $28^{\circ}51'41''$

Obs. dist. ☾'s rem. limb = $61^{\circ}41'58''$
 Index error + 0.10
 Moon's semidiameter — 15.46

Log. difference = 9.996770
 See Method VI., page 488.

Appar. central dist. = $61^{\circ}26'22''$ Half $30^{\circ}43'11''$ Log. diff. 9.996770
 Sum of app. alts. . . . $67^{\circ}53'34''$ Half $33^{\circ}56'47''$ Const. 1. 6.301030

Sum = $64^{\circ}39'58''$ L.co-sine 9.631335
 Difference = $3.13.36$ L.co-sine 9.999311

Natural number = 848098 Log. = 5.928446
 Sum of true alts. $68^{\circ}41'21''$ N.V.S. sup. 1. 363427

True central dist. $61^{\circ}0'32''$ N. V. S. . 515329

Nr. prec. dis. at 6^h = $61.46.27$ Prop. log. 2931 Diff. 10 *decr.* — See Rule, page 383.

Difference of ditto $0^{\circ}45'55''$ Prop. log. 5933

Portion of time . $1^h30^m10^s$ Prop. log. = 3002

Ti. at near. prec. dist. 6. 0. 0

Approx. ti. at Gr. $7^h30^m10^s$

Correc. Table A. + 3 — See Articles 80 and 81, p. 335 and 337.

Cor. mn. ti. at Green. $7^h30^m13^s$

Since the obstruction of the horizon prevented the altitudes of the objects from being taken to that degree of accuracy which is so essentially necessary to be observed when the mean time is to be inferred from either of them (though sufficiently exact to be employed in the reduction of the apparent to the true central distance); therefore at 20^h 20^m 0^s past noon of the same day, and *per* the same watch, the sun's altitude was observed, and when reduced to the *true* was found to be 14° 12' 33"; at which time the latitude of the ship was 35° 15' south.—Now, from these, the mean time at ship and the longitude are to be obtained in the following manner, viz. :—

Time of observing the sun's altitude per watch	. . . 20 ^h 20 ^m 0 ^s
Time of observing the lunar distance 10.28. 0

Interval between the times of observation 9 ^h 52 ^m 0 ^s
Mean time of lunar observation at Greenwich 7.30.13

Mean time at Greenw. of observing the sun's alt. = 17^h 22^m 13^s

Now, the sun's declination reduced to this time is 20° 35' 25" north; and the equation of time 3^m 35^s, subtractive from apparent time.

Sun's true central altitude	. . . 14° 12' 33"	
Sun's south polar distance	. . . 110.35.29	Log. co-secant 0.028672
Latitude of the ship	. . . 35.15. 0	Log. secant . 0.087968

Sum = 160° 3' 2"	Constant log. 6.301030
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Half sum = 80° 1' 31"	Log. co-sine 9.238582
Remainder = 65.48.58	Log. sine . . 9.960107

Sun's horary dist. from the mer. 3^h 36^m 20^s Log. rising = 5.616359

Apparent time of observation =	. 20 ^h 23 ^m 40 ^s
Reduced equation of time -3.35

Mean time of observation at ship =	20 ^h 20 ^m 5 ^s
Mean time of ditto at Greenwich	17.22.13

Longitude of the ship in time = 2^h 57^m 52^s = 44° 28' 0" east; which is the correct longitude of the place where the sun's altitude was observed for determining the mean time.

Remark.—Instead of finding the interval between the time of observing the lunar distance and that of taking the sun's altitude, as above: this part of the operation may be performed as follows; which perhaps may be more intelligible to those who are not very conversant with this subject.

Mean time deduced from the sun's altitude	20 ^h 20 ^m 5 ^s
Time per watch of observing the sun's altitude	20. 20. 0
<hr/>	
Error of the watch, which is also for mean time	+ 0 ^m 5 ^s
Time per watch of observing the lunar distance	10. 28. 0
<hr/>	
Correct mean time of observing ditto	10 ^h 28 ^m 5 ^s
Correct mean time at Greenwich, per lunar distance	7. 30. 13
<hr/>	
Longitude, in time, the same as above =	2 ^h 57 ^m 52 ^s

Note.—The young navigator must bear in mind that the longitude determined in this manner will *not* be for the place where the lunar distance was observed; but for *that in which the sun's altitude was taken* for the purpose of finding the correct mean time at ship, or the error of the watch.

Remark.—In taking a *lunar observation*, it is customary to have three assistants, two of whom are to observe the altitudes of the objects at the moment that the principal observer measures the distance; the third is to be provided with a watch, showing seconds, and to note down carefully the respective times of observation, with the corresponding distances and altitudes, as expressly pointed out in Article 79, page 335. But, since it sometimes happens, particularly in small ships, that the necessary assistant observers cannot be in readiness, or at liberty to attend, the following instance is given, by which it will be seen how one person may take the whole of the observations himself, without any other assistant than merely a person to note down the times of observation, per watch, with their respective distances and altitudes.

Example 7.

August 24th, 1836, in latitude 43[°] 23' south, and longitude by account 46[°] 20' west, the following observations were made for the purpose of determining the true longitude: the index error of the sextant used in measuring the distance was 1' 40" *subtractive*; and the height of the eye above the level of the sea 17 feet.

Cor. Mn. Time.			Means of the Times.	Means of the Alts. and Dist.
7 ^h 20 ^m 20 ^s	Altitude of Fomalhaut =	27°45'40"	7 ^h 20 ^m 40 ^s	27°51' 0"
7.20.40	Ditto	27.51. 0		
7.21. 0	Ditto	27.56.20		
7.21.30	Alt. of γ 's lower limb =	43.42.30	7.22. 0	43.45.20
7.22. 0	Ditto	43.45.20		
7.22.30	Ditto	43.48.10		
7.23. 0	Dist. of γ 's remote limb =	26.51.10	7.23.50	26.50.40
7.23.50	Ditto	26.50.40		
7.24.40	Ditto	26.50.10		
7.25.10	Alt. of γ 's lower limb =	44.13.10	7.25.40	44.16. 0
7.25.40	Ditto	44.16. 0		
7.26.10	Ditto	44.18.50		
7.26.40	Altitude of Fomalhaut =	28.53.10	7.27. 0	28.58.30
7.27. 0	Ditto	28.58.30		
7.27.20	Ditto	29. 3.50		

To find the Star's Altitude at the Time of taking the mean Distance.

1st time 7^h20^m40^s 1st alt. 27°51' 0" 1st time 7^h20^m40^s 1st alt. 27°51' 0"
 2d time 7.27. 0 2d alt. 28.58.30 $\left. \begin{array}{l} \text{Time of} \\ \text{mean dist.} \end{array} \right\} 7.23.50$

As 0^h 6^m20^s are to 1° 7'30" so are 0^h 3^m10^s to + 33.45

Star's observed alt. at time of taking the mean distance = 28°24'45"

To find the Moon's Altitude at the Time of taking the mean Distance.

1st time 7^h22^m 0^s 1st alt. 43°45'20" 1st time 7^h22^m 0^s 1st alt. 43°45'20"
 2d time 7.25.40 2d alt. 44.16. 0 $\left. \begin{array}{l} \text{Time of} \\ \text{mean dist.} \end{array} \right\} 7.23.50$

As 0^h 3^m40^s are to 0°30'40" so are 0^h 1^m50^s to + 15.20

Moon's altitude at time of taking the mean distance = 44° 0'40"

Computation, &c.

Mean time of obs.	7 ^h 23 ^m 50 ^s	☾'s aug. semidiameter =	16'. 35"
Long. 46° 20' W., in ti. + 3.	5. 20	☾'s red. horiz. parallax =	60. 8
Greenwich time =	<u>10^h 29^m 10^s</u>		
The obs. alt. of Fomal-		Alt. of ☾'s lower limb	
haut red. to app. alt. is	28° 20' 48"	red. to the app. alt. is	44° 13' 18"
Correction of ditto . . .	-1. 45	Correction of ditto . . .	+42. 8
Star's true altitude =	28° 19' 3"	Moon's true altitude =	44° 55' 26"

Logarithmic difference = 9. 994877.

Obs. dist. ☾'s rem. li. 26° 50' 40"	} See Method VII., page 489.
Ind. error of the sext. = -1. 40	
Moon's semi-diam. = -16. 35	

Appar. cent. dist. = 26° 32' 25" Half = 13° 16' 12½"
 Half difference of apparent altitudes = 7. 56. 15

Sum of ditto =	21° 12' 27½"	L. sine =	9. 558407
Difference =	5. 19. 57½	L. sine =	8. 968192
Logarithmic difference =			<u>9. 994877</u>

Sum, abating 10 in the index = 18. 521476

Arch = 10° 30' 9" L. sine = 9. 260738
 Half difference of the true altitudes = 8. 18. 11½

Sum of ditto =	18° 48' 20½"	L. co-s. =	9. 976175
Difference =	2. 11. 57½	L. co-s. =	9. 999680

Sum = 19. 975855

Half the distance = 13° 26' 50" . . . Log. co-sine = 9. 987927½

True central distance = 26° 53' 40"

Near. prec. dist. at 9^h = 27. 36. 59 Prop. log. 3183 Diff. 155 *incr.*—See Rule, page 383.

Difference of ditto = 0° 43' 19" Prop. log. 6186

Portion of time = . . . 1^h 30^m 9^s Prop. log. 3003

Portion of time = . . 1^h 30^m 9^s

Time at near. prec. dist. = 9. 0. 0

Approx. ti. at Greenw. = 10^h 30^m 9^s

Correction, Table A. = — 49 : See Arts. 80 and 81, p. 335 & 337.

Corr. mn. ti. at Greenw. = 10^h 29^m 20^s

Mn. time at pl. of obs. = 7. 23. 50

Long. of ditto in time = 3^h 5^m 30^s = 46° 22' 30" West.

Note.—In this Example we see the necessity of attending to the correction contained in Table A : the neglecting of *that* would, in the present instance, produce an error of 12½ miles in the longitude.

Remarks.—1. Proportional logarithms will be found very convenient in the reduction of the altitudes of the objects to the time of taking the mean lunar distance : thus, to the arithmetical complement of the proportional logarithm of the first term, add the proportional logarithms of the second and third terms ; and the sum, abating 10 in the index, will be the proportional logarithm of the reduction of altitude.—See Example, page 75 or 76.

2. In taking the means of the several observations, those which are evidently doubtful or erroneous ought to be rejected. A doubtful altitude or distance may be readily discovered, by observing if the successive differences of altitude or distance be proportional to those of the times of observation. If, however, the time (which is supposed to be accurately noted) and two of the observations be correct, the erroneous observation may be easily rectified by the rule of proportion.

In order to attain the greatest accuracy in deducing the mean from a series of observations, these ought to be taken at equal intervals of time, as nearly as possible ; such as, *one minute, one minute and a half, or two minutes.*

Example 8.

January 21st, 1835, in latitude 14° 10' ³~~north~~, and longitude by account 94° 30' east ; at 18^h 5^m 17^s, *correct mean time*, the mean of several observed distances between the moon's bright limb and that of Venus was 51° 0' 3" ; at the same time the mean of an equal number of altitudes of the moon's upper limb was 85° 12' 29", and that of the

lower limb of Venus $54^{\circ}31'32''$ * the height of the eye above the level of the sea was 20 feet, and *no* error in the sextant; required the true longitude of the place of observation?

Mean time of observ. = $18^h 5^m 17^s$	$\text{D's aug. semidiameter} = 16'23''$
Longitude $94^{\circ}30'$ east,	$\text{D's red. horiz. parallax} = 59'10''$
in time = . . . $-6.18.0$	Venus's semidiameter = . $22'$
	Venus's horiz. parallax = $22'$
Greenwich time = . $11^h 47^m 17^s$	
Ob. alt. of D's sup. limb = $85^{\circ}12'29''$	Observed alt. Venus's
Semidiameter = . . -16.23	lower limb = . . $54^{\circ}31'32''$
Dip of the hor. for 20 ft. = -4.17	Semidiam. of Venus = $+0.22$
	Dip of hor. for 20 feet = -4.17
Moon's apparent alt. = $84^{\circ}51'49''$	Venus's apparent alt. = $54^{\circ}27'37''$
Correction of ditto = . $+5.14$	Correction of ditto = -0.28
Moon's true altitude = $84^{\circ}57'3''$	Venus's true altitude = $54^{\circ}27'9''$
Auxiliary angle = $60^{\circ}33'1''$	Observed distance = $51^{\circ}0'3''$
	D's semidiam., nearest
Difference of ap. alts. = $30^{\circ}24'2''$	limb = . . . $+16.23$
Auxiliary angle = . $60.33.1$	Venus's do., rem. limb = -0.22
Apparent distance = $51.16.4$	
	App. central distance = $51^{\circ}16'4''$
	See Method I., p. 481.
Sum of two first terms = $90^{\circ}57'3''$	Nat. versed sine sup. . 0.983405
Difference of ditto = $30.8.59$	Nat. versed sine sup. . 1.864715
Sum of two last terms = $111.49.5$	Natural versed sine . . 1.371661
Difference of ditto = $9.16.57$	Natural versed sine . . 0.013095
Dif. of the true alts. = $30.29.44$	Natural versed sine . . 0.138332
True central dist. = $51^{\circ}2'20''$	Natural versed sine . . 0.371208
Near. preced. dis. at 9^h = $52.37.13$	Prop. log. 2467.—Diff. 3 <i>increasing</i> ;
	see Rule, p. 383.
Difference of ditto = $1^{\circ}34'53''$	Prop. log. 2781
Portion of time = . . $2^h 47^m 26^s$	Prop. log. 0314
Ti. at near. preced. dist. = $9.0.0$	
Meantime at Greenw. = $11^h 47^m 26^s$; because the cor. in Table A is = 0.	
Mn. ti. at place of obs. = $18.5.17$	
Long. of do., in time = $6^h 17^m 51^s$ = $94^{\circ}27'45''$ East.	

* This alt. is assumed $14^{\circ}16'57''$ more than the truth for the purpose of shewing the advantage which the modes of computation referred to in the 2^d paragraph p. 517 possesses over the generality of methods for clearing the distance.

Note.—As Venus was *east* of the moon, and a *morning star*, January 21st, 1835, her enlightened hemisphere lay to the eastward: consequently, since the moon was brought over the *dark part* of the planet to the remote or eastern point of her enlightened limb, the semidiameter of Venus becomes *subtractive* from the observed distance:—the omission of this would, in the present instance, produce an error of about 10 miles in the longitude of the place of observation. But, when Venus's semidiameter is 31" or 32", its being neglected would affect the longitude to the value of a *quarter of a degree*.

The reader will please to observe that, in the above Example, the triangle which is formed by the moon and Venus with respect to the zenith is an *imperfect one*: for, since the moon's apparent zenith distance is $5^{\circ}8'11''$, and that of Venus $35^{\circ}32'13''$; and since the sum of these is only $40^{\circ}40'24''$, which is $10^{\circ}35'40''$ *less* than the apparent central distance, or third side; the triangle, therefore, becomes *impossible*, according to the principles of spherical trigonometry:—But, its being of this description does not at all affect the resulting longitude; so long as the operation is performed as above, or by Method II., page 483.—See Remark 9, page 500.

Remarks to be observed in Measuring a Distance between the Moon and Venus.

Venus, as seen through a telescope, exhibits all the various phases of the moon, from the fine thin crescent to the enlightened hemisphere; except that she is never seen completely full: for, even at the time of her greatest *elongation*, her disc only assumes a certain degree of gibbosity, similar to that of the moon about the 12th or the 17th day of her age.—Now, since it is the *limb which is next to the sun that is always enlightened*, and that the other limb, or the one which is farthest from the sun, is dark or opaque, like the unenlightened hemisphere of the moon; therefore, when Venus is to the eastward of the sun,—that is, when her right ascension is greater than the sun's,—it is her *western limb* that will be enlightened: the converse of this takes place when she is to the westward of the sun; because, in this case, as her right ascension will be less than that of the sun, it is her *eastern limb* that will be illuminated.—Hence, when the moon is betwixt the sun and Venus in the interval which takes place between the time of conjunction and *the fifth day* of the moon's age; as Venus will then be to the eastward of the moon, and as the enlightened limbs of both objects will be turned to the westward, and that the bright limb of the moon is always to be brought in contact with that of Venus: therefore, as

the moon's bright limb, which is *now the most remote*, must be made to touch the *nearest* limb of Venus; it is evident that the moon's semidiameter becomes *subtractive* from, and that of Venus *additive* to, the observed angular distance.—And, in like manner, when the moon is betwixt Venus and the sun, in the interval which is between the *twenty-fourth day* of the moon's age and the period of her conjunction, as Venus will then be to the westward of the moon, and as it is the *eastern* limbs of both objects that will then be illuminated; therefore, since the *remote* limb of the moon must be brought in contact with the nearest limb of Venus, the moon's semidiameter becomes *subtractive* from, and that of Venus *additive* to, the observed distance; *the same as in the preceding instance*.—But, when Venus is to the westward of the moon, at any period between the *new moon* and the opposition, it is the western limbs of both objects that will be enlightened.—Hence, as the *nearest* limb of the moon is to be brought in contact with the *remote* limb of Venus; it is manifest that the moon's semidiameter is *additive* to, and that of Venus *subtractive* from, the observed angular distance. And when Venus is to the eastward of the moon at any period between the opposition and the conjunction, or new moon; as it is the *eastern* limbs of both objects that will then be illuminated, and that the *nearest* limb of the moon is to be brought in contact with the *remote* limb of Venus; therefore, the moon's semidiameter becomes *additive* to, and that of Venus *subtractive* from, the observed angular distance, the same as in the last case; in order to obtain the apparent central distance.

The young navigator is particularly cautioned against adopting the mistaken practice of making “the moon's limb pass through the middle of the visible disc of a planet;”—for, since Venus is actually the *brightest* when her enlightened limb is only in the form of a *fine crescent*, like that of the moon about three days old;—therefore, if the distance were measured between the moon's bright limb and *the middle of the visible disc of that planet*, it might produce an error of 8 or 10 miles in the longitude:—this, of course, would not be of much moment on *the high seas*; but still it is *an error*, and as such it ought to be avoided.

In measuring a distance between the moon and Mars or Saturn;—as the semidiameters of these two planets are not of much value, they need not be noticed;—hence it will be quite sufficient to make the moon's bright limb pass through the apparent centre of either of their discs. But, in observing the angular distance between the moon and Jupiter: since the semidiameter of this planet is *considerable*; and since, when visible, it always appears to be full and perfectly round, with its eastern and western limbs truly defined: therefore, in general, it will be advisable to bring the bright limb of the moon in contact

with the nearest limb of Jupiter : then, his semidiameter will be always *additive* to the observed distance ; whilst that of the moon is to be applied by *addition* or *subtraction*, according as her *nearest* or *remote* limb may have been observed.—Here it may be necessary to observe that, as the planets are always in motion, sometimes with, and at others against, the course of the moon ; and as their motions are subject to so many inequalities, as frequently to affect the accuracy of the lunar distance to the value of *a few seconds* ; therefore, in all cases, the angular distance between *the moon and a fixed star should be preferred*, to that betwixt the moon and a planet : for, the fixed stars are as immovable points in the heavens, the places of which are correctly known ; and from which the moon's distance may be determined to the most rigid degree of astronomical exactness.—See Article 78, page 334.

PROBLEM X.

Given the Mean Time at Ship or Place, the observed Distance between the Moon and Sun, a fixed Star, or a Planet, the true Latitude, and the Longitude by Account ; to find the correct Longitude.—Or, to deduce the Longitude from a Lunar Distance, when the Altitudes are determined by Calculation.

RULE.

Compute the true and the apparent altitude of each object's centre, by Problem I., II., III., or IV., between pages 445 and 449 ; according as the moon may be compared with the sun, a fixed star, or a planet,

Reduce the observed to the apparent central distance, agreeably to the directions contained in the third paragraph of the Rule to Problem IX., page 501 ; with which and the computed altitudes of the objects, let the true central distance be determined by any of the Methods given in Problem VII., between pages 481 and 495. Now, from the true central distance thus found, and the given mean time at place of observation, the longitude is to be determined by Problem XXX., page 383.

Example 1.

August 20th, 1836, in latitude $40^{\circ}25'$ north, and longitude by account $56^{\circ}36'$ west, at $3^h43^m36^s$ correct mean time, the mean of several observed distances between the sun and moon was $101^{\circ}34'16''$; the index error of the sextant was $0^{\circ}20''$ *subtractive* ; required the true longitude of the place of observation ?

Mean time of observ. = $3^h 43^m 36^s$
 Longitude $56^{\circ} 36'$ west,
 in time = . . . +3.46.24

Greenwich time = . $7^h 30^m 0^s$

To find the Moon's altitudes.

Mean time of observ. = $3^h 43^m 36^s$
Mean sun's right ascn. = 9.56.38

Right ascen. of merid. = $13^h 40^m 14^s$
 Moon's right ascen. = 16.28.53

Mean sun's red. R.A. = $9^h 56^m 38^s$
 Sun's red. declin. = $12^{\circ} 16' 25''$ N.
 Red. equation of time $3^m 2^s$ *subtrac.*
 ☽'s aug. semidiam. = $16' 12''$
 ☽'s horiz. parallax = $59' 12''$
 ☽'s red. right ascn. = $16^h 28^m 53^s$
 Ditto, declination = $23^{\circ} 58' 28''$ S.

Moon's hor. dis. fr. mer. $2^h 48^m 39^s$ Log. rising . 5.412890
 Moon's declination = $23^{\circ} 58' 28''$ South. Log. co-sine . 9.960817
 Lat. of the ship, or place $40.25.0$ North. Log. co-sine . 9.881584

☽'s merid. zen. dist. = $64^{\circ} 23' 28''$ Nat. ver. sine 567774
 Natural number = 180008 Log. 5.255291

☽'s true central alt. = $14^{\circ} 36' 32''$ N. co-v. sine 747782
 Red. of do., Table XIX. = -53.41

Logarithmic difference = 9.998400

Moon's appar. alt. = $13^{\circ} 42' 51''$

To find the Sun's True and Apparent Altitude.

Mean time of observation = $3^h 43^m 36^s$
 Equation of time = . . . - 3.2

Sun's meridian distance = $3^h 40^m 34^s$. . Log. rising 5.631900
 Sun's declination = . . . $12^{\circ} 16' 25''$ North—Log. co-sine 9.989958
 Latitude of the ship, or place = $40.25.0$ North—Log. co-sine 9.881584

Sun's merid. zenith dist. = $28^{\circ} 8' 35''$ N. V. S. 118228
 Natural number = 318744 Log. 5.503442

Sun's true central altitude = $34^{\circ} 15' 56''$ N. co-v. S. 436972
 Red. of ditto, Table XIX. = + 1.16

Sun's apparent altitude = $34^{\circ} 17' 12''$

To find the True Central Distance and the Longitude.

Observed distance =	101° 34' 16"	} See Method VIII., page 491.
Index error =	. . . - 0.20	
Sun's semidiameter =	+ 15.50	
Moon's semidiameter =	+ 16.12	

Apparent cent. dist. = 102° 5' 58" half = 51° 2' 59"
 Half sum of apparent altitudes = . . 24. 0. 1½

Sum of ditto and half the distance = 75° 3' 0½" L. diff. 9.998400
 Difference = 27. 2. 58½ L. co-si. 9.949689

Sum, abating 10 in the index = 19.359664

Arch = 61° 24' 58" L. co-si. 9.679832
 Half sum of the true altitudes = . . 24. 26. 14

Sum of ditto = 85° 51' 12" L. sine 9.998861
 Difference = 36. 58. 44 L. sine 9.779250

Sum = 19.778111

Half the true distance = . 50° 45' 55" . . L. sine 9.889055½

True central distance = . . 101° 31' 50"
 Nearest preced. dist. at 6^h = 100. 42. 33 P.L. 2615. Diff. 11 dec.; see
 Rule, p. 383.

Difference of ditto = . . . 0° 49' 17" P.L. 5626

Portion of time = 1^h 29^m 59^s P.L. 3011
 Time at nearest preced. dist. = 6. 0. 0

Approx. time at Greenwich = 7^h 29^m 59^s
 Correction, Table A = . . + 4: See Arts. 80 & 81, p. 335 & 337.

Cor. mean time at Greenwich = 7^h 30^m 3^s
 Mean time at place of observ. = 3. 43. 36

Longitude of ditto, in time = 3^h 46^m 27^s = 56° 36' 45" West.

Example 2.

January 10th, 1836, in latitude $36^{\circ}15'$ south, and longitude by account $47^{\circ}30'$ east, at $13^h40^m0^s$ correct mean time, the mean of several observed distances between the moon's *remote* limb and the *nearest* limb of Jupiter was $91^{\circ}14'46''$; at the same time the mean of an equal number of altitudes of the moon's lower limb was $24^{\circ}57'38''$; but, for want of the necessary assistants, the altitude of the planet could not be taken:—the index error of the sextant used in measuring the distance was $1'10''$ *additive*; and the height of the eye above the level of the sea 18 feet; required the true longitude of the place of observation?

Mean time of observ. = $13^h40^m0^s$
 Longitude $47^{\circ}30'$ east,
 in time = . . . $-3.10.0$

Greenwich time = . $10^h30^m0^s$

To find Jupiter's Altitudes.

Mean time of observ. = $13^h40^m0^s$

Ma. sun's right ascen. = $19.17.56$

Right ascen. of merid. = $8^h57^m56^s$

Jupiter's right ascen. = $6.42.14$

Ma. sun's red. R.A. = $19^h17^m56^s$

☽'s aug. semidiam. = 15.52

☽'s red, hor. paral. = 57.50

☽'s red, right ascn. = $12^h44^m9^s$

Ditto, declination = $0^{\circ}37'50''S$

Jupiter's right asc. = $6^h42^m14^s$

Ditto, declination = $23^{\circ}11'14''N$

Jupiter's hor. paral. = 2

Ditto, semidiameter = 24

Jupiter's merid. dist. = $2^h15^m42^s$ Log. rising 5.231040

Jupiter's declination = $23^{\circ}11'14''$ North Log. co-sine 9.963421

Latitude of the place = $36.15.0$ South Log. co-sine 9.906575

Jupiter's mer. zen. dis. = $59^{\circ}26'14''$ Nat. ver. sine 491518

Natural number = 126193 Log. 5.101096

Jupiter's true altitude = $22^{\circ}28'32''$ Nat. co-v. si. 617711

Reduction of ditto = $+2.15$

Jupiter's appar. alt. = $22^{\circ}30'47''$

Observed distance = $91^{\circ}14'46''$

Index error = . . . $+1.10$

Jupiter's semidiam. = $+0.24$

Moon's semidiam. = -15.52

The observed altitude of the ☽'s lower limb reduced to the apparent altitude is , , . . . $25^{\circ}59'26''$
 Correction of ditto = $+50.4$

Moon's true altitude = $26^{\circ}49'30''$

Logarith. difference = 9.996979

Apparent cent. dist. = $91^{\circ}0'28''$

Apparent cent. dist. = $91^{\circ} 0' 28''$; half = $45^{\circ} 30' 14''$: See M. IX., p. 492.
 Half difference of the apparent altitudes = 1. 44. 19½

————— L. diff. 19. 996979

Sum of ditto and half apparent distance = $47^{\circ} 14' 33\frac{1}{2}''$ L. sine 9. 865835

Difference = 43. 45. 54½ L. sine 9. 839920

Sum, the index of the log. difference increased by 10 = 39. 702734

Half sum = + 19. 851367

Half diff. of the true alts. = $2^{\circ} 10' 29''$, , Log. sine 8. 579177

Arch = $86^{\circ} 56' 29''$. . Log. tangent 11. 272190

Logarithmic sine of arch = - 9. 999381

Half the distance = . 45:19:55" . . . Log. sine 9. 851986

True central distance = $90^{\circ} 39' 50''$

Nearest preced. dis. at 9^h = 89. 48. 52 P. L. 2471, — Diff. 12 *decreasing* ;
 see Rule, p. 383.

Difference of ditto = . 0:50:58" P. L. 5480

Portion of time = . . 1^h 30^m 1^s P. L. 3009

Time at nearest preced. dis. 9. 0. 0

Appro. time at Greenw. = 10^h 30^m 1^s

Correc. Table A = . . + 4 : See Arts. 80 and 81, p. 335 & 337.

Cor. mn. tl. at Greenw. = 10^h 30^m 5^s

Mn. time at place of obs. = 13. 40. 0

Long. of ditto, in time = 3^h 9^m 55^s = $47^{\circ} 28' 45''$ East.

Note.—If Jupiter's semidiameter were not applied to the observed distance, it would produce an *error* of about 11 miles in the resulting longitude.

Example 3.

September 25th, 1836, in latitude $39^{\circ} 13'$ north, and longitude by account $42^{\circ} 56'$ west, at 16^h 42^m 20^s correct mean time, the mean of several observed distances between the moon's *nearest* limb and the centre of Mars was $93^{\circ} 31' 34''$; the index error of the sextant was 1' 10" *subtractive*; required the true longitude of the place of observation?

Mean time of observ. = $16^{\circ}42'20''$	Mn. sun's red. R.A. = $12^{\circ}20'33''$
Longitude $42^{\circ}56'$ west,	D's aug. semidiam. = $15'47''$
in time = . . . + 2. 51. 44	D's red. hor. par. = $57'29''$
<hr/>	D's red. right ascn. = $1^{\circ}15'53''$
Greenwich time = . $19^{\circ}34'4''$	D's red. declination = $5^{\circ}58'17''$
<hr/>	Mars' right ascen. = $7^{\circ}39'10''$
Mean time of observ. = $16^{\circ}42'20''$	Mars' declination = $22^{\circ}20'46''$
Mean sun's right ascn. = 12. 20. 33	Mars' hor. parallax = $6''$
<hr/>	Computed altitude of
Right ascn. of the merid. $5^{\circ}2'53''$	Mars = . . . $52^{\circ}49'14''$
<hr/>	Reduction of ditto = + 0.39
Computed altitude of	<hr/>
D's centre = . . $29^{\circ}13'35''$	Mar's appar. alt. = $52^{\circ}49'53''$
Red. of do. Table XIX. = -48. 50	
<hr/>	
Moon's appar. alt. = $28^{\circ}24'45''$	Logarithmic difference = 9. 996727

To find the true Central Distance, and the Longitude.

Obs. dist. D's n. limb = $93^{\circ}35'34''$
 Index error = . . - 1. 10
 Moon's semidiam. = + 15. 47—See Method X., page 493.

Appar. central dist. = $93^{\circ}50'11''$; half = $46^{\circ}55'5\frac{1}{2}''$
 Half sum of apparent altitudes = . . 40. 37. 19

—————L. diff. 19. 996727

Sum of ditto, and half the apparent dist. = $87^{\circ}32'24\frac{1}{2}''$ L. co-si. 8. 632658
 Difference = 6. 17. 46 $\frac{1}{2}$ L. co-si. 9. 997372

Sum, the index of the log. difference being increased by 10 = 38. 626757

Half sum = + 19. 313378
 Half sum of the true alts. = $41^{\circ}1'34\frac{1}{2}''$ Log. co-sine = . 9. 877607

Arch = $15^{\circ}49'36\frac{1}{2}''$ Log. sine = . . 9. 435771 $\frac{1}{2}$

Logarithmic tangent of arch = -9. 452516

Half the distance = . $46^{\circ}32'30''$ Log. sine = . . 9. 860862 $\frac{1}{2}$

True central distance = $93^{\circ}5'0''$

True central distance = $93^{\circ} 5' 0''$
 Nearest prec. dist. at 18^h = $93.54.25$ Prop. log. 2767.—Diff. 15 *increas.*
See Rule, p. 383.

Difference of ditto = $0^{\circ} 49' 25''$ Prop. log. 5614

Portion of time = $1^{\text{h}} 33^{\text{m}} 27^{\text{s}}$ Prop. log. 2847

Time at nearest prec. dis. = 18. 0. 0

Approx. time at Greenw. = $19^{\text{h}} 33^{\text{m}} 27^{\text{s}}$

Correction, Table A = $- 4$: See Arts. 80 & 81, p. 335 & 337.

Cor. mn. time at Greenw. = $19^{\text{h}} 33^{\text{m}} 23^{\text{s}}$

Mean time at place of obs. = 16. 42. 20

Long. of ditto, in time = $2^{\text{h}} 51^{\text{m}} 3^{\text{s}} = 42^{\circ} 45' 45''$ West.

Note.—The *two last* Examples and that given in page 516 comprehend the principal varieties to be found in a lunar observation when a planet is in question:—but, as hinted at in the last paragraph of page 519, the angular distance between the moon and a fixed star should *be always taken in preference* to the moon's distance from a planet:—*or*, whilst the planets are moving with unequal velocities, the fixed stars are as motionless points in the heavens, from which the moon's distance can be correctly, and *easily* determined; and to which distance we can always refer with an unbounded degree of confidence.

PROBLEM XI.

Given the observed Altitudes and Distance of the Moon and Sun, the Time, per Watch, NOT REGULATED, and the Longitude by Account; to find the Latitude, the correct Mean Time, and the Longitude of the Place of Observation, viz., to determine both Latitude and Longitude from the same set of Observations.

RULE.

Reduce the mean time of observation, per watch, to the meridian of Greenwich, by Problem III., page 342; to which let the moon's semidiameter and horizontal parallax be reduced by Problem XV., page 561, and let the reduced semidiameter be increased by the augmenta-

tion in Table IV. Find the apparent and the true altitude of the object's centre by the respective Problems for that purpose, between pages 374 and 378.

Correct the observed distance for index error, if any; to the respective semidiameters of the objects be *added*, and the result will be the apparent central distance. Then, with the apparent altitudes, the apparent central distance, and the true altitude of the sun, find the true central distance by any of the Methods given in Problem VII., between pages 481 and 495; and find the correct mean time at Greenwich corresponding thereto, by Problem XXX., page 500.

To the correct mean time at Greenwich, thus found, reduce the sun's right ascension and declination, and also the equation of time, by Problem XIV., page 357; and the moon's right ascension and declination, by Problem XVI., page 364; and find the difference of the right ascensions.—Then,

With the true central distance between the two objects, the altitudes, reduced declinations, and difference of right ascensions, the latitude be determined by the *General Rule* in page 409, or by Problem VIII.—Now, with the latitude, the altitude of the sun, and the declination of the sun, compute the mean time of observation by Problem II., page 435; the difference between which and the mean time at Greenwich will be the longitude of the place of observation in time. *East*, if the time at ship be the *greatest*; *west*, if the time at Greenwich be the *greatest*.

Note.—Should the sun be too near the meridian, let the difference of the right ascensions be deduced from the moon's true altitude, by Problem III., page 354.

Example

At sea, in south latitude, January 22nd, 1836, at 3^h 40^m time per watch *not regulated*, the mean of several observed altitudes between the moon and sun was 56° 56' 17"; at the same time the mean of an equal number of altitudes of the sun's lower limb was 40° 37' 22", and that of the moon's lower limb 50° 11' 54"; the height of the eye above the level of the sea was 16 feet; the instruments were free from errors, and the longitude by account 35° 6' west. Required the latitude, the correct mean time, and the true longitude of the place of observation?

Time of obs. per. watch = $3^h 40^m 0^s$ | D's aug. semidiameter = $15' 42''$
 Longitude $35^\circ 6'$ west, | D's reduced horiz. par. = $56' 53''$
 in time = . . . + $2. 20. 24$

Greenwich time = . $6^h 0^m 24^s$ | The moon's observ. alt. red. to the
 The sun's observed alt. red. to the | Apparent altitude is . $50^\circ 23' 46''$
 Apparent altitude is . $40^\circ 49' 48''$ | Correction of ditto = . + $35. 29$
 Correction of ditto = . - $0. 59$ | Moon's true altitude = $50^\circ 59' 15''$

Sun's true altitude = . $40^\circ 48' 49''$ | Observed distance = $56^\circ 56' 17''$
 To find the true central distance. | Sun's semidiameter = + $16. 16$
 Difference of ap. alts. = $9^\circ 33' 58''$ | Moon's ditto = . . + $15. 42$
 Auxiliary angle = . $60. 24. 19$ | Apparent central dist. = $57^\circ 28' 15''$
 Apparent central dist. = $57. 28. 15$

Sum of two first terms = $69^\circ 58' 17''$ Nat. ver. sine sup. . . . 1.342490
 Difference of ditto = $50. 50. 21$ Nat. ver. sine sup. . . . 1.631499
 Sum of two last terms = $117. 52. 34$ Nat. ver. sine 1.467561
 Difference of ditto = $2. 56. 4$ Nat. ver. sine 0.001311
 Diff. of true alts. = . $10. 10. 26$ Nat. ver. sine 0.015724

True central dist. = . $57^\circ 13' 12''$ Nat. ver. sine 0.458585
 Near. precd. dist. at 6^h = $57. 12. 47$ Prop. log. 3037. — Diff. 18 increasing;
 see Rule, p. 383.

Difference of ditto = $0^\circ 0' 25''$ P. log. 2.6355

Portion of time = . . $0^h 0^m 50^s$ P. log. 2.3318

Time at near. prec. dis. = $6. 0. 0$

Mean time at Greenw. = $6^h 0^m 50^s$; which is to be considered as the correct mean time, because the portion of time is so small that the equation in Table A vanishes, or becomes nearly insensible.

Now, the right ascensions and declinations of the objects being reduced to the correct mean time at Greenwich, thus found, the results will be as follows, viz. :—

Sun's R. A. = . $20^h 16^m 19^s 67$, and dec. $19^\circ 46' 19''$ south = A
 Moon's ditto = . . $0. 2. 58. 79$, and dec. $4. 21. 57$ south = R

Difference of R. A. = $3^h 46^m 39^s 12$ Log. half elapsed time = 0.078078
 Declination of A = $19. 46. 19$ Log. secant = . . . 0.026389
 True central dist. = $57. 13. 12$ Log. sine = . . . 9.924670

Arch the first = . . $4^h 36^m 59^s$ Log. half elapsed time = 0.029137

True central dist. = $57^{\circ}13'12''$ Log. co-secant = . . . 0.075330
 True altitude of R. = 50.59.15 Log. secant = . . . 0.201011

Sum of ditto = . $108^{\circ}12'27''$ Nat.co-V.S. 050069
 True alt. of A = . 40.48.49 Nat.co-V.S. 346180

Difference = 296111 Log.=5.471455

Arch the second = . $4^{\circ}15'27''$ Log. rising = . . . 5.747796
Arch the first = . 4.36.59

Arch the third = . $0^{\circ}21'32''$ Log. rising = . . . 3.644500
 Declination of R = $4^{\circ}21'57''$ Log. co-sine = . . . 9.998738
 True altitude of R = 50.59.15 Log. co-sine = . . . 9.798989

Difference of ditto = $46^{\circ}37'18''$ Nat. V. S. 313187
 Natural number = 2768 Log.=3.442227

True lat. of the place = $43^{\circ}9'38''$ Nat.co-V.S. 315955 ; which is South.

To find the Mean Time and the Longitude.

Sun's true central altitude = $40^{\circ}48'49''$
 Sun's south polar distance = 70.13.41 Log. co-secant = 0.026389
 Latitude of the ship = . . 43. 9.38 Log. secant = 0.137010

Sum = . . . $154^{\circ}12'8''$ Constant log = 6.301030

Half sum = . . $77^{\circ}6'4''$ Log. co-sine = 9.348755
 Remainder = . . 36.17.15 Log. sine = 9.772202

Apparent time = . . . $3^{\circ}28'10''$ Log. rising = 5.585386
 Equation of time = . . . + 11.50

Mean time at ship = . . $3^{\circ}40'0''$
 Cor. mean time at Greenw. = 6. 0.50

Long. of the ship, in time = $2^{\circ}20'50''$ = $35^{\circ}12'30''$ West.

Hence the latitude of the place of observation is $43^{\circ}9'38''$ south ; the correct mean time $3^{\circ}40'0''$, and the true longitude $35^{\circ}12'30''$ west ; as required.

Note.—From the above Example the Method of deducing the latitude, the mean time, and the longitude, from the same set of observations, when a fixed star, or a planet, is one of the objects, will appear obvious.

In taking leave of *the lunar observations*, I am desirous of intimating to the young navigator that, whenever it can be conveniently done, the longitude should be inferred from *two distances on opposite sides* of the principal object, viz., from an observed angular distance between the moon's enlightened limb and a star east of her; and, also, from one to the westward of her: for, in this instance, as the imperceptible errors of the sextant (and the *very best is not perfect*), and the unavoidable errors in its use, would have a mutual tendency to correct each other; half the sum of the two longitudes would be more entitled to confidence than either longitude considered singly.

PROBLEM XII.

To find the Longitude of a Place by the Eclipses of Jupiter's Satellites.

The eclipses of Jupiter's satellites are distinguished by the appellations of *immersions*, or *emersions*. An *immersion* of a satellite signifies the instant of its entrance into the shadow of Jupiter; and an *emersion*, that of its re-appearance out of the shadow. The instant of an immersion is known by the *last appearance of the satellite*; and that of an emersion, by its *first appearance* out of the shadow of the planet.

FIRST,

To know if an Eclipse will be visible at a given Place.

RULE.

Reduce the mean time of the eclipse at Greenwich (as given in page XX. of the month in the Nautical Almanac) to the meridian of the place of observation, by Problem IV., page 343. Then, if at this reduced time Jupiter be not less than 8 degrees above the horizon of the given place, and the sun about as many degrees below it, or stars of *the third magnitude*, to be visible to the naked eye, the eclipse will be visible at such place: this, it is presumed, does *not* require to be illustrated by an example.

M M

SECOND,

To find the Longitude of the Place of Observation.

RULE.

To the observed time of the eclipse per watch, at the given place, apply the error of the machine for mean time, deduced from observations of the sun's altitude, or from those of the moon, a planet, or a fixed star: hence the correct mean time at the place of observation will be known.—Now, the difference between this and the mean time of *immersion* or *emersion* at Greenwich, will be the longitude of the place of observation, in time:—*east*, if the time at the given place be the *greatest*; otherwise, *west*.

Example.

January 7th, 1836, at Trincomalee, in latitude° 8:33' north, and longitude, by account, 81:22' east, an *emersion* of the first satellite of Jupiter was observed to take place at 13^h 18^m 54^s per watch, the *error* of which was 1^m 16^s *slow* for mean time; required the true longitude of the place of observation?

Time of emersion per watch = 13^h 18^m 54^s

Error of the watch, *slow* = + 1. 16

Mean time at place of observation = . . 13^h 20^m 10^s

Mean time of emersion at Greenwich = 7. 54. 42

Longitude of the given place, in time = 5^h 25^m 28^s = 81:22' East.

Remarks.—1. If Jupiter be far enough from the meridian at the time of observing an immersion or an emersion of one of his satellites, and his altitude to be taken at the exact moment of the satellite's disappearance or re-appearance; the correct mean time of observation may be inferred therefrom by Problem IV., page 439; and thus any irregularity in the going of the watch, betwixt the time of finding its *error* and the moment of observation, will be provided against or obviated.

2. The eclipses of Jupiter's satellites afford the readiest means of determining the longitudes of places *on shore*; but, since those eclipses cannot be distinctly seen, except through telescopes of a high magnifying power,—and since glasses of this description cannot be used at sea, on account of the incessant motion of the vessel, which continually throws the planet out of the field of view; the above method of find-

ing the longitude is, therefore, of little use, if any, to the practical navigator:—moreover, it is *not always available*; because Jupiter passes so *apparently* close to the sun at certain intervals, that, for about six weeks in every year, that planet and its satellites are entirely lost in the refulgent splendour of the solar rays.

PROBLEM XIII.

To find the Longitude of a Place by an Eclipse of the Moon.

RULE.

Observe the moments, per watch (*duly regulated to mean time*), of the beginning and the end of the eclipse: then, half the sum of the observed times will be the mean time of *the middle of the eclipse*; the difference between which and that given in the Nautical Almanac, will be the longitude of the place of observation, in time: *east*, if the time at such place be the greatest; otherwise, west.

Note.—If only the beginning or the end of the eclipse be observed, the mean time of observation must be compared with the time answering to *the corresponding phase* in the Nautical Almanac; but, it must be remembered, that it will always be conducive to greater accuracy to observe the instants of both phases.

Example.

April 30th, 1836, in latitude $38^{\circ}24'$ north, and longitude, by account, $99^{\circ}12'$ west, the beginning of a lunar eclipse was observed at $12^{\text{h}}28^{\text{m}}33^{\text{s}}$, and the end thereof at $14^{\text{h}}33^{\text{m}}21^{\text{s}}$; the error of the watch was $1^{\text{m}}13'$ *fast* for mean time; required the true longitude of the place of observation?

Obs. beginning of the eclipse, per watch = $12^{\text{h}}28^{\text{m}}33^{\text{s}}$

Observed end of ditto = $14.33.21$

Sum of the observed times = $27^{\text{h}}1^{\text{m}}54^{\text{s}}$

Middle of the eclipse, per watch = . . . $13^{\text{h}}30^{\text{m}}57^{\text{s}}$

Error of the watch, *fast* = -1.13

Mean time of the middle of the eclipse = $13^{\text{h}}29^{\text{m}}44^{\text{s}}$

Mean time of ditto at Greenwich = . $20.6.24$

Longitude of the given place in time = $6^{\text{h}}36^{\text{m}}40^{\text{s}} = 99^{\circ}10'$ West.

M M 2

Note.—As the beginning and the end of a lunar eclipse takes place at the same instant on all parts of the earth, where the moon is above the horizon; the above Method would afford an easy means of finding the longitude, provided the eclipses were very frequent, and that the times of the phases could be observed to the necessary degree of exactness: but, since there are seldom more than two lunar eclipses visible in the course of the year, and since the precise instants of *the first contact* with the earth's *real dark shadow*, and of *the last contact* with the same, cannot be easily determined, on account of the uncertainty of the earth's *penumbra*; the Method in question is, therefore, of no practical utility.

PROBLEM XIV.

Given the observed Mean Time of Transit of the Moon's enlightened Limb; to find the Longitude of the Place of Observation.

Although the proposed method of finding the longitude cannot be of much use to the practical navigator, because of the extreme difficulty of determining the correct mean time of the moon's bright limb passing over the meridian of a ship that is *under way*, even though equal altitudes were employed; yet, to such maritime surveyors as are furnished with *transit instruments*, it will be found of very considerable utility in settling the longitudes of places on shore: for which purpose its conciseness and *correctness* are strong recommendations in favour of its adoption.—The celebrated Dr. Maskelyne speaks of this Method in his Instructions relative to the Transit of Venus over the sun's disc in the year 1769; and it is also spoken of by many other eminent astronomers and mathematicians.—Mr. Pigott very strongly recommends it, in the Philosophical Transactions for the years 1786 and 1790:—"Being convinced," as he says, "that in a short time it must be universally adopted, having every advantage over Jupiter's first satellite, and but little inferior in precision to occultations."

All that is necessary for determining the longitude by the Method in question is, to note the exact mean time of the moon's enlightened limb passing over the meridian of the observer: this is done by noting the instant, per watch, *well regulated to mean time*, that the bright limb of the moon *appulses each of the wires* in the transit instrument:—the sum of the appulses divided by their number will be the mean time of transit of the moon's bright limb over the meridian of the

place of observation. Now, the mean time of transit being thus known, the longitude is to be found by the following

RULE.

Reduce the correct mean time of observation to the meridian of Greenwich, by Problem III., page 342; to which let the *mean* sun's right ascension be reduced by Problem V., page 344; and, also, the moon's semidiameter and declination, the former by Problem XV., page 361, and the latter by Problem XVI., page 364.—Convert the moon's semidiameter into seconds.

To the mean time of observation, add the *mean* sun's reduced right ascension, and the sum, diminished by 24 hours if necessary, will be the right ascension of the meridian of the given place.

Now, on the principles of similar triangles, as explained between the *last* line of page 236, and the fifth line of page 237; and, also, in the *final* analogy commencing at line 5, page 238:—if A B be esteemed as the moon's polar distance at the time of observation, and B C her reduced semidiameter, and if A D be taken as the moon's polar distance extended to the equator; then, it will be,

As the moon's polar distance at time of observation is to her reduced semidiameter, so is the moon's polar distance of 90 degrees* to her semidiameter (D E) in equatorial seconds of motion; which, being converted *into time*, will be the moon's semidiameter in *right ascension*. This proportion may be reduced to a concise logarithmical expression, as thus,—

To the log. secant, less radius, of the moon's declination, add the logarithm of her semidiameter, and the constant logarithm 8.823909;† the sum, abating 10 in the index, will be the logarithm of the moon's semidiameter in *right ascension*; the natural number of which must be taken out to two places of decimals, viz., to *hundredths* of a second.

To the right ascension of the meridian apply the moon's semidiameter in *right ascension*, by addition when the enlightened limb is to the westward, but by subtraction when it is to the eastward; the sum, or difference, will be the correct right ascension of the moon's centre at the moment of observation.

Enter the Nautical Almanac, under the given day, and find the difference between the computed right ascension and the tabular right ascension which is *next less to it*; find also the difference between the two tabular right ascensions which are next less and next greater than the computed one: then,

When referred to

* ~~At~~ the equator the polar distance of a celestial object is always equal to 90 degrees.

† The log. ar. comp. of 15, the common divisor for turning motion into time.

From the sum of the first difference and the constant log. 3. 556303,* subtract the logarithm of the last difference, and the remainder will be the logarithm of a portion of time in seconds : raise this to minutes, if necessary, to which prefix the hour corresponding to the *next less* right ascension, and the result will be the correct mean time at Greenwich :—The difference between which and the observed mean time of transit will be the longitude of the place of observation, in time : *east*, if the time at such place be the greatest ; otherwise, west.

Example.

January 26th, 1836, in latitude $33^{\circ}50'$ north, and longitude, by account, $35^{\circ}30'$ east, the enlightened *western* limb of the moon was observed to transit over the meridian at $6^h39^m54^s$ correct mean time ; required the true longitude of the place of observation ?

Mean time of observ.= $6^h39^m54^s$	Mn. sun's red. R.A.= $20^h19^m59^s35$
Longitude $35^{\circ}30'$ east,	Δ 's red. semidi.= $14^{\circ}48'7''=888^s7$
in time = . . . $-2.22.0$	Δ 's red. declin.= $16^{\circ}16'29''N.$
Greenwich time = . $4^h17^m54^s$	

Mean time of observation = . . . $6^h39^m54^s0$
Mean sun's reduced right ascension= $20.19.59.35$

Right ascension of the meridian= . $2^h59^m53^s35$

Moon's reduced declination= . . $16^{\circ}16'29''$ Log. secant 0.017760
Moon's reduced semidiameter= . 888^s7 Logarithm= 2.948755
Constant logarithm= 8.823909

Δ 's semidiam. in right ascen.= $+1^m1^s72$ Logarithm= 1.790424
Right ascen. of the meridian= $2.59.53.35$

Right ascen. of moon's centre= $3^h0^m55^s07$	} Diff.= $0^m34^s38=34^s38$
Next less right asc. at 4 hours= $3.0.20.69$	
Next greater ditto at 5 hours= $3.2.16.30$	

Diff.= $1.55.61=115.61$

* The logarithm of one hour in *seconds*, viz., of 3600 seconds.

First difference, in seconds=	34'38	Logarithm=	1.536306
Constant logarithm =	.	.	3.556303
Sum =	.	.	5.092609
Last difference, in seconds=	115'61	Logarithm=	2.062955
Portion of time=	17 ^m 51 ^s = 1071'	Logarithm=	3.029654
Cor. mean time at Greenwich=	4 ^h 17 ^m 51 ^s		
Mean time at place of observ.=	6.39.54		
Longitude of ditto, in time =	2 ^h 22 ^m 3 ^s = 35 ^h 30 ^m 45 ^s	East.	

Remarks.—1. When the transit of the moon's *western* limb is observed, her centre will then be to the *eastward* of the meridian; and as the right ascension is reckoned from west to east, it is therefore manifest that the value of the moon's semidiameter becomes *additive*, as in the above Example.—The converse of this takes place when the transit of the moon's *eastern* limb is observed; because, in this case, since her centre will be to the *westward* of the meridian, the value of her semidiameter becomes *subtractive* from the right ascension of the meridian.

2. The above is a very concise and practicable method of settling the longitude of places on *shore*; but, it is to be observed, that the *transit instrument* must be so carefully adjusted as to move exactly in the plane of the meridian; for any trifling deviation therefrom, either to the eastward or westward, would sensibly affect the result of the observation. And, since the moon's right ascension is to be deduced, in a great measure, from the right ascension of the meridian, the time of transit, therefore, ought to be noted to the decimal part of a second:—an error to the value of *one second* in the observed mean time of transit would, at a *mean rate*, produce an error of about 29½ seconds in the computed right ascension of the moon's centre, and this would affect the longitude *nearly* 7½ miles. But, since those who are furnished with transit instruments are also provided with good chronometers; there cannot be any difficulty in knowing the exact moment, in *true mean time*, that the moon's enlightened limb will come to the meridian of the place of observation, viz., where the transit instrument is fixed.

• PROBLEM XV.

To deduce the Longitude from an Occultation of a fixed Star by the Moon.

The solution of the present Problem was originally intended for the instruction of Mr. Edmund Mowbray Lyons, a midshipman belonging to Her Majesty's ship Portland, and second son of Captain Sir Edmund Lyons, of the Royal Navy; a Knight Commander of the Royal Hanoverian Guelphic Order, Knight of the Cross of St. Louis, in France, and of the Cross Redeemer of Greece, and now the resident British Minister at Athens; under whose judicious command I had the honour of serving, during a period of three years, on board the Blonde frigate.—And, as Mr. E. M. Lyons, like the generality of young gentlemen in the Royal Navy, cannot be supposed to have entered very deeply into the elements of Astronomy, I have, therefore, endeavoured to divest this important Problem of all the mystical abstruseness of analytical expressions; to remove the dark veil from the *occult* doctrine of the occultations, and thus to reduce the whole to such a perfect state of simplicity, as not only to be adapted to juvenile capacities, but also to be peculiarly suited to the comprehension of every person whose early entrance into the nautical world may have precluded his making any thing like a great proficiency in the elementary parts of the sciences.

Of all the various methods which astronomers have devised, from time to time, for the solution of the important problem of the longitude, that which has been proposed by means of an occultation is, by far, the most correct, the most easy of observation, and the most unsusceptible of instrumental errors.

It is considerably more exact than the common method by *the lunar distances*: for, since an immersion, or an emersion, can be strictly observed to the *nearest second of time* with a good night-glass, such as those generally used by the captains of Her Majesty's ships of war; it, therefore, excludes the possible admission of any of those unavoidable errors to which *the lunar distances* are subject; as well from imperceptible defects in the sextants and quadrants employed in taking the distance and altitudes of the objects, as from an imperfect manner of using those instruments on the part of the observers.

The great Dr. Halley, in treating of this momentous subject, ob-

serves :—“ Of all the methods hitherto proposed for finding the longitude, none seems more adapted to the purpose than that by the occultations of the fixed stars by the moon. For those *immersions* of the stars which happen on the dark semicircle of the moon, and their *emersions* from the same, are perfectly momentaneous, without that ambiguity to which observations of the eclipses of the moon, and those of Jupiter’s satellites are subject.”— And Dr. Maskelyne, the late *Astronomer Royal*, in touching upon the same sublime point, expresses himself thus :—“ The conjunction of the moon with the planets, or fixed stars not less than the fourth magnitude, which may prove occultations in some inhabitable parts of the globe, are evidently designed to instruct mariners or travellers to look out frequently for such observations ; which, if they happen to prove occultations, and are carefully observed, will afford a certain means of determining the longitude of the place of observation.”

The method of finding the longitude by means of an occultation was proposed by *Cassini*, the celebrated continental astronomer, in the History of the Royal Academy of Sciences for the year 1700 : it has been since treated of by many other eminent mathematicians and astronomers, English as well as French ; but, since these have handled the subject in such a very learned manner as to carry their researches far above the range of ordinary ideas, the demonstrations which they have given are, therefore, so highly sublimed in science as to be perfectly incomprehensible to every person who is unacquainted with *Analytical Investigations*. And hence it is, that the very best of all methods for settling the longitude is still a *desideratum* in the nautical world. For the purpose of doing away with that *desideratum*, I shall adopt such plain and familiar illustrations as cannot fail of being intelligible to all who are acquainted with the trigonometrical canon, and who have a slight knowledge of the doctrine of spherics. In doing which, I shall make use of a little circumlocution, so as to reduce every elementary point that comes under consideration to the comprehension of school-boys, and to the standard of common capacities.

Before entering upon the proposed method of computation, it may be advisable to make a few *general observations*, for the guidance of such persons as have not had an opportunity of going through a regular course of the elementary branches of Astronomy.

1. An *occultation* signifies the time that a fixed star, or planet, is hid from our sight, when eclipsed by the interposition of the moon.

2. An *immersion* signifies the exact moment that a fixed star, or

planet, disappears behind the eastern limb of the moon ; and an *emersion*, the instant of the star's re-appearance, or of its coming out from behind the moon's western limb.

3. While the moon is performing her periodical revolutions round the heavens, she frequently passes between the earth and certain fixed stars or planets that lie in the plane of her track ; and thus intercepts them, for a short space of time, from the view of an observer. When a star, or planet, is so intercepted or hid from our sight, it may be said to be *eclipsed* ; but, as this species of eclipse, like that of the sun, is subject to the effects of the moon's horizontal parallax, it is, therefore, not universal : and thus a star may be occulted by the moon at London, which will not prove to be an occultation at Paris, and conversely. For, since the apparent place of the moon in the firmament is *depressed by parallax*,* this element, therefore, diminishes her apparent declination when it is of the same name with the latitude, and increases her declination when it is of a denomination contrary to the latitude ; that is, the horizontal parallax always increases the moon's apparent polar distance. Hence, in north latitude, the interposition of the body of the moon can only intercept from the view of an observer on the earth or sea such star, or planet, whose northern declination is some minutes *less* than the moon's true declination of the same name, or whose southern declination is some minutes *greater* than her true declination of the same denomination ; and *vice versa*, in south latitude.

This, perhaps, may be rendered more intelligible by observing that, when the latitude and the declination of the objects are of the same name, the declination of the moon must be some minutes greater than that of a star : but, if of contrary names, the moon's declination must be some minutes less than that of a fixed star or planet. Or, in other terms, *universally* the polar distance of the star or planet must be some minutes greater than that of the moon to produce an occultation.

4. It frequently happens that there is a perfect conjunction of the moon and a fixed star both in right ascension and declination, as seen from the centre of the earth ; and yet, to an observer on the face of the globe, such conjunction may not prove to be an occultation : for, owing to the depressing nature of parallax,* the moon will appear to pass under, or over the star, according as the latitude of the place of observation and the declination of the objects are of the same or of contrary denominations. To this there may be an exception in certain cases (particularly in latitudes between the tropics), when the moon,

* See the explanation of the moon's horizontal parallax, between pages 29 and 32.

at the time of conjunction with a fixed star, approaches so near to the zenith as to cause the value of her parallax *in altitude* to be reduced below the double of her true semidiameter:—this may happen when the moon is in *apogee*, or at her greatest distance from the earth, provided she passes within 34 degrees of the zenith: but, in general, it may happen whenever the moon approaches within $28^{\circ}30'$ of the zenith; because, betwixt this point and the zenith, the amount of her parallax will be always less than twice the value of her true semidiameter, viz., less than the full measure of her *whole diameter*.

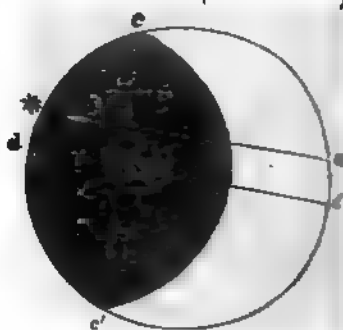
5. The *observation part* of an occultation is extremely simple; for it consists of no more than merely noting the exact moment, by means of a well regulated watch, or chronometer, that the moon's eastern limb covers a fixed star, or the moment that a star re-appears or emerges from behind the moon's western limb. The *immersion* of a fixed star is easily observed at sea; because the sight may be directed to the star by means of a good night-glass, or even a common telescope, until it is occulted by the moon. In case of an *emersion*, the observer should direct his attention to that part of the moon's western limb from which the star is expected to emerge, some minutes before the estimated time of emersion, and continue looking until the instant of the star's re-appearance. The *approximate time* of a star's emersion may be found by adding to the observed instant of *immersion* the time required by the moon to pass over an arc of her orbit, equal to the chord of the segment of her disc cut off by the *apparent passage of the star*. This time, and the length of the chord of the segment so cut off, may be nearly guessed at by attending to the following considerations and general precepts.

6. When the moon is in or near the equinoctial, which she must be twice in every lunation, the line of her path will be about east 25° north, or east 25° south, according to the direction of her declination. When she is at the northern or southern limits of the zodiac, her track will be very nearly due east: in other parts of the zodiac the average course of her track is about 13° to the northward or southward of east, according to the tendency of her declination. But, in general, when the enlightened part of the moon is in the form of a *crescent*, and also when it is *gibbous*, the diameter which is perpendicular to the imaginary line that connects the extremities of her *cusps* or horns, or that joins the points which divide the light from the dark on her upper and lower limbs, will indicate the course of her apparent path in the heavens, with respect to the position of the observer.

7. Take the moon's semidiameter in the compasses, from any scale

of equal parts, and with it describe a circle to represent the lunar disc; on which note, as near as can be estimated by the eye, the points of her cusps, when the enlightened part is less than a semicircle; or the points which separate light from dark on her limbs, when the enlightened hemisphere is more than a semicircle: and, note also the point of her eastern limb which was in contact with the star, and behind which the star may have disappeared.

Connect the cusps or points $c\ c'$ by a right line, as in the annexed diagram: bisect this line; and through the point of bisection at b , draw the diameter $d\ b\ d'$ at right angles thereto, and it will indicate the apparent line of the moon's path at the moment of observing the star's immersion.*



From the point of contact at the mark $*$, draw a chord line *parallel* to the diameter $d\ b\ d'$, and it will show the track of the star's occultation. The point a represents the part of the moon's western limb from which the star will make its re-appearance. Take the chord line $*\ a$ in the compasses, and apply it to the same scale of equal parts that the semidiameter was taken from; and it will give the probable length of the chord of the segment $*\ c\ a$, cut off by the *apparent track* of the star.—Now, since the chord line, thus determined, bears the same proportion to the moon's relative hourly motion in *space*,* that this motion does to one hour: therefore, as the moon's relative hourly motion in right ascension* (converted into degrees, or parts of a degree) is to 60 minutes, so is the measure of the chord line $*\ a$, to the approximate interval of time betwixt the instants of the star's disappearance behind the moon's eastern, and of its re-appearance from behind her western limb. But, as the time thus estimated only amounts to a probability, therefore, to make certain of detecting the star at the exact moment of its re-appearance, the sight should be directed to that part of the moon's western limb from which

* If the moon's relative path be wanted, it may be readily determined by the following formula, viz.:—To the log. co-secant of the moon's relative motion in right ascension,* add the log. sine of her relative motion in declination,* and the log. secant of her correct declination; the sum, abating 20 in the index, will be the orbital angle, or the correct angle of the moon's relative path in the heavens. However, for purposes like the above, such exactness as this becomes quite unnecessary.

* The excess of the moon's motion over the sun's during the space of one hour.

it is expected to emerge, a *few minutes* before the expiration of the computed time of emersion.

8. The most favourable season for observing an *emersion* is betwixt the full moon and the new: for, since the sight will then be directed to the unenlightened hemisphere of the moon, it can easily detect the star, at the very instant of its appearing to emerge from behind her dark western limb.—And, on this principle, it would appear that the best season, or period, for observing an *immersion*, is between the new moon and the full; because, as the moon's unenlightened limb will then approach the star, the moment of contact may be observed to the decimal part of a second. But, as to an *immersion*, it may always be observed, in every stage of the moon, whether its eastern limb be dark or bright, within one second of the truth, which is near enough for most purposes; because an error of 4 seconds in the observed time of an *immersion*, or an *emersion*, would only affect the longitude to the value of one mile.

9. The weariness occasioned by the prolixity of the necessary calculations for clearing the moon's apparent place from the effects of her horizontal parallax, has been hitherto considered as an *almost insuperable* objection against having recourse to the occultations of the fixed stars for the determination of a ship's longitude:—and the *over-scientific* manner in which the subject has been treated by elementary writers, which to the nautical world is quite *enigmatical*, has only tended to render that objection the more unconquerable.

But, since the elements of the moon, in right ascension and declination, are now given in the Nautical Almanac to *every hour* (which does away with the trouble of equating those elements on account of *second differences*), a considerable portion of the objection just mentioned has been removed; and it is presumed that the whole will be done away with, and for ever obviated, by the simplicity of the proposed method of computation; for, it is so plain, and so easy of attainment, as to be within the reach of every person who understands the use of the common logarithmical tables.

10. The proposed method of deducing the longitude from an occultation of a fixed star, or planet, by the moon, may be illustrated in the following manner, agreeably to the principles of spherical trigonometry.

In the annexed diagram let the section of a great circle H E Z P, represent an arc of the meridian, in which Z P represents the co-latitude of the place of observation. Let the point *t*, be the true

is in conjunction with the star ; and hence the apparent zenith distance $Z a$ is known.

Since the azimuth angle, viz., $t Z P$, just found, is supposed to be *unaffected by parallax* ; therefore, in the triangle $P Z a$, the co-latitude $P Z$, and the apparent zenith distance $a Z$, with the included angle Z , are given ; to find the apparent polar distance $P a$: the difference between which and the true polar distance $P t =$ the arc $a r$, is the effects of parallax on the polar distance.—Again, in the same triangle, viz., in $P Z a$, the sides and the angle at the zenith are given ; to find the angle $Z P a$: the difference between this angle and the true hour angle $Z P t$, gives the polar angle $t P a$; which determines the parallax in right ascension, measured by the arc of the equator $d c$.

Let the mark \star , represent a fixed star that is about to be occulted by the moon ; the line of the occultation being in the direction of $\star b$: bisect this line ; and between the point of bisection and the moon's centre (her disc being represented by M) draw the line $M S$, and it will be *perpendicular* to the track of the occultation $\star S b$.—Then $M S$, will express the value of the difference between the approximate polar distance $P M$, and the apparent polar distance $P a$.—Draw the line $M \star$, and it will represent the moon's true semidiameter.—Then, in the right-angled triangle $M S \star$; *right angled at S*, and which may be considered rectilineal ; the hypotenuse $M \star$, and the leg $M S$ are given ; to find the leg $\star S$.—Now, this leg being reduced to the equator in the ratio of the sine of *half the sum* of the approximate and apparent polar distances to radius *unity* ; the result will be the correct value of $\star S$ in right ascension, or the measure of half the chord line, *in right ascension*, of the segment $\star e b$, which is cut off by the apparent passage of the star.—Hence all the elements of the occultation are known.

11. Having thus explained the principles upon which the computation of an occultation is founded ; I have now to observe, that the above solution, though strictly correct in theory, becomes often *impracticable* at sea, owing to the extreme difficulty of finding the correct astronomical values of the apparent zenith distance $Z a$, and the azimuth angle $a Z P$:—hence, we shall now treat of a more practicable mode of determining the parallaxes of the moon, to the most rigorous degree of mathematical exactness.

12. Let the mark \star in the diagram, represent a fixed star or planet, which is about to be occulted by the moon M .—Then it will appear manifest that the point of the moon's limb which is in contact with the star, has the same *apparent* right ascension, and the same *apparent*

declination that the star itself has. And, if the apparent value of that point be accurately reduced to the centre at M, it will show the correct central right ascension and declination of the moon at the instant of observation; the reduction of the point of contact depends upon the following considerations:—

13. Since the horizontal parallax of the moon causes her to appear further from the zenith than she really is; it therefore affects her meridian distance, by making it more than the truth; and thus the *apparent* hour angle must be diminished by a quantity which is equal to the sine of the co-latitude multiplied by the sines of the hour angle and the horizontal parallax; divided by the rectangle of the sine of the apparent polar distance and the secant of 60 degrees.

14. The horizontal parallax of the moon increases her *apparent* distance from the elevated pole of the heavens, with respect to the place of an observer on the earth; and hence it is subject to a reduction: but, as the amount of this reduction is affected by the hour angle, according as its value is more or less than 90 degrees; it therefore consists of *two distinct parts*; one of which arises from the multiplication of the co-sines of the latitude and the corrected hour angle by the sines of the apparent declination and horizontal parallax; and the other, from the multiplication of the sines of the latitude and the horizontal parallax by the co-sine of the *apparent* declination. The manner of applying those parts to the *apparent* declination of the star will be explained in the *Rule*.

15. The moon's right ascension is affected by horizontal parallax in such a manner as to be increased in the eastern hemisphere, and decreased in the western: and hence, at an *immersion* the contact will take place a little sooner than it ought when the moon is east of the meridian, and not so soon as it ought when she is west of the meridian. For, as the horizontal parallax always projects the lunar right ascension towards the prime vertical; that is, towards the azimuth circle which cuts, *at right angles*, the point of intersection that is made by the equator and the horizon, and which is always 90 degrees distant from the meridian; it therefore causes the moon, when in contact with a star in the eastern hemisphere, to appear a little to the eastward of her true place; and thus, the parallax in right ascension will be *subtractive* from the *apparent* right ascension of the star. The converse of this takes place when the moon is in contact with a star in the western hemisphere; because, then, as the effects of parallax will cause her to appear a little to the westward of her true place, which

diminishes the value of her right ascension, the amount or correction of parallax will be *additive* to the *apparent* right ascension of the star, or of any other celestial object with which she may appear to be in contact. From this it is evident that the parallax in right ascension becomes subtractive when the moon is approaching, and additive when she is receding from, the meridian.

The parallax in right ascension *decreases* from the east and west points of the horizon to the meridian, where it *entirely vanishes*; and thus, should the moon's meridional passage chance to take place at the precise moment of an *immersion* or an *emersion*, there will *not* be any parallax in her right ascension; for, since the plane of the meridian will, at that moment, intersect the moon's polar distance at *right angles*, the whole effects of her horizontal parallax will be confined to her declination and zenith distance, without at all affecting her right ascension: for, as the circles of right ascension $P a c$, and $P t d$ (in the diagram), will then be in the same plane with the meridian $E Z P$; the polar angle $t P a$, and the corresponding arc of the equator $d c$, will disappear or become imperceptible.—The amount of parallax in the moon's right ascension is equal to the product arising from the sines of the co-latitude, the apparent hour angle, and the horizontal parallax; divided by the rectangle of radius and the co-sine of the *correct* declination of the moon's centre.

16. Since the apparent point of contact at $*$, is to be reduced to the centre of the moon at M ; it is therefore manifest, as appears by the diagram, that the value of the moon's semi-segment in right ascension, *viz.*, the *semi-chord* $* S$, becomes subtractive at an *immersion*, and additive at an *emersion*: because, in the first instance, the apparent right ascension of the point of contact, at $*$, is greater, and in the latter instance less, than the right ascension of the moon's centre.—The moon's semi-chord in *right ascension* is equal to *the square root* of the product arising from the sines of the sum and difference of her true semidiameter and the difference between her true central declination and *approximate* declination; multiplied by the square of radius, and divided by the co-sines of those declinations. This concise solution determines the semi-chord of the segment, *viz.*, the leg $* S$, independent of the angles; agreeably to the *formula* given under the *Remark* in page 175. And since small arcs are very nearly as their sines; and since the right angled triangle $M S *$, may be esteemed, at the option of the computer, as being either spherical or rectilineal; in which the hypotenuse, or longest side, *viz.*, the moon's semidiameter, can never exceed $16'.46''$; therefore, for the sake of uniformity in the calculations, the

semi-chord, which can *never be greater* than that side, is determined by the Table of Logarithmical Sines.

17. Since the point of the moon's limb which comes in contact with a fixed star or planet, at an *immersion* or an *emersion*, has the same *apparent* place in right ascension and declination that the star or planet has ; such object may be conceived, by analogy, to have the same parallax that the point of the moon's limb does.—Because the parallax of the moon's limb may be thus transferred to the star for the convenience of calculation ; therefore, in reducing the point of contact to the centre, the augmentation of the moon's semi-diameter is *not* to be taken into account ; the true semidiameter, as given in the Nautical Almanac, and properly reduced to the meridian of Greenwich, is that which is to be made use of in the computation, since the point of the moon's limb and the object with which it comes in contact are at the same degree of elevation above the horizon, and are, therefore, equally affected by refraction, and thus any correction on account of this element becomes unnecessary.

18. Having thus taken a cursory view of the various subjects connected with an *occultation*, and of the principles upon which the proposed method of computation is founded ; I have now to observe that the correct determination of the moon's central right ascension and the precise moment of an *immersion* or an *emersion*, that determine the chiefest attention ; and that this important element, together with the longitude of the place of observation, may be easily determined by the following

General Rule.

19. Reduce the mean time of observing the *immersion* or *emersion* to the meridian of Greenwich, by Problem III., page 342, and it will express the assumed time of observation at Greenwich : to which let the sun's right ascension be reduced by Problem V., page 344 ; the moon's semidiameter* and horizontal parallax by Problem XV., page 361 : let her declination be reduced by Problem XVI., page 361 : it will express her *approximate* declination. In the three last elements, decimals are quite unnecessary for practical purposes : take the *nearest second* in each, and it will be sufficiently exact, as stated in pages 363 and 365 : this will conduce much to the expeditiousness of the operation, without sensibly affecting its correctness.

* This element is *not* to be corrected for augmentation.— See Article 17, *as above*.

Diminish the latitude by the equation in Table B, and the result will be the geocentric, or reduced latitude: to the logarithmic radius corresponding to this in Table C, add the log. sine of the moon's reduced horizontal parallax; the sum, abating 10 in the index, will be the log. sine of the moon's horizontal parallax *adapted to the oblate spheroidal figure of the earth*.

Take the star's right ascension and declination from the Nautical Almanac under the head "Occultations," between pages 455 and 465; and let these elements be considered as the *apparent* right ascension, and the *apparent* declination of that point of the moon's limb which is in contact with a fixed star or planet at the exact instant of an *immersion* or an *emersion*.

To the mean time of observation add the *mean sun's* corrected right ascension; and the sum will be the right ascension of the meridian: the difference between which and the *apparent* right ascension will be the *apparent horary* distance of the moon's limb from the meridian. Convert this into motion by Problem II., page 342, and it will express the value of the *apparent* hour angle. Now,

To the sine of the *apparent* hour angle add the co-sine of the reduced latitude, and the sine of the *adapted* horizontal parallax; call the sum, abating 20 in the index, the *reserved logarithm*. To this add the secant, less radius, of the *apparent* declination, and the constant logarithm 9.698970;* the sum, abating 10 in the index, will be the sine of the parallax in the *apparent* hour angle;† which being subtracted therefrom, the result will be the moon's corrected hour angle.—See Article 13, page 544.

To the co-sine of the moon's corrected hour angle, add the co-sine of the reduced latitude, the sine of the *apparent* declination and the sine of the *adapted* horizontal parallax; the sum, abating 30 in the index, will be the sine of the minor part of the parallax in the moon's declination;† which is to have the same name as the *apparent* declination when the corrected hour angle is more than 90 degrees; but, a different name, if it be less than 90 degrees.—See Article 14, page 544.

To the sine of the reduced latitude, add the co-sine of the *apparent* declination, and the sine of the *adapted* horizontal parallax; the sum, abating 20 in the index, will be the sine of the major part of the parallax in the moon's declination;† which is to have the same name as

* The logarithmic versed sine or *co-sine* of 60 degrees.

† The arc is to be taken which corresponds to the tabular log. sine that comes nearest to the computed log. sine, whether the tabular one be the greatest or the least.

the *apparent* declination when *this* and the latitude are both north, or both south ; but, a contrary name, if one be north and the other south. —See Article 14, page 544.

Now the sum or the difference of the minor and major parts of parallax, according as they may be of the same or of different denominations, will be the effects of parallax in the moon's declination, with the name of the major part : then, this being applied to the *apparent* declination by addition, when the names are alike, or by *subtraction if unlike*, the result will be the correct declination of the moon's centre.

To the secant, less radius, of the correct declination of the moon's centre, thus found, add the *reserved logarithm* ; and the sum will be the sine of the parallax in right ascension ;* to which prefix the sign +, when the moon is *west*, or the sign —, when she is *east*, of the meridian. The reader would do well to turn back and re-peruse the whole of Article 15, page 544.

Find the difference between the correct declination of the moon's centre and her *approximate* declination ; then, to the sines of the sum and the difference of this and the moon's true semidiameter, add the secants, less radius, of those declinations : half the sum of these four terms will be the sine of the *semi-chord* of the occultation in right ascension ;* to which prefix the sign +, at an *emersion* ; but the sign —, at an *immersion*.—See Article 16, page 545 ; and the paragraph above Article 11, page 543.—Now, if the moon's parallax in right ascension, and the semi-chord of *the segment of occultation* in right ascension have signs *alike*, take their *sum* ; but, if *unlike*, their difference, with the *sign of the greater term*. Multiply this sum, or difference, as the case may be, by 4 ; and it will be converted into time ; observing that seconds of a degree produce *thirds* of time, and minutes of a degree, *seconds* of time. Annex a cipher to the thirds ; then divide by 6, and they will be reduced to two places of decimals, or to *hundredths* of a second. Apply the minutes and seconds, thus found, to the *apparent* right ascension by addition or subtraction, according as its sign may be either affirmative or negative ; and the result will be the correct right ascension of the moon's centre.

Enter the Nautical Almanac and find, *under the given day*, the difference between the right ascension of the moon's centre, determined as above, and her *next less* right ascension ; and find, also, the difference between the *next less* and the *next greater* right ascension ; which dif-

* The arc is to be taken which corresponds to the tabular log. sine that comes *nearest* to the computed log. sine, whether the tabular one be the greatest or the least.

ferences convert into seconds. Then, from the sum of the logarithm of the first difference and the constant logarithm 3.556303,* subtract the logarithm of the last difference; and the remainder will be the logarithm of a portion of time in *seconds*: raise this to minutes &c., to which prefix the hour corresponding to the *next less* right ascension, and it will express the mean time of the immersion, or emersion, at Greenwich. Now, the difference between the mean time at Greenwich, thus found, and the mean time of observation, being converted into motion, will be the correct longitude of the ship or place of observation; which will be *east*, if the mean time at ship be the *greatest*; otherwise, it will be west.

Remark.—The above *General Rule* is adapted to the second volume of this work; in which the *sines* and the *secants* of all arcs are given to every second in the semicircle: and as it departs from the customary modes of computation, by determining the different values of the moon's horizontal parallax in the *sines* of arcs instead of in their *lineal* measures; I think it right to observe, that this is done for the purposes of simplifying and expediting the numerical calculations:—and I shall now show, that in thus studying the accomplishments of these points, I have not sensibly departed from the strict line of mathematical correctness. The greatest value of the moon's horizontal parallax is 61'32"; beyond this point it can never pass, so long as the ordinary laws of nature remain unchanged. Now, since the Trigonometrical Tables are adapted to a circle whose radius is *unity*, or 1; and that it is *not* necessary, at least for practical purposes, to extend the numbers in those Tables beyond *six places* of decimals: and since the absolute length of the arc of 61'32" is 0.017608; and that the *sine* of the same arc is also 0.017608, as may be readily proved by calculation; therefore, the one may be substituted for the other, and so may all other small arcs and their sines that correspond to six places of decimals. But, since the different *values* of the moon's horizontal parallax will always be considerably below 60 minutes; therefore, it is clearly manifest, that for the important purpose of facilitating the calculations, the measures of the sines may be safely substituted for the lengths of the arcs.—See the latter part of Article 16, page 545.

* This is the logarithm of 3600', or of one hour reduced to seconds.

Mean time at ship = $8^h 58^m 58^s$
 Longitude $5^\circ 22' 45''$ west = .. $+21.31$

 Assumed time at Greenwich = $9^h 20^m 29^s$

 Mean sun's R. A. at noon = . $2^h 14^m 6^s 89$
 Equa., Tab. XLVI. to $9^h 20^m 29^s$ = $+1.32.08$

 Mean sun's cor. R. A. = $2^h 15^m 38^s 97$
 Mean time at ship or place = $8.58.58$ —

 R. A. of the meridian = $11^h 14^m 36^s 97$
 Apparent R. A. = $9.58.23.77$

 Appar. hor. dist., west = $1^h 16^m 13^s 20$
 Ditto, converted into motion, or
 Ap. h. a. = $19^\circ 3' 18''$ Sine 9.513851
 Red. lat. = $39.17.9$ Co-sine 9.888739
 Log. s. h. p. = 8.211969

 Reserved logarithm = 7.614559
 Ap. dec. = $17^\circ 33' 37''$ Secant 0.020724
 Constant logarithm = 9.698970

 Par. in h. a. = $-7' 25''$ Sine = 7.334253

 Cor. h. a. = $18^\circ 55' 53''$ Co-sine 9.975848
 Red. lat. = $39.17.9$ Co-sine 9.888739
 Ap. dec. = $17.33.37$ Sine 9.479588
 Log. s. h. p. = 8.211969

 Part I = S. $12' 22''$ Sine = 7.556144

 Red. lat. = $39^\circ 17' 9''$ Sine 9.801533
 Ap. dec. = $17.33.37$ Co-sine 9.979276
 Log. s. h. p. = 8.211969

 Part 2 = N. $33' 49''$ Sine = 7.992778
 Part 1 = S. 12.22

 Par. in dec. = $21' 27''$ North.
 Ap. dec. = $17.33.37$ North.

 γ 's cor. dec. $17^\circ 55' 4''$ Secant 0.021591
 Reserved logarithm = 7.614559

 Par. R. A. = $+14' 52''$ Sine = 7.636150

γ Leonis, apparent R. A. = $9^h 58^m 23^s 77$
 and apparent declin. = .. $17^\circ 33' 36'' 7$ N.
 γ 's approximate declin. = $17^\circ 48' 17''$ N.
 γ 's reduced semidiam. = $15' 17''$
 Ditto, horizontal par. = .. 56.5
 Latitude of ship or place = $39^\circ 28' 14''$ N.
 Equation, Table B = -11.5

 Reduced latitude = $39^\circ 17' 9''$ N.
 Log. radius of ditto in Table C. = 9.999428
 γ 's red. hor. par. $56'' 5$ Sine 8.212541

 Log. sine of γ 's hor. parallax = 8.211969 ;
 adapted to the oblate figure of the earth.

 γ 's cor. dec. = $17^\circ 55' 4''$ Secant 0.021591
 Ap. d. = $17.48.17$ Secant 0.021315

 Difference = $6' 47''$
 γ 's semidr. = 15.17

 Sum of ditto = $22' 4''$ Sine 7.807460
 Difference = 8.30 Sine 7.393145

 Sum = 15.243511

 Semichord = $+14' 23''$ Sine = $7.621755\frac{1}{2}$
 Par. R. A. = $+14.52$

 Sum = $+29' 15''$

 Ditto in time = $+1^m 57^s 0$
 Ap. R. A. = $9.58.23.77$

 γ 's cor. R. A. = $10^h 0^m 20^s 77$ } $-0^m 42^s 91$
 γ 's R. A. at 9^h = $9.59.37.86$ }
 Ditto at 10^h = $10. 1.42.05$ } $-2. 4.19$

 First diff. $0^m 42^s 91 = 42^s 91$ Log. = 1.632559
 Constant logarithm = 3.556303

 Sum = 5.188862
 Last diff. $2^m 4^s 19 = 124^s 19$ Log. = -2.094087

 Portion of time = 1244^s Log. = 3.094775

 Greenw. time = $9^h 20^m 44^s$
 Time at ship = $8.58.58$

 Long. in time = $6^h 21^m 46^s = 5^\circ 26' 30''$ W.;
 which is the correct longitude of the place
 of observation.

Remark.—Although the apparent horary distance, *in time*, is determined to hundredths of a second; yet, this is not absolutely necessary; for, in every instance, the nearest second will prove to be sufficiently exact,

RULE.

22. For the preliminary steps or preparatory part of the operation, follow the directions contained in the leading part of the General Rule, page 546 ; except that, in this instance, instead of taking out the elements of the moon to the *nearest second* ; her semidiameter, horizontal parallax, and declination, *must be taken out to decimals*, and then carefully reduced to the assumed mean time of observation at Greenwich. Let the moon's reduced semidiameter be converted into seconds, and, also, her horizontal parallax : to the logarithm of the latter, add the logarithmic radius in Table C* ; the sum, abating 10 in the index, will be the logarithm of the moon's horizontal parallax *adapted to the oblate spheroidal figure of the earth*.—Then,

To the sine of the *apparent* hour angle, add the co-sine of the reduced latitude, and the logarithm of the *adapted* horizontal parallax ; call the sum, abating 20 in the index, the *reserved logarithm*. To this add the secant, less radius, of the *apparent* declination, and the constant logarithm 9.698970 ; the sum, abating 10 in the index, will be the logarithm of the seconds of parallax in the *apparent* hour angle ; which being raised to minutes (if more than 60") and then subtracted from that angle ; the result will be the moon's corrected hour angle.—See Article 13, page 544 ; and the fourth paragraph in page 547.

To the co-sine of the moon's corrected hour angle, add the co-sine of the reduced latitude, the sine of the *apparent* declination, and the logarithm of the *adapted* horizontal parallax ; the sum, abating 30 in the index, will be the logarithm of the seconds in the *minor* part of the parallax in the moon's declination ; which is to have the same name as the *apparent* declination, when the corrected hour angle is more than 90° ; but, a different name, if it be less than 90 degrees.—See Article 14, page 544 ; and the fifth paragraph in page 547.

To the sine of the reduced latitude, add the co-sine of the *apparent* declination, and the logarithm of the *adapted* horizontal parallax ; the sum, abating 20 in the index, will be the logarithm of the seconds in the major part of the parallax in the moon's declination ; which is to have the same name as the *apparent* declination, when *this* and the latitude are both north or both south ; but, a contrary name, if one be north and the other south. Now, the sum or the difference of the minor and major parts of parallax, according as they may be of the same or of different denominations, will be the effects of parallax in the moon's declination, with the name of the major part. Then, this being raised to minutes, if necessary, and applied to the *apparent* declination by addition, when the names are alike, or by *subtraction* if *unlike* ; the sum

* Corresponding to the reduced latitude.

A
as
is

the correct declination of the moon's centre at the
—See Article 14, page 544; and the last para-
and the first in page 548.

less radius, of the correct declination of the moon's
add the reserved logarithm, and the constant loga-
the sum, abating 10 in the index, will be the
of the seconds of time in the parallax in right ascension;
which must be taken out to two places of decimals, or to hundredths of
a second: to this prefix the sign +, when the moon is west; but the
sign —, when she is east of the meridian.—See Article 15, page 544,
and the second paragraph in page 548.

Find the difference between the correct declination of the moon's
centre and her approximate declination at the assumed Greenwich time,
and turn it into seconds:—then, to the logarithms of the sum and the
difference of this and the moon's semidiameter in seconds, add the se-
cants, less radius, of those declinations, and the constant logarithm
7.647818;† half the sum of these four terms will be the logarithm of
the seconds in the semi-chord of the occultation in right ascension;
which must be taken out to two places of decimals, or to the hundredths
of a second:—to this prefix the sign + at an emersion, or the sign —
at an immersion.—See Article 16, page 542, and the paragraph which
immediately precedes Art. 11 in p. 543.—See, also, the third paragraph
in page 548. Now, if the moon's parallax in right ascension, and the
semi-chord of the segment of the occultation in right ascension, have
signs alike, let their sum be taken; but, if unlike, their difference,
with the sign of the greater term. Raise this sum, or difference, as
the case may be, to minutes, &c. Apply these to the apparent right
ascension by addition, or subtraction, according as the sign may be
either affirmative or negative; and the result will be the correct right
ascension of the moon's centre. Now, the moon's correct central de-
clination being known, the longitude is to be determined agreeably to the
instructions given in the last paragraph, p. 548, and the first in p. 549.

We shall now reduce the above Rule to practice.

Example.

April 3rd, 1836, in latitude $35^{\circ}12'$ north, and longitude, by account,
 $24^{\circ}37'$ west, the immersion of α° Libræ behind the moon (east of the
meridian) was observed at $9^{\text{h}}56^{\text{m}}52^{\text{s}}$ mean time; required the longi-
tude of the place of observation?

* The log. ar. comp. of 15; the common divisor for converting motion into time.

† Twice the log. ar. comp. of 15; it is doubled, so that the root of all the terms may
come out the log. of seconds of time.

Mean time at ship = $9^h 56^m 52^s$
 Longitude $24^{\circ} 37'$ west = $+ 1.38.28$

Assumed time at Greenwich = $11^h 35^m 20^s$

Mean sun's R. A. at noon = $0^h 47^m 22^s.67$
 Equ., Tab. XLVI. to $11^h 35^m 20^s$ = $+ 1.54.22$

Mean sun's cor. R. A. = $0^h 49^m 16^s.89$
 Mean time at ship = $9.56.52$

R. A. of the meridian = $10^h 46^m 8^s.89$
 Apparent R. A. = $14.41.50.21$

Appar. hor. dist., east = $3^h 55^m 41^s.32$
 Ditto converted into motion, or
 Ap. h. a. = $58^{\circ} 55' 20''$ Sine 9.932711
 Red. lat. = $35.1.22$ Co-sine 9.913244
 Log. adapted h. p. = 3.553631

Reserved log. = 3.399586
 Ap. dec. = .. $15^{\circ} 21' 26''$ Secant 0.015791
 Constant log. = 9.698970

Par. in h. a. = $-1301''$ Log. = 3.114347

Ditto = $-21' 41''$

Cor. h. a. = .. $58^{\circ} 33' 39''$ Co sine 9.717331
 Red. lat. = .. $35.1.22$ Co-sine 9.913244
 Ap. dec. = .. $15.21.26$ Sine 9.422978
 Log. adapted h. p. = 3.553631

Part 1 = N. $404'' 8$ Log. = 2.607184

Red. lat. = .. $35^{\circ} 1' 22''$ Sine 9.758838
 Ap. dec. = .. $15.21.26$ Co-sine 9.984209
 Log. adapted h. p. = 3.553631

Part 2 N. $1980'' 1$ Log. = 3.296678

Part 1 N. 404.8

Sum of ditto = .. $2384'' 9$

Par. in dec. = .. $39' 44'' 9$ North.

Ap. dec. = .. $15.21.26.4$ South.

p's cor. dec. = $14^{\circ} 41' 41'' 5$ Secant 0.014443
 Reserved logarithm = 3.399586
 Constant logarithm = 8.823909

Par. R. A. p .. -172.96 Log = 2.237938

α Libra, apparent R. A. = $14^h 41^m 50^s.21$
 and apparent declin. = $15^{\circ} 21' 26'' 4$ S.
 p's approximate declin. = $14^{\circ} 35' 43'' 3$ S.
 p's red. semidr. .. $16' 16'' = 976''$
 Ditto horizontal paral. = $59' 41'' 8 = 3581'' 8$
 Latitude of ship = $35^{\circ} 12' 0''$ North.
 Equation, Table B = -10.38 North.

Reduced latitude = $35^{\circ} 1' 22''$ North.
 Log. radius of ditto in Table C = 9.999530
 p's red. hor. par. = $3581'' 8$ Log. 3.554101

Log. of p's hor. parallax = 3.553631 ;
 adapted to the oblate figure of the earth.

p's cor. dec. = $14^{\circ} 41' 41'' 5$ Secant 0.014443
 Ap. d. = $14.35.43.3$ Secant 0.014246

Difference .. = $5' 58'' 2$

Do., in seconds = $358'' 2$
 p's semidr. = 976.0

Sum = $1334'' 2$ Log. 3.125221
 Difference = 617.8 Log. 2.790848
 Constant logarithm = 7.647818

Sum = 3.592576

Semichord = $-62' 56$ Log. = 1.796288

Par. R. A. = -172.96

Sum = $-235' 52$

Ditto, raised to
 min. &c. = $-3^m 55^s.32$
 Ap. R. A. = $14.41.50.21$

p's cor. R. A. = $14^h 37^m 54^s.69$ }
 p's R. A. at 11h. $14.36.34.12$ } $-1^m 20^s.57$
 Do. at 12 hrs. $14.38.51.83$ } $-2.17.71$

First diff. = $1^m 20^s.57 = 80^s.57$ Log. 1.906173
 Constant logarithm = 3.556383

Sum = 5.462476

Last diff. = $2^m 17^s.71 = 137^s.71$ Log. 2.138966

Portion of time = 2106^s Log. = 3.323511

Greenwich time = $11^h 35^m 6^s$
 Time at ship = $9.56.52$

Long. in time = $1^h 38^m 14^s = 24^{\circ} 33' 38''$ W.
 which is the correct longitude of the place
 of observation.

Note.—If the above Example be computed agreeably to the *General Rule*, Article 19, page 546, the resulting longitude will be *almost* precisely the same ; the difference will not amount to more than about *the twelfth part* of a mile.

22. Should the mean time at Greenwich, deduced from calculation, differ considerably from the *assumed time* at Greenwich ; it will be necessary to repeat the operation for finding the *semi-chord of the occultation* in right ascension. For this purpose, the moon's semidiameter and declination, as given in the Nautical Almanac, must be reduced to the computed Greenwich time. Find the difference between this new *approximate* declination and the correct declination of the moon's centre, as already determined by computation ; with which difference and the moon's new reduced semidiameter, re-compute the *semi-chord* of the segment of the occultation *in right ascension* ; as directed in the third paragraph of page 548, or the third in page 554. Then, this re-computed *semi-chord in right ascension*, and the parallax *in right ascension*, already found, being applied to the *apparent* right ascension, the result will be the correct declination of the moon's centre ; from which a new Greenwich time is to be found ; and hence the longitude. But, since all Her Majesty's *sea-going* ships are furnished with chronometers, and since the *assumed* mean time at Greenwich may be known thereby within *one or two minutes* of the truth ; the elements of the moon, properly reduced to that time, will, in general, be sufficiently correct for all practical purposes at sea ; and thus, a repetition of the above-named operation will be very rarely called for ; provided that due care be taken in the rigid performance of the different calculations. Here it may be right to observe, that an error of a few minutes of a *degree* in the *apparent* hour angle will not sensibly affect any of the resulting elements of the occultation.

23. Since the moon's daily motion in her orbit is at the *mean* rate of $13^{\circ}10'35''.02$; therefore, her mean hourly motion is $32'56''.46$:—and since the sun's daily *mean* motion is $59'8''.33$, its hourly motion is $2'27''.85$.—Then, $32'56''.46 - 2'27''.85 = 30'28''.61$, is the relative mean motion of the moon in one hour ; which, in right ascension, amounts to $2^{\text{h}}1'907 = 121'907$.—Now, as $121'907$ are to one hour, or 3600 seconds of mean time, so is 1' of right ascension to $29'53$ of time ; which, therefore, is the correct value, in *mean time*, of one second in the moon's right ascension. And hence it is manifest, that an error of *one second* in the computed right ascension of the moon's centre would produce, at a mean rate, an error of about $29\frac{1}{2}$ seconds in the mean time at Greenwich, or an error of $7'22''$ in the longitude

of the place of observation.—This is exactly on the principle of an error in the method of determining the longitude by *the lunar observations* :—for, since an error of *one minute* in a lunar distance would affect the longitude to the value of 29.53 miles (see *Remark 8*, page 499) ; therefore, an error of 15 seconds in a lunar distance would produce an error of 7'.22" in the longitude : for 15 seconds, or *a quarter of a minute*, in distance is precisely the same as *one second* in right ascension.

PROBLEM XVI.

To deduce the Longitude from an Eclipse of the Sun, or from an Occultation of a Planet by the Moon.

Were it not for the horizontal parallax and the semidiameter of the sun, the deduction of the longitude from a solar eclipse would be precisely the same as that from a stellar occultation :—but, in consequence of those two elements, *the preliminary parts* of the operation differ a little from the same parts in relation to a fixed star ; and this I shall endeavour to explain, so as to render the whole subject perfectly familiar to the reader.

At the moment of an *external* contact of the limbs of the sun and moon, both objects appear to be placed at the same distance from the earth ; though, at the same time, the sun is actually *four hundred times* farther off than the moon is ; and, therefore, the sun's semidiameter, as seen from the surface of the earth, must be reduced to what it would appear if seen, at the same moment, from the earth's centre, so as to adapt it to that of the moon, which is always adapted to the centre.

Now, let the sun be in the zenith, and let it be vertical to two observers, the one placed at the centre, and the other on the surface, of the earth : then, it will appear manifest, that the sun must be farther from the observer at the centre than from the one at the surface, by the full measure of the earth's semidiameter. And since the angle under which an object is seen, diminishes in proportion to the increase of its distance from the eye of an observer ; therefore, the semidiameter of the sun will appear under a smaller angle to an observer at the earth's centre than to one on its surface.

Now, since the moon's mean distance from the earth is 236692 miles, and since the sun's distance* (supposed in this instance to be

* The sun's mean distance from the earth is 94546196 English miles.

the same as the moon's) is equal to the moon's mean distance increased by the earth's semidiameter, viz., $236692 + 3959 = 240651$ English miles; therefore, if the sun's mean semidiameter be taken at $16'.2''$, as seen from the earth's surface, its value at the centre of the earth will be as the *ratio* of those distances :— hence, by logarithms

As the sun's appar. distance=	240651 miles,	Log.ar.comp.=	4.618612
Is to the moon's real distance=	236692 miles,	Logarithm=	5.374184
So is the sun's true semidiam.=	$16'.2''$ —	Log. sine =	. . 7.668748
<hr/>			
To sun's semi. at earth's cen.=	$15'.46''$ —	Log. sine =	. . 7.661544
<hr/>			
Difference =		0'.16"

Thus, it appears manifest that, under the optical illusion arising from our not being able to see very remote objects in their true places, the diminution of the sun's semidiameter, seen from the centre of the earth and at the apparent distance of the moon, and in the zenith of an observer on the earth's surface, amounts to 16 seconds of a degree.

Now, the diminution of the semidiameter at the zenith being known, that corresponding to any degree of elevation above the horizon will be as the vertical or zenith diminution multiplied by the sine of the apparent altitude, as thus :—Let the sun's apparent altitude be 40 degrees ;—then, the

Diminution at the zenith =	16 seconds—	Logarithm =	1.204120
Sun's apparent altitude =	40° —	Log. sine =	9.808068
<hr/>			
Diminution of sun's semidiameter=	10 seconds—	Log.=	. 1.012188

But, as the diminution of the sun's semidiameter is the *converse* of "the augmentation of the moon's semidiameter," which is geometrically illustrated in page 9, and as the real value of that augmentation is given in Table IV.; if, therefore, that table be entered with the sun's semidiameter at top, and its altitude (the *observed altitude* will answer) in the side column, in the angle of meeting will be found the required diminution; which, being subtracted from the sun's semidiameter, as given in the Nautical Almanac, will reduce it to what it would appear if seen from the centre of the earth and at the distance of the moon.

Having thus established the *necessity* of diminishing the sun's semidiameter, we must now proceed to the preparatory parts of the operation for determining the longitude by an eclipse of the sun.

To the assumed mean time at Greenwich (found as directed in Article 19, page 546) reduce the *true* sun's right ascension and declination; and, also, the equation of time, as given in the first, second, and fourth *elementary columns* of page II. of the month in the Nautical Almanac:—those reductions, particularly of the sun's right ascension, must be made to *the greatest exactness*.

To the mean time of observation apply the reduced equation of time; according to its sign, and the result will be the *apparent* horary distance; which, being converted into degrees, will be the *apparent* hour angle.

To the assumed time at Greenwich reduce the moon's semidiameter, horizontal parallax, and declination. To the sun's semidiameter, *diminished by the equation* in Table IV., as directed above, add the moon's *true* semidiameter;* the sum will be the apparent central distance of the objects, which call *the semidiameter for calculation*.—From the moon's horizontal parallax subtract that of the sun; the remainder will be the moon's diminished horizontal parallax. From the given latitude subtract the equation in Table B, the result will be the geocentric, or reduced latitude: to the logarithmic radius corresponding to this in Table C, add the log. sine of the moon's diminished horizontal parallax; the sum, abating 10 in the index, will be the log. sine of the moon's diminished horizontal parallax *adapted* to the oblate spheroidal figure of the earth.

Now, having duly corrected and *arranged* all the necessary elements, the right ascension of the moon's centre is to be deduced therefrom, and hence the longitude of the place of observation, in the same manner as if it were the occultation of a fixed star that were under consideration. The computation may be performed agreeably to the instructions contained between the fourth paragraph in page 547, and the last in page 548, or in conformity with those given between the second paragraph in page 553, and the second in page 554, according to the fancy of the computer.

Example.

May 15th, 1836, in latitude $55^{\circ}57'20''$ north, and longitude, by account, $3^{\circ}11'$ west, the termination, or *ending*, of a solar eclipse was observed at $4^h19^m20^s$ mean time. The altitude of the sun was about $29^{\circ}3'$; required the true longitude of the place of observation?

We shall work this Example by the *General Rule*, Article 19.

* See Article 17, page 546.

Mean time of observation = .. $4^h 19^m 20^s$
 Longitude $3^\circ 11'$ west, in time = $+12.44$

Assumed time at Greenwich = $4^h 32^m 4^s$

True sun's corrected right ascension, or
 Apparent right ascension = $3^h 29^m 46^s 77$
 True sun's corrected decl., or
 Apparent declination = $18^\circ 59' 13'' 8$ N.

Sun's semidiameter = $15' 49'' 9$
 Diminution of do., Table IV. = -8.0

☉'s diminished semidiam. = $15' 41'' 9$
 ☉'s reduced semidiameter = $14.49.1$

Semidiameter for calcula. = $30' 31'' 0$

Mean time of observation = $4^h 19^m 20^s 0$
 Equation of time corrected = $+3.55.90$

Appar. hor. distance, *west* = $4^h 23^m 15^s 90$
 Ditto converted into motion, or
 Ap. h. a. = .. $65^\circ 48' 58\frac{1}{2}''$ Sine 9.960107
 Red. lat. = .. $55.46.52$ Co-sine 9.750012
 Log. s. h. p. = 8.196998

Reserved logarithm = 7.907117
 Ap. dec. = .. $18^\circ 59' 14''$ Secant 0.024296
 Constant logarithm = 9.698970

Par. in h. a. = $-14' 41''$ Sine = 7.630383

Cor. h. a. = .. $65^\circ 34' 17\frac{1}{2}''$ Co-sine 9.616536
 Red. lat. = .. $55.46.52$ Co-sine 9.750012
 Ap. dec. = .. $18.59.14$ Sine 9.512361
 Log. s. h. p. = 8.196998

Part 1 = S. $4' 6''$ Sine = 7.075907

Red. lat. = .. $55^\circ 46' 52''$ Sine 9.917451
 Ap. dec. = .. $18.59.14$ Co-sine 9.975704
 Log. s. h. p. = 8.196998

Part 2 = N. $42' 19''$ Sine = 8.090153

Part 1 = S. 4.6

Par. in dec. = $38' 13''$ North.
 Ap. dec. = .. $18.59.14$ North.

☉'s cor. dec. = $19^\circ 37' 27''$ Secant 0.025988
 Reserved logarithm = 7.907117

Par. R.A. = .. $+29' 28''$ Sine = 7.933105

☉'s reduced horizontal parallax = $54' 22'' 7$
 ☉'s horizontal parallax = -8.5

☉'s diminished hor. par. = $54' 14'' 2$

☉'s approximate declination = $19^\circ 46' 40''$ N.
 Latitude of the place = $55^\circ 57' 20''$ N.
 Equation, Table B = -10.28

Reduced latitude = $55^\circ 46' 52''$ N.
 Log. radius of do., in Table C = 9.999024
 ☉'s dimin. hor. par. $54' 14''$ Sine = 8.197974

Log. sine of ☉'s dim. hor. par. = 8.196998 ;
adapted to the oblate figure of the earth.
 ☉'s cor. dec. = .. $19^\circ 37' 27''$ Sec. 0.025988
 Apx. d. = $19.36.40$ Sec. 0.026405

Difference = $9' 13''$
 Semidia. for calcul. = 30.31

Sum of ditto = $39' 44''$ Sine 8.062871
 Difference = 21.18 Sine 7.792103

Sum = 15.907367

Semichord = .. $+30' 54''$ Sine $7.953683\frac{1}{2}$
 Par. R.A. = $+29.28$

Sum = $+60' 22''$

Ditto in time = .. $+4^m 1' 47^s$
 Ap. R. A. = $3.29.46.77$

☉'s cor. R. A. = $3^h 33^m 48^s 24$ } $-1^m 4' 62$
 ☉'s R.A. at 4 hours = $3.32.43.62$ }
 Ditto at 5 hours = $3.34.44.60$ } $-2.0.98$

First diff. = $1^m 4' 62 = 64' 62$ — Log. 1.810367
 Constant logarithm = 3.556303

Sum = 5.366670
 Last diff. = $2^m 0.98 = 120' 98$ — Log. 2.082714

Portion of time = .. $1923'$ — Log. 3.283956

Greenwich time = $4^h 32^m 3^s$
 Ti at given place = $4.19.20$

Long. in time = $0^h 12^m 43^s = 3^\circ 10' 45''$ West;
 which is the true longitude of the place of
 observation.

* See Table of Conversions, page 552.

Note.—If the above Example be computed according to the most rigorous mode of calculation, as in Article 22, page 553, the moon's right ascension will come out $3^{\text{h}}33^{\text{m}}48^{\text{s}}.2$, which differs but the *one hundredth* part of a second from the above ; and, hence it is manifest that the familiar method, by means of the logarithmic sines, comes as near to the truth as the ordinary purposes of navigation require.

By computing, agreeably to the *General Rule*, in Article 19, page 546, the young navigator is saved the trouble of finding the natural number, corresponding to a given logarithm, to four places of *integers*, and two of *decimals*, and he is also saved the trouble of raising the *seconds* to minutes, &c. But, when rigid exactness becomes indispensably necessary, as in the case of settling the geographical positions of places on shore, or in finding the *error of a chronometer*, the Rule in Article 22, page 553, *should have the preference*.

The longitude may be deduced in a similar manner from an occultation of a planet by the moon. But, as the planets have both semidiameters and parallaxes, therefore, the *sum* of the semidiameters of the moon and planet is to be taken at an *external* contact, or their *difference* at an *internal* contact :—this sum, or difference, as the case may be, is to be considered as *the semidiameter for calculation*.

The horizontal parallax of the planet is always to be *subtracted* from that of the moon, and the remainder to be esteemed as *the horizontal parallax for calculation*.

But, since the logarithmical part of the operation is precisely the same as that for a fixed star, or the sun, as shown in pages 551, 555, and 560 ; it is, therefore, presumed that any illustration thereof by an example would be quite superfluous.

Remarks relative to the Contacts of the Planets at the time of an Occultation ; and to the application of their Semidiameters to that of the Moon.

1. Since the disc of Mars may always be considered as being *nearly full*, the observation of its *immersion* or *emersion*, may be made at the moment of the external, or internal contact of either limb of that

planet with the moon, the same as if it were Jupiter or Saturn, according as it may suit the convenience of the observer. But, it must be borne in mind, that each of the above-named planets is susceptible of two contacts on the same limb of the moon, viz., an external and an internal contact on the east limb, and an internal and an external contact on the west limb of the moon. The external contact on the east side of the moon takes place the instant that the moon's eastern limb appears to touch the west limb of the planet; the internal contact, on the same side, takes place the moment that the east limb of the planet touches the east limb of the moon; at which moment the disc of the planet will be *immersed* behind the body of the moon. The converse of this takes place on the west limb of the moon; that is, the internal contact takes place the moment that the west limb of the planet appears to emerge from behind the west limb of the moon; and the external contact, the moment that the planet's eastern limb is about to be separated from the moon's west limb.—Hence, it is manifest that, at an external contact, it is the sum of the semidiameters that expresses the apparent central distance between the objects; but, at an internal contact, as the body of the planet is behind the moon, it is the difference of their semidiameters that will express the apparent central distance, or the value of the *semidiameter* for calculation:—this is general for Mars, Jupiter, and Saturn.

2. Since the disc of Venus is *never seen full*, or never forms a complete circle; it will be advisable to make use of a little precaution in observing the moment of an external or of an internal contact:—As thus, when Venus is an *evening star*; that is, when her longitude is greater than the sun's, it is her western limb that will be enlightened. In this case the external contact will take place betwixt the nearest limbs of the objects, which is to be noted the instant that the western limb of Venus touches the moon's eastern limb; and the internal contact, the moment that the same limb appears from behind the moon's western limb. The converse of this takes place when Venus is a *morning star*; that is, when her longitude is less than the sun's; because, then, it is her eastern limb that will be illuminated:—in this case, it is the *internal* contact of the planet's *farthest* or east limb that is to be noted (which amounts to an immersion of her whole disc); and the external contact of the same limb at the precise moment of its appearing to emerge from behind the body of the moon. At an external contact, the sum of the semidiameters is to be taken; but, at an internal contact, the difference of the semidiameters:—in either case, the result will express the apparent central distance, or the *semidia-*

meter for calculation, with which, and the horizontal parallax for calculation, proceed as if it were a fixed star that were under consideration.

Since writing the two last Problems, a friend has kindly supplied me with a copy of the Nautical Almanac for 1837, in the *Appendix* to which I perceive that Mr. W. S. B. Woolhouse has turned his attention to "the determination of the longitude from an observed solar eclipse or an occultation."—As I am, in common with all lovers of astronomy, already much indebted to the writings of that learned gentleman for a great deal of valuable information relative to *eclipses* (see "Appendix to the Ephemeris for 1836"); I shall add another link to the chain of obligation, by making use of one of his examples (given in page 181 of the *Appendix*), for the purpose of showing with what manifest *ease and conciseness* the longitude may be determined by either of my methods of calculation.

Example.

Suppose, at Bedford, on January 7th, 1836, in latitude $52^{\circ}8'28''$ north, and longitude by account $0^{\circ}28'$ west, the immersion of ϵ Leonis to be observed at $10^{\text{h}}39^{\text{m}}22^{\text{s}}.4$, apparent time; required the true longitude?

Since the *apparent time* of observation was $10^{\text{h}}39^{\text{m}}22^{\text{s}}.4$, the *mean time* was $10^{\text{h}}45^{\text{m}}53^{\text{s}}.3$:—the moment of an immersion, or an emersion, should be always noted in mean time, and *not* in apparent time.

Mean time of observation = .. $10^h 45^m 53^s$ 3
 Long. of Bedford, in time = .. $+ 15^m 2^s$ 0

Assumed time at Greenwich = $10^h 47^m 45^s$ 3

Mean sun's R. A. at noon = .. $19^h 4^m 22^s$ 41
 Equation, Table XLVI. = $+ 1.46.40$

Mean sun's correct R. A. = .. $19^h 6^m 8^s$ 81
 Mean time of observation = .. $10.45.53.$ 3

R. A. of the meridian = $5^h 52^m 2^s$ 11
 Apparent R. A. = $10.23.26.39$

Apparent hor. dist., east = .. $4^h 31^m 24^s$ 28
 Ditto converted into motion, or
 Ap. h. a. = .. $67^{\circ} 51' 4''$ Sine 9.966708
 Red lat. = .. $51.57.32$ Co-sine 9.789741
 Log. adapted h. p. = 3.525708

Reserved logarithm = 3.282157
 Ap. dec. = $14^{\circ} 58' 39''$ Secant 0.015010
 Constant logarithm = 9.698970

Par. in h. a. = .. 991' Log. = 2.996137

Ditto = $-16' 31''$

Cor. h. a. = .. $67^{\circ} 34' 33''$ Co-sine 9.581449
 Red. lat. = .. $51.57.32$ Co-sine 9.789741
 Ap. dec. = .. $14.58.39$ Sine 9.412359
 Log. adapted h. p. = 3.525708

Part 1 = S. $203'' 8$ Log. 2.309257

Red. lat. = $51^{\circ} 57' 32''$ Sine 9.896288
 Ap. dec. = $14.58.39$ Co-sine 9.984990
 Log. adapted h. p. = 3.525708

Part 2 = N. $2552'' 6$ Log. = 3.406986
 Part 1 = S. 203.8

Difference = .. $2348'' 8$ North.

Par. in dec. = .. $39' 8'' 8$ North.

Ap. dec. = $14.58.38.8$ North.

D's cor. dec. = $15^{\circ} 37' 47'' 6$ Sec. 0.016364
 Reserved logarithm = 3.282157
 Constant logarithm = 8.823909

Par. R. A. = $-132' 56$ Log. = 2.122430

i Leonis, apparent R. A. = $10^h 23^m 26^s$ 39
 And apparent declination = $14^{\circ} 58' 38''$ 8N.
 D's approximate declin. = $15^{\circ} 49' 34''$ N.
 D's red. semidiameter = $15' 16'' 2 = 916'' 2$
 D's red. hor. par. = $56' 2'' = 3362''$
 Latitude of the place = $52^{\circ} 8' 28''$ N.
 Equation, Table B = -10.56

Reduced latitude = $51^{\circ} 57' 32''$ N.
 Log. radius of ditto, Table C = .. 9.999110
 D's hor. parallax $3362''$ Log. 3.526598

Log. of D's hor. parallax = 3.525708;
 adapted to the oblate figure of the earth.

D's cor. dec. = $15^{\circ} 37' 47'' 6$ Secant 0.016364
 Ap. d. = $15.49.34.0$ Secant 0.016782

Difference = .. $11' 46'' 4$

Ditto in seconds = $706'' 4$ Co-log. 7.647818
 D's semidiameter = 916.2

Sum = $1622'' 6$ Log. 3.210211
 Difference = 209.8 Log. 2.321806

Sum = 3.212981

Semichord = .. $-40' 41$ Log. 1.6064904
 Par. R. A. = -132.56

Sum = $-172' 97$

Do. raised to mi. = $2^m 52^s$ 97

Ap. R. A. = $10^h 23^m 26^s$ 39

D's cor. R. A. = $10^h 20^m 33.42$ } $-1^m 37^s$ 90
 D's R. A. at 10h. $10.18.55.52$ }
 Do. at 11 hours $10.20.58.47$ } $-2. 2.95$

First diff. = $1^m 37^s$ 90 = $97' 90$ Log. 1.990783
 Constant logarithm = 3.556303

Sum = 5.547086

Last diff. = $2^m 2.95 = 122' 95$ Log. 2.089728

Portion of time = 2866.5 Log. 3.457358

Greenw. time = $10^h 47^m 46.5$

Time of obs. = $10.45.53.3$

Longitude, in time = $1^h 53' 2 = 0^{\circ} 28' 18''$ W.

Remark.—The Greenwich time thus found, differs $1' 2$ from the time as determined in page 182 of the *Appendix* to the Nautical Almanac for 1837:—this difference is owing to the moon's hourly motion

in declination having been taken in the *Appendix*, last line, page 181, as $11^{\circ}41'5''$ instead of $11^{\circ}39'1''$:—as thus,—the occultation took place between the hours of 10 and 11; at the former hour the moon's declination was $15^{\circ}58'50''1$, and at the latter $15^{\circ}47'11''0$; the difference of these is $11^{\circ}39'1''$; which, therefore, is the *true variation* of the moon's declination during the intervening hour: and, if this *hourly motion* be made use of in that part of the calculation which stands in the right hand compartment of page 182 of the *Appendix*, it will cause the *final* correction to amount to $19^{\circ}43'7''$, instead of $19^{\circ}44'$; and thus the mean time at Greenwich will be $10^{\text{h}}47^{\text{m}}46^{\text{s}}.2$, being in almost perfect accordance with the mean time at Greenwich, found as above.

Having thus shown with what evident facility the longitude may be deduced from an occultation, by adopting the familiar modes of calculation laid down in pages 546 and 553, I shall now close this subject by recommending the young navigator to turn back and give the Articles between pages 537 and 546 *another patient perusal*.

SOLUTION OF PROBLEMS RELATIVE TO THE VARIATION OF THE COMPASS.

Definitions.

1. The *variation of the compass* is the deviation of the points of the mariner's compass from the corresponding points of the horizon, and is denominated east or west variation accordingly.

2. *East variation* is, when the north point of the compass is to the eastward of the true north point of the horizon; *west variation* is, when the north point of the compass is to the westward of the true north point of the horizon.

The variation of the compass may be found by various methods, such as amplitudes, azimuths, transits, equal altitudes, rising and setting of the celestial objects, &c.

3. The *true amplitude* of any celestial object is, an arch of the horizon intercepted between the true east or west point thereof, and the object's centre at the time of its rising or setting.

4. The *magnetic amplitude* of an object is, the arch of the horizon that is intercepted between its centre, and the east or west point of the compass, at the time of its rising or setting; or, it is the compass bearing of the object when in the horizon of the eastern or western hemisphere.

The true amplitude of a celestial object is found by calculation; and the magnetic amplitude is found by an azimuth compass.

5. The *true azimuth* of a celestial object is, the angle contained between the true meridian and the vertical circle passing through the object's centre.

6. The *magnetic azimuth* is, the angle contained between the magnetic meridian and the azimuth, or vertical circle passing through the centre of the object; or, in other words, it is the compass bearing of the object, at any given elevation above the horizon.

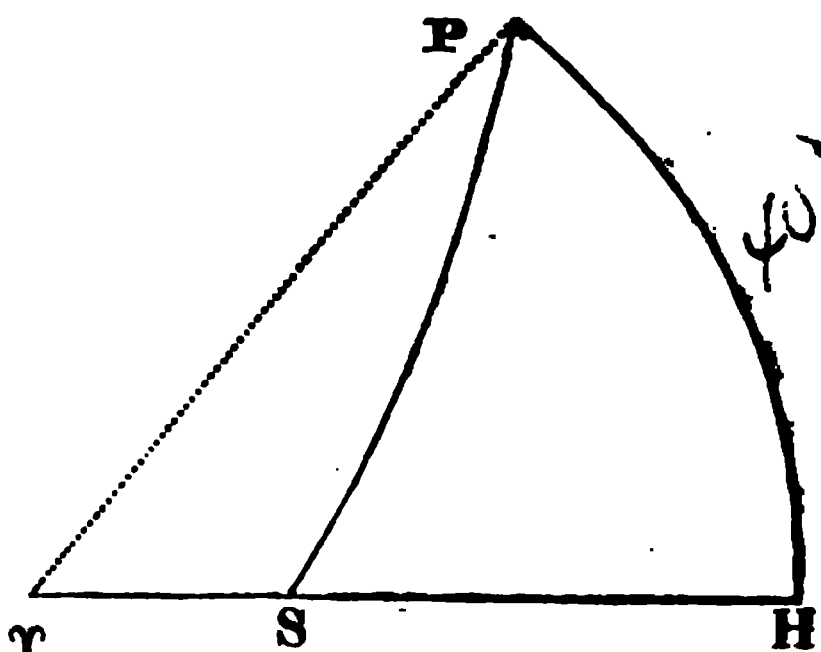
The true azimuth of a celestial object is found by calculation; and the magnetic azimuth by an azimuth compass.

PROBLEM I.

Given the Latitude of a place, the Sun's Declination, and his Magnetic Amplitude; to find the true Amplitude, and the Variation of the Compass.

The computation of an amplitude is involved in a right angled spherical triangle, the principles of which may be familiarly illustrated; as thus:—

In the annexed diagram let γ H be an arc of the horizon equal to 90 degrees, in which the point γ represents the prime vertical, or the true east, or west, point of the horizon. Let P represent the elevated pole of the heavens, and H P an arc of the meridian equal to the latitude of the place of observation: and,



let P S be the polar distance of a celestial object, and S the point of the horizon in which it rises or sets: then, the arc γ S represents the true amplitude, and the arc S H the *complement* of the amplitude of the celestial object at the moment of its rising or setting.—Now, in the right angled spherical triangle P H S, given the leg H P = the latitude, and the hypotenuse P S = the polar distance of the object; to find the leg H S = the complement of the true amplitude; which leg is to be found by spherical trigonometry, Problem I., page 184, reading H S for B C in the operation for finding the leg in the upper part of

page 185 :—then, $H S$ being found, its difference to 90 degrees = the arc γS will be the true amplitude of the celestial object.

Example.

Let the latitude of a ship, or place, be $48^{\circ}50'$ north, and the declination of a celestial object $20^{\circ}7'5''$ north; required the true amplitude of the object?

Latitude of the ship, or leg $H P = 48^{\circ}50' 0''$ L.co-si.ar.comp. 0.181608
Polar distance, or hypotenuse = 69.52.55 Log. co-sine . 9.586502

Co-amplitude, or leg $H S = . 58^{\circ}29'53''$ Log. co-sine = 9.718110

True amplitude, or arc $\gamma S = . 31^{\circ}30' 7''$; as required.

Now, since the *log. sine of the declination* is the same as the *log. co-sine of the polar distance*, and the *log. sine of the true amplitude* the same as the *log. co-sine of the co-amplitude*; therefore, if the sine of the declination be added to the secant of the latitude, the sum will be the sine of the true amplitude :—and hence the following

RULE.

Reduce the mean time of the sun's rising or setting to the meridian of Greenwich, by Problem III., page 342; to which time let the sun's declination at noon of the given day be reduced, by Problem XIV., page 357. Then, to the logarithmic secant of the latitude, add the logarithmic sine of the sun's reduced declination; and the sum, abating 10 in the index, will be the logarithmic sine of the true amplitude,—to be reckoned north or south of the true east or west point of the horizon, according to the name of the declination. Now, if the true amplitude, thus found, and the magnetic amplitude, observed per azimuth compass, be both north or both south, their *difference* is the variation; but if one be north and the other south, their *sum* is the variation :—and to know whether it be east or west, let the observer look directly towards that point of the compass representing the *true amplitude*; then, if the magnetic amplitude be to the *left hand* of this, the variation is easterly; but if to the *right hand*, it is westerly.

Example 1.

May 20th, 1836, in latitude $48^{\circ}50'$ north, and longitude $6^{\circ}30'$ west, at about 7^h40^m mean time, the sun was observed to set W. $56^{\circ}42'$ N.; required the variation of the compass?

Mean time of observation =	7 ^h 40 ^m	Sun's decl. at noon =	20° 2' 57" N.
Long. 6° 30' W., in time =	+ 26	Correction for 8 ^h 6 ^m =	+ 4. 8

Greenwich time =	8 ^h 6 ^m	Sun's true declin. =	20° 7' 5" N.
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Latitude of the ship, or place =	48° 50' 0"	Log. secant	. 10. 181608
Sun's reduced declination =	20. 7. 5	Log. sine	. 9. 536502

Sun's true amplitude =	. W. 31° 30' 7" N.	Log. sine =	9. 718110
Magnetic amplitude =	. W. 56. 42. 0 N.		

Variation of the compass = 25° 11' 53" ; which is *west*, because the magnetic amplitude is to the *right hand* of the true amplitude.

Example 2.

July 10th, 1836, in latitude 18° 40' north, and longitude 73° 45' west, at about 17^h 34^m mean time, the sun was observed to rise E. 30° 12' N. ; required the variation of the compass ?

Mean time of obs. =	17 ^h 34 ^m	Sun's dec. at noon =	22° 13' 43" N.
Long. 73° 45' W., in ti. =	+ 4. 55	Cor. for 22 ^h 29 ^m =	- 7. 22

Greenwich time =	22 ^h 29 ^m	Sun's true declin. =	22° 6' 21" N.
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Latitude of the ship or place =	18° 40' 0"	Log. secant =	10. 023468
Sun's reduced declination =	22° 6' 21"	Log. sine =	9. 575556

Sun's true amplitude =	. E. 23° 24' 15" N.	Log. sine =	. 9. 599024
Magnetic amplitude =	. E. 30. 12. 0 N.		

Variation of the compass = 6° 47' 45" ; which is *east*, because the magnetic amplitude is to the *left hand* of the true amplitude.

Example 3.

October 17th, 1836, in latitude 42° 10' north, and longitude 14° 30' west, at about 5^h 12^m mean time, the sun was observed to set W. 7° 33' N. ; required the variation of the compass ?

Mean time of observ. =	5 ^h 12 ^m	Sun's declin. at noon =	9° 22' 45" S.
Long. 14° 30' W. in time =	+ 58	Correction for 6 ^h 10 ^m =	+ 5. 37

Greenwich time =	6 ^h 10 ^m	Sun's true declin. =	9° 28' 22" S.
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Lat. of the ship or place = $42^{\circ}10' 0''$ N. Log. secant = 10.130067
 Sun's reduced declination = $9.28.22$ S. Log. sine = 9.216374

Sun's true amplitude = $W. 12^{\circ}49'45''$ S. Log. sine = 9.346441
 Magnetic amplitude = $W. 7.33. 0$ N.

Variation of the compass = $20^{\circ}22'45''$; which is *west*, because the magnetic amplitude is to the *right hand* of the true amplitude.

Remarks.

In finding the variation of the compass by this method, the sun's amplitude should be taken, with an azimuth compass, when the altitude of his lower limb is equal to the sum of his semidiameter and the dip of the horizon. Thus, if the sun's semidiameter be $16'5''$, and the dip of the horizon $4'17''$ (for 20 feet), the sum = $20'22''$ is the height which the lower limb of that object should be above the horizon, at the time of observing its amplitude.

If the index of the quadrant be set to the altitude, thus determined, the sun's magnetic amplitude may be taken when his lower limb attains that altitude, either at rising or setting; for, although the sun is apparently so elevated, yet, on account of the atmospherical refraction, his centre is actually then in the horizon of the place of observation.

From the above Examples the method of finding the variation of the compass by an observed amplitude of the moon, a planet, or a fixed star, will appear so obvious as not to require any illustration.

PROBLEM II.

Given the Latitude of a Place, the Sun's Altitude, and his Magnetic Azimuth; to find the true Azimuth, and the Variation of the Compass.

—See Definitions 5 and 6, page 566.

The principles upon which the computation of an azimuth is founded may be readily perceived by referring to the diagram, Problem I., page 428; as thus:—In the oblique-angled spherical triangle $Z \triangleright S$, the three sides are given, to find the angle at the zenith, *viz.*, the co-latitude SZ ; the co-altitude or zenith distance $\triangleright Z$, and the polar distance $S \triangleright$; to find the angle comprehended between the arc of the meridian SZ , and the arc of the vertical circle $\triangleright Z$, which angle is the true azimuth of the celestial object; and which is to be found by oblique-angled spherical trigonometry, Problem V., page 207.

Example.

Let the latitude be $39^{\circ}40'$ north; the true altitude of the sun's centre $27^{\circ}25'52''$, and his declination $9^{\circ}57'21''$ north; required the true azimuth?

Sun's north polar dist., or $S \ P = 80^{\circ} 2'39''$

Sun's zenith distance, or $P \ Z = 62.34. 8$ Log. co-sec. = 0.05179

Co-latitude of the ship, or $S \ Z = 50.20. 0$ Log. co-sec. = 0.11368

Sum = $192^{\circ}56'47''$

Half sum = $96^{\circ}28'23\frac{1}{2}''$ Log. sine = . 9.99722

Remainder = $16.25.44\frac{1}{2}$ Log. sine = . 9.45152

Sum = 19.64874

Arch = $50^{\circ} 6'29''$ Log. co-sine = 9.807094

Angle at the zen. or true azim. = $100^{\circ}12'58''$; as required.

Now, from the above, the following method of computation will appear manifest.

RULE.

Reduce the mean time of observation to the meridian of Greenwich, by Problem III., page 342; to which time let the sun's declination, at noon of the given day, be reduced, by Problem XIV., page 357.

Find the true central altitude of the sun, by Problem XXIII., page 374.—Now,

To the sun's polar distance, add its true central altitude and the latitude of the place of observation; take half their sum, and call the difference between it and the polar distance *the remainder*.

Then, to the logarithmic secants, less radius, of the true central altitude and the latitude, add the logarithmic co-sines of the half sum and the remainder: half the sum of these four logarithms will be the logarithmic co-sine of an arch; which, being doubled, will be the true azimuth, to be reckoned from the north in north latitude, but from the south in south latitude; towards the east in the forenoon, and towards the west in the afternoon.

Now, if the true azimuth, thus found, and the magnetic azimuth, observed per azimuth compass, are on the *same side of the meridian*, their *difference* is the variation; but if *on different sides*, their *sum* is

the variation :—and to know whether it be east or west, let the observer look directly towards that point of the compass which represents the *true azimuth*; then, if the magnetic azimuth be to the *left hand* of this, the variation is easterly; but if to the *right hand*, it is westerly.

Example 1.

April 15th, 1836, in latitude $39^{\circ}40'$ north, and longitude $14^{\circ}0'$ west, at $4^h 10^m$ mean time, the observed altitude of the sun's lower limb was $27^{\circ}16'20''$, and the bearing of his centre, by azimuth compass, N. $80^{\circ}37'30''$ W.; the height of the eye above the level of the sea was 24 feet, and no error in the sextant; required the variation of the compass?

Mean time of observ. =	$4^h 10^m$	Obs. alt. \odot 's low. limb =	$27^{\circ}16'20''$
Long. $14^{\circ}0'$ W., in time =	$+56$	\odot 's semid. $15'57''$	} Dif. $+11.13$
		Dip. for 24ft. 4.42	
Greenwich time =	$5^h 6^m$		
Sun's declin. at noon =	$9^{\circ}52'49''$ N.	Sun's apparent alt. =	$27^{\circ}27'33''$
Correction for $5^h 6^m$ =	$+4.52$	Refrac. $1'49''$ par. $8''$ diff.	-1.41
Sun's true declin. =	$9^{\circ}57'41''$ N.	Sun's true central alt. =	$27^{\circ}25'52''$

Sun's north polar distance =	$80^{\circ} 2'39''$
Sun's true central altitude =	$27.25.52$ Log. secant = 0.051799
Lat. of the ship or place =	$39.40.0$ Log. secant = 0.113638

$$\text{Sum} = \quad . \quad . \quad . \quad . \quad 147^{\circ} 8'31''$$

$$\begin{aligned} \text{Half sum} &= \quad . \quad . \quad . \quad 73^{\circ}34'15\frac{1}{2}'' \text{ Log. co-sine} = 9.451522 \\ \text{Remainder} &= \quad . \quad . \quad 6^{\circ}28'23\frac{1}{2}'' \text{ Log. co-sine} = 9.997222 \end{aligned}$$

$$\text{Sum} = \quad . \quad . \quad . \quad . \quad 19.614181$$

$$\text{Arch} = \quad . \quad . \quad . \quad . \quad 50^{\circ} 6'29'' \text{ Log. co-sine} = 9.807090\frac{1}{2}$$

$$\text{Sun's true azimuth} = \quad . \quad \text{N. } 100^{\circ}12'58'' \text{ W.}$$

$$\text{Magnetic azimuth} = \quad . \quad \text{N. } 80.37.30 \text{ W.}$$

Variation of the compass = $19^{\circ}35'28''$; which is *west*, because the magnetic azimuth is to the *right hand* of the true azimuth.

Example 2.

March 10th, 1836, in latitude $42^{\circ}41'$ south, and longitude $148^{\circ}5'$ east, at $19^{\text{h}}35^{\text{m}}25^{\text{s}}$ mean time, the observed altitude of the sun's lower limb was $17^{\circ}57'40''$, and the bearing of his centre, by azimuth compass, $S. 108^{\circ}37'30'' E.$; the height of the eye above the level of the sea was 19 feet, and no error in the sextant; required the variation of the compass?

Mean time of obs = $19^{\text{h}}35^{\text{m}}25^{\text{s}}$	Obs. alt. \odot 's lower li. = $17^{\circ}57'40''$
Long. $148^{\circ}5' E.$, in ti. = $9.52.20$	\odot 's semid. $16' 7''$
	Dip. for 19 ft. 4.11 } Diff. $+11.56$
Greenwich time = . $9^{\text{h}}43^{\text{m}} 5^{\text{s}}$	
Sun's dec. at noon = $3^{\circ}58'18'' S.$	Sun's apparent alt. = $18^{\circ} 9'36''$
Corr. for $9^{\text{h}}43^{\text{m}}5^{\text{s}}$ = -9.32	Refrac. $2'52''$ — paral. $8''$ = -2.4
Sun's true dec. = $3^{\circ}48'46'' S.$	Sun's true central alt. = $18^{\circ} 6'52''$

Sun's south polar distance = . $86^{\circ}11'14''$	
Sun's true central altitude = . $18. 6.52$	Log. secant = 0.022071
Latitude of the ship or place = $42.41. 0$	Log. secant = 0.13364

$$\text{Sum} = 146^{\circ}59' 6''$$

Half sum = . . .	$73^{\circ}29'33''$	Log. co-sine = 9.453534
Remainder = . . .	$12.41.41$	Log. co-sine = 9.989232

$$\text{Sum} = 19.598510$$

$$\text{Arch} = 50^{\circ}57'33'' \text{ Log. co-sine} = 9.799255$$

$$\text{Sun's true azimuth} = . . S. 101^{\circ}55' 6'' E.$$

$$\text{Magnetic azimuth} = . . S. 108.37.30 E.$$

Variation of the compass = . $6^{\circ}42'24''$; which is *east*, because the magnetic azimuth is to the *left hand* of the true azimuth.

Note.—The variation may be deduced in a similar manner from the true altitude and the magnetic bearing of a fixed star, a planet, or the moon, as may be seen by referring to “the Young Navigator’s Guide to the Sidereal and Planetary parts of Nautical Astronomy,” page 263; where the principles of the above method are familiarly explained by a stereographic projection.

PROBLEM III.

To find the Variation of the Compass by Observations of a Circumpolar Star.

Definition.—A star, or other celestial object is said to be circumpolar when its distance from the elevated pole of the heavens is less than the latitude of the place of observation (the star's declination and the latitude being of the same denomination); because, under such circumstances, the object comes within the circle of *perpetual apparition*, and revolves round the celestial pole without ever setting, or going below the horizon of the given place.

RULE.

From the log. co-sine of the star's declination (the index being increased by 10), subtract the logarithmic co-sine of the latitude: the remainder will be the logarithmic sine of the star's greatest eastern or western azimuth (according as it may be situated with respect to the meridian); to be reckoned from the north in north latitude, but from the south in south latitude. Then,

From the logarithmic sine of the latitude (the index being increased by 10), subtract the logarithmic sine of the star's declination, and the remainder will be the logarithmic sine of the star's true altitude when at its greatest eastern or western azimuth. Set the index of the quadrant to this altitude, and, when the star has attained it, let its bearing be taken by the azimuth compass; the difference between which and the computed azimuth, when they are of the same name, or their sum when of contrary names, will be the variation; which will be *east*, if the observed or magnetic azimuth be to the *left* of the computed azimuth; otherwise, *west*.

Example.

January 1st, 1825, in latitude $41^{\circ}53' \text{ S.}$, the greatest eastern azimuth of the star Canopus, by azimuth compass, was $\text{S. } 72^{\circ}50' \text{ E.}$; required the variation of the compass?

To find the Star's Altitude when at its greatest Azimuth.

Latitude of the place of observ.	= $41^{\circ}53' 0'' \text{ S.}$	Log. sine	= 9.824527
Reduced declination of Canopus	= $52.36.10 \text{ S.}$	Log. sine	= 9.900063
			—————
Star's alt. at greatest azimuth	= $57^{\circ}10'40''$	Log. sine	= 9.924464

To find the Star's greatest eastern Azimuth.

Reduced declin. of Canopus = $52^{\circ}36'10''$ S. Log. co-sine = 9.78308
 Lat. of the place of observation = $41.53. 0$ S. Log. co-sine = 9.87188

Greatest eastern azimuth = $S. 54^{\circ}39'45''$ E. Log. sine = 9.91158
 Magnetic azimuth = . . $S. 72.50. 0$ E.

Variation = $18^{\circ}10'15''$; which is *east*, because the magnetic azimuth is to the *left hand* of the computed azimuth.

Remark.—The variation of the compass may be found by equal altitudes of the fixed stars; as thus:—

Let the star's altitude be observed in the eastern hemisphere, when it is at least two hours distant from the meridian; and, at the same instant, let its bearing be taken with an azimuth compass: then, when the star comes to the same altitude in the western hemisphere, let its azimuth be again taken. Now, half the difference between the eastern and western azimuths will be the variation; which, when the observations are reckoned from the *south point* of the compass, will be east or west according as the eastern or western azimuth is the greatest; but if they be reckoned from the *north point* of the compass, a contrary process is to be observed: that is, the variation is to be called *east*, if the *western azimuth* be the greatest; but *west*, if the *eastern azimuth* be the greatest. The variation also may be found, by observing the points of the compass upon which a fixed star rises and sets; then, half the difference between those points will be the variation of the compass, as before.

Note.—The above method of finding the variation of the compass by observations of a circumpolar star, is clearly illustrated in “The Young Navigator's Guide to the Sidereal and Planetary Parts of Nautical Astronomy,” between pages 267 and 271.

PROBLEM IV.

To find the Variation of the Compass by the Magnetic Bearing of a fixed Star, or Planet, taken at the time of its Transit, or Passage over any known Meridian.

RULE.

Find the mean time of the star's transit or passage over the meridian of the given place, by Problem VIII., page 348; but if the object

selected for observation be a planet, its mean time of transit, as given in the Nautical Almanac, is to be reduced to the meridian of the place of observation, by Problem XIII., page 355. Let the watch be well regulated to mean time under the meridian of the given place, and it will show the instant of the star's or planet's transit over that meridian; at which instant its bearing, by azimuth compass, is to be carefully taken: the difference between which and the north or south point of the compass (according to the hemisphere in which the star may be posited), will show the deviation of the needle from the true corresponding point of the horizon; then, if the observed or magnetic azimuth be to the *left hand* of the meridian, the variation is easterly; but if to the *right hand*, it is westerly.*

Example 1.

January 2nd, 1836, in latitude $20^{\circ}15'$ north, and longitude $165^{\circ}30'$ east, at $11^h35^m44^s$ mean time, the star Canopus was on the meridian, and bore by azimuth compass, $S. 9^{\circ}30' E.$; required the variation?

Solution.—The observed or magnetic bearing of the star, viz., $9^{\circ}30'$ is the variation of the compass; and it is *easterly*, because it is to the *left hand* of the meridian of the given place.

Example 2.

January 2nd, 1836, in latitude $34^{\circ}25'$ south, and longitude $18^{\circ}52'$ east, at $11^h10^m10^s$ mean time, the moon was on the meridian, at which moment her centre bore, by azimuth compass, $N. 25^{\circ}36' E.$; required the variation?

Solution.—The observed bearing of the moon's centre, viz., $25^{\circ}36'$, is the variation of the compass; and it is *westerly*, because the magnetic bearing or azimuth is to the right hand of the meridian.

* The variation may be determined in a similar manner at noon, by observing the magnetic bearing of the sun at the time of its transit over the meridian: and, if the place of observation be considerably distant from the equator, a very rigid degree of accuracy is not necessary in the moment of observing the sun's bearing; since, in such a place, an error of 5 minutes in the time, before or after noon, will only produce an error of about *half a quarter of a point* in the variation; which comes sufficiently near the truth for most nautical purposes. Hence, this method may often prove useful in cases where the mariner is prevented, by clouds or other unavoidable causes, from ascertaining, in the forenoon, the true value of the magnetic variation.

Remarks.

The less the altitude of the star or planet, and the greater its declination, the more accurately will the variation be obtained. When the north polar star is in the same vertical circle with the star Alioth or the star Cor Caroli, it will be on the meridian, or nearly so; and if its azimuth be observed at that time, the variation will be obtained as before. If two stars be observed to be vertical, whose right ascensions are either equal or differ 180 degrees, they will be on the meridian: the azimuth of either may then be taken, but that which is nearest to the elevated pole should be preferred; whence the variation may be inferred in the same manner as if its mean time of transit had been computed.

The variation may also be deduced from the magnetic azimuth of a fixed star at the mean time of its transit below the pole: this time may be always known, by adding 12 hours (*diminished by 1^h 58^m, viz., half the variation of the mean sun's right ascension*), to the computed apparent time of the star's superior transit above the pole.—See Remark 2, page 349.

The number of brilliant stars which pass over the meridian of a ship at night, and the readiness with which their respective times of transit may be found, render the above method of finding the variation of the compass at sea both desirable and convenient to the practical navigator.

PROBLEM V.

Given the true Course between two Places, and the Variation of the Compass; to find the Magnetic or Compass Course.

RULE.

When the variation is westerly, let it be allowed to the right hand of the true course; but when easterly, to the left hand: in either case, the magnetic or compass course will be obtained.

Example 1.

Required the course, per compass, from Scilly to Cape Clear, the true course being N. 52° 55' W., or N.W. $\frac{3}{4}$ W. near'y, and the variation 2½ points westerly?

Solution.—The variation 2½ points, being allowed to the right hand of the true course, because it is westerly, shows the magnetic course

to be N.N.W. $\frac{1}{4}$ W. ; which, therefore, is the course which a ship must steer by compass from Scilly to Cape Clear, provided the variation be as above.

Example 2.

Required the course, per compass, from Port Royal, Jamaica, to Santa Martha, Columbia ; the true course being S. $21^{\circ}42'$ E., or S.S.E. nearly, and the variation about half a point easterly ?

Solution.—The variation $\frac{1}{2}$ a point, being allowed to the left hand of the true course, because it is easterly, shows the magnetic course to be S.S.E. $\frac{1}{2}$ E. ; which, therefore, is the course which a ship must steer by compass from Port Royal to Santa Martha, provided the variation be as above,—and independent of currents.

PROBLEM VI.

Given the Magnetic Course, or that steered by Compass, and the Variation ; to find the true Course.

RULE.

If the variation be westerly, it is to be allowed to the left hand of the course steered by compass ; but if easterly, to the right hand : in either case, the true course will be obtained.

Example 1.

Let the magnetic, or course steered by compass, be E. by N. $\frac{1}{4}$ N., and the variation $1\frac{1}{4}$ point westerly ; required the true course ?

Solution.—The variation $1\frac{1}{4}$ point, being allowed to the left hand of the compass course, because it is west, shows the true course to be N.E. by E. $\frac{1}{4}$ E.

Example 2.

Let the course steered by compass be N.W. $\frac{3}{4}$ W., and the variation one point and three-quarters easterly ; required the true course ?

Solution.—The variation $1\frac{3}{4}$ point, being allowed to the right hand of the magnetic or compass course, because it is easterly, shows the true course to be N.W. by N.

AZIMUTH COMPASS;

The card being graduated on an improved principle, so as to be more particularly adapted to the taking of amplitudes and azimuths, the measuring of horizontal angles, &c. &c.; being thus rendered far more applicable to nautical purposes in general than that which is now in common use at sea.

The azimuth compass, as well as the mariner's compass, is an artificial representation of the horizon of any place on the terrestrial globe: it consists of a circular card, divided into 32 equal parts, called points or rhumbs; and, since the circle contains 360° , each point is equal to $11^\circ 15'$: for $360^\circ \div 32 = 11^\circ 15'$.* The four principal points of the compass, viz., N., E., S., and W., are called *cardinal points*; the others are compounded of these, and are named according to the quarter in which they are situated.

To the under side of the card, and in the direction of its north and south line, a bar of hardened steel is attached, called the **NEEDLE**, which, being touched by a load-stone, acquires the peculiar property of pointing north and south, and thus directs the different points on the card to the correspondent points of the horizon. In the centre of the needle there is a small socket, by means of which it is placed, with its attached card, on an upright pin called the *pivot* or *supporter*, which is fixed in the bottom of a circular or conical brass box: on this pin the needle turns freely, and, by its magnetic property, the several points of the compass card keep always in the same direction, very nearly; though these do not always indicate the true correspondent points of the horizon, because of the aberration which the needle suffers, owing to that secret and unknown agency which causes its north and south poles to deviate more or less from the respective correspondent poles of the world.

However, since the compass is an instrument with which mariners are well acquainted, it is not deemed necessary, in this place, to enter any farther into its description. Hence, I shall merely point out some of the many advantages which a compass card, graduated on the above principle, possesses over those now in general use at sea. In this card, the circular ring of silvered brass is to be sufficiently broad to admit of four concentric spaces. The outer edge of the ring is to be graduated, *mathematically correct*, to every 20th minute of a degree

* Table XXXIII. contains the different angles which every point and quarter-point of the compass makes with the meridian; and Table XXXIV. contains the logarithmic sines, tangents, and secants of every point and quarter-point of the compass.

(though, for want of room, the present card is only graduated to every 30th minute of a degree), to which a vernier is to be adapted, containing 20 divisions on each side of its *nonius*, for the purpose of subdividing the divisions on the card into minutes of a degree.

The interior surface of the vernier should be ground concave to the segment of a circle, whose radius is equal to that of the card. The remote edge of the inner concentric space, on the silvered brass flat ring, may be graduated similarly to that of the outer edge, so as to render it more convenient in reading off amplitudes, according as they may be reckoned from the prime vertical, or from the meridian.

The first space on the broad ring of silvered brass, viz., that next the points of the compass, is particularly adapted to taking amplitudes, when the observations are reckoned from the east or the west points of the horizon; and, therefore, it is numbered both ways, from those points, towards the meridian: that is, from 0° to 90° . The second space being adapted to *horizontal azimuths*, viz., to amplitudes reckoned from the meridian, is therefore numbered both ways, from the north and south points of the horizon towards the east and west points thereof: that is, from 0° to 90° , in a contrary order to the last. The third space is intended for the accommodation of an azimuth when the observation is reckoned from the south in north latitude, or from the south in south latitude: hence, it is numbered both ways from the south to the north point of the compass, or from 0° to 180° . The fourth, or outer space, is designed for azimuths reckoned from the north in north latitude, or from the north in south latitude, according to the will of the observer; and, therefore, it is numbered both ways from the north to the south, or from 0° to 180° , &c.—See the Frontispiece to this volume.

Besides the evident uses of a compass card, graduated after this manner, in observing amplitudes and azimuths, it will also be found of the greatest utility in taking correct surveys of coasts and harbours, and in settling the *true positions* of places on shore from a *known position at sea*. It may, moreover, be applied successfully to many astronomical purposes; nay, it may even be applied to the determination of the longitude by lunar observations, as thus:—Let two observers, with two good compasses of the above description, take the azimuths of the moon and sun, or a fixed star, &c., at the same instant; then, if those two azimuths be reckoned from the same point of the horizon, their *sum*, subtracted from 360° , will be the angle at the zenith comprehended between the zenith distances of the objects; with which, and the true zenith distances of the objects, the true central distance may be found by oblique-angled spherical trigonometry, Problem III., Remark 1 or 2, page 203 or 204; and, hence, the longitude of the place of observation, by Problem XXX., page 388.

Take the star's right ascension, viz., $29^{\circ}20'47''.64$, in the compasses, from the scale of semi-tangents, and lay it off on the Equator from γ to R , and with the secant of its complement, viz. $60^{\circ}39'12''.36$, describe the circle of right ascension $P R S$; the intersection of which with the parallel circle $a b$, at $*$, will be the apparent place of the star in the heavens.—Make PN , $SO=23^{\circ}27'42''.875$ the obliquity of the ecliptic:—draw the polar line NO , and, at right angles thereto, the ecliptic line $\wp \gamma \varpi$:—through the intersecting point $*$, draw the circle of longitude $N * O$, cutting the ecliptic in L ;—then, the arc γL , will be the longitude of the star, and the arc $L *$, its latitude; the former being taken in the compasses, and applied to the scale of semi-tangents, will be found to measure about $35\frac{1}{4}$ degrees:—the angle $P N *$ (measured by the arc $L \varpi = 54\frac{3}{4}$ degrees), represents the co-longitude of the star, and the arc $N *$ its co-latitude; the latter being reduced to the primitive circle will be found to measure about 80 degrees.

Now, in the oblique angled spherical triangle $P N *$, given the side $P N = 23^{\circ}27'42''.875$, the obliquity of the ecliptic; the side $P * = 67^{\circ}21'55''.56$, the star's north polar distance, and the included angle $N P * = 119^{\circ}20'47''.24$; to find the side $N *$ = the co-latitude of the star, or its distance from the north pole of the ecliptic, and the angle $P N *$ = its co-longitude.

Note.—The circle of right ascension which passes through \wp , viz., $P Q S$, is always equal to, or expressed by 270 degrees; and that which passes through γ , or ϖ , by 0, or 360 degrees; the difference, therefore, between \wp and γ , or, which is the same, between Q and γ , is 90 degrees; which being added to the arc γR , $29^{\circ}20'47''.64$, makes the whole arc $Q R = 119^{\circ}20'47''.64$, which is the true measure of the angle $R P Q$; that is, the angle $N P *$, comprehended between the two given sides.

Hence, by spherical trigonometry, Problem III., Remark 1, p. 203,

An. $NP * = 119^{\circ}20'47''.64 + 2 = 59^{\circ}40'23''.82$ tw. L. si. 19.8721827.68
 Side PN = obl. of the ecliptic = 23.27.42 .875 L. si. 9.6000350.88
 Side $P *$ = star's N. polar dist. = 67.21.55 .56 L. si. 9.9651914.48

Sum = 39.4374093.04

Diff. of the two sides = . . . $43^{\circ}54'12''.685$ Hf. S. + 19.7187046.52

Diff. of the two sides = $43^{\circ}54'12''.685$ H.S. of logs + $19.7187046.52$

Half difference = $21^{\circ}57'6''.3425$ Log. sine = $9.5726693.15$

Arch = $54^{\circ}27'23''.53$ Log. tang. = $10.1460353.37$

Log. sine of arch, subtract from half sum of logs = $-9.9104508.95$

Half the side P * = $40^{\circ}1'14''.26$ Log. sine = $9.8082537.57$

The whole side P * = $80^{\circ}2'28''.52$; which is the co-latitude of the given star, or its distance from the north pole of the ecliptic; hence the latitude of α Arietis is $9^{\circ}57'31''.48$ north,

Now, in the oblique angled spherical triangle P N *, the three sides are given, and the angle P; to find the angle N = the co-longitude of the star.—Hence, by spherical trigonometry, Problem I., page 198,

As the side N * = $80^{\circ}2'28''.52$ Log. co-secant = $10.0065934.92$

Is to the ang. NP * = $119.20.47.64'$ Log. sine = $9.9403527.32$

So is the side P * = $67.21.55.56$ Log. sine = $9.9651914.48$

To the angle PN * = $54^{\circ}46'11''.38$ Log. sine = $9.9121376.72$

The angle N, thus found = $54^{\circ}46'11''.38$, which is the co-longitude of the star, and which is measured by the arc of the ecliptic L \odot , being taken from 90 degrees; that is, from $\gamma \odot$, leaves the arc γL = $35^{\circ}13'48''.62$; which, therefore, is the apparent longitude of the star α Arietis.—Hence, the apparent longitude of the given star is $1^h5^m13^s48''.62$, and its apparent latitude $9^{\circ}57'31''.48$ north.

Now, from the above Problem we obtain the following

General Rule

for computing the latitude and longitude of a celestial object, viz. :—

Find the difference between the object's right ascension, expressed in degrees, and 270 degrees;* to twice the logarithmic sine of half this difference, add the logarithmic sines of the object's north polar distance, and of the obliquity of the ecliptic: from half the sum of these three logarithms subtract the logarithmic sine of half the dif-

* In all cases, whenever this difference exceeds 180 degrees, it must be subtracted from 360 degrees.

ference between the object's north polar distance and the obliquity of the ecliptic ; the remainder will be the logarithmic tangent of an arch, the logarithmic sine of which being subtracted from the half sum of the three logarithms will leave the logarithmic sine of an arc, which, being doubled, will give the object's distance from the north pole of the ecliptic ; the difference between which and 90 degrees will be the latitude of the object, which will be north when the ecliptic polar distance is the least ; otherwise, south.

To find the Longitude :—

To the logarithmic secant of the object's latitude, add the logarithmic sine of the difference between its right ascension and 270 degrees,* and the logarithmic sine of its north polar distance ; the sum of these three logarithms, abating 20 in the index, will be the logarithmic sine of an arch, which being subtracted from 90 degrees, will leave the object's true longitude when its right ascension is less than 6 hours or 90 degrees ; but which is to be increased by 6 signs when the right ascension is between 12 and 18 hours, that is, between 180 and 270 degrees.

Again,—If the right ascension lies between 6 and 12 hours ; that is, between 90 and 180 degrees, the arch, so found, is to be augmented by 3 signs ; but if the right ascension is between 18 and 24 hours, viz., between 270 and 360 degrees, it is to be augmented by 9 signs ; in either case the true longitude of the object will be obtained.

Example.

The apparent right ascension of Aldebaran, August 3rd, 1825, was $4^h 25^m 56^s \cdot 115$, and its declination $16^\circ 9' 5'' \cdot 35$ north ; required its apparent latitude and longitude, the obliquity of the ecliptic being $23^\circ 27' 42'' \cdot 875$?

$$\text{Aldeb's R.A.} = 4^h 25^m 56^s \cdot 115 = 66^\circ 29' 1'' \cdot 725$$

$$\text{Diff. between R.A. and } 270^\circ = 203^\circ 30' 58'' \cdot 275$$

$$\text{And } 360^\circ - 203^\circ 30' 58'' \cdot 275 = 156^\circ 29' 1'' \cdot 725$$

$$\text{The half of which is} = \quad \quad 78^\circ 14' 30'' \cdot 862$$

* See Note, page 582.

The half of which is= . . . $78^{\circ}14'30''.862$ Tw. L.si. 19.981580
 Aldebaran's N. polar distance= $73.50.54.650$ Log. si.= 9.982511
 Obliquity of ecliptic= . . . $23.27.42.875$ Log. si.= 9.600035

 Sum = 39.564126

 Diff. bet. P. dist. and ob. of eclip.= $50^{\circ}23'11''.775$ Half S.=19.782063+
 Half ditto= $25^{\circ}11'35''.887$ Log. si.= 9.629077

 Arch= $54^{\circ}53'21''.0$ Log. tang.= 10.152986
 Log. sine of arch, subtract from
 half sum of logs.= 9.912775-

 Arc= $47^{\circ}44'22''.5$ Log. sine= 9.869288

 Twice the arc= $95^{\circ}28'45''.0$; which is Aldebaran's
 distance from the north pole of the ecliptic; its latitude, therefore, is
 $5^{\circ}28'45''$ south.

To find Aldebaran's Longitude:—

Lat. of Aldebaran= $5^{\circ}28'45''$ S. Log. sec.= 10.001989
 Diff. betw. R.A. and 270 deg.= $156.29.1.725$ Log. sine= 9.600982
 Aldebaran's N. polar distance= $73.50.54.65$ Log. sine= 9.982511

 Aldebaran's co-longitude= . . . $22^{\circ}38'41''.6$ Log. sine= 9.585482

 Aldebaran's longitude= . . . $67^{\circ}21'18''.4$; or $2^{\circ}7^{\circ}21'18''.4$.

Hence, the latitude of Aldebaran is $5^{\circ}28'45''$ south, and its longitude $2^{\circ}7^{\circ}21'18''.4$, as required.

Note.—In like manner may the latitudes and longitudes of the moon and planets be deduced from their respective right ascensions and declinations.

PROBLEM II.

Given the Latitude and Longitude of a Celestial Object; to find its Right Ascension and Declination.

Example.

The apparent long. of α Arietis, August 1st, 1825, was $1^{\circ}5^{\circ}13'48''.62$, and its latitude $9^{\circ}57'31''.48$ north; required its right ascension and declination, the obliquity of the ecliptic being $23^{\circ}27'42''.875$?

The construction of this Problem is exactly like that of the preceding: thus, lay the longitude of the given star off on the ecliptic line from γ to L, and draw the circle of longitude N L O.—Make γd , ϖc = the star's latitude, and draw the parallel circle $c d$; the intersection of which with the circle of longitude, at $*$, will represent the apparent place of the star in the heavens.—See the last *projection*.

Now, in the oblique angled spherical triangle N P $*$; given the side P N = $23^{\circ}27'42''.875$, the obliquity of the ecliptic; the side N $*$ = $80^{\circ}2'28''.52$, the star's distance from the north pole of the ecliptic, and the included angle P N $*$ = $54^{\circ}46'11''.38$, the complement of the star's longitude (measured by the arc L ϖ); to find the side P $*$ = the star's north polar distance, and the angle N P $*$ (measured by the arc R Q); the difference between which and γ Q, viz., 90 degrees, expressed by the arc γ R = will be the star's right ascension.

Hence, by spherical trigonometry, Problem III., Remark 1, p. 203,

$$\text{Angle P N } * = \quad . \quad . \quad . \quad 54^{\circ}46'11''.38$$

$$\text{Half ditto} = \quad . \quad . \quad . \quad 27^{\circ}23'5''.69 \quad \text{Tw.L.si.} = 19.3254514.58$$

Side P N = obliquity of the

$$\text{ecliptic} = \quad . \quad . \quad . \quad 23.27.42.875 \quad \text{Log. si.} = 9.6000350.88$$

Side N $*$ = star's ecliptic

$$\text{polar distance} = \quad . \quad . \quad 80.2.28.250 \quad \text{Log. si.} = 9.9934065.08$$

$$\text{Sum} = 38.9188930.54$$

$$\text{Difference of the two sides} = 56^{\circ}34'45''.645 \quad \text{Half S.} = 19.4594465.27 +$$

$$\text{Half difference} = \quad . \quad . \quad 28^{\circ}17'22''.822 \quad \text{Log. si.} = 9.6757137.08$$

$$\text{Arch} = \quad . \quad . \quad . \quad 31^{\circ}17'22''.56 \quad \text{Log. T.} = 9.7837328.19$$

$$\text{Logarithmic sine of arch} = \quad . \quad . \quad . \quad . \quad . \quad . \quad 9.7154718.60 -$$

$$\text{Half the side P } * = \quad . \quad 33^{\circ}40'57''.76 \quad \text{Log. si.} = 9.7439746.67$$

The whole side P $*$ = $67^{\circ}21'55''.52$; which is the co-declination of the given star, or its north polar distance; hence, the declination of α Arietis is $22^{\circ}38'4''.48$, north.

Now, in the oblique angled spherical triangle N P $*$; the three sides are given, and the angle N; to find the angle P = the arc R Q; or the sum of the star's right ascension and 90 degrees.

Hence, by spherical trigonometry, Problem I., page 198,

As the side $P \star = \quad . \quad 67^{\circ}21'55''.52$ Log. co-sec. 10.0348085.84
 Is to the angle $P N \star = \quad 54.46.11 .38$ Log. sine = 9.9121376.70
 So is the side $N \star = \quad . \quad 80.2.28 .52$ Log. sine = 9.9934065.08

To the sup. of ang. $N P \star = 60^{\circ}39'12''.38$ Log. sine = 9.9403527.62

Hence, the ang. $N P \star$ is $= 119^{\circ}20'47''.62 =$ the arc $R Q$; from which take the arc γQ , 90 degrees; and the remaining arc $\gamma R = 29^{\circ}20'47''.62$, is the star's right ascension.—The apparent right ascension of α Arietis, on the given day was, therefore, $29^{\circ}20'47''.62$, and its declination $22^{\circ}38'4''.48$ north.

Now, from the above Problem, the following General Rule is deduced for computing the right ascension and declination of a celestial object, viz.:—

Find the difference between the object's longitude and three signs (\odot) or 90 degrees.* Then, to twice the logarithmic sine of half this difference, add the logarithmic sines of the object's distance from the north pole of the ecliptic, and of the obliquity of the ecliptic; from half the sum of these three logarithms subtract the logarithmic sine of half the difference between the obliquity of the ecliptic and the object's ecliptic polar distance, and the remainder will be the logarithmic tang. of an arch; the log. sine of which being subtracted from the half sum of the three logarithms, will leave the logarithmic sine of an arc, the double of which will be the object's distance from the north pole of the equator:—now, the difference between this distance and 90 degrees will be the declination of the object; which will be north when the first term is less than the latter; otherwise south.

To find the Right Ascension:—

To the logarithmic secant of the object's declination, add the logarithmic sine of the difference between its longitude and 90 degrees,* and the logarithmic sine of its distance from the north pole of the ecliptic; the sum of these three logarithms, abating 20 in the index, will be the logarithmic sine of an arch, which being subtracted from 90 degrees, will leave the object's correct right ascension when its longi-

* In all cases, whenever this difference exceeds 180 degrees, it must be subtracted from 360 degrees.

tude is less than 3 signs or 90 degrees; but which is to be increased by 180 degrees when the longitude is between 6 and 9 signs, or between 180 and 270 degrees. Again, if the longitude is between 3 signs and 6 signs, that is, between 90 and 180 degrees, the arch, so found, is to be augmented by 90 degrees; but, if the longitude lies between 9 and 12 signs, viz., between 270 and 360 degrees, it is to be augmented by 270 degrees; in either case the correct right ascension of the object will be obtained, which may be converted into time, if necessary; by Problem I., page 342.

Example.

The apparent longitude of Aldebaran, August 3rd, 1825, was $2^{\circ}7'21''18''.4$, and its latitude $5^{\circ}28'45''$ south; required its apparent right ascension and declination, the obliquity of the ecliptic being $23^{\circ}27'42''.875$?

Aldebaran's longitude = . . $67^{\circ}21'18''.4$

Difference to 90 degrees = . $22^{\circ}38'41''.6$

The half of which is = . . $11^{\circ}19'20''.8$ Tw.L.sine 18.585974

Aldebaran's ecliptic polardis. = $95.28.45.0$ Log. sine 9.998011

Obliquity of ecliptic = . . $23.27.42.875$ Log. sine 9.600035

Sum = 38.184020

Difference betw. ob. of eclip. and

star's eclip. polar distance = $72^{\circ}1'2''.125$ Half S. = 19.092010 +

Half ditto = $36^{\circ}0'31''.062$ Log. sine . 9.769309

Arch = $11.52.21.27$ Log. tang. = 9.322701

Log. sine of arch, subtract from

half sum of logs. = 9.313310 -

Are = $36^{\circ}55'27''.3$ Log. sine = 9.778700

Twice the arc = $73^{\circ}50'54''.6$; which is Aldebaran's distance from the north pole of the equator; hence its declination is $16^{\circ}9'5''.4$ north.

To find the Right Ascension :—

Declination of Aldebaran=	16° 9' 5".4	Log. secant=	10.017490
Diff. bet. long. and 90 deg.=	22.38.41 .6	Log. sine =	9.585481
Aldebaran's ecliptic P. dist.=	95.28.45 .0	Log. sine =	9.998011
<hr/>			
Arch=	23°30'58".3	Log. sine =	9.600982

Aldebaran's right ascension= 66°29' 1".7, or 4^h25^m56^s.1.

Hence, the right ascension of the star Aldebaran is 4^h25^m56^s.1, and its declination 16°9'5".4 north, as required.

Note.—In the same manner may the right ascensions and declinations of the moon and planets be deduced from their respective latitudes and longitudes.

PROBLEM III.

Given the Latitudes and Longitudes of the Moon and Sun, Moon and fixed Star, or Moon and Planet; to find the true Central Distance between them.

Note.—If this Problem be projected stereographically on the plane of the circle of longitude passing through one of the objects, it will be found, in every respect, like Problem XXIV., page 273, of “The Young Navigator’s Guide to the Sidereal and Planetary Parts of Nautical Astronomy;” reading, however, difference of longitude for difference of right ascension, and ecliptic polar distances for polar distances:—hence, it is evident that there is a spherical triangle to work in, where two sides and the included angle are given to find the third side, or central distance between the objects, and which may be computed by the following

General Rule.

To twice the logarithmic sine of half the difference of longitude between the two objects, add the logarithmic co-sines of their latitudes; from half the sum of these three logarithms subtract the logarithmic sine of half the difference or half the sum of the latitudes, according as they are of the same or of contrary names; the remainder will be the logarithmic tangent of an arch, the logarithmic sine of which being

subtracted from the half sum of the three logarithms, will leave the logarithmic sine of half the true central distance between the two given objects.

Example 1.

September 3rd, 1825, the moon's apparent longitude, at noon, was $1^{\circ}16'19''.29$, and her latitude $2^{\circ}33'30''$ north; at the same time the apparent longitude of Pollux was $3^{\circ}20'48''.38$, and its latitude $6^{\circ}40'19''$ north; required the true central distance between those two objects?

Longitude of Pollux =	. $110^{\circ}48'38''$	
Longitude of the moon =	$46.19.29$	
	<hr/>	
Difference of longitude =	$64^{\circ}29'9''$	
	<hr/>	
The half of which is =	. $32^{\circ}14'34\frac{1}{2}''$	Tw. the log. si. = 19.454285
Latitude of Pollux =	. . $6.40.19$ N.	Log. co-sine = 9.997049
Latitude of the moon =	. . $2.33.30$ N.	Log. co-sine = 9.999567
	<hr/>	
		Sum = 39.450901
	<hr/>	
Difference of latitude =	. $4^{\circ}6'49''$	Half sum = $19.725450\frac{1}{2} +$
	<hr/>	
Half difference =	. . . $2^{\circ}3'24\frac{1}{2}''$	Log. sine = $8.554977\frac{1}{2}$
		<hr/>
Arch = $86^{\circ}8'11''$	Log. tang. = 11.170473
Log. sine of arch, subtract		<hr/>
from half sum of logs. =	$9.999012 -$
		<hr/>
Half true distance =	. . $32^{\circ}11'4''$	Log. sine = $9.726438\frac{1}{2}$

Hence the true central distance between the moon and Pollux, at the given time, was $64^{\circ}22'8''$; which corresponds exactly with the computed distance in the Nautical Almanac.

Note.—It is evident that the same result will be obtained by making use of the right ascensions and declinations of the objects.

The true central distance may be readily found by the formula under *Remark 2*, page 204.

Example 2.

August 4th, 1825, the moon's apparent longitude, at noon, was $0^{\circ}14'13''.32$, and her latitude $4^{\circ}38'41''$ north; at the same time the sun's longitude was $4^{\circ}11'43''.46$; required the true central distance?

Note.—Since the sun apparently moves in the ecliptic, he has, therefore, very little, or *no* latitude.

Moon's longitude = $14^{\circ}13'32''$

Sun's longitude = $131.43.46$

Difference of long. = $117^{\circ}30'14''$ this divided

by 2 gives $58^{\circ}45'7''$ Twice the L.sine = 19.863860

Moon's lat. $4.38.41$ N. Log. co-sine = 9.998571

Sun's lat. = $0.0.0$ Log. co-sine = 10.000000

Sum = . . . 39.862431

Diff. of lat. = $4^{\circ}38'41''$ Half sum = . 19.931215½ 19.931215½

Half ditto = $2^{\circ}19'20\frac{1}{2}''$ Log. sine = . 8.607688

Arch = . $87^{\circ}16'55''$ Log. tang. = . 11.323527½ L.sin. 9.999511

Hf. req. dis. $58^{\circ}42'10\frac{1}{2}''$ Log. sine = 9.931704½

Hence, the true central distance between moon and sun is $117^{\circ}24'21''$; which corresponds with that in the Nautical Almanac.

Remark.—Since the co-latitudes of the sun and moon and the comprehended angle (expressed by their difference of longitude,) form a quadrantal spherical triangle; therefore the true central distance between these particular objects may be more readily determined by the following concise method than by the above general Rule, viz.,

To the logarithmic co-sine of the difference of longitude, add the logarithmic co-sine of the moon's latitude; the sum of these two logarithms, abating 10 in the index, will be the logarithmic co-sine of the true central distance between the sun and moon.

Example 1.

August 6th, 1825, the moon's longitude, at noon, was $1^{\circ}8'0'34''$, and her latitude $3^{\circ}23'20''$ north; at the same time, the sun's longitude was $4^{\circ}13'38'46''$; required the true central distance?

Moon's longitude = $38^{\circ} 0' 34''$

Sun's longitude = $133.38.46$

Difference of long. = $95^{\circ} 38' 12''$ Log. co-sine = . . . 8.992199

Moon's latitude = . $3.23.20$ Log. co-sine = . . . 9.999240

True central distance = $95^{\circ} 37' 36''$ Log. co-sine = . . . 8.991439

which is precisely the same as that given in the Nautical Almanac.

Example 2.

August 7th, 1825, the moon's longitude, at noon, was $1^{\circ} 20' 4.42''$, and her latitude $2^{\circ} 30' 42''$ north; at the same time the sun's longitude was $4^{\circ} 14' 36.18''$; required the true central distance?

Moon's longitude = $50^{\circ} 4' 42''$

Sun's longitude = $134.36.18$

Difference of long. = $84^{\circ} 31' 36''$ Log. co-sine = . . . 8.979468

Moon's latitude = . $2.30.42$ Log. co-sine = . . . 9.999583

True central dist. = $84^{\circ} 31' 55''$ Log. co-sine = . . . 8.979051

which exactly corresponds with the computed distance in the Nautical Almanac.

Example 3.

Required the true central distance between the moon and sun at noon, August 8th; at midnight, August 8th; at noon, August 9th, and at midnight, August 9th, 1825?

Moon's long. noon Aug. 8th. = $62^{\circ} 22' 38''$

Sun's longitude ditto = . . $135.33.51$

Difference of longitude = . . $73^{\circ} 11' 13''$ Log. co-sine = 9.461273

Moon's latitude = . . . $1.30.9$ Log. co-sine = 9.999851

Distance at noon, Aug. 8th = $73^{\circ} 11' 34''$ Log. co-sine = 9.461124

Moon's long. mid. Aug. 8th = $68^{\circ} 38' 20''$

Sun's longitude ditto = . . $136.2.38\frac{1}{2}$

Difference of longitude = . $67^{\circ} 24' 18\frac{1}{2}''$ Log. co-sine = 9.584572

Moon's latitude = . . . $0.57.33$ Log. co-sine = 9.999939

Dist. at midnight, Aug. 8th. = $67^{\circ} 24' 31''$ Log. co-sine = 9.584511

Moon's long. noon, Aug. 9th = $74^{\circ}59'18''$

Sun's longitude ditto = $136.31.26$

Difference of longitude = $61^{\circ}32'8''$ Log. co-sine = 9.678166

Moon's latitude = $0.23.49$ Log. co-sine = 9.999990

Distance at noon, Aug. 9th = $61^{\circ}32'11''$ Log. co-sine = 9.678156

Moon's long. at mid. Aug. 9th = $81^{\circ}25'59''$

Sun's longitude at ditto = $137.0.14$

Difference of longitude = $55^{\circ}34'15''$ Log. co-sine = 9.752346

Moon's latitude = $0.10.42$ Log. co-sine = 9.999998

Distance at midnight, Aug. 9th = $55^{\circ}34'16''$ Log. co-sine = 9.752344

Now, from these four consecutive lunar distances, the distances at the intermediate periods, or every third hour, may be readily determined in the following manner, viz. :—

Find the proportional parts of the difference at the middle interval between the four distances (that is, between the second and third distances,) answering to 3 hours, 6 hours, and 9 hours : correct these proportional parts by the equation of second differences, agreeably to the rule given, for that purpose, in page 35 ;—then, these corrected proportional parts being applied to the second lunar distance by addition or subtraction, according as the distances are increasing or decreasing, the sum or difference will be the true distances at the given periods :—thus,

Aug. 8, 1825, dis. at N. = $73^{\circ}11'34''$	} $5^{\circ}47'3''$	} $0^{\circ}5'17''$	} $5'26''$
Ditto . . . do. at M. = $67.24.31.$			
Aug. 9, . . . do. at N. = $61.32.11.$	} $5.52.20$	} $0.5.35$	
Ditto . . . do. at M. = $55.34.16.$			

The proportional parts of $5^{\circ}52'20''$ (the middle first difference) answering to the intermediate periods, viz. :—

To 3 hours it is = $1^{\circ}28'5''$;
Equation of second difference = -31

Proportional part corrected = $1^{\circ}27'34''$
Distance at midnight, Aug. 8th = $67.24.31$

Distance at 15 hours, Aug. 8th = $65^{\circ}56'57''$

$$\begin{array}{rcl} \text{To 6 hours it is} & = & 2^{\circ}56'10'' \\ \text{Equation of second difference} & = & - 41 \end{array}$$

$$\begin{array}{rcl} \text{Proportional part corrected} & = & 2^{\circ}55'29'' \\ \text{Distance at midnight, Aug. 8th} & = & 67.24.31 \end{array}$$

$$\text{Distance at 18 hours, Aug. 8th} = 64^{\circ}29' 2''$$

$$\begin{array}{rcl} \text{And to 9 hours it is} & = & 4^{\circ}24'15'' \\ \text{Equation of second difference} & = & - 31 \end{array}$$

$$\begin{array}{rcl} \text{Proportional part corrected} & = & 4^{\circ}23'44'' \\ \text{Distance at midnight, Aug. 8th} & = & 67.24.31 \end{array}$$

$$\text{Distance at 21 hours, Aug. 8th} = 63^{\circ} 0'47''$$

The distances for the intermediate periods corresponding to the first and to the last 12 hours ; that is, for every third hour between the first and second distances, and between the third and fourth distances, may be also very readily determined by means of the Formulæ which are given in page 117 of the Nautical Almanac for 1825.

SOLUTION OF PROBLEMS RELATIVE TO FINDING THE MEAN TIMES OF THE RISING AND SETTING OF THE CELESTIAL BODIES.

PROBLEM I.

Given the Day of the Month, the Latitude of a Place, and the Height of the Eye above the Level of the Horizon : to find the mean Times of the Sun's Rising and Setting.

RULE.

Let the sun's declination, at noon of the given day, be reduced to the meridian of the given place, by Problem XIV., page 357, then, to the logarithmic tangent of this reduced declination, add the logarithmic tangent of the latitude ; and the sum (abating 10 in the index) will be the logarithmic co-sine of an arch ; which, being converted into time, will be the approximate time of the sun's rising, and its difference to 12 hours will be that of the sun's setting, the latitude and the declination being of the same name ; but if these elements be of contrary

names, the above arch, reduced into time, will be the approximate time of the sun's setting, and its complement to 12 hours that of the sun's rising. Increase the time of rising by 12 hours, and it will be the time of rising past noon of the preceding day.

Reduce the approximate times of rising and setting, thus found, to the correspondent times at Greenwich, by Problem III., page 342; to which times, respectively, let the sun's declination be reduced, by Problem XIV., page 357, then,

To the aggregate of 90 degrees,* the horizontal refraction,† and the dip of the horizon, diminished by the sun's horizontal parallax,‡ add the sun's polar distance, and the co-latitude of the place of observation: take half the sum; the difference between which and the first term, call the *remainder*.

Now, to the logarithmic co-secants, less radius, of the polar distance, and the co-latitude, add the logarithmic sines of the half sum, and of the remainder: half the sum of these four logarithms will be the logarithmic sine of an arch; which, being doubled, and converted into time, will be the apparent time of the sun's rising. In the same manner the apparent time of the sun's setting is to be computed; but, in this case, the half sum of the four logarithms is to be considered as a logarithmic co-sine.

Note.—Since the refraction and the dip of the horizon affect the rising and setting of a celestial object in *the same manner*, by causing the object to appear a *little earlier*, and to continue a *little later*, above the *true* horizon; and since the parallax affects the object in a *contrary* manner, by depressing it below the *true* horizon; therefore, the sum of the two first terms, *minus* the last term added to 90° , gives the zenith distance of the sun at the moment of its rising or setting. Now, the sun's zenith distance, thus found, his polar distance, and the co-latitude of the place of observation, form the three sides of a spherical triangle; in which the angle opposite to the zenith distance is required; and which is to be found by Problem V., page 207:—then, this being converted into time by Problem II., page 342, the result will be the apparent time of the sun's rising or setting, as the case may be; which is to be converted into mean time by applying thereto the equation of time, according to the directions given in page I. of the Month in the Nautical Almanac.

* The sun's distance from the zenith when his centre is in the horizon.

† The horizontal refraction of a celestial object is 33 minutes of a degree.

‡ The sun's horizontal parallax is about 9 seconds.

Example.

Required the mean times of the sun's rising and setting, July 13th, 1836, in latitude $50^{\circ}48'$ north, and longitude 120° west; the height of the eye above the level of the sea being 30 feet?

Sun's declin. at noon,

July 13th = . . . $21^{\circ}49' 0''$ N.

Cor. for lon. 120° W. Ta. XV. — 3. 0

Sun's reduced declin. = $21^{\circ}46' 0''$ N. Log. tangent = . . 9.601296

Lat of the ship or place = $50.48. 0$ N. Log. tangent = . 10.088533

Arch = $60^{\circ}41'12''$ Log. co-sine = . . 9.689829

Approximate time of \odot 's

rising, A. M. = . . . $4^h 2^m 45^s$

Do. past noon, July 12th $16^h 2^m 45^s$

Long. 120° W., in ti. = +8. 0. 0

Greenwich time past

noon, July 13th = . $0^h 2^m 45^s$

Sun's declin. at noon,

July 13th = . . . $21^{\circ}49' 0''$ N.

Corr. for $0^h 2^m 45^s$ = — 0. 1

\odot 's dec. at ti. of ris. = $21^{\circ}48'59''$ N.

Equation of time at rising = $5^m 22^s$ additive.

Apx. ti. of sun's setting

past noon, July 13th = $7^h 57^m 15^s$

Long. 120° W. in ti. = +8. 0. 0

Greenwich time past

noon, July 13th = $15^h 57^m 15^s$

Sun's declin. at noon,

July 13th = . . . $21^{\circ}49' 0''$ N.

Corr. for $15^h 57^m 15^s$ = — 5. 59

\odot 's dec. at ti. of sett. = $21^{\circ}43' 1''$ N.

Equa. of time at setting = $5^m 27^s$ additive.

To find the Mean Time of the Sun's Rising.

$90^{\circ} + 33' + 5' 15'' - 9'' = \odot$'s zen. dist. $90^{\circ}38' 6''$

Sun's north polar distance = . . 68. 11. 1 Log. co-sec. = 0.032274

Co-latitude of the ship = . . . 39. 12. 0 Log. co-sec. = 0.199263

Sum = $198^{\circ} 1' 7''$

Half sum = $99^{\circ} 0' 33\frac{1}{2}''$ Log. sine = 9.994609

Remainder = $8.22.27\frac{1}{2}$ Log. sine = 9.163279

Sum = . . . 19.389425

Arch = $29^{\circ}40'40''$ Log. sine = 9.694712 $\frac{1}{2}$

$$\text{Arch} = \dots\dots\dots 29^{\circ}40'40''$$

$$\text{Arch doubled} = \dots\dots\dots 59^{\circ}21'20'' = 3^h57^m25^s \text{ app. ti. A.M.}$$

$$\text{Equation of time at sun's rising} = \dots\dots\dots +5.22$$

$$\text{Mean time of the sun's rising} = \dots\dots\dots 4^h2^m47^s \text{ A.M.}$$

To find the Mean Time of the Sun's Setting.

$$\text{Sun's zenith distance} = \dots\dots 90^{\circ}38'6''$$

$$\text{Sun's north polar distance} = 68.16.59 \quad \text{Log. co-secant} = 0.031974$$

$$\text{Co-latitude of the ship} = \dots\dots 39.12.0 \quad \text{Log. co-secant} = 0.199263$$

$$\text{Sum} = \dots\dots\dots 198^{\circ}7'5''$$

$$\text{Half sum} = \dots\dots\dots 99^{\circ}3'32\frac{1}{2}'' \quad \text{Log. sine} = \dots\dots 9.994549$$

$$\text{Remainder} = \dots\dots\dots 8.25.26\frac{1}{2}'' \quad \text{Log. sine} = \dots\dots 9.165831$$

$$\text{Sum} = \dots\dots\dots 19.391617$$

$$\text{Arch} = \dots\dots\dots 60^{\circ}14'23'' \quad \text{Log. co-sine} = 9.695808\frac{1}{2}$$

$$\text{Arch doubled} = \dots\dots\dots 120^{\circ}28'46'' = 8^h1^m55^s \text{ appar. time P.M.}$$

$$\text{Equation of time at sun's setting} = \dots\dots\dots +5.27$$

$$\text{Mean time of the sun's setting} = \dots\dots\dots 8^h7^m22^s \text{ P.M.}$$

Hence the correct mean time of the sun's rising is $4^h2^m47^s$ A.M., nautical time, or $16^h2^m47^s$ past noon, July 12th; and the mean time of the sun's setting is $8^h7^m22^s$ past noon, July 13th, as required.

See *Remark*, page 126, relative to the effects of Refraction, &c. &c.

PROBLEM II.

Given the Latitude of a Place, and the Height of the Eye above the Level of the Horizon; to find the mean Times of the Moon's Rising and Setting.

RULE.

Reduce the mean time of the moon's transit at Greenwich to the meridian of the given place, by Problem XI., page 352.—Convert

this into Greenwich time by Problem III., page 342; to which let the moon's horizontal parallax be reduced, by Problem XV., page 361, and her declination by Problem XVI., page 364.

To 90 degrees, diminished by the difference between the moon's horizontal parallax and the sum of the horizontal refraction and the dip of the horizon, add the moon's polar distance and the co-latitude of the given place. Find the difference between half the sum and the first term, which call the *remainder*.

Now, to the logarithmic co-secants, less radius, of the polar distance, and the co-latitude, add the logarithmic sines of the half sum and of the remainder: half the sum of these four logarithms will be the logarithmic co-sine of an arch; which, being doubled, and converted into time, will be the moon's approximate semi-diurnal arc: this being subtracted from and added to the reduced time of the moon's transit, the respective approximate times of her rising and setting will be obtained.

Reduce the approximate times of the moon's rising and setting, thus found, to the correspondent times at Greenwich, by Problem III., page 342; to which times, respectively, let the moon's horizontal parallax and declination be again reduced by the above-mentioned Problems.—Then,

With 90 degrees, *diminished as before*, the moon's respective polar distances, and the co-latitude, compute the approximate semi-diurnal arcs corresponding to the times of rising and setting.

Find the proportional part of the daily variation of the moon's transit answering to *each semi-diurnal arc*, and 24 hours augmented by the variation of transit, by Problem XI., page 352, in the same manner as if it were the reduction of transit to a different meridian that was under consideration. Now, these proportional parts, being *added* to their corresponding semi-diurnal arcs, will give the true semi-diurnal arcs at the times of the moon's rising and setting: the former being subtracted from the reduced time of transit, and the latter added thereto, the respective mean times of the moon's rising and setting will be obtained.

Note.—Since the moon's horizontal parallax, which depresses her below the level of the true horizontal plane, causes her to be several minutes (at certain times and places *nearly half an hour*) *later in rising and earlier in setting*; and since the value of the horizontal parallax is always greater than the combined effects of refraction and dip of the horizon, which have a direct tendency to make her *rise a little earlier and to set a few minutes later*; therefore, if 90° be augmented by the horizontal refraction (33') and the value of the dip of the horizon;

and then diminished by the horizontal parallax; the result will be the moon's correct zenith distance at the instant of her rising or setting.

Hence, we have an oblique-angled spherical triangle, in which the three sides are given; to find the angle opposite to the zenith distance, as stated in the *Note* to Problem I., page 594.

Respecting the nature and the effects of the moon's horizontal parallax, see the Articles between pages 29 and 34.

Example.

Required the mean times of the moon's rising and setting on the 1st of January, 1836, in latitude $51^{\circ}28'40''$ north, and longitude 75° west; the height of the eye above the level of the horizon being 30 feet.

To reduce the Time of the Moon's Transit to the given Meridian.

24 hours + retardation $50''$	=	$24^{\circ}50''$	Prop. log. ar. comp. =	9.1398
Daily retardation =		0.50	Prop. logarithm =	0.5563
Longitude 75° W., in time =		5. 0	Prop. logarithm =	1.5563
<hr/>				
Correction =		$+10^{\circ} 4'$	Prop. log. =	1.2524
Moon's transit, per Naut. Alm. =		10.22.42		

Time of μ 's trans. at given place =	$10^{\circ}32'46''$:—See <i>Example</i> , p. 358.
Long. of given place, in time =	$+5. 0. 0$	

Greenwich time = $15^{\circ}32'46''$.—Now, the moon's horizontal parallax reduced to this time is $54'3''$, and her declination $24^{\circ}58'18''$ North.

To find the Approximate Times of Rising and Setting.

$90^{\circ} - 54'3'' + 33'0'' + 5'15''$	=	$89^{\circ}44'12''$		
Moon's north polar distance =	65. 1.42	Log. co-secant =	0.042694	
Co-latitude of the ship = . . .	38.31.20	Log. co-secant =	0.206639	
<hr/>				
Sum =		$193^{\circ}17'14''$		
<hr/>				
Half sum =	$96^{\circ}38'37''$	Log. sine =	9.997074	
Remainder =	6.54.25	Log. sine =	9.080111	
<hr/>				
		Sum =	19.325448	
<hr/>				
Arch =	$62^{\circ}36'55''$	Log. co-sine =	9.682794	

$$h = \dots \dots \dots 62^{\circ}36'55''$$

$$\begin{aligned} \text{approx. semi-diurnal arc} &= 125^{\circ}13'50'' = 8^{\text{h}}20^{\text{m}}55^{\text{s}} & . & . & 8^{\text{h}}20^{\text{m}}55^{\text{s}} \\ \text{ae of } D \text{'s transit at given place} &= & . & . & 10.32.46 & . & 10.32.44 \end{aligned}$$

$$\begin{aligned} \text{proximate time of moon's rising} &= & . & . & 2^{\text{h}}11^{\text{m}}49^{\text{s}} & \text{Setg.} & 18^{\text{h}}53^{\text{m}}39^{\text{s}} \\ \text{altitude of the given place, in time} &= & . & . & 5.0.0 & & 5.0.0 \end{aligned}$$

$$\text{Greenwich time of moon's rising} = \dots \dots 7^{\text{h}}11^{\text{m}}49^{\text{s}} \text{ Setg. } 23^{\text{h}}53^{\text{m}}39^{\text{s}}$$

Now, the moon's horizontal parallax reduced to the Greenwich approximate time of rising is $54'0''$; and her declination $24^{\circ}14'53''$ th; and the same elements reduced to the approximate Greenwich time of setting are $54'6''$, and $25^{\circ}33'40''$ respectively.

To find the correct Mean Time of the Moon's Rising.

$$-54'0'' + 33'0'' + 5'15'' = 89^{\circ}44'15''$$

$$\begin{aligned} \text{moon's north polar distance} &= 65.45.7 & \text{Log. co-secant} &= 0.040112 \\ \text{latitude of the ship} &= 38.31.20 & \text{Log. co-secant} &= 0.205639 \end{aligned}$$

$$D = \dots \dots \dots 194^{\circ}0'42''$$

$$\begin{aligned} \text{f sum} &= \dots \dots \dots 97^{\circ}0'21'' & \text{Log. sine} &= \dots \dots 9.996745 \\ \text{remainder} &= \dots \dots \dots 7.16.6 & \text{Log. sine} &= \dots \dots 9.102146 \end{aligned}$$

$$\text{Sum} = \dots \dots \dots 19.344642$$

$$h = \dots \dots \dots 61^{\circ}57'0'' \text{ Log. co-sine} = 9.672321$$

$$\begin{aligned} \text{semi-diurnal arc} &= \dots \dots 123^{\circ}54'0''; \text{ in time} = \dots \dots 8^{\text{h}}15^{\text{m}}36^{\text{s}} \\ \text{duration of transit in 24 hours is } 50''; \text{ the proportional part} \\ \text{of which answering to } 8^{\text{h}}15^{\text{m}}36^{\text{s}} \text{ is} &\dots \dots \dots +16^{\text{m}}38^{\text{s}} \end{aligned}$$

$$\begin{aligned} \text{correct semi-diurnal arc, in mean time} &= \dots \dots \dots 8^{\text{h}}32^{\text{m}}14^{\text{s}} \\ \text{mean time of moon's transit at given place} &= \dots \dots \dots 10.32.46 \end{aligned}$$

$$\text{correct mean time of the moon's rising, as required} = \dots \dots 2^{\text{h}}0^{\text{m}}32^{\text{s}}$$

To find the correct Mean Time of the Sun's Setting.

$$90^{\circ} - 54^{\circ} 6' + 33^{\circ} 0' + 5^{\circ} 15' = 89^{\circ} 44' 9''$$

$$\text{Moon's north polar distance} = 64.26.20 \quad \text{Log. co-secant} = 0.044733$$

$$\text{Co-latitude of the ship} = .38.31.20 \quad \text{Log. co-secant} = 0.205639$$

$$\text{Sum} = 192^{\circ} 41' 49''$$

$$\text{Half sum} = 96^{\circ} 20' 54\frac{1}{2}'' \quad \text{Log. sine} = . 9.997329$$

$$\text{Remainder} = 6.36.45\frac{1}{2}'' \quad \text{Log. sine} = . 9.061288$$

$$\text{Sum} = 19.308989$$

$$\text{Arch} = 63^{\circ} 10' 15'' \quad \text{Log. co-sine} = . 9.654494$$

$$\text{Semi-diurnal arc} = 126^{\circ} 20' 30'' ; \text{ in time} = . 8^{\text{h}} 25^{\text{m}} 22^{\text{s}}$$

$$\text{Variation of } D \text{'s transit in 24 hours is } 50'' ; \text{ the proportional part of which answering to } 8^{\text{h}} 25^{\text{m}} 22^{\text{s}} \text{ is} = + 16^{\text{s}} 57'$$

$$\text{Correct semi-diurnal arc, in mean time} = 8^{\text{h}} 42^{\text{m}} 19'$$

$$\text{Mean time of moon's transit at given place} = 10.32.46$$

$$\text{Correct mean time of the moon's setting, as required} = . 19^{\text{h}} 15^{\text{m}} 5'$$

Remark.—From the above Problem the manner of finding the mean times of the rising and setting of a planet will appear so obvious as not to require being elucidated by an example ;—and so will the method of finding the mean times of the rising and setting of a fixed star ; omitting, however, the parts which relate to horizontal parallax, and to the reduction of declination at the *approximate times* of rising and setting :—for the fixed stars have not any sensible parallax ; and their declinations may be esteemed as being invariable during the interval betwixt the times of rising and setting in any part of the world.

PROBLEM III.

Given the Latitude and Longitude of a Place, and the Day of the Month ; to find the Time of the Beginning and of the End of Twilight, and the Length of its Duration.

RULE.

Reduce the sun's declination, at the midnights preceding and following the noon of the given day, to the meridian of the given place, by Problem XIV., page 357, then,

Add together the constant quantity 108 degrees,* the sun's polar distance, and the co-latitude of the given place: take half the sum; the difference between which and the constant quantity call the *remainder*. Now,

To the logarithmic co-secants, less radius, of the polar distance, and the co-latitude, add the logarithmic sines of the half sum and of the remainder: half the sum of these four logarithms will be the logarithmic sine or logarithmic co-sine of an arch; which, being doubled, and converted into time, will be the apparent time of the beginning or of the end of twilight accordingly.

Compute the apparent times of the sun's rising and setting, by Problem I., page 593; then, the interval between the time of the commencement of twilight and that of sun rising, will be the duration of the morning twilight; and the interval between the time of sun setting and the end of twilight, will be the duration of the evening twilight.

Note.—If much accuracy be required, the sun's declination must be reduced to the meridian of the given place, at the respective times of the commencement and of the end of twilight, found as above; then, the operations being repeated, the correct apparent times of the beginning and of the end of twilight will be obtained. This degree of accuracy may, however, be dispensed with,—unless in cases of mere speculative inquiry, or where some philosophical subject is under consideration.

Example.

Required the apparent times of the beginning and of the end of twilight, and its duration, October 1st, 1824, in latitude $40^{\circ}30'$ north, and longitude 105° east?

* $90^{\circ} + 18^{\circ} = 108^{\circ}$.—See Remarks at the end of the Example.

To find the Beginning of Twilight.

Sun's declination at midnt. September 30th = $3^{\circ} 4' 26''$ S.Reduction of ditto for longitude 105° E. = -6.48 Sun's reduced declination = $2^{\circ} 57' 38''$ S.Constant quantity = $108^{\circ} 0' 0''$ Sun's polar distance = $92.57.38$ Log. co-secant = 0.000580 Co-latitude = $49.30.0$ Log. co-secant = 0.118954 Sum = $250^{\circ} 27' 38''$ Half sum = $125^{\circ} 13' 49''$ Log. sine = 9.912137 Remainder = $17.13.49$ Log. sine = 9.471604 19.503741 Arch = $34^{\circ} 21' 54\frac{1}{2}''$ Log. sine = $9.751637\frac{1}{2}$ Beginning of twilight = $68^{\circ} 43' 40''$, in time = $4^h 34^m 55^s$ Apparent time of sun-rising on the given day = $6.5.8$ Duration of morning twilight = $1^h 30^m 13^s$

To find the End of Twilight.

Sun's declination at midnight, October 1st = $3^{\circ} 27' 45''$ S.Reduction of ditto for longitude 105° E. = -6.48 Sun's reduced declination = $3^{\circ} 20' 57''$ S.Constant quantity = $108^{\circ} 0' 0''$ Sun's polar distance = $93.20.57$ Log. co-secant = 0.000743 Co-latitude = $49.30.0$ Log. co-secant = 0.118954 Sum = $250^{\circ} 50' 57''$ Half sum = $125^{\circ} 25' 28\frac{1}{2}''$ Log. sine = 9.911093 Remainder = $17.25.28\frac{1}{2}$ Log. sine = 9.476324 19.507417 Arch = $55^{\circ} 27' 40''$ Log. co-sine = 9.753557

Arch =	55°27'40"	
End of twilight =	110°55'20", in time = 7 ^h 23 ^m 41 ^s
Apparent time of sun-setting on the given day =		5.52.13
Duration of evening twilight		1 ^h 31 ^m 28 ^s

Remarks.

Twilight, technically called the *crepusculum*, is that faint light which we perceive before the sun rises and after he sets. It is produced by the rays of light being refracted in their passage through the earth's atmosphere, and reflected from the different particles thereof.

The morning twilight commences when the sun wants 18 degrees of appearing in the horizon of the eastern hemisphere, and the evening twilight ends when he is depressed 18 degrees below the horizon of the western hemisphere.

When the sun's declination exceeds the difference between the co-latitude of any given place and 18 degrees, there will be no *real darkness* or night at that place, but continual day and twilight; as is the case at London, from the 22nd of May to the 21st of July.

When the sun is on the same side of the equinoctial with the elevated pole, the duration of twilight will constantly increase as he approaches that pole, till he enters the tropic; at which time the duration of twilight will be the longest. It will then decrease until some time after the sun passes the equinox, but will increase again before he arrives at the opposite tropic: hence, there must be a point within the tropics where the duration of twilight is the shortest. This point may be found by the following Problem.

PROBLEM IV.

Given the Latitude of a Place; to find the Time of the shortest Twilight, and its duration.

RULE.

To the logarithmic tangent of the half of 18 degrees, add the logarithmic sine of the latitude; and the sum (abating 10 in the index) will be the logarithmic sine of the sun's declination at the time of the shortest twilight, of a *contrary name to the latitude*: the day corresponding to this declination will be that required.

Again, to the logarithmic sine of the half of 18 degrees, add the logarithmic secant of the latitude; and the sum (abating 10 in the index) will be the logarithmic sine of an arch, which, being doubled and converted into time, will be the duration of the shortest twilight.

Example.

Required the time of the shortest twilight, and its duration, in the year 1824, in latitude $50^{\circ}48'$ N.?

Half of 18 degrees = $9^{\circ} 0' 0''$	Log. tangent = 9.199713
Latitude of the place = $50.48.0$	Log. sine = 9.889271

Sun's declination = $7^{\circ} 3' 1''$	Log. sine = 9.088984;
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which is *south*, of a contrary name to the latitude.

Half of 18 degrees = $9^{\circ} 0' 0''$	Log. sine = 9.194332
Latitude of the place = $50.48.0$	Log. secant = 10.199263

Arch = $14^{\circ}19'49''$	Log. sine = 9.393595
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Duration of twilight = $28^{\circ}39'38''$, in time = $1^{\text{h}}54^{\text{m}}39^{\text{s}}$.

The days in the Nautical Almanac, corresponding to the sun's declination $7^{\circ}3'1''$ S., are March 2nd and October 11th, which, therefore, are the days of the shortest twilight in the year 1824, in latitude $50^{\circ}48'$ north; and the duration of the twilight, on those days, is $1^{\text{h}}54^{\text{m}}39^{\text{s}}$.

PROBLEM V.

Given the Latitude of a Place between $48^{\circ}32'$ and $66^{\circ}32'$ (the Limits of regular Twilight); to find when real Night or Darkness ceases, and when it commences.

RULE.

The complement of the latitude, diminished by 18 degrees, will be the declination of the sun, of the *same name as the latitude*, at the time when it ceases to be real night, and also when real night commences.

Example.

Required the interval of time, in the year 1824, during which there will be no real darkness or night, in latitude $50^{\circ}48'$ north?

Solution.—The complement of the latitude $39^{\circ}12'N.$ — $18^{\circ}=21^{\circ}12'N.$ = the sun's declination. Now, the days answering to $21^{\circ}12'$ of north declination are, May 26th and July 17th. Upon the first of these days, therefore, real night ceases, and it commences upon the last. During this interval there is no real darkness, because the sun is less than 18 degrees below the horizon; and so on for any other latitude within the limits.

PROBLEM VI.

Given the Sun's Declination and Semidiameter; to find the Interval between the Instants of his Lower and Upper Limbs being in the Horizon of a known Place.

RULE.

Find the approximate time of the sun's rising or setting, by Problem I., page 124; to which time let the sun's declination be reduced, by Problem XIV., page 357.

To the logarithm of the sun's semidiameter, expressed in seconds, add the constant logarithm 9.124939, and call the sum a *reserved logarithm*; then,

To the logarithmic co-sine of the sum of the latitude and declination, add the logarithmic co-sine of their difference: half the sum of these two logarithms, being subtracted from the *reserved logarithm*, will leave the logarithm of the interval of time, in seconds, between the instants of the sun's lower and upper limbs being in the horizon of the given place.

◆ *Example.*

Required the interval between the instants of the sun's lower and upper limbs being in the horizon, at the time of its setting, July 13th, 1824, in latitude $50^{\circ}48'N.$, and longitude $120^{\circ}W.$?

Apparent time of setting in Table L., to latitude $50^{\circ}48'N.$,

and declination $21^{\circ}49'51''N.$ = $7^h57^m12^s$

Longitude 120° west, in time = $8. 0. 0$

Greenwich time of sun's setting = $15^h57^m12^s$

Sun's declination at noon, July 13th, 1824 = . . . $21^{\circ}49'51''N.$

Correction of ditto for $15^h57^m12^s$ = $- 5.58$

Sun's reduced declination = $21^{\circ}43'53''N.$

Sun's semidiameter $15'.45''.8 = 945''.8$ Log. = 2.975799

Constant logarithm = 9.124939

Reserved logarithm = 12.100738 . . 12:100738

Sun's red. dec. = $21^{\circ}43'53''$ N.

Lat. of the place = $50.48.0$ N.

Sum = . . . $72^{\circ}31'53''$ Log. co-sine = 9.477387

Difference = . $29.4.7$ Log. co-sine = 9.941531

Sum = 19.418918

Half sum = 9.709459 . . 9.709459

Interval, in seconds = $246.195 =$ Log. = 2.391279

Hence, the interval between the instants of the sun's limbs touching the horizon, is 4 minutes and 6 seconds.

Note.—The constant logarithm made use of in this Problem is the arithmetical complement of the proportional logarithm of 24 hours esteemed as minutes. If the sun's diameter be used, instead of its semidiameter, it must be expressed in minutes and decimal parts of a minute: in this case, the same result will be obtained by employing the constant logarithm 8.823909; viz., the arithmetical complement of the common logarithm of 15 degrees, or the motion corresponding to one hour of time.

SOLUTION OF PROBLEMS IN GNOMONICS OR DIALLING.

Dialling, or *Gnomonics*, is a branch of mixed mathematics, which depends partly on the principles of geometry and partly on those of astronomy; and it may be defined as being the method of projecting on the surface of any given body, whether plane or otherwise, a figure called a *sun-dial*,—the different lines of which indicate, by the shadow of a style or gnomon, when the sun shines thereon, the apparent time of the day.

The upper edge of the *style* or *gnomon*, which projects the sun's shadow on the plane of the dial, must be parallel to the earth's axis: hence, it is sometimes called the *axis of the dial*.

The plane of the *gnomon* must be perpendicular to that of the dial. The plane on which it is erected is called the *sub-style*: in horizontal dials it may be called the meridian, or 12 o'clock line.

The angle comprehended between the *style* and the *sub-style*, is called the *elevation of the style*: this angle, in horizontal dials, is always equal to the elevation of the pole, or the latitude of the place for which it is computed; but, in erect direct north or south dials, it is equal to the complement of the latitude of such place.

Those dials whose planes are parallel to the plane of the horizon, are called *horizontal dials*; but such as have their planes perpendicular to the plane of the horizon, are called *vertical* or *erect* dials.

Those vertical dials whose planes are either parallel or perpendicular to the plane of the meridian, are called *direct erect dials*. One of these must always face one of the cardinal points of the horizon, according as it may be a north, south, east, or west, *erect dial*.

All other erect dials are called *declining dials*. Those dials whose planes are neither parallel nor perpendicular to the plane of the horizon, are called *reclining dials*.

In this place, however, we shall only show the method of constructing a horizontal dial, and, also, that of a north or south erect direct dial; these being by far the most useful, and, indeed, the most common of all the varieties in dialling.

PROBLEM I.

Given the Latitude of a Place; to find the Angles which the Hour Lines make with the Sub-Style or Meridian Line of a Horizontal Sun-Dial.

GENERAL PROPOSITION.

In every right angled spherical triangle, ~~radius is to the sine of one of the legs containing the right angle, as the tangent of the angle adjacent to that side is to the tangent of the other containing side of the triangle.~~ This is merely a variation of the equation for finding the leg B C, in Problem IV., page 189; hence, the following

RULE.

To the logarithmic sine of the latitude, add the logarithmic tangent of the sun's horary angle from noon; and the sum (abating 10 in the index), will be the logarithmic tangent of the angle comprehended between the corresponding hour line and the sub-style, at the centre of the dial.

Note.—Since the sun's apparent motion in the ecliptic is at the rate of 15 degrees to an hour, therefore, at one hour from noon the sun's horary angle is 15° ; at two hours from noon it is 30° ; and so on.

Example.

Required the angles which the hour lines make with the sub-style, or meridian line of a horizontal dial, in a place situated in $50^\circ 48' 15''$ north latitude:

To find the Angle at one Hour from Noon.

Latitude of the place = . . . $50^\circ 48' 15''$ Log. sine = 9.889296
 Sun's horary ang. at 1^h from noon = 15° . 0. 0 Log. tangent = 9.428053

Hour line of 1, or 11 o'clock = $11^\circ 43' 52''$ Log. tangent = 9.317349

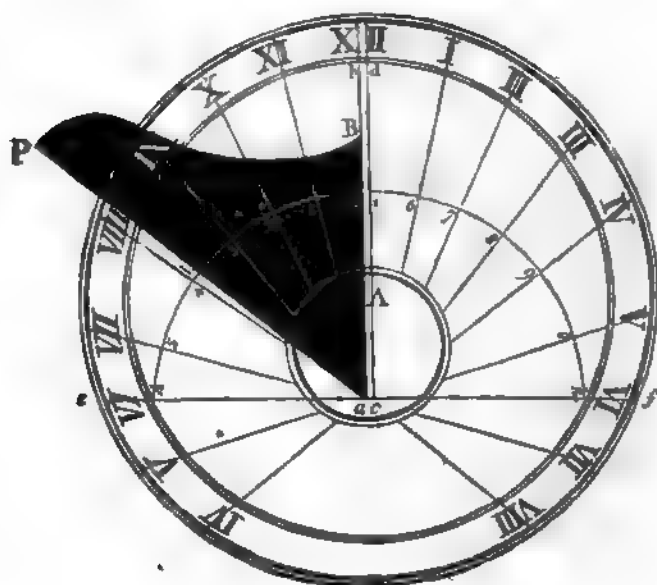
To find the Angle at two Hours from Noon.

Latitude of the place = . . . $50^\circ 48' 15''$ Log. sine = 9.889296
 Sun's horary ang. at 2^h from noon = 30° . 0. 0 Log. tangent = 9.761439

Hour line of 2, or 10 o'clock = . $24^\circ 6' 20''$ Log. tangent = 9.650735

Proceeding in this manner, the several angles which the respective hour lines make with the meridian will be found to be as follows; viz.,

Hour lines of	I. and	XI.	=	$11^\circ 43' 52''$
Ditto	II. and	X.	=	$24. 6. 20$
Ditto	III. and	IX.	=	$37. 46. 31$
Ditto	IV. and	VIII.	=	$53. 18. 53$
Ditto	V. and	VII.	=	$70. 55. 39$
Ditto	VI. and	VI.	=	$90. 0. 0$



The hour lines of VII. in the evening and V. in the morning, make the same angles with the meridian, on *the opposite side of the VI. o'clock hour line*, as the hour lines of VII. in the morning and V. in the evening. In the same manner the hour lines of VIII. in the evening and IV. in the morning make the same angles with the meridian as the hour lines of VIII. in the forenoon and IV. in the afternoon; and so on.

The angles for the halves, quarters, or other subdivisions of the hours, are to be determined in the above manner.

The angles which the different hour lines, &c. make with the meridian, being thus determined, the dial may then be very readily constructed, by means of a pair of compasses, and the line of chords on a common Gunter's scale, or of that on a *Sector*: the latter, however, should be preferred, because the degrees thereon are generally divided into halves, and sometimes quarters, which gives it a decided advantage, in point of accuracy, over that on Gunter's scale.

CONSTRUCTION.

On the proposed plane draw the meridian, or XII. o'clock hour line, *a b*; parallel to which, at a distance equal to the intended thickness of the gnomon or style, draw the line *c d*: perpendicularly to these draw the VI. o'clock hour line *e f*. Open the Sector to any convenient extent, and take the transverse distance 60° to 60° (on the line of chords) as a radius in the compasses, and, from *a* as a centre, describe the arc *g h*: with the same radius, and from *c* as a centre, describe the arc *i k*; and, since the hour lines are less distant from each other about noon than in any other part of the day, it is advisable to have the centres of those quadrants or arcs at a little distance from the centre of the plane of the dial, on the side opposite to XII., so as to allow of the hour distances being enlarged near the meridian under the same angles in the plane of the dial: thus, the centre of the plane is at *A*; but the centres of the quadrants or arcs are taken a little below it, at the points *a* and *c*.

Take the transverse distance $11^\circ 43' 52''$ to $11^\circ 43' 52''$, in the compasses, from the line of chords, and set it off from *g* to 1, and, also, from *i* to 6: take the transverse distance $24^\circ 6' 20''$, in the compasses, and set it off from *g* to 2, and from *i* to 7; and proceed in the same manner with the remaining horary angles.

Now, from the centre *a* draw the forenoon hour lines *a 1 XI.*, *a 2 X.*, *a 3 IX.*, *a 4 VIII.*, *a 5 VII.*; and, from *c* as a centre, draw the afternoon hour lines *c 6 I.*, *c 7 II.*, *c 8 III.*, *c 9 IV.*, *c 0 V.*: produce *a 5 VII.* and *a 4 VIII.* for the hour lines of VII. and VIII. o'clock in the

evening; and produce $c\ 9\ IV.$ and $c\ 0\ V.$ for the hour lines of IV. and V. in the morning. In the same manner may the quarter and half-hour lines be drawn (and minutes if necessary), by setting off the computed corresponding angles from the meridian: these, however, have been omitted in the above diagram, with the view of preventing embarrassment.

Take the latitude $50^{\circ}48'15''$ in the compasses, viz., the transverse distance $50^{\circ}48'15''$ to $50^{\circ}48'15''$, and set it off from g to L , and draw the hypothenuse line $a\ L\ P$ for the axis of the style or gnomon.

The style may have any shape the artist pleases, provided its edge $a\ L\ P$ be a perfectly straight line. It should be a metallic substance, and must be of an equal thickness with the breadth of the space comprehended between the two parallel straight lines $a\ b$ and $c\ d$; in which space it must be erected truly perpendicular to the plane of the dial: then, since the angle $B\ a\ P$ is equal to the latitude, the straight edge of the style $= a\ L\ P$ will be directed to the elevated pole of the world, and, hence, parallel to the earth's axis when the dial is truly set; the shadow of which, when the sun shines, will indicate the hour of the day.

Note.—Since the hour of the day indicated by a sun-dial is expressed in apparent solar time, it must be reduced to *mean time*, by Problem XIX., page 369, so as to make it correspond with that shown by a well-regulated watch or clock.

PROBLEM II.

To find the Angles on the Plane of an erect direct south Dial for any proposed north Latitude, or on that of an erect direct north Dial for any proposed south Latitude.

RULE.

To the logarithmic co-sine of the latitude, add the logarithmic tangent of the sun's horary angle from noon; and the sum (abating 10 in the index), will be the logarithmic tangent of the angle comprehended between the corresponding hour line and the sub-style, at the centre of the dial.

Example.

Required the angles which the hour lines on an erect direct south dial make with the sub-style or 12 o'clock line, in latitude $50^{\circ}48'15''$ north?

To find the Angle at one Hour from Noon.

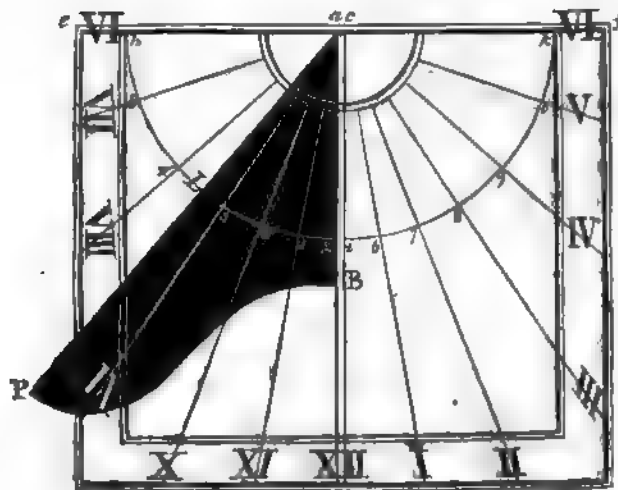
Latitude of the place = . . . $50^{\circ}48'15''$ Log. co-sine = 9.800699
 Sun's horary ang. at 1^h from noon = 15. 0. 0 Log. tangent = 9.428053
 Sum angle of 1, or 11 o'clock = $9^{\circ}36'40''$ Log. tangent = 9.928752

To find the Angle at two Hours from Noon.

Latitude of the place = . . . $50^{\circ}48'15''$ Log. co-sine = 9.800699
 Sun's horary ang. at 2^h from noon = 30. 0. 0 Log. tangent = 9.761429
 Sum angle of 2, or 10 o'clock = $20^{\circ}2'44''$ Log. tangent = 9.562138

Proceeding in this manner, the several angles which the respective hour lines make with the meridian will be found to be as follows; viz.,

Hour lines of	I.	and	XI.	=	$9^{\circ}36'40''$
Ditto	II.	and	X.	=	$20. 2. 44$
Ditto	III.	and	IX.	=	$32. 17. 30$
Ditto	IV.	and	VIII.	=	$47. 35. 10$
Ditto	V.	and	VII.	=	$67. 1. 25$
Ditto	VI.	and	VI.	=	$90. 0. 0$



CONSTRUCTION.

On the proposed plane draw the XII. o'clock hour line ab ; parallel to which, at a distance equal to the intended thickness of the style, draw the line cd : at right angles to the sub-style, or XII. o'clock line, draw the VI. o'clock hour line ef . Open the sector to any convenient extent, and take the transverse distance 60° to 60° (on the line of chords) as a radius in the compasses, and, from a as a centre, describe the arc gh ; with the same radius, and from c as a centre, describe the arc ik . Take the transverse distance $9^\circ 36' 40''$ to $9^\circ 36' 40''$ in the compasses, and set it off from g to 1, and, also, from i to 6. Take the transverse distance $20^\circ 2' 44''$ to $20^\circ 2' 44''$ in the compasses, and set it off from g to 2, and from i to 7; and proceed in the same manner with the remaining horary angles. Then, from the centre a , draw the forenoon hour lines a 1 XI., a 2 X., &c. &c.; and, from c as a centre, draw the afternoon hour lines c 6 I., c 7 II., &c. &c.

Take the complement of the latitude in the compasses, viz., the transverse distance $39^\circ 11' 45''$ to $39^\circ 11' 45''$; set it off from g to L, and draw the hypotenuse line a L P for the axis of the style or gnomon.

Now, when the dial is placed vertically, with its plane duly facing the south, the VI. o'clock hour line ef will be parallel to the plane of the horizon; and the style B a L P, directed downwards, making an angle with the sub-style or XII. o'clock hour line equal to the complement of the latitude, will be truly parallel to the earth's axis.

Since the sun cannot shine any longer on a dial of this description than from VI. in the morning until VI. in the evening, it is not necessary to describe hour lines upon it before or after those periods of time.

Note.—An erect direct north dial for a place in north latitude, is constructed exactly in the same manner as an erect direct south dial; but the position of the dial must be reversed: that is, the VI. o'clock hour line must be at the bottom instead of the top of the dial; and the style or gnomon must be directed upwards instead of downwards.

SOLUTION OF PROBLEMS RELATIVE TO THE MENSURATION OF HEIGHTS AND DISTANCES.

Since it is frequently of the greatest importance to the mariner, but *at all times to the engineer or other military officer*, to be able to ascertain the heights and distances of remote objects with precision, the following

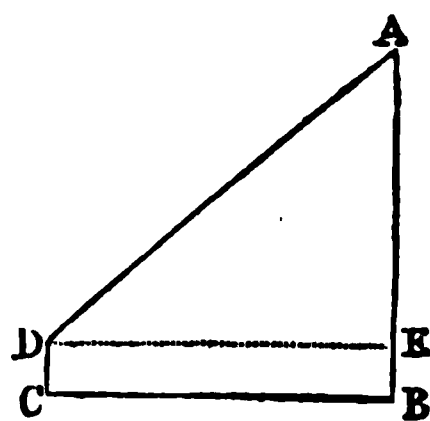
problems are given for their general guidance in such cases. In solving these problems, it is the logarithmical mode of calculation that will be attended to, with the view of showing the direct application of the principles of plane trigonometry to such cases. To the imagination of the ingenious, however, many other modes of obtaining an approximate value for the heights and distances of remote objects will soon present themselves: such as, by means of shadows, mirrors, unequal vertical staves, &c. &c.; but, since these methods entirely depend upon the principles of similar triangles (as demonstrated in Euclid, Book VI., Prop. 4), they admit of direct solutions without the assistance of trigonometrical tables: hence, no notice can be taken of them in this work.

PROBLEM I.

To find the Height of an accessible Object.

RULE.

Let A B, in the annexed diagram, be the object: from B measure any convenient distance to C; take, at C, with a quadrant or other instrument, the angle A D E; then, in the triangle A D E, given the side D E = B C, and the angle at D; to find the side A E: to which let the height of the observer's eye above the horizontal plane = C D or B E be added, and the sum will be the true height of the object A B.



Example.

Let the horizontal distance B C be 250 feet, the angle of elevation A D E = $41^{\circ}45'$, and the height of the eye C D = 5 feet; required the height of the object A B?

This comes under Problem II. of right angled plane trigonometry, page 172; and by making D E radius, it will be

As radius = 90°	Log. co-secant =	10.000000
Is to the distance D E = C B =	250 feet	Log. =	. . . 2.397940
So is the angle of elevation A D E =	$41^{\circ}45'$	Log. tangent =	. 9.950625

To the part A E =	. . . 223.13	Log. =	. . . 2.348565
Height of the eye C D =	. . . 5.		

Height of the object A B = . 228.13 feet, as required.

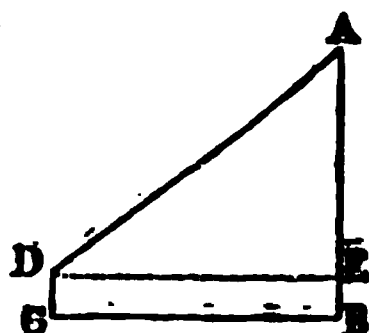
Remark.—By removing either towards or from the object, until the quadrant shows the angle of altitude to be 45 degrees, the measure of the distance between the foot of the observer and that of the object, augmented by the height of the eye, will become the altitude or height of that object.

PROBLEM II.

Given the Angle of Elevation, and the Height of an Object ; to find the Observer's horizontal Distance from that Object.

RULE.

At any convenient distance, as at C, let the angle of elevation A D E be taken ; then, in the triangle A D E, given A E = the height of the object A B, diminished by the height of the eye C D, or its equal B E, and the angle at D ; to find the horizontal distance D E = C B.



Example.

Let the height of the object A B be 175 feet, the angle of elevation A D E $37^{\circ}20'$, and the height of the observer's eye C D = 5 feet; required the horizontal distance B C ?

This falls under Problem II., of right angled plane trigonometry, page 172 ; and by making A B radius, the proportion will be

As radius = 90° Log. co-secant = 10.00000
 Is to height of the object AB 175 ft.—BE 5 ft.=170 Log.= 2.23049
 So is the angle of elevation ADE= $37^{\circ}20'$ Log. co-tangent = 10.117637

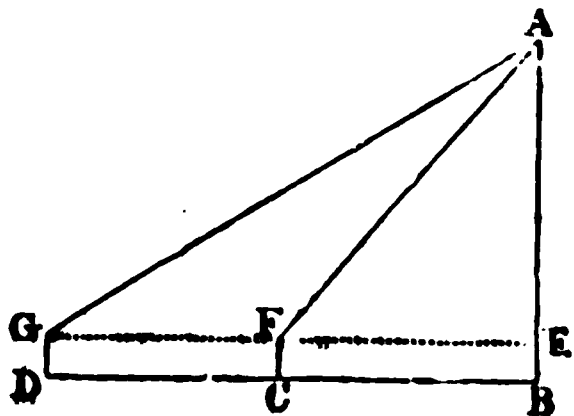
To the horizon. dist. D E = C B = 222.89 Log. = . . . 2.348088

PROBLEM III.

To find the Height of an Inaccessible Object, as A B.

RULE.

At any convenient points, as C and D (these being in the same vertical plane with A B), observe the angles of elevation A F E and A G E; and measure the distance C D: then because the exterior angle A F E is equal to the two interior and opposite angles A G F and G A F (Euclid, Book I., Prop. 32), if from the angle A F E the angle A G E be subtracted, the remainder will be the angle G A F. Now, in the oblique angled triangle A G F, given the side G F = D C, and the angles A and G; to find the side A F: and, in the right angled triangle A F E, given the hypotenuse A F, found as above, and the angle A F E; to find the perpendicular A E: to which let the height of the observer's eye above the horizontal plane be added, and the sum will be the height of the object A B.



Example.

In the above diagram let the angle of elevation at C = A F E be $49^{\circ}28'$, and, after receding 200 feet in the same vertical plane, to the point D, let the angle of elevation A G E be $31^{\circ}20'$; now, admitting the height of the observer's eye above the horizontal plane = D G or B E to be 5 feet, it is required to determine the height of the object A B?

The angle A F E $49^{\circ}28'$ — the angle A G F $31^{\circ}20'$ = the angle G A F $18^{\circ}8'$.

Now, in the oblique angled triangle A G F, since the angles and one side are given, the side A F is found by oblique angled plane trigonometry, Problem I., page 177; and, in the right angled triangle A E F, since the hypotenuse A F is now known, and the angle at F given, the perpendicular A E is found by right-angled plane trigonometry, Problem I., page 171. Hence,

To find the Side A F.

As the angle G A F = . . . $18^{\circ} 8'$	Log. co-secant = 10.506919
Is to the side G F = D C = . 200	Log. = . . . 2.301030
So is the angle A G F = . . . $31^{\circ}20'$	Log. sine = . . . 9.716017
<hr/>	
To the side A F = . . . 834.17	Log. = . . . 2.523966

To find the Perpendicular A E.

As radius = 90° Log. co-secant = 10.00000
 Is to the hypotenuse A F = . 334.17 Log. = 2.52396
 So is the angle A F E = $49^\circ 28'$ Log. sine = 9.880830

To the perpendicular A E = . . . 253.98 Log. = 2.404796
 Height of the eye B E = 5.

Height of the object A B = . . . 258.98 feet, as required.

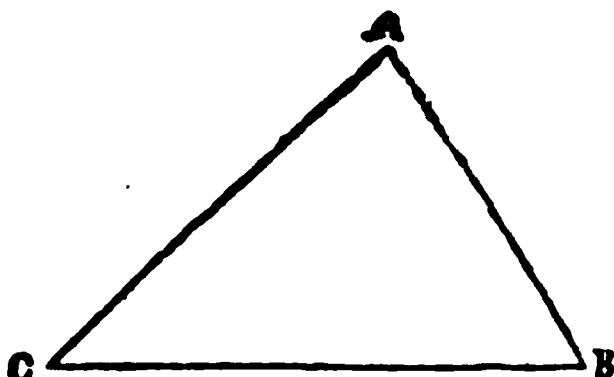
Remark.—If it be required to know the horizontal distance B C or B D, it may be readily determined by means of the last Problem.

PROBLEM IV.

To find the Distance of an Inaccessible Object which the Observer can neither advance towards nor recede from in its vertical Line of Direction.

RULE.

Let the point A be any inaccessible object, and B and C two stations from which the distance of that object is to be determined: measure the distance B C, and, with a sextant or other instrument, observe the horizontal angles A B C and A C B; then, in the triangle A B C, the angles and the side B C are given; to find the other two sides, viz., A B and A C.



Example.

Let the horizontal angle A B C, measured with a sextant, be $59^\circ 15'$, the angle A C B $42^\circ 45'$, and the measured base line B C 350 yards; required the respective distances A B and A C?

The angle A B C $59^\circ 15'$ + the angle A C B $42^\circ 45'$ = 102° ; and
 $180^\circ - 102^\circ = 78^\circ$, the angle C A B.

Now, the angles and one side being thus known, the remaining sides are to be determined by oblique-angled trigonometry, Problem I., page 177. Hence the following proportions:—

To find the Distance A C

As the angle C A B =	. . . 78°	Log. co-secant=	10.009596
Is to the side B C =	. . . 350	Log. =	. . . 2.544068
So is the angle A B C =	. . . 59°15'	Log. sine =	. . . 9.934199
<hr/>			
To the distance A C =	. . . 307.51	Log. =	. . . 2.487863

To find the Distance A B.

As the angle C A B =	. . . 78°	Log. co-secant=	10.009596
Is to the side B C =	. . . 350	Log. =	. . . 2.544068
So is the angle A C B =	. . . 42°45'	Log. sine =	. . . 9.831742
<hr/>			
To the distance A B =	. . . 242.89	Log. =	. . . 2.385406

Remark.—This problem will be found of very essential service to Her Majesty's ships and vessels of war, on many hostile occasions: for, when it is intended that a squadron of ships should cannonade a fort to effect, or batter a breach in the sea-defences of a town, the distance at which the ships should be placed, abreast of such fort or town, with the view of opening their fire to the greatest advantage, may be readily determined in the above manner. Thus, let two competent persons, provided with sextants, in two ships, observe the angles subtended between the fort and each ship respectively; and let the distance between the two ships be carefully ascertained, which is readily done by Problem II., page 614, provided the height of the masts be known; or it may be found by means of a boat sent from one ship to the other, with instructions to pull at an uniform rate: then, if the interval, per watch, be noted between the time of the boat's pulling off from one ship and that of her arrival at the other, and her velocity or hourly rate of sailing be duly determined by the log, the distance between those ships may be easily obtained by the rule of proportion.

Now, with the distance between the two ships as a base line, thus found, and the angles subtended between the fort and each ship, the respective distances of those ships from the fort may be very readily computed, agreeably to the principles of the present Problem.

Note.—The most convenient distance for commencing a cannonade, is about 300 yards; that is, about a cable and a quarter's length from the object at which the guns are directed. On such occasions, however, the captains of Her Majesty's ships of war always make choice of a much closer position, provided there be a sufficient depth of water.

This problem is also extremely useful in military movements: because, when a general is determined on the reduction of a town or garrison, his engineer is thus enabled to apprise him of his absolute distance from any point of the enemy's defences against which he may be desirous of commencing operations, and of the most advantageous position for throwing up batteries which may produce the greatest possible effect on the fortified works of the besieged.

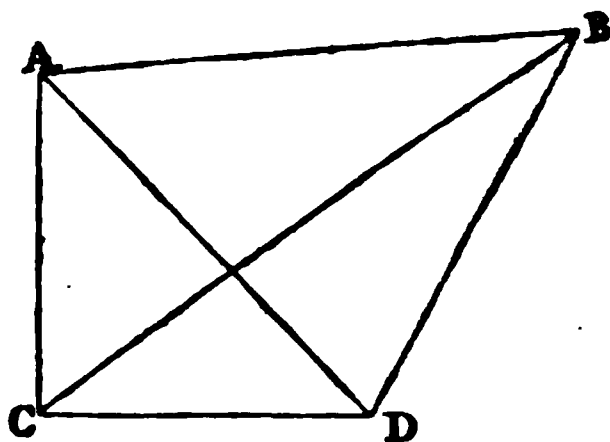
In military operations, the battering guns are generally placed at about 375 paces ($312\frac{1}{2}$ yards) from the works intended to be breached. —A military pace is reckoned at 30 inches.

PROBLEM V.

To find the Distance between two Inaccessible Objects.

RULE.

Let A and B be any two inaccessible objects, the distance between which is required. Measure any base line, as C D; at the point C observe the angles A C B, B C D; and, at the point D, observe the angles B D A, A D C. Now, in the triangle A C D, in which the angles and the side C D are given, compute the side A D, by oblique-angled trigonometry, Problem I., page 177. In like manner, in the triangle B C D, where the angles and the side C D are given, compute the side B D by the above-mentioned problem. Now, in the triangle A B D, the sides A D and B D, and the included angle A D B, are given; with which the distance A B is to be computed, by oblique angled trigonometry, Problem III., page 179.



Example.

Wanting to know the distance between the two inaccessible objects A and B, in the above diagram, I measured a base line C D of 360 yards: at C, the horizontal angle A C B was observed with a sextant, and found to be $53^{\circ}30'$, and the angle B C D $38^{\circ}45'$; at D, the horizontal angle B D A was $67^{\circ}20'$, and the angle A D C $44^{\circ}30'$; required the distance between A and B?

Angle A C B $53^{\circ}30'$ + angle B C D $38^{\circ}45'$ = angle A C D $92^{\circ}15'$; and angle A C D $92^{\circ}15'$ + angle A D C $44^{\circ}30'$ = $136^{\circ}45'$. Now, $180^{\circ} - 136^{\circ}45' =$ the angle C A D $43^{\circ}15'$.

Again: angle B D A $67^{\circ}20'$ + angle A D C $44^{\circ}30'$ = angle B D C $111^{\circ}50'$; and angle B D C $111^{\circ}50'$ + B C D $38^{\circ}45'$ = $150^{\circ}35'$.
Now, $180^{\circ} - 150^{\circ}35' =$ the angle C B D $29^{\circ}25'$.

In the Triangle A C D, to find the Side A D.

As the angle C A D = . . .	$43^{\circ}15'$	Log. co-secant =	10.164193
Is to the side C D = . . .	360	Log. = . . .	2.556303
So is the angle A C D = . . .	$92^{\circ}15'$	Log. sine = . . .	9.999665
<hr/>			
To the side A D = . . .	525.0	Log. = . . .	2.720161

In the Triangle B C D, to find the Side B D.

As the angle C B D = . . .	$29^{\circ}25'$	Log. co-secant =	10.308779
Is to the side C D = . . .	360	Log. = . . .	2.556303
So is the angle B C D = . . .	$38^{\circ}45'$	Log. sine = . . .	9.796521
<hr/>			
To the side B D = . . .	458.78	Log. = . . .	2.661603

In the Triangle A B D, to find the Angle D A B or D B A, and the Side A B.

$180^{\circ} -$ the angle B D A $67^{\circ}20' = 112^{\circ}40' \div 2 = 56^{\circ}20' =$ half the sum of the angles D B A and D A B.

As the sum of the sides AD and DB =	983.78	Log. ar. comp. =	7.007102
Is to their difference = . . .	66.28	Log. = . . .	1.821383
So is $\frac{1}{2}$ sum of angles DBA and DAB =	$56^{\circ}20'$	Log. tangent =	10.176476
<hr/>			

To half difference of ditto = . . .	$5^{\circ}46'33''$	Log. tang. =	9.004961
<hr/>			

Angle D B A = . . .	$62^{\circ}6'33''$
---------------------	--------------------

Angle D A B = . . .	$50.33.27$
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To find the Distance A B.

As the angle D A B = . . .	$50^{\circ}33'27''$	Log. co-secant =	10.112218
Is to the side B D = . . .	458.78	Log. = . . .	2.661603
So is the angle A D B = . . .	$67^{\circ}20'0''$	Log. sine = . . .	9.965090
<hr/>			

To the side A B = . . .	548.16	Log. = . . .	2.738911
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which, therefore, is the required distance.

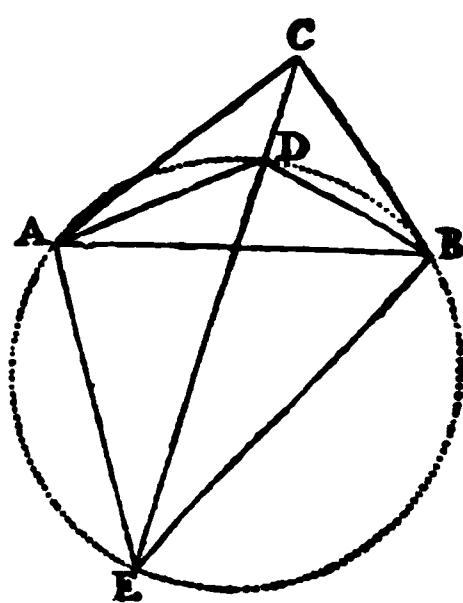
Note.—This problem is very useful in taking surveys of coasts, harbours, bays, islands, &c.

PROBLEM VI.

Given the Distances between three Objects, and the Angular Distances between these Objects taken at any Point in the same horizontal Plane; to find the Distance between that Point and each of the Objects.

RULE.

Let A , B , and C , be any three objects whose distances from each other are given, and E the place of the observer: at E , observe the angles CEA and CEB ; connect the points A , B , and C , by right lines; make the angle ABD equal to the observed angle CEA , and make the angle DAB equal to the angle CEB : hence the point D is found; then, through the three points A , D , and B , describe the circle $ADBE$; join CD , and produce this line till it meet the circle at the point E , the place of the observer; join EA and EB .



Now, in the triangle ABC , of which the three sides are given, find the angle BAC . In the triangle ABD , in which the angles and the side AB are known, find the side AD . In the triangle ACD , of which two sides, AC and AD , and the included angle CAD are known, find the angle ACD . In the triangle AEC , of which the angles and the side AC are given, find the sides EA and EC . And in the triangle ABE , the sides AB , EA , and the angles AEB and EAB are given; to find the side EB .

Example.

Let the points A , B , and C , in the above diagram, be three known objects: the distance between A and B , 290 yards; between B and C , 195 yards; and between A and C , 240 yards: let E be the place of an observer, where the angle CEA was measured with a sextant and found to be $30^{\circ}5'$, and the angle CEB $25^{\circ}45'$; required the distances EA , EC , and ED ?

In the triangle ABC , the three sides are given; to find the angle BAC . Hence, by oblique angled plane trigonometry, Problem IV., page 180,

Side B C (opposite the required angle) = 195
 Side A C (containing the required angle) = 240 Log. ar. co. = 7.619789
 Side A B (containing the required angle) = 290 Log. ar. co. = 7.537602

Sum = 725

Half sum = 362.5 Log. = . 2.559308

Remainder = 167.5 Log. = . 2.224015

Sum = 19.940714

Arch = 20°55'46" Log. co-sine = 9.970357

Angle B A C = 41°51'32"

Angle B A D = the angle C E B = 25.45. 0

Angle D A C = 16° 6'32"

In the triangle A B D, the angles and the side A B are given ; to find the side A D : thus, the angle A B D (= the angle C E A) = 30°5' + the angle D A B (= the angle C E B) = 25°45' = the angle A E B 55°50' ; and 180° - 55°50' = the angle A D B = 124°10' : for, the angle A D B is evidently the supplement of the angle A E B ; because the opposite angles of every quadrilateral figure described in a circle are equal to two right angles.—Euclid, Book III., Prop. 22. Hence, by trigonometry,

As the angle A D B = 124°10' Log. co-secant = 10.082281

Is to the side A B = 290 Log. = 2.462398

So is the angle A B D = 30° 5' Log. sine = . 9.700062

To the side A D = . 175.69 Log. = 2.244741

In the triangle A D C, the two sides A C, A D, and the included angle D A C, are given ; to find the angle A C D : hence, by oblique angled trigonometry, Problem III. page 179,

As side A C 240 + side A D 175.69 = 415.69 Log. ar. co. 7.381230

Is to side A C 240 - side A D 175.69 = 64.31 Log. = . 1.808279

So is 180° - angle D A C 16°6'32" } 81°56'44" Log. tang. 10.849213
 = 163°53'28" ÷ 2 =

To half diff. of angles A D C and A C D = 47° 35' 5" Log. tang. 10.038722

Angle A C D = 34°23'41"

In the triangle A E C, the angles and the side A C are given ; to find the sides E A and E C : thus, the angle C E A $30^{\circ}5' +$ the angle A C E $34^{\circ}23'41'' = 64^{\circ}28'41''$; and $180^{\circ} - 64^{\circ}28'41'' =$ the angle E A C $115^{\circ}31'19''$.—Hence, by oblique angled trigonometry, Problem I., page 177,

To find the Side E A.

As the angle A E C =	. . .	$30^{\circ} 5' 0''$	Log. co-secant =	10.299938
Is to the side A C =	. . .	240	Log. =	. . . 2.380211
So is the angle A C E =	. . .	$34^{\circ}23'41''$	Log. sine =	. . . 9.751965
<hr/>				
To the side E A =	. . .	270.47	Log. =	. . . 2.432114

To find the Side E C.

As the angle A E C =	. . .	$30^{\circ} 5' 0''$	Log. =	. . . 10.299938
Is to the side A C =	. . .	240	Log. =	. . . 2.380211
So is the angle E A C =	. . .	$115^{\circ}31'19''$	Log. sine =	9.955407
<hr/>				
To the side E C =	. . .	432.07	Log. =	. . . 2.635556

In the triangle A B E, the sides A B, A E, and the angles A E B, E A B, are given ; to find the side E B : thus, from the angle E A C $115^{\circ}31'19''$, take the angle B A C $41^{\circ}51'32''$, and the remainder is the angle E A B $= 73^{\circ}39'47''$. Hence, by trigonometry,

As the angle A E B =	. . .	$55^{\circ}50' 0''$	Log. co-secant =	10.082281
Is to the side A B =	. . .	290	Log. =	. . . 2.462398
So is the angle E A B =	. . .	$73^{\circ}39'47''$	Log. sine =	. . . 9.982101
<hr/>				
To the side E B =	. . .	336.34	Log. =	. . . 2.526780

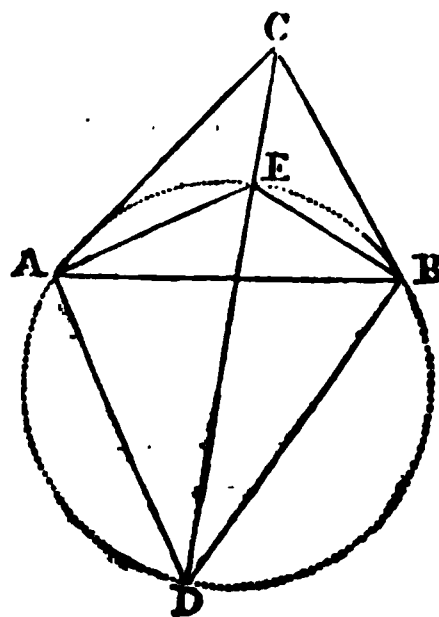
Hence the distance of the object A from the observer at E, is 270.47 yards ; that of C, 432.07 yards ; and that of B, 336.34 yards.

PROBLEM VII.

Given the Distances between three Objects, and the Angular Distances between these Objects, taken at any Point within the Triangle formed by the right Lines connecting the Objects; to find the Distance between that Point and each of the Objects.

RULE.

Let A, B, and C, be any three objects whose distances from each other are given, and E the place of the observer: complete the triangle A B C; at E, observe the angles A E C, A E B, and B E C; make the angle B A D equal to the supplement of the angle B E C; in like manner, make the angle A B D equal to the supplement of the angle A E C: hence the point D is found. Through the three points A, B, and D, describe a circle; join D C, and it will cut the circle in E, the place of the observer; connect the points A E, B E, and the construction will be completed; the calculations in which will be nearly similar to those in the preceding Problem.



Example.

Let A, B, and C, in the above diagram, be any three known objects whose distances from each other are as follow: viz., A B, 620 yards; A C, 570 yards; and B C, 460 yards. At a point E, within the triangle formed by those objects, the angle A E C was measured with a circle, and found to be $125^{\circ}15'$; the angle A E B, $124^{\circ}15'$; and the angle B E C, $110^{\circ}30'$; required the distances E A, E C, and E B?

In the triangle A B D, the angles and the side A B are given; to find the side A D: thus, the angle B A D $69^{\circ}30' =$ the supplement of the angle B E C; the angle A B D $54^{\circ}45' =$ the supplement of the angle A E C; and the angle A D B $55^{\circ}45' =$ the supplement of the angle A E B. Hence, by trigonometry,

As the angle A D B =	$55^{\circ}45'$	Log. co-secant =	10.082710
Is to the side A B =	620	Log. = 2.792392
So is the angle A B D =	$54^{\circ}45'$	Log. sine = 9.912032
To the side A D =	612.54	Log. = 2.787134

In the triangle A B C, all the sides are given; to find the angle B A C: which, being added to the angle B A D, will give the obtuse angle C A D. Hence, by trigonometry, Problem IV., page 180.

$$\begin{array}{rcl}
 \text{Side } B C = & . & 460 \\
 \text{Side } A C = & . & 570 \quad \text{Log. ar. comp.} = 7.244125 \\
 \text{Side } A B = & . & 620 \quad \text{Log. ar. comp.} = 7.207608
 \end{array}$$

$$\text{Sum} = . . . 1650$$

$$\text{Half sum} = . \quad 825 \quad \text{Log.} = . . . 2.916454$$

$$\text{Remainder} = . \quad 365 \quad \text{Log.} = . . . 2.562293$$

$$\text{Sum} = 19.930480$$

$$\text{Arch} = . . \quad 22^{\circ} 37' 9'' \quad \text{Log. co-sine} = 9.965240$$

$$\text{Angle } C A B = 45^{\circ} 14' 18'' + \text{angle } B A D = 69^{\circ} 30' = \text{angle } C A D 114^{\circ} 44' 18''$$

In the triangle $A C D$, the sides $A C$, $A D$, and the included angle $C A D$ are given; to find the angle $A C D$: hence, by oblique angled trigonometry, Problem III., page 179,

$$\begin{array}{l}
 \text{As the side } A D 612.54 + \text{the side } A C 570 = 1182.54 \quad \text{Log. ar. co.} 6.927184 \\
 \text{Is to the side } A D 612.54 - \text{the side } A C 570 = 42.54 \quad \text{Log.} = 1.628798 \\
 \text{So is } 180^{\circ} - \text{angle } C A D 114^{\circ} 44' 18'' \left. \vphantom{\begin{array}{l} \text{As the side } A D 612.54 + \text{the side } A C 570 = 1182.54 \\ \text{Is to the side } A D 612.54 - \text{the side } A C 570 = 42.54 \end{array}} \right\} 32^{\circ} 37' 51'' \quad \text{Log. tang.} 9.806374 \\
 = 65^{\circ} 15' 42'' \div 2 =
 \end{array}$$

$$\text{To half diff. of angles } A C D \text{ and } A D C = 1^{\circ} 19' 10'' \quad \text{Log. tang.} 8.362356$$

$$\text{Angle } A C D = 33^{\circ} 57' 1''$$

¶ In the triangle $A D C$, all the angles and the side $A C$ are given; to find the sides $A E$ and $E C$: thus, the angle $A E C 125^{\circ} 15' + \text{angle } A C E 33^{\circ} 57' 1'' = 159^{\circ} 12' 1''$; and $180^{\circ} - 159^{\circ} 12' 1'' = \text{the angle } C A E 20^{\circ} 47' 59''$. Hence, by oblique angled trigonometry, Problem I., page 177,

To find the Side $A E$.

$$\begin{array}{rcl}
 \text{As the angle } A E C = & . & 125^{\circ} 15' 0'' \quad \text{Log. co-secant} = 10.087968 \\
 \text{Is to the side } A C = & . & 570 \quad \text{Log.} = . . . 2.755875 \\
 \text{So is the angle } A C E = & . & 33^{\circ} 57' 1'' \quad \text{Log. sine} = . \quad 9.747002
 \end{array}$$

$$\text{To the side } A E = . . . 389.80 \quad \text{Log.} = . . . 2.590845$$

To find the Side $E C$.

$$\begin{array}{rcl}
 \text{As the angle } A E C = & . & 125^{\circ} 15' 0'' \quad \text{Log. co-secant} = 10.087968 \\
 \text{Is to the side } A C = & . & 570 \quad \text{Log.} = . . . 2.755875 \\
 \text{So is the angle } C A E = & . & 20^{\circ} 47' 59'' \quad \text{Log. sine} = . \quad 9.550359
 \end{array}$$

$$\text{To the side } E C = . . . 247.86 \quad \text{Log.} = . . . 2.394202$$

In the triangle BEC, given the sides BC, CE, and the angle BEC; to find the angle BCE, and, thence, the side BE: the angle BCE is found by oblique angled trigonometry, Problem II., page 178; and the side BE by Problem I., page 177. Hence,

To find the angle BCE.

As the side BC =	. . . 460	Log. ar. comp. =	7.337242
Is to the angle BEC =	. . 110°30'	Log. sine =	. 9.971588
So is the side EC =	. . . 247.86	Log. =	. . . 2.394202
To the angle CBE =	. . 30°18'41"	Log. sine =	. 9.703032
Angle BEC =	. . . 110.30. 0		
Sum = 140°48'41";	and 180° - 140°48'41" =	
the angle BCE =	39°11'19"		

To find the Side BE.

As the angle BEC =	. . 110°30' 0"	Log. co-secant =	10.028412
Is to the side BC =	. . 460	Log. =	. . . 2.662752
So is the angle BCE =	. 39°11'19"	Log. sine =	. 9.800631
To the side BE =	. . . 310.31	Log. =	. . . 2.491795

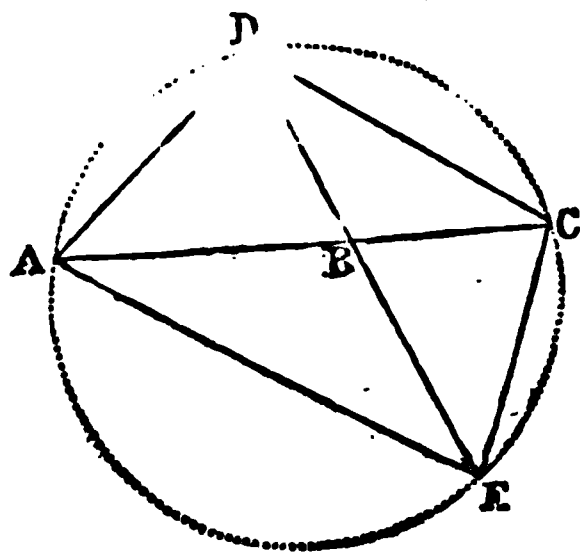
Hence the required distances are, EA, 389.80 yards; EB, 310.31 yards; and EC, 247.86 yards.

PROBLEM VIII.

Given the Distances between three Objects situated in a straight Line, and the Angular Distances of these Objects taken at any Point in the same horizontal Plane; to find the Distance between that Point and each of the Objects.

RULE.

Let the points A, B, and C, be any three objects situated in a straight line: make the angle ACD equal to the observed angle AEB, and make the angle DAC equal to the observed angle BEC: hence the point D is found. Through the three points A, D, and C, describe a circle; join DB, and produce it till it cuts the circle in E: then E will be the place of the observer, and EA, EB, and EC, the required distances.



Example.

Let A, B, and C, in the above diagram, be any three known objects situated in a straight line, whose distances from each other are as follow : viz., A B, 490 yards ; B C, 300 yards ; and A C, 790 yards : at a point E, the angle B E C was observed, and found to be 43° , and the angle B E A $33^\circ 45'$; required the distances E A, E B, and E C ?

In the triangle A D C, all the angles and the side A C are given ; to find the side A D : thus, the angle D A C 43° = the observed angle B E C ; the angle A C D $33^\circ 45'$ = the observed angle B E A ; and, consequently, the angle A D C = $103^\circ 15'$: hence the side A D may be found.

As the angle A D C =	. . .	$103^\circ 15'$	Log. co-secant =	10.011718
Is to the side A C =	. . .	790	Log. =	. . . 2.897627
So is the angle A C D =	. . .	$33^\circ 45'$	Log. sine =	. . . 9.744739

To the side A D =	. . .	450.94	Log. =	. . . 2.654084
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In the triangle A B D, given the sides A B, A D, and the included angle D A B ; to find the angle A B D : hence, by trigonometry, Problem III., page 179,

As the side A B 490 + the side A D 450.94 = 940.94	Log.ar.co.	7.026438
Is to the side A B 490 - side A D 450.94 = 39.06	Log. =	1.591732
So is 180° - angle D A B 43° = 137° $\div 2$ = $68^\circ 30'$ 0"	Log.tang.	10.404602

To half diff. of angles A D B and A B D =	$6^\circ 0' 57''$	Log.tang.	9.022772
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Angle A B D = $62^\circ 29' 3''$; and, since the two straight lines A C and D E intersect each other in the point B, the opposite angles are equal to one another (Euclid, Book I., Prop. 15) : therefore the angle E B C is $62^\circ 29' 3''$, equal to the angle A B D. In like manner, the angles D B C and A B E are equal to one another ; and because D B C is the supplement of the angle D B A, it is equal to $117^\circ 30' 57''$: hence the angle A B E is also equal to $117^\circ 30' 57''$

In the triangle A B E, all the angles and the side A B are given ; to find the sides E A and E B : thus, the angle B E A $33^\circ 45'$ + the angle A B E $117^\circ 30' 57''$ = $151^\circ 15' 57''$; and $180^\circ - 151^\circ 15' 57''$ = the angle B A E $28^\circ 44' 3''$ Hence, by trigonometry, Problem I., page 177,

To find the Side E A.

As the angle B E A = . . .	33° 45' 0"	Log. co-secant =	10.255261
Is to the side A B = . . .	490	Log. = . . .	2.690196
So is the angle A B E = . . .	117° 30' 57"	Log. sine = . . .	9.947867
To the side E A = . . .	782.21	Log. = . . .	2.893324

To find the Side E B.

As the angle B E A = . . .	33° 45' 0"	Log. co-secant =	10.255261
Is to the side A B = . . .	490	Log. = . . .	2.690196
So is the angle B A E = . . .	28° 44' 3"	Log. sine = . . .	9.681917
To the side E B = . . .	424.01	Log. = . . .	2.627374

In the triangle E B C, given the sides E B, B C, and all the angles ; to find the side E C.

As the angle B E C = . . .	43° 0' 0"	Log. co-secant =	10.166217
Is to the side B C = . . .	300	Log. = . . .	2.477121
So is the angle E B C = . . .	62° 29' 3"	Log. sine = . . .	9.947866
To the side E C = . . .	390.13	Log. = . . .	2.591204

Hence the required distances are, E A, 782.21 yards ; E B, 424.01 yards ; and E C, 390.13 yards.

Remark.—The above Problem, together with that given in page 620, will be found exceedingly useful to a general or other officer employed in conducting the military operations of a *siege* ; because, if he can only procure a correct map of the town or garrison which he may have occasion to invest, so as to ascertain the relative distances between any three desirable positions, the above Problems will enable him to find his absolute distance from those positions without the trouble of measuring a base line : nor is it necessary to resort to trigonometrical calculation for this particular purpose, since the distances may be readily determined by geometrical projection, to every degree of accuracy desirable in such operations.

PROBLEM IX.

Given the Height of the Eye ; to find the Distance of the visible Horizon.

RULE.

Let the earth's diameter, *in feet*, be augmented by the height of the eye ; then, to the logarithm thereof, add the logarithm of the height of

the eye; from half the sum of these two logarithms subtract the constant logarithm 3.788904,* and the remainder will be the logarithm of the distance in nautical miles; which is to be increased by a twelfth part of itself, on account of terrestrial refraction.

Example.

Chimborazo, the highest part of the Andes, is said to be 20633 feet above the level of the sea: now, admitting that an observer be placed upon its summit, at what distance can he see the visible horizon, allowing a twelfth part of that distance for the effects of refraction?

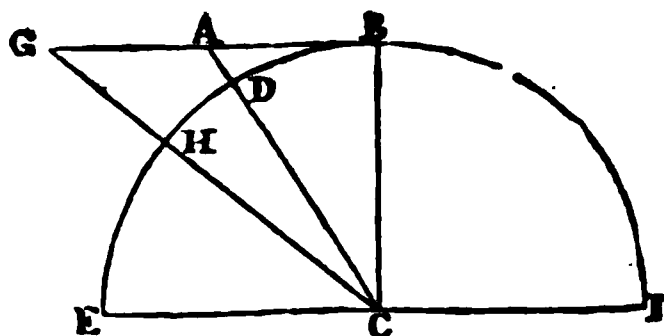
Diameter of the earth, in feet =	41804400	
Elevation of Chimborazo 20633 + 5 feet, }	20638	Log. = 4.314668
the height of the observer's eye =		
Sum =	41825038	Log. = 7.621436
Sum =		11.936104
Half sum =		5.968052
Constant log. =		3.788904
Distance uncorrected by refraction =	152.81	Log. = 2.184148
Allowance for terrestrial refraction =	12.40	
Dist. at which the visible horizon may be seen =	165.21	miles.

PROBLEM X.

Given the measured Length of a base Line; to find the Allowance for the Curvature or Spherical Figure of the Earth.

RULE.

Let EBF represent the arc of a great circle on the earth; C, the earth's centre; CB its semidiameter; and AB the measure of a base line, on an apparent level or horizontal plane on the earth's surface: join CA, and it will cut the arc of the great circle in D; then AD will be the excess of the apparent level of the horizon above its true level.



* This is the logarithm of 6080, the number of feet in a nautical mile.

Now, in the right angled plane triangle $A B C$, given the perpendicular $B C$ and the base $A B$; to find the hypotenuse $A C$: which is readily determined by Euclid, Book I., Prop. 47. Then, the difference between $A C$, thus found, and $C D = C B$, will be equal to $D A$, or the absolute value of the true level *below* the apparent level: and, if this value be expressed in miles and decimal parts of a mile, it may be reduced to inches, if necessary, by being multiplied by 63360 = the number of inches in an English mile.

Example.

Let the base line A B, in the above diagram, be 1 English mile, and the earth's semidiameter B C=3958.75 miles ; required the allowance for the earth's curvature answering to that base line, or the difference between the true and apparent levels on the earth's surface expressed by the measure of the line A D ?

$$\mathbf{B\ C\ 3958.75 \times B\ C\ 3958.75 = 15671701.5625}$$

$$\mathbf{A} \mathbf{B} = \mathbf{1} \times \mathbf{A} \mathbf{B} = \mathbf{1} = \quad . \quad . \quad . \quad \mathbf{1}.$$

Sum of the squares= . . 15671702.5625; the square

root of which = C A, is 3958.7501263

Subtract C D=C B, the earth's semidiameter= . . 3958.7500000

Remainder=the line A D, the allowance for curvature=0000.0001263

Multiply by the number of inches in an English mile = 63360,

Number of inches which the true level is below the

apparent level in one mile = 8.0023680

Now, since the curvature answering to A B is known, that corresponding to any other base line on the earth's surface may be readily determined by the following proportion :—

As the square of AB , is to AD ; so is the square of BG , to GH : whence it is manifest, that the curvature answering to any given distance, as BG , is in the *duplicate ratio* of that distance to AB :

And, since AB is expressed by unity or 1, and that AD is a constant quantity, the proportion may be reduced to a logarithmical expression ; as thus :—

To twice the logarithm of the given base line, expressed in miles and decimal parts of a mile, add the constant logarithm 0.903219 (the log. of 8.002368 inches); and the sum will be the logarithm of the number of *inches and decimal parts of an inch* which the true horizontal level at sea is below its apparent level,

Example.

Required the curvature of the earth answering to a distance of 2 miles on its surface ?

$$\begin{array}{rcl}
 \text{Distance}=2 \text{ miles ; twice the log.} & = & . \quad 0.602060 \\
 \text{Constant log.} & = & . \quad . \quad . \quad . \quad . \quad . \quad . \quad 0.903219 \\
 \hline
 \text{Curvature, in inches}=32.009 & . \quad \text{Log.} & = 1.505279
 \end{array}$$

Hence, the curvature answering to a distance of 2 miles on the surface of the earth, is 32.009 inches ; or $2\frac{2}{3}$ feet, nearly.

Remark.—If to twice the logarithm of the given base line, in miles, the constant logarithm 9.824037 be added, the sum (abating 10 in the index) will be the logarithm of the excess of the apparent above the true level, in feet.

Example.

Required the curvature of the earth, or the excess of the apparent above the true level, answering to a base line of 15 English miles in length ?

$$\begin{array}{rcl}
 \text{Given base line}=15 \text{ miles ; twice the logarithm} & = & . \quad . \quad . \quad 2.352182 \\
 \text{Constant log.}=\text{log. of } 8.002368 \text{ inches, diminished by the} & & \\
 \text{log. of } 12 \text{ inches} & = & . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 9.824037 \\
 \hline
 \text{Excess of the app. above the true level, in ft.} & = & 150.044 \quad \text{Log.} = 2.176219
 \end{array}$$

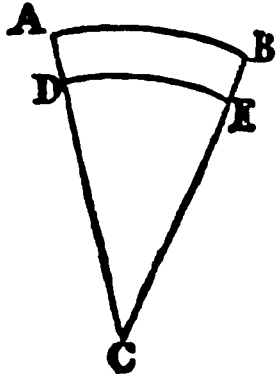
Note.—This Problem will be found useful to land-surveyors, engineers, and others employed in the art of levelling, cutting canals, and conducting water (by means of pipes, &c.) from one place to another.

PROBLEM XI.

Given the measured Length of a Base Line on any elevated Level ; to find its true Measure, when referred to the Level of the Sea.

RULE.

In the annexed diagram, let the arc A B represent the measured length of a base line, at any given elevation above the level of the sea expressed by the arc D E ; let C D be the radius of the earth, or the distance from its centre to the surface of the sea ; and let C A be the earth's radius referred to the level of the measured base line A B.



Now, because the arcs A B and D E are concentric and similar, and that similar arcs of spheres are to each other as their radii, we have the following analogy; viz.,

As the radius C A, is to the radius C D; so is the arc A B, to the arc D E: that is, as the earth's semidiameter, augmented by the height of the base line above the level of the sea, is to the earth's true semidiameter; so is the measured length of the given base line, to the true measure of that line at the surface of the sea.

Example.

Given a base line of 36960 feet in length, measured on a horizontal plane which is elevated 120 feet above the level of the sea; required the measure of that base line at the surface of the sea?

As $C D = 20902200 + D A = 120 = C A$ $20902320 = A B$ $36960 :: C D$ $20902200 : D E = 36959.787813$. Hence the given base line, reduced to the level of the sea, is 36959.787813 feet; which is about $2\frac{1}{2}$ inches less than the measure on the elevated horizontal plane.

But, since the probable elevation of any horizontal plane on the earth, above the level of the sea, can bear but a very insignificant proportion to the earth's semidiameter,—if, therefore, the product of the measured base line by its height above the level of the sea be divided by the earth's radius, the quotient will be the excess of the measured base above the corresponding arc at the surface of the sea. This may be reduced to a logarithmical expression, in the following manner; viz., to the constant logarithm 2.679808, add the logarithms of the base line and of its elevation above the level of the sea, both expressed in feet: the sum will be the logarithm of a natural number, which, being taken from the measured base line, will leave the measure of that line at the surface of the sea, sufficiently near the truth for all practical purposes. Thus, to work the last Example,

Constant log.=ar.co.of the log.of the earth's semidiam.in ft.=	2.679808
Eleva.of given base line above level of sea=	120 feet. Log.=2.079181
Measured length of the given base line=	36960 feet. Log.=4.567732

Excess of the given base line above the	
arc at the surface of the sea =	. . . -0.212188 Log.=9.326721

Given base line, red. to level of sea = 36959.787812; which approximates so very closely to the true result by the direct method of computation, as scarcely to admit of any sensible difference.

Remark.—In consequence of the spherical figure of the earth, no two points on its surface can be situated exactly on the same horizontal plane; for it is the chord of the arc, and not the arc itself, that measures the horizontal distance between two points. Hence, when philosophical inquiries are under consideration, it becomes necessary to apply a small correction to the measured base line on a horizontal plane, so as to reduce it to the corresponding terrestrial arc; though, in general, this correction is so very inconsiderable, that, even in the most extensive trigonometrical surveys, it may be safely disregarded. If, however, it be deemed necessary to find its value, or (which amounts to the same thing) if the excess of the arc over its chord be required, it may be very readily determined by the following Rule, to every desirable degree of accuracy; viz.,

From thrice the measured length of the base line, in feet, subtract the constant logarithm 16.020595: the remainder will be the logarithm of the excess of the arc over its corresponding chord, expressed by the given base line.

Example.

Let it be required to find the excess of the terrestrial arc over its chord, answering to a measured base line of 36960 feet in length, or seven English miles?

Given base line=36960 feet; thrice its logarithm=	13.703196
Constant log.=log. of 24 times the square of the earth's	
radius, in feet=	16.020595
	—————

Excess of the arc over its chord, in feet=0.004815 Log.=−7.682601

Hence it is evident, that the extent by which a terrestrial arc of 36960 feet exceeds the chord of the same arc, is only the small decimal fraction .004815 of a foot,—an excess so very trivial, as to be scarcely worth taking into account, even where the greatest accuracy is required: for, in the present instance, though the base line is 7 English miles in length, it amounts to no more than about the *two hundred and seventh part of a foot*.

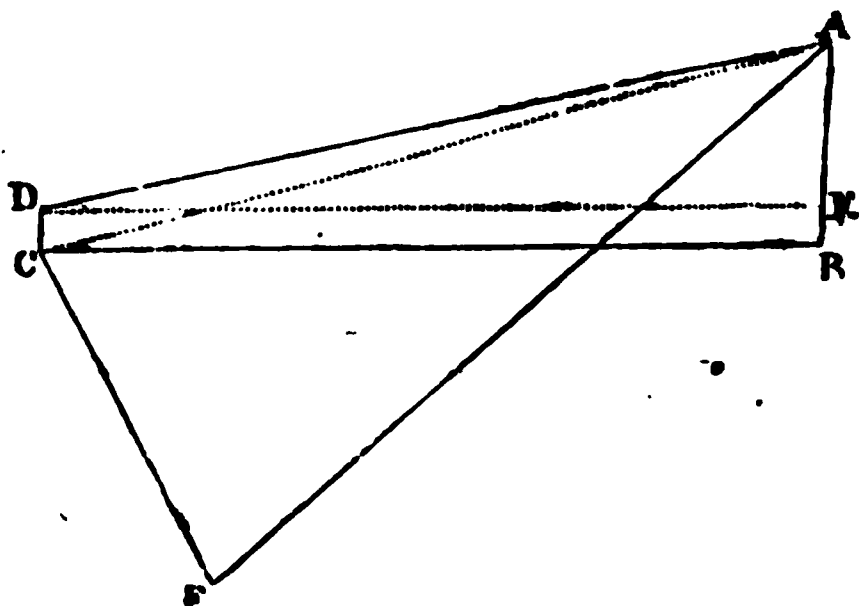
Note.—The index of the logarithm of the excess comes out a negative quantity, because the index of the constant logarithm is greater than that of the term from which it is subtracted.

PROBLEM XII.

To find the Height and Distance of a Hill or Mountain.

RULE.

Let the point A, in the annexed diagram, be the summit of a hill, the height of which, AB, is to be determined; and let the point C be the place from which its distance is to be found: at C, observe the vertical angle ADE; then measure any convenient distance for a base line, as CF; at the point C, observe the inclined angle ACF, and, at F, observe the angle AFC. Now, in the inclined triangle ACF, given the angles and the side FC; to find the side AC, which may be considered as being essentially equal to the side AD: and, in the vertical or right angled triangle AED, the angle ADE and the hypotenuse or side AD are given; to find the perpendicular AE: to which, the height of the eye BE=CD being added, gives the required height AB.



Example.

Wanting to know the height of the hill AB, and the distance of its summit A from the point C, the vertical angle ADE was observed, and found to be $14^{\circ}30'$; at the points C and F, 500 feet asunder, the inclined angles ACF and AFC were measured, and found to be $80^{\circ}5'$, and $73^{\circ}30'$ respectively; required the distance of the point A from the observer at C, and its height above the level of the horizontal plane CB?

In the inclined triangle ACF, the angles are given, and the side $FC=500$ feet; to find the side or distance AC: thus, the angle ACE $80^{\circ}5' +$ the angle AFC $73^{\circ}30' = 153^{\circ}35'$; and $180^{\circ} - 153^{\circ}35' = 26^{\circ}25'$, the measure of the angle CAF. Hence, by trigonometry,

To find the Distance AC=the side AD.

As the angle CAF =	. . .	$26^{\circ}25'$	Log. co-secant =	10.351742
Is to the side CF =	. . .	500	Log. =	2.698970
So is the angle CFA =	. . .	$73^{\circ}30'$	Log. sine =	9.981737

To the distance AC =	. . .	1077.55	Log. =	8.032449;
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and, since AC and AD are essentially equal, the side AD is also 1077.55 feet.

To find the Height A B.

As radius =	90°	Log. co-secant =	10.000000
Is to the side A D = A C . . .	1077.55	Log. = . . .	3.032449
So is the angle A D E = . . .	14°30'	Log. sine = . . .	9.398600

To the perpendicular A E = .	269.84	Log. = . . .	2.431049
Height of the eye B E = C D .	5.		

Height of A B = 274.84 feet.

PROBLEM XIII.

To find the Height of a Mountain, by means of two Barometers and Thermometers.

RULE.

Let two observers (provided with barometers and thermometers of equal construction), carefully note down, at the same instant, the respective heights of the barometers at the top and bottom of the mountain, or other eminence intended to be measured, with the temperature of the quicksilver in each instrument by means of attached thermometers, and also the temperature of the air, in the *shade*, by means of detached thermometers; then,

Find the difference of the logarithms of the observed heights of the barometers, the first four figures of which, besides the index, are to be considered as whole numbers. To this difference apply the product of 0.454, by the difference of the altitudes of the two attached thermometers, by subtraction if the temperature of the quicksilver at the bottom station exceed that at top; otherwise, by addition: and the sum or difference will be the approximate height, in fathoms, English measure.

Now, to the logarithm of the approximate height, thus found, add the logarithm of the difference between the mean of the two temperatures of the detached thermometers and 32°, and the constant logarithm 7.387390: the sum of these three logarithms will be the logarithm of a correction, which being added to the approximate height when the mean temperature exceeds 32°, but subtracted if it be less, the sum or difference will be the true elevation of the mountain, expressed in fathoms; which may be reduced to feet, if necessary, by being multiplied by 6.

Note.—This Rule is deduced from that given by Dr. Hutton, in the second volume of his “Course of Mathematics,” page 255.

Example.

Let the observations at the top and bottom of a mountain be as follow ; required its height ?

	Attached thermometer.	Detached thermo.	Barometer.	
Obs. at bottom=	57	57	29.68	Log.=1.472464
Ditto at top=	43	42	25.28	Log.=1.402777
	—	—		—
Difference=	14	Sum=99	Difference=	0.069687
	—	—		—
Multiply by	.454	Mean=49½	Product=	— 6.36
	—			—
Product=	—6.356	32	Approx. alt.=	690.51 Log. 2.839170
		—		
		Diff.= 17½ or 17.5	Log.= 1.243038
		Constant log.=	 7.387390
				—
Correction of the approximate altitude=			+29.48	Log. 1.469598
				—
True altitude of the mountain, in fathoms=			719.99,	or 4319.94 ft.

PROBLEM XIV.

To find the Distance of an Object, by observing the Interval of Time between seeing the Flash and hearing the Report of a Gun or of a Thunder-Cloud.

RULE.

To the logarithm of the number of seconds elapsed between seeing the flash and hearing the report, add the constant logarithm 9.273762;* and the sum (abating 10 in the index) will be the logarithm of the distance in nautical miles ; or, if the constant logarithm 9.335032† be made use of, it will give the distance in English statute miles.

* This is the sum of the arithmetical complement of the logarithm of 6080, the number of feet in a nautical mile, and the logarithm of 1142, the number of feet which sound travels in one second of time.

† This is the sum of the arithmetical complement of the logarithm of 5280, the number of feet in an English mile, and the logarithm of 1142 feet, the established velocity of sound.

Example 1.

A ship at sea was observed to fire a gun, and 43 seconds afterwards the report was heard; required the distance of the ship in nautical miles?

Interval betw. seeing the flash and hearing the gun = 43' Log. = 1.633469
 Constant logarithm = 9.273762

Distance in nautical miles = 8.076 Log. = 0.907231

Example 2.

A flash of lightning was seen, and after a lapse of 18 seconds the report reached the ear of the observer; required the distance of the thunder-cloud in English miles?

Inter. betw. seeing the flash & hearing the thunder = 18' Log. = 1.255273
 Constant logarithm = 9.335032

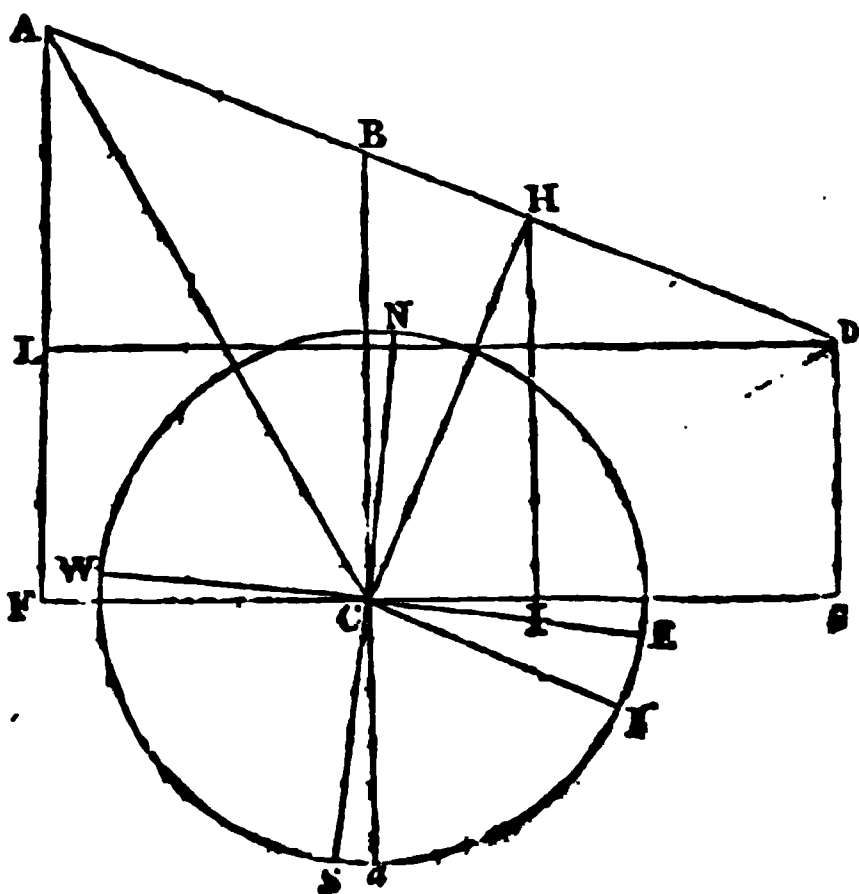
Distance of the thunder-cloud in English miles = 3.894 Log. = 0.590305

PROBLEM XV.

Given three Bearings of a Ship sailing upon a direct Course, and the Intervals of Time between those Bearings; to find the Course steered by that Ship, and the Time of her nearest Distance from the Observer.

RULE.

Describe the circle **NESW**, and from the centre **C**, the place of the observer, draw the lines **CA**, **CB**, and **CD**, to represent the three bearings of the ship: through the centre **C** draw the line **FG**, at right angles to the *second bearing* **CB**; make **CF** equal to the first interval, and **CG** equal to the second, each being expressed in minutes, and taken from any convenient scale of equal parts; from the points **F**



and G draw the lines F A, G D, parallel to the second bearing C B, and meeting C A and C D in the points A and D; join A D, and it will represent the ship's track; through C draw C K, parallel to A D, and the arch S K will be the measure of the ship's course. From C let fall the perpendicular C H upon the line A D, produced if necessary; and from H let fall the perpendicular H I upon the line F G, produced also, if necessary; then the measure of C I will give the interval between the time of the second bearing and that when the ship was nearest to the observer.

Make A L equal to the difference between the perpendiculars A F and D G; then, in the right angled triangle A L G, given the perpendicular A L and the base L D; to find the angle L A D, which is evidently equal to the angle a C K; to this let the inclination of C B to a parallel be applied, and the result will be the apparent course of the ship.

Example.

At 1^h20^m past noon a ship, sailing upon a direct course, was observed to bear N.W. b. N.; at 2^h10^m, she bore N. $\frac{1}{4}$ W.; and at 3^h25^m, the bearing was N.E. b. E.; required the apparent course steered by that ship, and the time when she was nearest to the observer?

Solution.—The circle being described and quartered, and the three given bearings laid down as above directed, through C draw F G perpendicular to the second bearing C B; make F C equal to 50 minutes, the interval between the first and second bearings, and C G equal to 75 minutes, the interval between the second and third bearings: these may be taken from any scale of equal parts. Then proceed with the other parts of the construction, agreeably to the rule; now, the ship's apparent course, represented by the angle S C K, being applied to the line of chords, will be found to measure $72\frac{1}{2}$ degrees: hence the course is S. $72^{\circ}30'$ E., or E. b. S. $\frac{1}{4}$ S. nearly. The perpendicular G D, being applied to the scale of equal parts from which the intervals were taken, will be found to measure 40, and the perpendicular F A $93\frac{1}{2}$; the difference between which $= 53\frac{1}{2}$, is the measure of A L. Then C I, measured upon the same scale, gives 26 minutes; which is evidently, by the construction, past the time of the second bearing: hence the time of the ship's nearest approach to the observer at C, is $2^h10^m + 26^m = 2^h36^m$ past noon. Now, the figure being thus completed, the required parts may be obtained by trigonometrical calculation, in the following manner:—

In the right angled triangle A F C, given the angles and the base

FC 50 minutes; to find the perpendicular **FA**. Thus, since the straight line **AC** falls upon the two parallel straight lines **CB** and **FA**, it makes the alternate angles equal to one another (Euclid, Book I, Prop. 29): therefore the angle **FAC** is equal to the angle **ACB**; but the angle **ACB** is given, being equal to $2\frac{1}{2}$ points, viz., the difference between N.W. b. N. and N. $\frac{1}{2}$ W.: hence the angle **FAC** is also equal to $2\frac{1}{2}$ points.

In the same manner it may be shown (in the right angled triangle **DGC**, where the angles and the base **CG** 75 minutes are given; to find the perpendicular **GD**), that the angle **GDC** is equal to the angle **BCD**; and since **BCD** is given, being equal to $5\frac{1}{2}$ points, viz., the sum of N.E. b. E., and N. $\frac{1}{2}$ W., therefore the angle **GDC** is also equal to $5\frac{1}{2}$ points. Hence,

To find the Perpendicular **GD**.

As radius = 90°	Log. co-secant = 10.000000
Is to the base CG = 75"	Log. = 1.875061
So is the angle GDC = $5\frac{1}{2}$ points	Log. co-tangent = 9.727957

To the perpendicular GD = 40.09	Log. = 1.603018
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To find the Perpendicular **FA**.

As radius = 90°	Log. co-secant = 10.000000
Is to the base FC 50"	Log. = 1.698970
So is the angle FAC = $2\frac{1}{2}$ points	Log. co-tangent = 10.272043

To the perpendicular FA = 93.54	Log. = 1.971013
Perpendicular GD = 40.09	

Difference = 53.45, which is equal to the part **AL**.

In the right angled triangle **ALD**, given the base **LD** = **FG** 125 minutes, and the perpendicular **AL** 53.45; to find the angle **LAD**: therefore,

As the perpendicular AL = 53.45 minutes	Log. ar. comp. = 8.272052
Is to radius = 90. 0	Log. sine = 10.000000
So is the base LD = 125 minutes	Log. = 2.096910

To the angle LAD = 66°50'54"	Log. tangent = 10.368962
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Now, since CK is parallel to AD , and Ca to AL , the angle aCK is equal to the angle LAD ; but the angle LAD is found, by computation, to be $66^{\circ}50'54''$; wherefore the angle aCK is also equal to $66^{\circ}50'54''$: to this let the angle $aCS =$ the angle NCB $0\frac{1}{2}$ point, or $5^{\circ}37'30''$, be added; and the sum $72^{\circ}28'24'' =$ the angle SK is the apparent course of the ship between the south and the east, viz., $S.72^{\circ}28'24'' E.$, or $E. b. S \frac{1}{2} S.$ nearly.

We have now to determine the measure of the base CI , in the right angled triangle CIH ; to do which, we must first find the value of the hypotenuse AC in the right angled triangle AFC , and that of the base CH in the right angled triangle AHC . Thus,

To find the Hypotenuse AC .

As radius = 90°	Log. co-secant =	10.000000
Is to the base $FC =$ $50''$	Log. = 1.698970
So is the angle $FAC = 2\frac{1}{2}$ points		Log. co-secant =	10.326613

To the hypotenuse $AC =$ 106.07 Log. = 2.025583

To find the Base CH .

As radius = 90°	Log. co-secant =	10.000000
Is to the hypotenuse $AC =$	106.07	Log. = 2.025583
So is $LAD - FAC = CAH = 38^{\circ}43'24''$		Log. sine =	9.796269

To the base $CH =$ 66.35 Log. = 1.821852

Now, in the right angled triangle CIH , given the hypotenuse $CH = 66.35$ minutes, and the angle CHI ; to find the base CI . The measure of the angle CHI is thus determined. In all quadrilateral or four-sided figures, the sum of the four angles is equal to four right angles, or 360 degrees. Now, in the quadrilateral figure $AHIF$, since three of the angles are given, the remaining or obtuse angle AHI is known by subtracting the sum of the given angles from 360 degrees: thus, the angle $HIF 90^{\circ} + IFA 90^{\circ} + FAH 66^{\circ}50'54'' = 246^{\circ}50'54''$; and $360^{\circ} - 246^{\circ}50'54'' = 113^{\circ}9.6''$, is the measure of the angle AHI ; from which take away the right angle $AHC 90^{\circ}$ and the remainder $= 23^{\circ}9'6''$ is the absolute measure of the angle CHI . Hence CI may be readily found; as thus:—

As radius = 90°	Log. co-secant =	10.00000
Is to the hypotenuse CH =	66.35 minutes	Log. =	. . . 1.821852
So is the angle C H I =	23° 9' 6"	Log. sine =	. . . 9.594572

To the interval or base CI = 26 . 09 minutes Log. = . . . 1.416424
 Time of second bearing = 2° 10' . 0

Sum = 2° 36' . 09; which is the time of the ship's nearest approach to the observer.

Note.—This interesting Problem is thus worked at length, trigonometrically, with the view of adapting it to the use of mariners in general; though, indeed, in such cases, calculation need not be resorted to, since the solution deduced from geometrical construction will always be sufficiently near the truth.

SOLUTION OF PROBLEMS IN PRACTICAL GUNNERY.

Gunnery is the art of projecting balls and shells from great guns and mortars; of finding the ranges and times of flight of shot and shells; and of determining the different degrees of elevation at which those bodies should be projected, so as to produce the greatest possible effect.

PROBLEM I.

Given the Diameter of an Iron Ball; to find its Weight.

RULE.

The diameter of an iron ball of 9 lbs. weight is 4 inches, very nearly; and, since the weights of spherical bodies, composed of the same materials, are as the cubes of their diameters (Euclid, Book XII., Prop. 18), it will be,—as the cube of 4, is to 9 lbs.; so is the cube of the diameter of any other iron ball, to its weight. Hence the following rule:—

To thrice the logarithm of the diameter of the given ball, add the constant logarithm 9.148063; and the sum (abating 10 in the index) will be the logarithm of the required weight in lbs.

Example.

Required the weight of an iron ball, the diameter of which is 6.7 in.?

Given diameter = 6.7 ; thrice its log. = 2.478225
 Constant log. = 9.148063

Weight in pounds = 42.295 Log. = . 1.626288

Note.—The constant logarithm used in this Problem is expressed by the arithmetical complement of the logarithm of the cube of 4, added to the logarithm of 9.

PROBLEM II.

Given the Weight of an Iron Ball ; to find its Diameter.

RULE.

This Problem being the converse of the last, we obtain the following logarithmical expression :—

To the logarithm of the weight of the given ball, add the constant logarithm 0.851937 ; divide the sum by 3, and the quotient will be the logarithm of the required diameter.

Note.—The constant logarithm given in this Rule is expressed by the arithmetical complement of the logarithm of 9, added to the logarithm of the cube of 4.

Example.

Required the diameter of a 42 lb. iron ball ?

Given weight = 42 lb. Log. = . . . 1.623249
 Constant log. = 0.851937

Divide by 3) 2.475186

Diameter in inches = 6.685 Log. = 0.825062

PROBLEM III.

Given the Diameter of a Leaden Ball ; to find its Weight.

RULE.

A leaden ball of 1 inch in diameter, weighs $\frac{3}{14}$ of a lb. ; which, reduced to a decimal fraction, is .2143, very nearly : and, as the weights

T T

of spherical bodies are as the cubes of their diameters, it will be,—as the cube of 1, is to .2143 ; so is the cube of the diameter of any other leaden ball, to its weight in lbs. Whence the following logarithmical rule :—

To thrice the logarithm of the diameter of the given leaden ball, add the constant logarithm 9.331022 ; and the sum (abating 10 in the index) will be the logarithm of the required weight.

Example.

Required the weight of a leaden ball, the diameter of which is 6.68 inches ?

$$\begin{array}{rcl} \text{Given diameter} = 6.68 ; \text{ thrice its log.} & = & 2.474331 \\ \text{Constant log.} & = & . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 9.331022 \\ & & \hline \text{Weight in pounds} = 63.88 & \text{Log.} = & . \quad 1.805353 \end{array}$$

Note.—The constant logarithm used in this Problem is the logarithm of the decimal fraction .2143.

PROBLEM IV.

Given the Weight of a Leaden Ball ; to find its Diameter.

RULE.

Since this Problem is merely the converse of the last, we obtain the following logarithmical expression ; viz., to the logarithm of the weight of the given leaden ball, add the constant logarithm 0.668978 ; divide the sum by 3, and the quotient will be the logarithm of the required diameter.

Example.

Required the diameter of a 64 lb. leaden ball ?

$$\begin{array}{rcl} \text{Given weight} = 64 \text{ lb.} & . \quad . \quad . & \text{Log.} = 1.806180 \\ \text{Constant log.} & = & . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 0.668978 \\ & & \hline \end{array}$$

$$\text{Divide by 3) } 2.475158$$

$$\text{Diameter in inches} = 6.68 \quad \text{Log.} = . \quad 0.825052\frac{1}{2}$$

Note.—The constant logarithm made use of in this Rule is the arithmetical complement of *the constant expression* which is used in the Rule to Problem III.

PROBLEM V.

Given the Internal and External Diameters of an Iron Shell, to find its Weight.

RULE.

Find the difference of the cubes of the internal and external diameters of the shell, to the logarithm of which add the constant logarithm 9.148063; and the sum (abating 10 in the index) will be the logarithm of the required weight in pounds.

Note.—The constant logarithm used in this Rule is the same as that given in Problem I., page 640.

Example.

Let the external diameter of an iron shell be 12.8 inches, and its internal diameter 9.1 inches; required its weight?

$12.8 \times 12.8 \times 12.8 = 2097.152$, cube of the external diameter.

$9.1 \times 9.1 \times 9.1 = 753.571$, cube of the internal diameter.

Difference =	. . 1343.581	Log. = 3.128264
Constant log. = 9.148063
Weight in pounds =	188.94	Log. = 2.276327

PROBLEM VI.

To find how much Powder will fill a Shell.

RULE.

To thrice the logarithm of the internal diameter of the shell, in inches, add the constant logarithm 8.241845; and the sum (abating 10 in the index) will be the logarithm of the pounds of powder.

Example.

How much powder will fill a shell, the internal diameter of which is 9.1 inches?

Internal diameter, 9.1 inches; thrice its log. = 2.877123
Constant log. = 8.241845

Powder, in pounds =	13.15	Log. = 1.118968
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Note.—The constant logarithm made use of in this Problem is the arithmetical complement of the logarithm of 57.3, the established divisor for filling shells.

PROBLEM VII.

To find the Size of a Shell to contain a given Weight of Powder.

RULE.

This Problem being the converse of the last, we obtain the following logarithmical expression :—

To the logarithm of the given weight of powder, in pounds, add the constant logarithm 1.758155 ; divide the sum by 3, and the quotient will be the logarithm of the internal diameter of the shell, in inches.

Example.

Required the internal diameter of a shell that will hold 13.15 lbs. of powder ?

Given weight = 13.15 lbs.	Log. = . .	1.118926
Constant log. =	1.758155
		<hr/>
	Divide by 3)	2.877081
		<hr/>
Internal diameter, in inches = 9.1	Log. =	0.959027

PROBLEM VIII.

To find how much Powder will fill a Rectangular Box.

RULE.

To the logarithms of the length, breadth, and depth of the box, in inches, add the constant logarithm 8.522879 ; and the sum (abating 10 in the index) will be the logarithm of the pounds of powder.

Example.

How much powder will fill a box, the length of which is 15, the breadth 12, and the depth 10 inches ?

Length = 15 inches.	Log. =	1.176091
Breadth = 12 do.	Log. =	1.079181
Depth = 10 do.	Log. =	1.000000
Constant log. =	8.522879
		<hr/>
Powder, in pounds = 60.0	Log. = .	1.778151

Note.—The constant logarithm made use of in this Problem is the arithmetical complement of the logarithm of 30, the established divisor for filling rectangular powder boxes.

PROBLEM IX.

To find the Size of a Cubical Box to contain a given Weight of Powder.

RULE.

To the logarithm of the given weight of powder, in pounds, add the constant logarithm 1.477121; divide the sum by 3, and the quotient will be the logarithm of the side of the box, in inches.

Example.

Required the side of a cubical box that will hold 60 lbs. of gunpowder?

Given weight = 60 lbs.	Log. = . . .	1.778151
Constant log. =	1.477121

Divide by 3) 3.255272

Side of the box, in inches, 12.16 Log. = 1.085090½

Note.—Since this Problem is the converse of the last, the constant logarithm made use of is the logarithm of 30, the established divisor and multiplier for filling rectangular boxes.

PROBLEM X.

To find how much Powder will fill a Cylinder.

RULE.

To twice the logarithm of the diameter of the cylinder, add the logarithm of its length and the constant logarithm 8.417937; the sum (abating 10 in the index) will be the logarithm of the pounds of powder.

Example.

How much powder will a cylinder hold, the diameter of which is 13 inches, and the length 26 inches?

Diameter of the cylinder = 13 inches; twice its log. = 2.227886
 Length of ditto = . . . 26 ditto. Log. = . 1.414978
 Constant log. = 8.417937

Powder, in pounds = 115.02 Log. = . 2.060796

Note.—The constant logarithm made use of in this Problem is the arithmetical complement of the logarithm of 38.2, the established divisor for filling cylinders with gunpowder.

PROBLEM XI.

To find what Length of a Cylinder will be filled with a given Weight of Gunpowder.

RULE.

To the arithmetical complement of twice the logarithm of the diameter of the cylinder, or caliber of the gun, add the logarithm of the given weight of powder in pounds, and the constant logarithm 1.582063: the sum (abating 10 in the index) will be the logarithm of the length of the cylinder, in inches.

Example.

What length of a 24-pounder gun, of 5.66 inches caliber, will be filled with 8 lbs. of gunpowder?

Caliber of the gun = 5.66 Ar. comp. of twice its log. = 8.494323
 Given weight of powder = 8 lbs. Log. = 0.903090
 Constant log. = 1.582063
 Length, in inches = 9.539 Log. = 0.979521

Note.—This Problem being the converse of the last, the constant logarithm is the logarithm of 38.2, the established divisor and multiplier for filling cylinders with gunpowder.

PROBLEM XII.

To find the Number of Balls in a Triangular Pile.

RULE.

To the logarithm of the number of balls in the bottom row, add the logarithm of that number increased by 1, and also its logarithm in-

creased by 2, and the constant logarithm 9.221849: the sum (rejecting 10 in the index) will be the logarithm of the required number of balls.

Example.

Required the number of balls in a triangular pile, each side of its base containing 30 balls?

Balls in one side of the base =	30	Log. =	1.477121
Ditto, increased by 1 =	31	Log. =	1.491362
Ditto, increased by 2 =	32	Log. =	1.505150
Constant log. =	.	.	9.221849

Number of balls = . . 4960 Log. = 3.695482

Note.—The constant logarithm employed in this Problem is the arithmetical complement of the logarithm of 6, the established divisor for triangular, square, and rectangular piles of shot.

PROBLEM XIII.

To find the Number of Balls in a Square Pile.

RULE.

To the logarithm of the number of balls in one side of the bottom row, add the logarithm of that number increased by 1, the logarithm of twice the same number increased by 1, and the constant logarithm 9.221849: the sum (abating 10 in the index) will be the logarithm of the required number of balls.

Example.

Required the number of balls in a square pile, each side of its base containing 30 balls?

Balls in one side of the base =	30	Log. =	1.477121
Ditto, increased by 1 =	31	Log. =	1.491362
Twice ditto, increased by 1 =	61	Log. =	1.785330
Constant log. =	.	.	9.221849

Number of balls = . . . 9455 Log. = 3.975662

Note.—The constant logarithm used in this Rule, is the same as that given in the Rule to Problem XII.

PROBLEM XIV.

To find the Number of Balls in a Rectangular Pile.

RULE.

From three times the number of balls contained in the length of the bottom row, subtract the number of balls, less by 1, contained in the breadth of that row; then, to the logarithm of the remainder, add the logarithm of the number of balls contained in the breadth of the bottom row, the logarithm of that number increased by 1, and the constant logarithm 9.221849: the sum (rejecting 10 in the index) will be the logarithm of the required number of balls.

Example.

Required the number of balls in a rectangular pile, which contains 46 balls in the base row of its longest side, and 15 balls in that of its shortest side?

Balls in length $46 \times 3 = 138$

Balls in breadth $15 - 1 = 14$

Remainder = . . . 124 Log. = 2.093422

Balls in breadth row = 15 Log. = 1.176091

Ditto, increased by 1 = 16 Log. = 1.204120

Constant log. = 9.221849

Number of balls = . 4960 Log. = 3.695482

Note.—The constant logarithm made use of in this Rule is the same as that which is given in the Rule to Problem XII.

PROBLEM XV.

To find the Number of Balls in an incomplete Triangular Pile.

RULE.

Find the number of balls in the whole pile, considered as complete, by Problem XII., page 646; and find also, by the same Problem, the number of balls answering to the triangular pile, the side of whose base is represented by the number of shot in the side of the top course of the incomplete pile diminished by 1; then, the difference of the two results will be the number of shot remaining in the pile.

Example.

Required the number of shot in an incomplete triangular pile; each side of its bottom course containing 40 balls, and each side of its top course containing 20 balls?

To find the Number of Balls in the complete Pile.

Balls in one side of bottom course =	40	Log. =	1.602060
Ditto, increased by 1 =	41	Log. =	1.612784
Ditto, increased by 2 =	42	Log. =	1.623249
Constant log. =			9.221849

Number of balls for the whole pile = 11480 Log. = 4.059942

To find the Number of Balls deficient.

Balls in each side of top course =	20 - 1 = 19	Log. =	1.278754
Diminished course, or 19, increased by 1 =	20	Log. =	1.301030
Ditto, increased by 2 =	22	Log. =	1.322219
Constant log. =			9.221849

Number of shot wanting = 1330 Log. = 3.123852

Now, 11480 - 1330 = 10150 is the number of shot in the incomplete pile.

PROBLEM XVI.

To find the Number of Balls in an incomplete Square Pile.

RULE.

Find the number of balls in the whole pile, considered as complete, by Problem XIII., page 647 ; and find also, by the same Problem, the number of balls answering to the square pile, each side of whose base is represented by the number of shot in each side of the top course of the incomplete pile diminished by 1 ; then, the difference of the two results will be the number of shot remaining in the pile.

Example.

Required the number of shot in an incomplete square pile ; each side of its bottom course containing 24 balls, and each side of its top course 8 balls ?

To find the Number of Balls in the complete Pile.

Balls in one side of the base =	24	Log. =	1.380211
Ditto, increased by 1 =	25	Log. =	1.397940
Twice ditto, increased by 1 =	49	Log. =	1.690196
Constant log. =			9.221849

Number of balls for the whole pile = 4900 Log. = 3.690196

To find the Number of Balls deficient.

Balls in each side of top course	$= 8 - 1 = 7$	Log. = 0.845098
Diminished course, or 7, increased by 1	$= 8$	Log. = 0.903090
Twice ditto, increased by 1	$= 15$	Log. = 1.176091
Constant log.		9.221849

Number of balls wanting = 140 Log. = 2.146128

Now, $4900 - 140 = 4760$ is the number of shot in the incomplete pile.

PROBLEM XVII.

To find the Number of Balls in an incomplete Rectangular Pile.

RULE.

Find the number of balls in the whole pile, considered as complete, by Problem XIV., page 648; and find also, by the same Problem, the number of balls answering to the rectangular pile, whose sides are represented by the respective sides of the top course of the incomplete pile, the number of shot in each side being diminished by 1; then, the difference of the two results will be the number of shot remaining in the pile.

Example.

Required the number of shot in an incomplete rectangular pile; the length of its bottom course being 40 balls, its breadth 20, and the length of its top course 29 balls, and its breadth 9?

To find the Number of Balls in the complete Pile.

Bottom course,	$40 \times 3 = 120$
Breadth,	$20 - 1 = 19$

Remainder =	101	Log. = 2.004321
Balls in breadth row =	20	Log. = 1.301030
Ditto, increased by 1 =	21	Log. = 1.322219
Constant log. =		9.221849

Numb. of balls for whole pile = 7070 Log. = 3.849419

To find the Number of Balls deficient.

$$\text{Top row, } 29 - 1 = 28 \times 3 = 84$$

$$\text{Breadth, } 9 - 1 = 8 - 1 = 7$$

$$\text{Remainder} = 77 \quad \text{Log.} = 1.886491$$

$$\text{Balls in breadth row} = . \quad 8 \quad \text{Log.} = 0.903090$$

$$\text{Ditto, increased by 1} = . \quad 9 \quad \text{Log.} = 0.954243$$

$$\text{Constant log.} = 9.221849$$

$$\text{Number of balls wanting} = 924 \quad \text{Log.} = 2.965673$$

Now, $7070 - 924 = 6146$ is the number of shot in the incomplete pile.

Note.— In triangular and square piles, the number of horizontal rows or courses is always equal to the number of balls in one side of the bottom row; and, in rectangular piles, the number of horizontal rows is equal to the number of balls in the *breadth* of the bottom row. In these piles, the number of balls in the top row or edge is always one more than the difference between the number of balls contained in the length and the breadth of the bottom row.

PROBLEM XVIII.

To find the Velocity of any Shot or Shell.

RULE.

From the logarithm of twice the weight of the charge or powder, in pounds, subtract the logarithm of the weight of the shot: to half the remainder add the constant logarithm 3.204120, and the sum (rejecting 5 in the index) will be the logarithm of the velocity in feet, or the number of feet which the shot or shell passes over in a second.

Example 1.

With what velocity will a 24-pounds ball be projected by 8 lbs. of powder?

$$\text{Twice the charge} = 16 \text{ lbs.} \quad \text{Log.} = 1.204120$$

$$\text{Weight of the shot} = 24 \text{ lbs.} \quad \text{Log.} = 1.380211$$

$$\text{Remainder} = 9.823909$$

$$\text{Half the remainder} = 4.911954\frac{1}{2}$$

$$\text{Constant log.} = 3.204120$$

$$\text{Velocity of shot, in ft.,} = 1306 \quad \text{Log.} = 3.110074\frac{1}{2}$$

Example 2.

With what velocity will a 13-inch shell, weighing 196 lbs., be discharged by 9 lbs. of powder?

Twice the charge =	18 lbs.	Log. =	1.255273
Weight of the shell =	196 lbs.	Log. =	2.292256
<hr/>			
Remainder =		8.963017
<hr/>			
Half the remainder =		4.481508½
Constant log. =		3.204120
<hr/>			
Velocity of shell in feet,	485	Log. =	2.685628½

Note.—The constant logarithm made use of in this Problem is the logarithm of 1600 feet, which is the velocity acquired by a 1 lb. ball, when fired with 8 ounces of powder.

PROBLEM XIX.

To find the terminal Velocity of a Shot or Shell; that is, the greatest Velocity it can acquire in descending through the Air by its own Weight.

RULE.

For *Balls*.—To half the logarithm of the diameter of the ball, in inches, add the constant logarithm 2.244277; and the sum will be the logarithm of the terminal velocity of the ball.

And, for *Shells*.—To half the logarithm of the external diameter of the shell, in inches, add the constant logarithm 2.168203; and the sum will be the logarithm of the terminal velocity of the shell.

Example 1.

Required the terminal velocity of a 24 lbs. ball, its diameter being 5.6 inches?

Diameter of the ball =	. 5.6	Log. =	0.748188
<hr/>			
Half the log. =		0.374094
Constant log. =		2.244277
<hr/>			
Terminal velocity =	. . 415	Log. =	2.618371

Example 2.

Required the terminal velocity of a shell weighing 196 lbs., its external diameter being 12.8 inches ?

Diameter of the shell =	12.8	Log. =	1.107210
Half the log. =		0.553605
Constant log. =		2.168203
Terminal velocity =	. . 527	Log. =	2.721808

Note.—The constant logarithms made use of in this Problem are the respective logarithms of 175.5 and 147.3, the established multipliers for shot and shells. It is by this Problem that the terminal velocities contained in Tables A and B, following, have been computed.

PROBLEM XX.

To find the Height from which a Body must fall, IN VACUO, in order to acquire a given Velocity.

RULE.

Since the spaces descended by falling bodies are as the squares of the velocities, and as a fall of $16\frac{1}{2}$ feet produces a velocity of $32\frac{1}{2}$ feet, —therefore, as the square of $32\frac{1}{2}$ feet, is to $16\frac{1}{2}$ feet ; so is the square of any other given velocity, to the altitude from which it must fall, to acquire such velocity. Hence the following logarithmical expression :—

To twice the logarithm of the given velocity, in feet, add the constant logarithm 8.191564 ; and the sum (abating 10 in the index) will be the logarithm of the required altitude, or height.

Example 1.

From what height must a body fall, in order to acquire a velocity of 1340 feet per second ?

Given velocity =	1340 ;	twice its log. =	6.254210
Constant log. =		8.191564
Altitude, or height =	27911	Log. =	4.445774

Example 2.

From what height must a body fall, in order to acquire a velocity of 1670 feet per second?

Given velocity = 1670 ; twice its log. = 6.445434

Constant log. = 8.191564

Altitude, or height = 43352 Log. = 4.636998

The constant log. is expressed by the sum of the arithmetical complement of twice the log. of 32½ and the log. of 16½.

Note.—It is by this Problem that the altitudes in Tables A and B, following, have been computed ; but, since the fractional parts beyond 16 and 32 were omitted, and the constant logarithm, in consequence thereof, assumed at 8.193820, the respective altitudes, in these Tables, are something beyond the truth.

CONCISE TABLES

FOR DETERMINING THE GREATEST HORIZONTAL RANGE OF A SHOT OR SHELL WHEN PROJECTED IN THE AIR WITH A GIVEN VELOCITY ;
TOGETHER WITH THE ELEVATION OF THE PIECE TO PRODUCE THAT RANGE.

TABLE A.—For Great Guns.

Weight of Shot.	Diameter, in inches.	Terminal Velocity.	Logarithm.	Altitude.	Logarithm.
1	1.94	244	7.612610	930	2.968483
2	2.45	275	7.560667	1182	3.072618
3	2.80	294	7.531653	1360	3.133539
4	3.08	308	7.511449	1482	3.170848
6	3.53	330	7.481486	1701	3.230704
9	4.04	353	7.452225	1958	3.291813
12	4.45	370	7.431798	2139	3.330211
18	5.09	396	7.402305	2450	3.389166
24	5.60	415	7.381952	2691	3.429914
32	6.17	436	7.360514	2970	3.472756
36	6.41	444	7.352617	3080	3.488551
42	6.75	456	7.341035	3249	3.511750

TABLE B.—For Mortars.

Size of Shell, in inches.	Weight of Shells filled.	Diameter, in inches.	Terminal Velocity.	Logarithm.	Altitude.	Logarithm.
4½	9	4.53	314	7.503070	1541	3.187803
5½	18	5.72	352	7.453457	1936	3.286905
8	47	7.90	414	7.383000	2678	3.427811
10	91½	9.84	462	7.335358	3335	3.523096
13	201	12.80	527	7.278189	4340	3.637490

TABLE C.—*For Great Guns and Mortars.*

Initial Velocity, divided by Terminal Velocity.	Logarithm.	Elevation.	Range divided by Altitude.	Logarithm.
0.6910	9.839478	44° 0'	0.3914	9.592621
0.9445	9.975202	43.15	0.5850	9.767156
1.1980	0.078457	42.30	0.7787	9.891370
1.4515	0.161817	41.45	0.9724	9.987845
1.7050	0.231724	41. 0	1.1661	0.066736
1.9585	0.291924	40.15	1.3598	0.133475
2.2120	0.344785	39.30	1.5535	0.191311
2.4655	0.391905	38.45	1.7472	0.242343
2.7190	0.434409	38. 0	1.9409	0.288903
2.9725	0.473122	37.15	2.1346	0.329317
3.2260	0.508664	36.30	2.3283	0.367039
3.4795	0.541517	35.45	2.5220	0.401745
3.7330	0.572058	35. 0	2.7157	0.433882
3.9865	0.600592	34.15	2.9094	0.463803
4.2400	0.627366	33.30	3.1031	0.491796
4.4935	0.652585	32.45	3.2968	0.518093
4.7470	0.676419	32. 0	3.4905	0.542888
5.0000	0.698970	31.15	3.6842	0.566343

Note.—These Tables are deduced from those given in the third volume of Dr. Hutton’s “Course of Mathematics.”

PROBLEM XXI.

To find the greatest Range of a Ball or Shell, and the Elevation of the Piece to produce that Range.

RULE.

Enter Table A or B, and take out the logarithm of the terminal velocity answering to the given ball or shell, as the case may be, and also the logarithm of the corresponding altitude ; then,

To the logarithm of the velocity with which the ball or shell is projected, add the logarithm of its terminal velocity ; and the sum (abating 10 in the index) will be the logarithm of the quotient of the initial velocity of the ball or shell, divided by its terminal velocity. With this logarithm, enter the second column of Table C, and in the adjoining or middle column will be found the corresponding degree of elevation to produce the greatest range ; abreast of which, in the last column of the same table, will be found the logarithm of the range divided by the altitude. Now, to this logarithm add the logarithm of the altitude taken from Table A or B, as above directed ; and the sum will be the logarithm of the greatest range.

Note.—If *great accuracy* be required, proportional parts must be taken for the excess of the given above the next less tabular numbers in Table C.

Example 1.

Let it be required to find the greatest range of a 24 lbs. ball, when discharged with a velocity of 1640 feet, and the elevation of the piece to produce that range?

Log. of terminal velocity of a 24 lbs. ball, Table A = . . 7.381952
Given velocity of the ball = 1640 Log. = 3.214844

Answering to which, in Table C, is $34^{\circ}15'$ = . . Log. = 0.596796

Log. of corresponding altitude, Table A = 3.429914
Abreast of $34^{\circ}15'$, in last column of Table C, stands . . 0.463803

Greatest range, in feet = 7829 Log. = 3.893717

Hence the greatest range of a 24 lbs. ball, when projected with a velocity of 1640 feet, is 7829 feet, which is nearly an English mile and a half; and the elevation to produce that range, is $34^{\circ}15'$.

Example 2.

Let it be required to find the greatest range of a 13-inch shell, when projected with a velocity of 2000 feet per second, and the elevation to produce that range; the diameter of the shell being 12.80 inches?

Log. of terminal velocity of a 13-inch shell, Table B = . 7.278189
Given velocity of the shell = 2000 Log. = 3.301030

Answering to which, in Table C, is $34^{\circ}49'$. . Log. = 0.579219

Log. of corresponding altitude, Table B = 3.637490
Corresponding to $34^{\circ}49'$, in Table C, is 0.441196

Greatest range, in feet = 11986 Log. = 4.078686

Hence the greatest range of a 13-inch shell, when projected with a velocity of 2000 feet, is 11986 feet, which is $2\frac{1}{4}$ miles and 106 feet; and the elevation to produce that range, is $34^{\circ}49'$.

Note.—In this Example, proportion is made for the excess of the given above the next less numbers in Table C.

PROBLEM XXII.

Given the Range at one Elevation ; to find the Range at another Elevation.

RULE.

As the logarithmic sine of twice the first elevation, is to the logarithm of its corresponding range ; so is the logarithmic sine of twice the other elevation, to the logarithm of its corresponding range.

Example 1.

If a 13-inch shell be found to range 11986 feet, when discharged at an elevation of $34^{\circ}49'$, how far will it range when the elevation is 45 degrees ; the charge of powder being the same at both elevations ?

As twice $34^{\circ}49' =$. . . $69^{\circ}38'$	Log. co-secant =	10.028036
Is to its range = 11986 feet	. . .	Log. =	. . . 4.078674
So is twice $45^{\circ}0' =$. . . $90^{\circ}0'$	Log. sine =	. . . 10.000000
To the required range, in feet = 12785		Log. =	. . . <u>4.106710</u>

Example 2.

If a shell be found to range 4760 feet, when discharged at an elevation of 45 degrees, how far will it range when the elevation is $30^{\circ}45'$; the charge of powder being the same at both elevations ?

As twice $45^{\circ} =$. . . $90^{\circ}0'$	Log. co-secant =	10.000000
Is to its range =	. . . 4760 feet	Log. =	. . . 3.677607
So is twice $30^{\circ}45' =$. . . $61^{\circ}30'$	Log. sine =	. . . 9.943899
To the required range, in feet = 4183		Log. =	. . . <u>3.621506</u>

PROBLEM XXIII.

Given the Elevation for one Range ; to find the Elevation for another Range.

RULE.

As the logarithm of the first range, is to the logarithmic sine of twice its corresponding elevation ; so is the logarithm of any other given range, to the logarithmic sine of an arch. Now, the half of this arch will be the elevation required.

“ ”

Example 1.

If a shell be found to range 11986 feet when projected at an elevation of $34^{\circ}49'$, at what elevation must it be discharged to strike an object at the distance of 12785 feet, with the same charge of powder?

As the first range = 11986	Log. ar. comp. =	5.921326
Is to twice $34^{\circ}49'$ = $69^{\circ}38'$	Log. sine =	9.971964
So is the other range = 12785	Log. =	4.106710

To arch = $90^{\circ} 0'$	Log. sine =	10.000000
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Half the arch = $45^{\circ} 0'$, the elevation required.

Example 2.

If a shell be found to range 4760 feet when discharged at an elevation of 45° , at what elevation must it be projected to strike an object at the distance of 4183 feet, with the same charge of powder?

As the first range = 4760	Log. ar. comp. =	6.322393
Is to twice 45° = 90°	Log. sine =	10.000000
So is the other range = 4183	Log. sine =	3.621488

To arch = $61^{\circ}29'45''$	Log. sine =	9.943881
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Half the arch = $30^{\circ}44'52\frac{1}{4}''$, the elevation required.

PROBLEM XXIV.

Given the Charge for one Range ; to find the Charge for another Range.

RULE.

Since the ranges at the same elevation are *nearly* proportional to the charges, therefore—As the logarithm of the first range, is to the logarithm of its corresponding charge ; so is the logarithm of the other range, to the logarithm of the charge corresponding thereto.

Example 1.

If, with a charge of 12 lbs. of powder, a shell range 5334 feet, what charge will be sufficient to throw it 2667 feet ; the elevation being 45° in both cases ?

As the first range = 5334	Log. ar. comp. =	6.272947
Is to its charge = 12lbs.	Log. =	1.079181
So is the other range = 2667	Log. =	3.426023
<hr/>			
To the required charge, in lbs. =	6.0	Log. =	0.778151

Example 2.

If, with a charge of 9 lbs. of powder, a shell range 4000 feet, what charge will be sufficient to throw it 3000 feet; the elevation being 45° in both cases?

As the first range = 4000	Log. ar. comp. =	6.397940
Is to its charge = 9 lbs.	Log. =	0.954243
So is the other range = 3000	Log. =	3.477121
<hr/>			
To the required charge, in lbs. =	6.75	Log. =	0.829304

PROBLEM XXV.

Given the Range for one Charge ; to find the Range for another Charge.

RULE.

As the logarithm of the first charge, is to the logarithm of its corresponding range ; so is the logarithm of the other charge, to the logarithm of its corresponding range ; the elevation being the same in both cases.

Example 1.

If a shell be projected 5334 feet by a charge of 12 lbs. of powder, at what distance will it strike an object when discharged with 6 lbs. of powder ; the elevation being the same in both cases?

As the first charge = 12 lbs.	Log. ar. comp. =	8.920819
Is to its range = 5334	Log. =	3.727053
So is the other charge = 6 lbs.	Log. =	0.778151
<hr/>			
To the required range, in feet =	2667	Log. =	3.426023

Example 2.

If a shell be projected 4000 feet by a charge of 9 lbs. of powder, at what distance will it strike an object when discharged with 6½ lbs. of powder ; the elevation being the same in both cases?

As the first charge = 9 lbs.	Log. ar. comp. =	9.045757
Is to its range = 4000	Log. =	. . . 3.602060
So is the other charge = 6.75	Log. =	. . . 0.829304
<hr/>			
To the required range, in feet =	3000	Log. =	. . . 3.477121

PROBLEM XXVI.

Given the Range and the Elevation ; to find the Impetus.

RULE.

As the logarithmic sine of twice the angle of elevation, is to the logarithm of half its corresponding range ; so is radius, or the logarithmic sine of 90° , to the impetus.

Example 1.

With what impetus must a shell be discharged at an elevation of $34^\circ 49'$, to strike an object at the distance of 2986 feet ?

As twice $34^\circ 49'$ = $69^\circ 38'$	Log. co-secant =	10.028036
Is to half the range = 1493	Log. =	. . . 3.174060
So is radius, or 90°	Log. sine =	. . . 10.000000
<hr/>			
To the impetus, in feet =	. . . 1592	Log. =	. . . 3.202096

Example 2.

With what impetus must a shell be discharged at an elevation of 25° , to strike an object at the distance of 2760 feet ?

As twice 25° = 50°	Log. co-secant =	10.115746
Is to half the range = 1380	Log. =	. . . 3.139879
So is radius, or 90°	Log. sine =	. . . 10.000000
<hr/>			
To the impetus, in feet =	. . . 1804	Log. =	. . . 3.255625

PROBLEM XXVII.

Given the Elevation and the Range ; to find the Time of the Flight.

RULE.

As radius, is to the logarithmic tangent of the elevation ; so is the logarithm of the range, in feet, to a logarithmic number ; which, being

divided by 2, will give the logarithm of 4 times the number of seconds taken up in the flight.

Example.

In what time will a shell range 11986 feet, at an elevation of 34°49' ?

As radius =	90° 0'	Log. co-secant =	10.000000
Is to the elevation =	34°49'	Log. tangent =	9.842266
So is the range =	11986	Log. =	4.078674

Divide by 2) 3.920940

Four times the flight =	91.30	Log. =	1.960470
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Number of seconds = 22.825 = the time of flight.

Note.—From this it is manifest that when the elevation of the mortar is 45 degrees, half the logarithm of the range will be the logarithm of 4 times the number of seconds taken up in the flight of the shell.

Remark.—The above Problem may be more readily solved in the following manner, viz.:—

To the log. tangent of the elevation, add the log. of the range, and the constant logarithm 8.795880 ;* half the sum, 20 being *previously rejected* from the index, will be the log. of the time of flight in seconds.

Example.

In what time will a shell range 3250 feet, at an elevation of 32 degrees ?

Elevation of the piece =	32°	Log. tangent =	9.795789
Range, in feet =	3250	Logarithm =	3.511883
Constant logarithm =			8.795880

Divide by 2) 2.103552

Time of flight in seconds =	11.26	Log. =	1.051776
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* This is the arithmetical complement of *twice* the logarithm of 4 ;—the log. is *doubled*, so that its root may be comprehended in the common extraction, or, division of the sum of the three terms by 2.

PROBLEM XXVIII.

Given the Range and the Elevation ; to find the greatest Altitude of the Shell.

RULE.

As radius, is to the logarithmic tangent of the elevation ; so is the logarithm of one-fourth of the range, to the logarithm of the required altitude.

Example 1.

If a shell range 11986 feet, when projected at an elevation of 34°49'; required the greatest altitude which it acquires during its flight ?

As radius =	90° 0'	Log. co-secant =	10.000000
Is to the elevation =	34°49'	Log. tangent =	9.842266
So is $\frac{1}{4}$ of the range =	2296.5	Log. =	3.476614
				<hr/>
To the altitude, in feet =	2084	Log. =	3.318880

Example 2.

If a shell range 4760 feet, when projected at an elevation of 45°; required the greatest altitude which it acquires during its flight ?

As radius =	90° 0'	Log. co-secant =	10.000000
Is to the elevation =	45° 0'	Log. tangent =	10.000000
So is $\frac{1}{4}$ of the range =	1190	Log. =	3.075547
				<hr/>
To the altitude, in feet =	1190	Log. =	3.075547

Note.—From this it is manifest, that when the elevation of the mortar is 45 degrees, one-fourth of the range will be equal to the greatest altitude at which the shell can arrive.

PROBLEM XXIX.

Given the Inclination of the Plane, the Elevation of the Piece, and the Impetus ; to find the Range.

RULE.

To twice the logarithmic secant of the inclination of the plane, add the logarithmic sine of the elevation of the piece above the plane, the

logarithmic co-sine of the elevation of the piece above the horizon, and the logarithm of 4 times the impetus: the sum of these four logarithms (rejecting 40 in the index) will be the logarithm of the required range.

Example.

How far will a shell range on a plane which ascends $10^{\circ}15'$, and also on another plane which descends $10^{\circ}15'$; the impetus being 2000 feet in both cases, and the elevation of the mortar $31^{\circ}45'$?

Solution. $31^{\circ}45' - 10^{\circ}15' = 21^{\circ}30'$, the elevation of the piece above the ascending plane;
and, $31^{\circ}45' + 10^{\circ}15' = 42^{\circ}0'$, the elevation of the piece above the descending plane.

To find the Range on the ascending Plane.

Inclination of the plane =	$10^{\circ}15'$	Twice the log. sec. =	20.013974
Elevation above the plane =	21.30	Log. sine =	. . . 9.564075
Elevation above the horizon =	31.45	Log. co-sine =	. . . 9.929599
Four times the impetus =	8000	Log. =	. . . 3.903090
Range, in feet =	. . . 2575	Log. =	. . . 3.410738

To find the Range on the descending Plane.

Inclination of the plane =	$10^{\circ}15'$	Twice the log. secant =	20.013974
Elevation above the plane =	42.0	Log. sine =	. . . 9.825511
Elevation above the horizon =	31.45	Log. co-sine =	. . . 9.929599
Four times the impetus =	8000	Log. =	. . . 3.903090
Range, in feet =	. . . 4701	Log. =	. . . 3.672174

PROBLEM XXX.

Given the Inclination of the Plane, the Elevation of the Piece, and the Range; to find the Impetus.

RULE.

To twice the logarithmic co-sine of the inclination of the plane, add the logarithmic co-secant of the elevation of the piece above the plane, the logarithmic secant of the elevation of the piece above the horizon, and the logarithm of the one-fourth of the range: the sum of these four logarithms (abating 40 in the index) will be the logarithm of the impetus.

Example.

With what impetus must a shell be discharged to strike an object at the distance of 2575 feet, on an inclined plane which ascends $10^{\circ}15'$, and, also, another object at the distance of 4701 feet, on an inclined plane which descends $10^{\circ}15'$; the elevation of the piece being $31^{\circ}45'$ in both cases?

Solution. $31^{\circ}45' - 10^{\circ}15' = 21^{\circ}30'$, is the elevation of the piece
above the ascending plane;

and, $31^{\circ}45' + 10^{\circ}15' = 42^{\circ}0'$, is the elevation of the piece
above the descending plane.

To find the Impetus on the ascending Plane.

Inclination of the plane =	$10^{\circ}15'$	Twice the log.co-sine =	19.986026
Elevation above the plane =	21.30	Log. co-secant =	. 10.435925
Elevation above the horizon =	31.45	Log. secant =	. . 10.070401
One-fourth of the range =	643.75	Log. =	. . . 2.808717
Impetus, in feet =	. . 2000	Log. =	. . . 3.301069

To find the Impetus on the descending Plane.

Inclination of the plane =	$10^{\circ}15'$	Twice the log.co-sine =	19.986026
Elevation above the plane =	42. 0	Log. co-secant =	. 10.174489
Elevation above the horizon =	31.45	Log. secant =	. . 10.070401
One-fourth of the range =	1175.25	Log. =	. . . 3.070130
Impetus, in feet =	. . 2000	Log. =	. . . 3.301046

PROBLEM XXXI.

Given the Weight of a Ball, the Charge of Powder with which it is fired, and its known Velocity; to find the Velocity of a Shell, when projected with a given Charge of Powder.

RULE.

To the arithmetical complement of half the logarithm of the weight of the shell, add half the logarithm of twice the weight of the charge, in pounds, and the constant logarithm 3.204120: the sum (abating 10 in the index) will be the velocity of the shell answering to the given charge.

Example.

If a ball of 1 lb. weight acquire a velocity of 1600 feet per second, when fired with 8 ounces of powder, it is required to find with what

velocities the several kinds of shells will be projected by the respective charges of powder expressed against them in the following table?

For the 13-inch Shell.

Weight of the shell = . . . 196	Ar.comp.of $\frac{1}{2}$ its log.=8.853872
Twice the weight of the charge= 18	Half its log.= . 0.627636 $\frac{1}{2}$
Constant log. =	3.204120
<hr/>	
Velocity, in feet = . . . 485	Log. = . . . 2.685628 $\frac{1}{2}$

For the 10-inch Shell.

Weight of the shell = . . . 90	Ar.comp.of $\frac{1}{2}$ its log.=9.022879
Twice the weight of the charge= 8	Half its log.= . 0.451545
Constant log. =	3.204120
<hr/>	
Velocity, in feet = 477	Log. = . . . 2.678544

For the 8-inch Shell.

Weight of the shell = . . . 48	Ar.comp.of $\frac{1}{2}$ its log.=9.159380
Twice the weight of the charge= 4	Half its log.= . 0.301030
Constant log. =	3.204120
<hr/>	
Velocity, in feet = 462	Log. = . . . 2.664530

Note.—The same results will be obtained by computing agreeably to the rule in Problem XVIII., page 651. For the constant logarithm see the *Note* to that Problem.

TABLE D.—*Showing the Velocities of the different sized Shells, when projected with given Charges of Powder,*

Size of Shell, in inches.	Weight of Shell, in pounds.	Charge of Powder, in lbs.	Logarithm.	Velocity, in feet.	Logarithm.
13	196	9	0.477121*	485	7.314258†
10	90	4	0.301030	477	7.321482
8	48	2	0.150515	462	7.335358
5 $\frac{1}{2}$	16	1	0.000000	566	7.247184
4 $\frac{3}{4}$	8	0 $\frac{1}{2}$	0.349485	566	7.247184

* The numbers in this column are the logarithms of the square roots of the respective charges.

† The numbers in this column are the arithmetical complements of the logarithms of the respective velocities.

PROBLEM XXXII.

Given the Elevation and the Range ; to find the Impetus, Velocity, and Charge of Powder.

RULE.

Find the impetus, by Problem XXVI., page 660 ; to the logarithm of which add the constant logarithm 1.808436 * : take half the sum, and it will be the logarithm of the required velocity. Now, to the logarithm of the velocity, thus found, add the logarithms from Table D answering to the charge and the velocity of the given shell : the sum of these three logarithms (abating 10 in the index) being doubled, will give the logarithm of the required charge of powder, in pounds.

Example.

With what impetus, velocity, and charge of powder, must a 13-inch shell be fired at an elevation of $34^{\circ}49'$, to strike an object at the distance of 11986 feet ?

To find the Impetus and the Velocity.

Twice the elevation = . . .	69°38'	Log. co-secant =	10.028036
Half the range = . . .	5993	Log. = . . .	3.777644
<hr/>			
Impetus, in feet = . . .	6392	Log. = . . .	3.805680
Constant log. = . . .			1.808436
<hr/>			
			Divide by 2) 5.614116
<hr/>			
Velocity in feet = . . .	642	Log. = . . .	2.807058

To find the Charge of Powder.

Velocity, in feet = . . .	642	Log. = . . .	2.807535
Log. of charge for a 13-inch shell, from Table D = . . .			0.477121
Log. of velocity for a 13-inch shell, from Table D = . . .			7.314258
<hr/>			
			Sum = . . .
			0.598914
<hr/>			
Charge, in pounds = . . .	15.77	Log. = . . .	1.197828

Hence the impetus is 6392 feet, the velocity 642 feet, and the charge of powder, 15.77 lbs., or 15 lbs. $12\frac{1}{2}$ oz. nearly.

* This is the logarithm of $16\frac{1}{2}$ feet, the descent of a falling body in the first second of time, increased by twice the logarithm of 2.

PROBLEM XXXIII.

Given the Inclination of the Plane, the Elevation of the Piece, and the Range ; to find the Charge of Powder.

RULE.

Find the impetus, by Problem XXX., page 663 ; with which proceed as directed in the last Problem.

Example 1.

How much powder will throw a 10-inch shell 6760 feet, on an inclined plane which ascends $7^{\circ}30'$; the elevation of the mortar being $33^{\circ}14'$?

Solution. $33^{\circ}14' - 7^{\circ}30' = 25^{\circ}44'$ is the elevation of the mortar above the ascending plane.

Inclination of the plane =	$7^{\circ}30'$	Twice the log.co-sine =	19.992538
Elevation above the plane =	25.44	Log. co-secant =	10.362327
Elevation above the horiz. =	33.14	Log. secant =	10.077562
One-fourth of the range =	1690	Log. =	3.227887

Impetus, in feet =	4574	Log. =	3.660314
Constant log. =			1.808436

Divide by 2) 5.468750

Velocity in feet =	542	Log. =	2.734375
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To find the Charge of Powder.

Velocity =	542 feet	Log. =	2.733999
Log. of charge for 10-inch shell, from Table D =			0.301030
Log. of velocity for 10-inch shell, from Table D =			7.321482

Sum = 0.356511

Charge, in pounds =	5.164	Log. =	0.713022
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Hence the charge of powder is 5.164 lbs., or 5 lbs. $2\frac{1}{2}$ oz.

Example 2.

How much powder will throw a 10-inch shell 6760 feet, on an inclined plane which descends $7^{\circ}30'$, the elevation of the mortar being $33^{\circ}14'$?

Solution. $33^{\circ}14' + 7^{\circ}30' = 40^{\circ}44'$ is the elevation of the mortar above the descending plane.

Inclination of the plane =	$7^{\circ}30'$	Twice the log. co-sine =	19.992538
Elevation above the plane =	40.44	Log. co-secant =	. . 10.185393
Elevation above the horiz. =	33.14	Log. secant =	. . 10.077562
One-fourth of the range =	1690	Log. =	. . . 3.227887

Impetus, in feet =	. . . 3044	Log. =	. . . 3.483380
Constant log. = 1.808436

Divide by 2) 5.291816

Velocity in feet =	. . . 442	Log. =	. . . 2.645908
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To find the Charge of Powder.

Velocity =	. . . 442	Log. =	. . . 2.645422
Log. of charge for 10-inch shell, from Table D =			. . . 0.301630
Log. of velocity for 10-inch shell, from Table D =			. . . 7.321482

Sum = . . 0.267934

Charge, in pounds =	. . 3.434	Log. =	. . . 0.535868
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Hence the charge is 3.434 lbs., or 3 lbs. 7 oz. nearly.

PROBLEM XXXIV.

Given the Inclination of the Plane, the Elevation of the Piece, and the Impetus ; to find the Time of Flight.

RULE.

To the logarithmic secant of the inclination of the plane add the logarithmic sine of the elevation above the plane, half the logarithm of the impetus and the constant log. 9.698970: the sum (abating 30 in the index) will be the logarithm of the time of flight, in seconds.

Example.

In what time will a 10-inch shell strike an object on an inclined plane which ascends $7^{\circ}30'$, when discharged with an impetus of 4574 feet, the elevation of the mortar being $33^{\circ}14'$; and in what time will

it strike another object on a descending plane, with the same impetus and elevation ?

Solution. $33^{\circ}14' - 7^{\circ}30' = 25^{\circ}44'$ is the elevation of the mortar
above the ascending plane ;
and, $33^{\circ}14' + 7^{\circ}30' = 40^{\circ}44'$ is the elevation of the mortar
above the descending plane.

To find the Time of Flight on the ascending Plane.

Inclination of the plane =	. . . $7^{\circ}30'$	Log. secant=	10.003731
Elevation above the plane =	. . 25.44	Log. sine =	9.637673
Impetus = 4574	Half its log.=	1.830148
Constant logarithm =		9.698970
<hr/>			
Time of flight, in seconds =	. . 14.81	Log. =	. . 1.170522

To find the Time of Flight on the descending Plane.

Inclination of the plane =	. . . $7^{\circ}30'$	Log. secant=	10.003731
Elevation above the plane =	. . 40.44	Log. sine =	9.814607
Impetus = 4574	Half its log.=	1.830148
Constant logarithm =		9.698970
<hr/>			
Time of flight, in seconds =	. . 22.256	Log. =	. . 1.347456

Note.—The constant logarithm made use of in this Rule, is the arithmetical complement of the log. of 2.

PROBLEM XXXV.

Given the Impetus and the Elevation ; to find the Horizontal Range.

RULE.

To the logarithm of the impetus add the logarithmic sine of twice the angle of elevation, and the constant logarithm 0.301030 ; the sum (abating 10 in the index) will be the logarithm of the required range on the horizontal plane.

Example 1.

Let a shell be discharged with an impetus of 1592 feet, at an elevation of $34^{\circ}49'$; required its range on the horizontal plane ?

Impetus =	1592	Log. = .	3.201943
Twice the elevation =	69°38'	Log. sine =	9.971964
Constant log. =			0.301030
<hr/>			
Horiz. range, in feet =	2985	Log. = .	3.474937

Example 2.

Let a shell be discharged with an impetus of 1804 feet, at an elevation of 25° ; required its range on the horizontal plane ?

Impetus =	1804	Log. = .	3.256237
Twice the elevation =	50°	Log. sine =	9.884254
Constant log. =			0.301030
<hr/>			
Horiz. range, in feet =	2764	Log. = .	3.441521

Note.—The constant logarithm used in this Rule is the logarithm of 2.

PROBLEM XXXVI.

Given the Impetus and the Elevation ; to find the Time of Flight on the Horizontal Plane.

RULE.

With the impetus and the elevation compute the horizontal range, by the last Problem ; then, with the horizontal range, thus found, and the elevation of the piece, compute the time of flight, by Problem XXVII., page 660. Or, the time of flight may be computed directly, by Problem XXXIV., page 668.

Example.

In what time will a 13-inch shell strike an object on a horizontal plane, when discharged with an impetus of 6392 feet, the elevation of the mortar being 34°49' ?

Impetus =	6392	Logarithm =	3.805637
Twice the elevation =	69°38'	Log. sine =	9.971964
Constant logarithm =			0.301030
<hr/>			
Horizontal range in feet =	11985	Log. = .	4.078631
Elevation of the mortar =	34°49'	Log. tangent =	9.842266
Constant logarithm =			8.795880
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		Divide by 2)	2.716777
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Time of flight, in seconds =	22.824	Log. = .	1.358388½
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small arc oo ; the intersection of which with nn , at V , will be the vertex of the *parabola* path of the shell, or the highest point in the *air* to which the projectile can ascend. Draw the dotted curves AV and VB ; the former will represent *the impetus* or *the ascending path* of the shell, and the latter its *descending path*. Now, since *those curves* may be considered as *right lines*; therefore, let the points A , V , and V, B , be connected by straight lines; and then we shall have the plane triangle AVB .—Take *half* the angle VAB in the compasses, and lay it off from b to e (that is, subtract *half the angle* at A from *half the complement* of the angle of inclination), and through the point e draw the line AD . Then the angle RAD will be the true elevation of the mortar above the plane of the horizon, and AD the correct line of direction for projecting a shell to the point B .

Now, from *the data* thus laid down, together with certain known elements that enter into the Problem, we obtain the following universal

Rule.

To the logarithm of the weight of the charge of powder in *lbs.*, add the log. ar. comp. of *half the weight* of the shell, and the constant logarithm 4.599804; the sum, abating 10 in the index, will be the logarithm of the impetus AV , or *the ascending path* of the shell.

In the right angled triangle ARB , the angle A , and the hypotenuse AB , are given; to find the perpendicular or side BR :—hence,

To the logarithm of the range AB , add the log. sine of the inclination of the plane (*viz.*, the angle RAB); the sum, abating 10 in the index, will be the logarithm of a natural number (equal BR in the diagram), which being *added* to the impetus AV , the result will be the value of VB , or *the descending path* of the shell.

Now, the range of the shell AB ; its impetus, or ascending path AV ; and *the descending path* VB , form the plane triangle AVB ; wherein all the sides are given to find *half the angle* VAB ; which is to be found by Problem IV., page 180:—then, this *half angle* being subtracted from *half the complement* of the angle of inclination, the result will be the correct angle above the horizontal plane, to which the mortar is to be elevated, so as to project a shell to the point B .

Note.—When the plane is *ascending*, the *half angle* becomes *additive*.

Example.

At what elevation must a 13-inch mortar be pointed, so as to strike an object at the distance of 6745 feet, on a plane which descends $8^{\circ}15'$; the charge of powder being $7\frac{3}{4}$ pounds?

$90^{\circ} + 8^{\circ}15' =$ the arc ad , $98^{\circ}15'$; the half of which, or $49^{\circ}7'30''$, is the angle $d A b$; then, $90^{\circ} - 49^{\circ}7'30'' = 40^{\circ}52'30'' = b A c$, or *half the complement* of the angle of inclination.

Numerical computation—*First Part.*

Weight of the given charge $7\frac{1}{2}$ lbs., or 7.375 Logarithm = 0.867762
 Weight of a 13-inch shell 196 lbs. $\frac{1}{2}$ ditto 98 Log.ar.comp. = 8.008774
 Constant logarithm* = 4.599804

Impetus or the ascend. path $A V = +2994.6$ ft. Logarithm = 3.476340

Second Part.

Given range of the shell = $A B$ 6745 feet Logarithm = 3.828982
 Inclination of the plane = $R A B$ $8^{\circ}15'$ Log. sine = 9.156830

Natural number, or part $B R = +967.9$ feet Logarithm = 2.985812

Descending path of the shell = VB 3962.5 feet

Ascending ditto, or *impetus* = AV 2994.6 Log.ar.comp. = 6.523660

Range of the shell = $A B$. 6745.0 Log.ar.comp. = 6.171018

Sum of the three sides = 13702.1

Half sum = 6851.05 Logarithm = 3.835757

Remainder = 2888.55 Logarithm = 3.460681

Sum of the four logarithms = 19.991116

Half the angle $V A B$, at the mortar = $8^{\circ}10'50''$ Co-sine = 9.995558

Half the comp. of the angle of inclin. = 40.52.30

Elevation of the mortar = . . . $32^{\circ}41'40''$ as required.

Note.—The elevation thus found is the correct answer to Example 11, in page 163, Volume II., of Dr. Hutton's "Course of Mathematics," Edition 1811.

See Table D, page 665, relative to the weights of shells; for it is the established weight of the *unfilled* shell that is to be used in the calculation.

* This expression comprehends the sum of twice the logarithm of 800 (*half* the experimented velocity of a ball) and the log. arith. comp. of $16\frac{1}{2}$ feet, the known descent of a heavy body at the earth's surface in one second of time.

PROBLEM XXXVIII.

Given the Time of Flight, and the Horizontal Range ; to find the Elevation of the Mortar.

RULE.

To twice the logarithm of the time of flight, add the log. ar. comp. of the horizontal range, *in feet*, and the constant logarithm 1.206376*; the sum will be the log. tangent of the elevation.

Example.

A shell was observed to take 16.32 seconds in ranging 6675 feet; required the elevation at which it was projected ?

Time of flight =	16.32	Twice its logarithm =	2.425510
Horizontal range =	6675 feet	Log. ar. comp. =	6.175549
Constant logarithm =			1.206376

Elevation of the mortar = 32°41'40" Log. tangent = . . . 9.807435

Note.—The horizontal range, as above, is deduced from the range on the descending plane in the preceding Example.—This is pointed out for the purpose of showing that the elevation found by the present concise Rule will be always as correct as that deduced from Problem 37.

PROBLEM XXXIX.

Given the Elevation, and the Time of Flight ; to find the Horizontal Range of a Shell.

RULE.

To twice the logarithm of the time of flight, add the log. co-tangent of the elevation, and the constant logarithm 1.206376; the sum, abating 10 in the index, will be the logarithm of the horizontal range, *in feet*.

Example.

A shell projected from a mortar at an elevation of 32°41'40", was observed to take 16.32 seconds in its flight; required the measure of its horizontal range, *in feet* ?

* This is the log. of 16 $\frac{1}{4}$ feet; the known fall of a heavy body in one second.

Time of flight = . . . 16:32 Twice its logarithm = 2.425510
 Elevation of the mortar = 32°41'40" Log. co-tangent = 10.192565
 Constant logarithm = 1.206376

Horizontal range, in feet = 6675 Logarithm = . . 3.824451

Note.—The force of gravity has a tendency to diminish the projectile velocity of a ball or shell from the instant of its exit at the muzzle of the piece : this force, which may be considered *uniform*, is exerted at the rate of $16\frac{1}{7}$, or 16.08333 feet in one second of time :—now, the logarithm of this is 1.206376 ; which, therefore, is the constant logarithm made use of in the present Problem.

Remark.—Since the above Problem is the converse of Problem XXVII. , page 660, it may be solved in the following manner, viz. :—

To twice the logarithm of quadruple the time of flight, add the log. co-tangent of the elevation ; the sum, abating 10 in the index, will be the logarithm of the horizontal range in feet. Thus :

Time of flight $16:32 \times 4 = 65:28$ Twice its logarithm = 3.629560
 Elevation of the mortar = 32°41'40" Log. co-tangent = 10.192565

Horizontal range, *in feet* = 6640 Logarithm = . . 3.822125

Note.—The horizontal range found in this manner will always be *less* than that deduced from the preceding Rule : in the present instance it is 35 feet less than the result determined by the foregoing method of calculation :—but, since that method is consonant to theory, it seems to deserve the preference.

PROBLEM XL.

Given the Elevation, and the Horizontal Range ; to find the Time of Flight.

RULE.

To the log. tangent of the elevation, add the logarithm of the horizontal range, *in feet*, and the constant logarithm 8.793624 ; take half the sum (20 being previously rejected from the index), and it will be the logarithm of the time of flight in seconds.

Example.

In what time will a shell range 6675 feet on a horizontal plane ; the mortar being elevated 32°41'40" ?

x x 2

Elevation of the mortar =	. . 32°41'40"	Log. tangent=	9.807435
Horizontal range, in feet =	. . 6675	Logarithm =	3.824451
Constant logarithm =		8.793624
Sum =			2.425510
Time of flight = 16.32 seconds	Log. =	1.212755

Note.—This Problem is similar to Problem XXVII~~4~~, page 662: the constant logarithm which is employed in its solution is the *arithmetical complement* of that made use of in Problem XXXIX.

PROBLEM XLI.

Given the Time of Flight of a Shell; to find the Length of the Fuze.

RULE.

To the logarithm of the time of flight add the constant logarithm 9.342423, for 13 and 10-inch shells,—or 9.380211, for 8, 5½, and 4½-inch shells: and the sum (abating 10 in the index) will be the logarithm of the length of the fuze, in inches.

Example 1.

Let the time of flight of a 13-inch shell be 31.75 seconds; required the length of the fuze?

Time of flight, in seconds =	. 31.75	Log. =	1.501744
Constant log. =		9.342423
Length of the fuze, in inches =	6.985	Log. =	0.844167

Example 2.

Let the time of flight of an 8-inch shell be 21.5 seconds; required the length of the fuze?

Time of flight, in seconds =	. 21. 5	Log. =	1.332438
Constant log. =		9.380211
Length of the fuze, in inches =	5.16	Log. =	0.712649

Note.—The fuzes for a 13 and a 10-inch shell are so constructed as to burn .22 of an inch in one second; and those for the smaller kind,

viz., 8, $5\frac{1}{2}$, and $4\frac{1}{2}$ -inch shells, .24 of an inch in the same space of time. Now, the logarithms of these two decimal numbers, viz., 9.342423 and 9.380211, are therefore the constant logarithms made use of in the above Rule.

Fuzes are generally marked off, by circular lines, into seconds and fractional parts of a second, so that no time may be lost in measuring and adapting them to the shells for which they are intended.

PROBLEM XLII.

To find the Time that a Red-hot Cannon Ball will take to Cool.

RULE.

If balls of iron be made *red-hot*, the times of cooling will be as the squares of their diameters. Now, it has been found *by experiment*, that an iron ball of 2 inches in diameter takes 60 minutes, or 1 hour to cool:—hence, as the square of 2 is to 1 hour; so is the square of the diameter of any *red-hot* ball, to the time, in hours, that it will take to cool;—or, as thus, by logarithms:

To twice the diameter of the given *red-hot* ball, add the constant logarithm 9.397940; the sum, abating 10 in the index, will be the logarithm of the time, *in hours*, that the ball will take to cool.

Example.

Required the time that a *red-hot* 24 lbs. ball will take to cool; its diameter being 5.6 inches?

Diameter of the ball =	5.6	Twice its log. =	1.496376
Constant logarithm =	.	.	9.397940

Required time, in hours =	7.84	Logarithm =	0.894316
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Hence, the time that a *red-hot* 24 lbs. ball will take to cool is 7.84, or 7 hours, 50 minutes, and 24 seconds.

Note.—The constant logarithm is expressed by the arithmetical complement of twice the logarithm of 2.

Remarks.

The velocities communicated to shot of the same weight, with different charges of powder, are *nearly* as the square roots of the respective weights of the charges,

When shot of different sizes are fired with the same charge of powder, their velocities are nearly in the inverse ratio of the square roots of their respective weights.

When shot of different sizes are fired with different charges of powder, the velocities which they acquire are directly as the square roots of the charge of powder, and inversely as the square roots of their respective weights.

When a shot is fired, it is never impelled by the full power of the charge; because a certain portion of the inflamed powder escapes at the vent or touch-hole, and a very considerable portion by *windage*: besides which, *some grains of the powder generally remain unignited*, and, consequently, of no expansive force in projecting the shot.

It is almost impossible to find a charge of powder in which the inflammatory matter of each grain is so well proportioned, and so uniformly active, as to make the whole susceptible of the same instantaneous degree of explosion; and hence it is that several grains always pass out of the bore, *and escape the fire*, uninjured! This assertion, paradoxical as it may appear, is supported by the unquestionable test of experiments, as thus:—

When the ground is covered with snow that is a little hardened by frost, let a person discharge a loaded musket, pointed at an object about 40 or 50 yards distant; then, if he walks slowly towards the object, and carefully examines the surface of the snow, he will be certain of finding several grains of powder, each of which will be so sound and perfect as to ignite at the touch of a spark.

Again,—When men are wounded in the face by the accidental firing of a blank cartridge from a musket, or by the unlucky blowing-up of a powder-monkey, several grains of powder will be seen buried, skin-deep, in their cheeks, &c. Now, on extracting those with a surgeon's instrument, or the point of a needle, it will be found that each grain is so perfect as to be susceptible of being inflamed on the application of a lighted match.

The expansive force of gunpowder is at the rate of about 6000 feet in one second; but, owing to the quantity lost by windage &c., as mentioned above, it is estimated that not more than three-fourths of that force, or about 4500 feet, are applied to the projection of the ball.

The windage of shot is generally about the $\frac{1}{16}$ of the calibre, or diameter of the bore; in consequence of this, balls are greatly deflected from the direction in which they are projected:—for, owing to windage, the ball, in its passage through the bore of the gun, seldom escapes without touching some point in its exit at the muzzle:—hence, if the ball strike against the right side, it will be deflected to *the left*; and, *vice versa*, should it strike against the left side, it will be deflected to

the right.—Again,—If the ball strike against the upper side, it will be forced *downwards*, so as to diminish its range; and, conversely, should it strike against the under side, it will be forced *upwards*, so as to give an increase to its customary range.—Hence, it frequently happens, that in ranges of about *half a mile*, balls are sometimes deflected to the value of 80 or 100 yards from the objects at which they are aimed.

When a ball passes so freely out of the bore as not to strike any part near the muzzle; then, it will constitute what is termed “a good shot;” but it is manifest that this is entirely owing to *chance*, and not to the skill of the gunner; for the most experienced artillerist cannot guard against the ordinary deflection of military projectiles.

It may be laid down as a *general rule*, that *the smaller the windage* of a gun is, the *less* will its shot deviate from the line of direction in which it is fired.—And, when one has the good fortune to be placed at a gun that possesses the *least* comparative degree of windage, he is sure to obtain the credit of being *the best marksman* in the ship, or in the corps to which he belongs; and thus instances frequently occur in which the commanding officer does not always extend the marks of his approbation to the most deserving gunner.

SOLUTION OF PROBLEMS IN GAUGING.

Gauging is the art of finding the number of gallons, &c., contained in any vessel.

By a Parliamentary statute, it is enacted that there is to be but one general standard gallon throughout Her Majesty's dominions of Great Britain and Ireland; which gallon is to contain 10 lbs. (avoirdupois weight) of distilled water, each pound of which is to weigh 7000 grains (troy weight): hence the standard gallon is to contain 70000 grains (troy weight) of distilled water. Now, since a cubic inch of distilled water weighs 252.458 grains (troy weight), the contents of the standard gallon may be readily reduced to cubic measure, by the following proportion; viz., as 252.458 grains : 1 inch :: 70000 grains : 277.27384357 inches; which, therefore, is the number of cubic inches in the standard gallon. And because the measure of the old standard wine gallon is 231 cubic inches, and that of the old standard ale gallon 282 such inches, we have sufficient data for obtaining proper multipliers for the reduction of the old standard wine and ale measure into the new general standard measure, and conversely. Hence,

$$277.27384357 \div 231 = 1.20031967$$
$$\text{Log.} = 0.079297$$

$$231 \div 277.27384357 = 0.83311140$$
$$\text{Log.} = 9.920703$$

$$277.27384357 \div 282 = 0.98324058$$
$$\text{Log.} = 9.992660$$

$$282 \div 277.27384357 = 1.01704508$$
$$\text{Log.} = 0.007340$$

$$\left. \begin{array}{l} \text{is the general multiplier} \\ \text{the new standard mea} \\ \text{old standard wine mea} \end{array} \right\}$$

$$\left. \begin{array}{l} \text{is the general multiplier} \\ \text{the old standard wine} \\ \text{the new standard mea} \end{array} \right\}$$

$$\left. \begin{array}{l} \text{is the general multiplier} \\ \text{the new standard mea} \\ \text{old standard ale measu} \end{array} \right\}$$

$$\left. \begin{array}{l} \text{is the general multiplier} \\ \text{the old standard ale m} \\ \text{the new standard mea} \end{array} \right\}$$

Now, the respective multipliers and their corresponding being thus obtained, the reduction of the old standard wine measure into the new general standard measure, and conversely, may be very readily performed, by means of the following Problem.

PROBLEM I.

To reduce the old standard Wine Measure into the new Imperial Measure.

RULE.

To the logarithm of the old standard wine gallons add the logarithm 9.920703, and the sum will be the logarithm of the new standard gallons.

Example.

Reduce 400 gallons of the old standard wine measure into the new general standard measure.

Given number of gallons = 400

Constant log. =

New standard gallons = 333.245

Log. = 2.60206

9.92070

Log. = 2.52276

PROBLEM II.

To reduce the new Imperial Measure into the old standard Wine Measure.

RULE.

To the logarithm of the new standard gallons add the constant logarithm 0.079297, and the sum will be the logarithm of the old standard wine gallons.

Example.

Given number of new standard gallons = 400 Log.=2.602060

Constant log. = 0.079297

Old standard wine gallons = . . 480.128 Log.=2.681357

Note.—From the above Problems it appears that the new general standard gallon is, very nearly, *one-fifth* greater than the old standard wine gallon.

PROBLEM III.

To reduce the old standard Ale Measure into the new Imperial Measure.

RULE.

To the logarithm of the old standard ale gallons add the constant logarithm 0.007340, and the sum will be the logarithm of the new general standard gallons.

Example.

Reduce 400 gallons of the old standard ale measure into the new general standard measure.

Given number of ale gallons = 400 Log. = 2.602060

Constant log. = 0.007340

New standard gallons = . 406.82 Log. = 2.609400

PROBLEM IV.

To reduce the new Imperial Measure into the old standard Ale Measure.

RULE.

To the logarithm of the new standard gallons add the constant logarithm 9.992660, and the sum will be the logarithm of the old standard ale gallons.

Example.

Reduce 400 gallons of the new general standard measure into the old standard ale measure.

$277.27384357 + 231 = 1.20031967$	}	is the general multiplier for reducing the new standard measure into the <i>old standard wine measure</i> ; and,
Log. = 0.079297		
$231 \div 277.27384357 = 0.83311140$	}	is the general multiplier for reducing the old standard wine measure into the <i>new standard measure</i> .
Log. = 9.920703		
$277.27384357 \div 282 = 0.98324058$	}	is the general multiplier for reducing the new standard measure into the <i>old standard ale measure</i> ; and,
Log. = 9.992660		
$282 + 277.27384357 = 1.01704508$	}	is the general multiplier for reducing the old standard ale measure into the <i>new standard measure</i> .
Log. = 0.007340		

Now, the respective multipliers and their corresponding logarithms being thus obtained, the reduction of the old standard wine and ale measure into the new general standard measure, and conversely, may be very readily performed, by means of the following Problems.

PROBLEM I.

To reduce the old standard Wine Measure into the new Imperial Measure.

RULE.

To the logarithm of the old standard wine gallons add the constant logarithm 9.920703, and the sum will be the logarithm of the new standard gallons.

Example.

Reduce 400 gallons of the old standard wine measure into the new general standard measure.

Given number of gallons =	400	Log. =	2.602060
Constant log. =		9.920703
New standard gallons =	333.245	Log. =	2.522763

PROBLEM II.

To reduce the new Imperial Measure into the old standard Wine Measure.

RULE.

To the logarithm of the new standard gallons add the constant logarithm 0.079297, and the sum will be the logarithm of the old standard wine gallons.

Example.

Given number of new standard gallons = 400 Log.=2.602060
 Constant log. = 0.079297

Old standard wine gallons = . . 480.128 Log.=2.681357

Note.—From the above Problems it appears that the new general standard gallon is, very nearly, *one-fifth* greater than the old standard wine gallon.

PROBLEM III.

To reduce the old standard Ale Measure into the new Imperial Measure.

RULE.

To the logarithm of the old standard ale gallons add the constant logarithm 0.007340, and the sum will be the logarithm of the new general standard gallons.

Example.

Reduce 400 gallons of the old standard ale measure into the new general standard measure.

Given number of ale gallons = 400 Log. = 2.602060
 Constant log. = 0.007340

New standard gallons = . 406.82 Log. = 2.609400

PROBLEM IV.

To reduce the new Imperial Measure into the old standard Ale Measure.

RULE.

To the logarithm of the new standard gallons add the constant logarithm 9.992660, and the sum will be the logarithm of the old standard ale gallons.

Example.

Reduce 400 gallons of the new general standard measure into the old standard ale measure.

Given number of new standard gallons = 400 Log. = 2.602060
 Constant log. = 9.992660

Old standard ale gallons = . . . 398.296 Log. = 2.594720

Note.—From the two last Problems it appears that the new general standard gallon is, very nearly, *one-sixtieth* less than the old standard ale gallon.

PROBLEM V.

Given the Dimensions of a circular-headed Cask ; to find its Content in Ale and in Wine Gallons, and also agreeably to the new general standard or Imperial Gallon.

RULE.

Divide the head diameter by the bung diameter, to two places of decimals in the quotient ; then,

Add together the logarithm for ale or wine gallons corresponding to this quotient in the first part of Table LVII., the logarithm of the bung diameter in the second part of that table, and the common logarithm of the length of the cask : the sum (abating 10 in the index) will be the logarithm of the content of the cask in ale or wine gallons. Now, to the logarithm, thus found, add the constant logarithm 0.007340 for ale gallons, or 9.920703 for wine gallons ; and the sum will be the logarithm of the true content of the cask in gallons, agreeably to the new general standard or imperial measure.

Example.

Let the bung diameter of a cask be 25 inches, its head diameter 19.5 inches, and length 31 inches ; required its content in ale and wine gallons, and also in gallons agreeably to the new general standard measure?

$19.50 \div 25 = 0.78$, quotient of the head diameter divided by the bung diameter.

First,—For Ale Gallons.

Quotient = 0.78	Log. for ale gallons =	7.362671
Bung diameter =	. . . 25 inches	Corresponding log. =	2.795880
Length of the cask =	. . . 31 inches	Common log. =	1.491362
			<hr/>
Content in ale gallons =	. . . 44.66	Log. = 1.649913
Constant log. = 0.007340
			<hr/>
Content in imperial gallons =	45.42	Log. = 1.657253

Second,—For Wine Gallons.

Quotient =	0.78	Log. for wine gallons =	7.449340
Bung diameter = . . .	25 inches	Corresponding log. =	2.795880
Length of the cask =	31 inches	Common log. = . . .	1.491362
<hr/>			
Content in wine gallons =	54.52	Log. =	1.736582
Constant log. =			9.920703
<hr/>			
Content in imperial galls. =	45.42	Log. =	1.657285

See the Example for illustrating the use of Table LVII., page 153, and also page 154.

Note.—In gauging a cask, it is to be remembered that the dimensions of the bung diameter, the head diameter, and the length of the cask, must be all taken within the cask. In measuring these dimensions, it is to be carefully observed that the bung-hole be in the middle of the cask, and that the bung-stave and the stave directly opposite thereto be both regular and even within the cask; also, that the heads of the cask be equal and truly circular: if so, the distance between the inside of the chimb to the outside of its opposite stave will be the *head diameter within the cask*, very nearly.

Remark.—The above Problem will be found exceedingly useful to Pursers in the Royal Navy, to Commissaries in the Army, and to other officers in charge of Government stores, who may have occasion to purchase beer, wine, or spirits, on Her Majesty's account, in foreign countries; because it enables them to ascertain, in a few minutes, the absolute number of gallons contained in any given quantity of liquor, of the old measure, agreeably to the established standard, or imperial measure.

PROBLEM VI.

Given the Content of a Cask lying in a horizontal Position, its Bung-Diameter, and the Depth of the Ullage or Wet Inches; to find the Quantity of Liquor in the Cask.

RULE.

Conceive the bung diameter to be represented by unity or 1 inch, and that it be divided into 10000 equal parts; then the half of this, viz., 5000, is to be considered as a *constant decimal*.

Divide the *wet inches*, or depth of the ullage, by the bung diameter, to four places of decimals in the quotient; find the difference between this quotient and the constant decimal. Now, one-fourth of this difference being subtracted from the quotient, if the latter be less than the constant decimal, or added thereto if it be more than that decimal, the difference or sum will be the *multiplier*.

Then, to the logarithm of the multiplier, thus found, add the logarithm of the content of the cask, in wine measure; and the sum will be the logarithm of the ullage, or number of gallons of liquor in the cask, in wine measure. And if to this logarithm the constant logarithm 9.920703 be added, the sum will be the logarithm of the ullage, agreeably to the imperial measure.

Note.—If the content of the cask be given in ale measure, the constant logarithm will be 0.007340.

Example 1.

Let the bung diameter of a cask be 31.25 inches, its content in wine measure 105.32 gallons, and the depth of the ullage, 11.5 inches; required the quantity of liquor in the cask?

Depth of the ullage, or wet inches, 11.5 ÷ 31.25

inches (B. D.) = .3680 quotient, .3680, which is less than the constant decimal = .5000

Difference = .1320 ÷ 4 = .330, subtractive.

Multiplier =3350 Log. = 9.525045

Content of cask, in wine measure = 105.32 gallons Log. = 2.022506

Content of ullage, in wine gallons = 35.28 Log. = 1.547551

Constant log. = 9.920703

Content of ullage, in imperial galls. = 29.39 Log. = 1.468254

Note.—If the content of the cask be given in imperial measure, let the logarithm thereof be added to the logarithm of the multiplier; and the sum will be the logarithm of the ullage.

Thus, in the above Example, let the content of the cask be given agreeably to the general standard or imperial measure; viz., 87.74 gallons; then,

Multiplier, as above =3350 Log. = 9.525045

Content of the cask, in impl. meas. = 87.74 gallons Log. = 1.943209

Content of ullage, in imperial galls. = 29.39 Log. = 1.468254

Example 2.

Let the bung diameter of a cask be 25 inches, its content in wine measure 54.52 gallons, and the depth of the ullage 15.75 inches; required the quantity of liquor in the cask?

- Depth of the ullage, or wet inches, $15.75 \div 25$
 inches (B. D.) = .6300 quotient, .6300, which is more than the
 Constant decimal = .5000 constant decimal.

Difference = . . . $1300 \div 4 =$. . . 325, additive.

Multiplier = 6625 Log.=9.821186

Content of the cask, in wine meas.= 54.52 gallons Log.=1.736582

Content of ullage, in wine gallons = 36.12 Log.=1.557768

Constant log.=9.920703

Content of ullage, in imperial galls.=30.09 Log.=1.478471

But if the content of the cask be given agreeably to the imperial standard measure, viz., 45.42 gallons, then the latter part of the operation will be as thus:—

Multiplier, as above = 6625 Log.=9.821186

Content of the cask, in impl. meas. = 45.42 gallons Log.=1.657285

Content of ullage, in imperial galls. = 30.09 Log.=1.478471

Remark.—If the dry inches of the bung diameter be made use of instead of the wet, the result of the operation will express the vacuity in the cask; and if this vacuity be added to the ullage, the sum will be the content of the cask, which will be a proof that the work is right.

Thus, in the last Example, where the bung diameter is 25 inches, and the depth of the ullage 15.75 inches, the difference of these is 9.25, which, therefore, is the number of dry inches.

Then, dry inches $9.25 \div 25$

inches (B. D.) = .3700 quotient, .3700, which is less than the con-
 Constant decimal = .5000 stant decimal.

Difference = . . $1300 \div 4 =$. . 325, subtractive.

Multiplier = 3375 Log.=9.528274

Content of the cask, in impl. meas.= 45.42 gallons Log.=1.657285

Vacuity in the cask = 15.33 Log.=1.185559

Content of the ullage = 30.09

Content of the cask = 45.42; which proves the work is
 right.

PROBLEM VII.

Given the Content of a Cask standing in a vertical or upright Position, its Length, and the Depth of the Ullage or Wet Inches ; to find the Quantity of Liquor in the Cask.

RULE.

Conceive the length of the cask to be represented by unity or 1 inch, and that it be divided into 10000 equal parts ; then, the half of this, viz., .5000, is to be considered as a *constant decimal*.

Divide the *wet inches*, or depth of the ullage, by the length of the cask, to four places of decimals in the quotient ; find the difference between this quotient and the constant decimal : now, one-tenth of this difference being subtracted from the quotient, if the latter be less than the constant decimal, or added thereto if it be more than that decimal, the difference or sum will be the multiplier.

Then, to the logarithm of the multiplier, thus found, add the logarithm of the content of the cask, in wine measure ; and the sum will be the logarithm of the ullage, or number of gallons of liquor in the cask, in wine measure. And if to this logarithm the constant logarithm 9.920703 be added, the sum will be the logarithm of the ullage agreeably to the imperial standard measure.

Note.—If the content of the cask be given in ale measure, the constant logarithm will be 0.007340.

Example 1.

Let the length of a cask, between the heads, be 39 inches, its content in wine measure 105.32 gallons, and the depth of the ullage 16.5 inches ; required the quantity of liquor in the cask ?

Depth of ullage, or wet inches, $16.5 \div 39$

inches (length) = .4231 quotient, .4231, which is less than the constant decimal = .5000

Difference =769 + 10 = . .77, subtractive.

Multiplier =4154 Log.=9.618467

Content of the cask, in wine meas.= 105.32 gallons Log.=2.022506

Content of ullage, in wine gallons = 43.75 Log.=1.640973

Constant log.=9.920703

Content of ullage, in imperial gails.= 36.45 Log.=1.561676

Note.—If the content of the cask be given agreeably to the imperial standard measure, let the logarithm thereof be added to the logarithm of the multiplier; and the sum will be the logarithm of the ullage. Thus, in the above Example, let the content of the cask be given in imperial measure; viz., 87.74 gallons; then,

Multiplier =4154	Log.=9.618467
Content of the cask, in impl. meas.=	87.74 gallons	Log.=1.943209
		<hr/>
Content of ullage, in imperial galls.=	36.45	Log.=1.561676

Example 2.

Let the length of a cask, between the heads, be 31 inches, its content in wine measure 54.52 gallons, and the depth of the ullage 18.5 inches; required the quantity of liquor in the cask?

Depth of ullage, or wet inches, 18.5+31

inches (length) = .5968 quotient, .5968, which is more than the
Constant decimal = .5000 constant decimal.

Difference =968 ÷ 10 = . . .97, additive.

Multiplier =6065	Log.=9.782831
Content of the cask, in wine galls. =	54.52	Log.=1.736582
		<hr/>

Content of ullage, in wine gallons = 33.07 Log.=1.519413
Constant log.=9.920703

Content of ullage, in imperial galls.= 27.55 Log.=1.440116

But if the content of the cask be given in imperial measure, viz., 45.42 gallons, then the latter part of the operation will be as thus:—

Multiplier, as above =6065	Log.=9.782831
Content of the cask, in impl. meas. =	45.42 gallons	Log.=1.657285
		<hr/>

Content of ullage, in imperial galls. = 27.55 Log.=1.440116

Remark.—If the dry inches of the length of the cask be made use of instead of the wet, the result of the operation will express the vacuity in the cask; and if this vacuity be added to the ullage, the sum will give the content of the cask: but this, it is presumed, does not need to be elucidated by an Example.

A TABLE

For readily finding the Number of Wine or Ale Gallons which is actually equivalent to any given Number of Gallons Imperial Standard Measure.

Imperial Measure.	Wine Measure.				Imperial Measure.	Ale Measure.				Wine Measure.	Imperial Measure.				Ale Measure.	Imperial Measure.			
	G.	Q.	P.	Gills.		G.	Q.	P.	Gills.		G.	Q.	P.	Gills.		G.	Q.	P.	Gills.
1 gill	0	0	0	1.200	1 gill	0	0	0	0.983	1 gill	0	0	0	0.833	1 gill	0	0	0	1.017
2 do.	0	0	0	2.401	2 do.	0	0	0	1.966	2 do.	0	0	0	1.666	2 do.	0	0	0	2.034
3 do.	0	0	0	3.601	3 do.	0	0	0	2.950	3 do.	0	0	0	2.499	3 do.	0	0	0	3.051
1 pt.	0	0	1	0.801	1 pt.	0	0	0	3.933	1 pt.	0	0	0	3.332	1 pt.	0	0	1	0.069
1 qt.	0	1	0	1.603	1 qt.	0	0	1	3.866	1 qt.	0	0	1	3.665	1 qt.	0	1	0	0.136
2 qts.	0	2	0	3.205	2 qts.	0	1	1	3.732	2 qts.	0	1	1	1.330	2 qts.	0	2	0	0.273
3 qts.	0	3	1	0.808	3 qts.	0	2	1	3.598	3 qts.	0	2	0	3.995	3 qts.	0	3	0	0.409
G. 1	1	0	1	2.410	G. 1	0	3	1	3.464	G. 1	0	3	0	2.660	G. 1	1	0	0	0.345
2	2	1	1	0.820	2	1	3	1	2.927	2	1	2	1	1.319	2	2	0	0	1.091
3	3	2	0	3.231	3	2	3	1	2.391	3	2	1	1	3.979	3	3	0	0	1.636
4	4	3	0	1.641	4	3	3	1	1.855	4	3	1	0	2.638	4	4	0	0	2.182
5	5	0	0	0.051	5	4	3	1	1.318	5	4	0	1	1.298	5	5	0	0	2.727
6	7	0	1	2.461	6	5	3	1	0.782	6	4	3	1	3.957	6	6	0	0	3.273
7	8	1	1	0.872	7	6	3	1	0.246	7	5	3	0	2.617	7	7	0	0	3.818
8	9	2	0	3.282	8	7	3	0	3.710	8	6	2	1	1.277	8	8	0	1	0.364
9	10	3	0	1.692	9	8	3	0	3.173	9	7	1	1	3.936	9	9	0	1	0.909
10	12	0	0	0.102	10	9	3	0	2.637	10	8	1	0	2.596	10	10	0	1	1.454
20	24	0	0	0.205	20	19	2	1	1.274	20	16	2	1	1.191	20	20	1	0	2.909
30	36	0	0	0.307	30	29	1	1	3.911	30	24	3	1	3.787	30	30	2	0	0.363
40	48	0	0	0.409	40	39	0	2	2.548	40	33	1	0	2.383	40	40	2	1	1.818
50	60	0	0	0.511	50	49	0	1	1.185	50	41	2	1	0.978	50	50	3	0	3.272
60	72	0	0	0.614	60	58	3	1	3.822	60	49	3	1	3.574	60	61	0	0	0.727
70	84	0	0	0.716	70	68	3	0	2.459	70	58	1	0	2.170	70	71	0	1	2.181
80	96	0	0	0.818	80	78	2	1	1.096	80	66	2	1	0.765	80	81	1	0	3.635
90	108	0	0	0.921	90	88	1	1	3.733	90	74	3	1	3.361	90	91	2	0	1.090
100	120	0	0	1.023	100	98	1	0	2.370	100	83	1	0	1.956	100	101	2	1	2.544
200	240	0	0	2.046	200	196	2	1	0.740	200	166	2	0	3.913	200	203	1	1	1.089
300	360	0	0	3.069	300	294	3	1	3.110	300	249	3	1	1.869	300	305	0	0	3.633
400	480	0	1	0.092	400	393	1	0	1.479	400	333	0	1	3.826	400	406	3	0	2.177
500	600	0	1	1.115	500	491	2	0	3.849	500	416	2	0	1.782	500	508	2	0	0.721
600	720	0	1	2.138	600	589	3	1	2.219	600	499	3	0	3.739	600	610	0	1	3.266
700	840	0	1	3.161	700	688	1	0	0.589	700	583	0	1	1.695	700	711	3	1	1.810
800	960	1	0	0.184	800	786	2	0	2.959	800	666	1	1	3.652	800	813	2	1	0.354
900	1080	1	0	1.207	900	884	3	1	1.329	900	749	3	0	1.608	900	915	1	0	2.898
1000	1200	1	0	2.229	1000	983	0	1	3.699	1000	833	0	0	3.565	1000	1017	0	0	1.443

Note.—In using the above Table, if the given number of gallons cannot be exactly found, or if it fall without the limits of the Table, the sum of the different quantities corresponding to the several terms which make up the given number of gallons is, in such cases, to be taken; as in the following Examples:—

Example 1.

In 1736 gallons, imperial measure, how many gallons of wine measure?

			G. Q. P. Gills.
1000	galls., impl. meas.,	are equal to	1200. 1. 0. 2. 229 W. M.
700	ditto	ditto	840. 0. 1. 3. 161 ditto.
30	ditto	ditto	36. 0. 0. 0. 307 ditto.
6	ditto	ditto	7. 0. 1. 2. 461 ditto.

Hence, 1736 galls., impl. meas., are equal to 2083. 3. 0. 0. 158 W. M.

Example 2.

In 1839 gallons, wine measure, how many gallons imperial measure?

			G. Q. P. Gills.
1000	galls., wine meas.,	are equal to	833. 0. 0. 3. 565 impl. meas.
800	ditto	ditto	666. 1. 1. 3. 652 ditto.
30	ditto	ditto	24. 3. 1. 3. 787 ditto.
9	ditto	ditto	7. 1. 1. 3. 936 ditto.

Hence, 1839 galls., wine meas., are equal to 1532. 0. 0. 2. 940 impl. meas.

Remark.—The preceding Table will be found useful to those who have occasion to purchase wine or spirits in places out of the British dominions.

A COMPENDIUM OF PRACTICAL NAVIGATION; including the direct manner of making out a *Day's Work* at sea; intended for the use of persons unacquainted with the elements of Geometry and Trigonometry.

PROBLEM I.

To reduce the Sun's Declination, as given in the Nautical Almanac, to the Time of mean Noon under any known Meridian.

RULE.

From page II., of the month in the Nautical Almanac, take out the sun's declination for noon of the given day, and note whether it is increasing or decreasing; and, at the same time, take out the variation of the sun's declination between the noons of the given and preceding

days if the longitude be east, but between those of the given and following days if the longitude be west. Then, with this variation, or difference of declination, enter Table XV., at top, and the longitude of the given meridian in the right hand column;—in the angle of meeting will be found a correction, which being applied to the declination, taken from the Nautical Almanac, agreeably to the directions expressed at the bottom of the Table, will give the sun's correct declination at the noon of the given place.

Note.—When the longitude of the given meridian, and the variation of declination cannot be exactly found in the Table; then, the sum of the proportional parts, corresponding to the several terms which make up the whole longitude and the whole variation, will be the correction of declination required.

Example 1.

Required the sun's declination at noon, August 10th, 1825*, in longitude 100°30' East?

Variation of declination between given and preceding noons (the longitude being east) is 17'26"

Sun's declination at noon of the given day per Nautical Almanac (<i>decreasing</i>) =	15°36'30" N.
Pro. pt. to lon. 90° 0' and var. 17' 0" =	4'15" 0"
Ditto . . 90. 0 ditto 0.26 =	0. 6.30
Ditto . . 10.30 ditto 17. 0 =	0.29.45
Ditto . . 10.30 ditto 0.26 =	0. 0.45.30

Correction of dec. additive =	4'52" 0"30" + 4'52"
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Sun's reduced, or corrected declination =	15°41'22" N.
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Example 2.

Required the sun's declination at noon, April 3rd, 1825*, in longitude 75°45' West?

* It is the nautical or sea day that is made use of in this and the following Examples:—this day, like the civil, begins at midnight, and ends at the following midnight:—it is divided into two parts, of 12 hours each; the first part or that contained between midnight and noon is called A.M. or ante meridiem, and the other part, or that between noon and midnight, P.M. or post meridiem.

Variation of declination between given and following noons (the longitude being west) is 22'.54"

Sun's declination at noon of the given day, per Nautical Almanac (<i>increasing</i>) =		5°18'40" N.
Pro. pt. to lon. 75° 0'	and var. 22' 0" =	4'35" 0"
Ditto 0.45	ditto 22. 0 =	0. 2.45
Ditto 75. 0	ditto 0.54 =	0.11.15
Ditto 0.45	ditto 0.54 =	0. 0. 6.45

Correction of declination, additive = 4'49" 6"45" + 4'49"

Sun's reduced, or corrected declination = 5°23'29" N.

PROBLEM II.

Given the Sun's Meridian Altitude, to find the Latitude of the Place of Observation.

RULE.

Reduce the sun's declination to the meridian of the given place by the preceding Problem.

Then, to the observed altitude of the *sun's lower limb* add the difference between its semidiameter (page II. of the month in the Nautical Almanac) and the dip of the horizon (Table II.), and the sum will be the apparent altitude of the sun's centre; from which, let the difference between the parallax and refraction answering thereto (Tables VII. and VIII.) be subtracted, and the remainder will be the sun's true central altitude; which being taken from 90 degrees will leave the sun's meridional zenith distance of a contrary denomination to that of its observed altitude. Now,

If the sun's meridian zenith distance and its reduced declination are both north, or both south, their sum will be the latitude of the place of observation: but if one be north and the other south, their difference will be the latitude, and always of the same name with the greater term.

Example 1.

April 10th, 1825, in longitude 75° west, the meridian altitude of the sun's lower limb was 57°40'30" south, and the height of the observer's eye above the level of the sea 22 feet; required the latitude of the place of observation?

Variation of the sun's declination between the given and following noons (the longitude being west) is 22'.6".

Sun's declination at noon of the given day, per Nautical

Almanac (*increasing*) = 7°56'42" N.
Propl. part to long. 75°0' and var. 22' 0" = 4'35" 0"
Ditto . . . 75.0 ditto 0. 6 = 0. 1.15

Correction of declination, additive = . . . 4'36"15" + 4'36"

Sun's reduced, or corrected declination = 8° 1'18" N.

Observed altitude of the sun's lower limb = 57°40'30" S.
Sun's semidiameter = . . 15'59" }
Dip of the hor. for 22 feet = 4.30 } difference, add 11'29"

Apparent altitude of the sun's centre = 57°51'59" S.
Parallax 0'5" refrac. 0'35", diff. = 0'30" subtractive = 0'30"

Sun's true central altitude = 57°51'29" S.

Sun's meridional zenith distance = 32° 8'31" N.
Sun's reduced declination = 8. 1.18 N.

Latitude of the place of observation = 40° 9'49" N.

Example 2.

September 21st, 1825, in longitude 60° east, the meridian altitude of the sun's lower limb was 56°26' north, and the height of the observer's eye above the level of the sea 26 feet; required the latitude of the place of observation?

Variation of the sun's declination between the given and preceding noons, the longitude being east, is 23'22".

Sun's declination at noon of the given day, per Nautical

Almanac (*decreasing*) = 0°43'34" N.
Propl. part to long. 60°0' and var. 23' 0" = 3'50" 0"
Ditto . . . 60.0 ditto 0.22 = 0. 3.40

Correction of declination, additive = . . . 3'53"40" + 3'54"

Sun's reduced or corrected declination = 0°47'28" N.

Observed altitude of the sun's lower limb =	. . .	56°26' 0" N.
Sun's semidiameter 15'58" dip of the horizon for 26		
feet = 4'52" difference =	+ 11' 6"
<hr/>		
Apparent altitude of the sun's centre =	56°37' 6" N.
Parallax 0'5" refrac. 0'37" diff. = 0'32" subtractive =		0'32"
<hr/>		
Sun's true central altitude =	56°36'34" N.
<hr/>		
Sun's meridional zenith distance =	33°23'26" S.
Sun's reduced declination =	0.47.28 N.
<hr/>		
Latitude of the place of observation =	32°35'58" S.

PROBLEM III.

Given the difference of Longitude between two Places, both under the same Parallel of Latitude ; to find their Distance.

RULE.

To the logarithmic co-sine of the latitude, add the logarithm of the difference of longitude, in miles ; and the sum, abating 10 in the index, will be the logarithm of the distance.

Example.

Required the distance between Portsmouth, in longitude 1°6' west, and Green Island, Newfoundland, in longitude 55°35' west, their common latitude being 50°47' north ?

Long. of Portsmouth = . . . 1° 6' W.
 Long. of Green Island = . . . 55.35 W.

Difference of longitude = . . . 54°29' = 3269 ms. Log. 3.514415
 Latitude of the parallel=50°47'N. Log. co-sine = . 9.800892

Distance, in miles = . . . 2066.8 Log. = . . . 3.315307

PROBLEM IV.

Given the Distance between two Places, both under the same Parallel of Latitude ; to find their Difference of Longitude.

RULE.

To the logarithmic secant of the latitude, add the logarithm of the distance, and the sum, abating 10 in the index, will be the logarithm of the difference of longitude.

Example.

A ship from Cape Clear, in latitude $51^{\circ}25'$ north, and longitude $9^{\circ}29'$ west, sailed due west 1040 miles ; required the longitude at which she then arrived ?

Lat. of the parallel =	. . .	$51^{\circ}25'$	Log. secant =	. . .	10. 205057
Distance sailed =	. . .	1040 miles	Log. =	. . .	3. 017033

Difference of long. =	. . .	$27^{\circ}48'$ W.	= 1667.6 miles	Log. 3. 222090
Longitude sailed from =	. . .	9.29 W.		

Longitude arrived at = . . . $37^{\circ}17'$ W.

Note.—The above two Problems are essentially useful when a ship sails upon a parallel of latitude ; that is, when she steers either due east or due west.

PROBLEM V.

Given the Latitudes and Longitudes of two Places ; to find the Course and Distance.

RULE.

From the logarithm of the difference of longitude, the index being augmented by 10, subtract the logarithm of the meridional difference of latitude ; the remainder will be the logarithmic tangent of the course :—then, to the logarithmic secant of the course, thus found, add the logarithm of the difference of latitude, and the sum, abating 10 in the index, will be the logarithm of the distance.

Example.

Required the course and distance between Cape Bajoli, in latitude $40^{\circ}3'$ north, longitude $3^{\circ}52'$ east, and Cape Sicie, in latitude $43^{\circ}2'$ north, and longitude $5^{\circ}58'$ east ?

Lat. of C. Bajoli $40^{\circ} 3' N.$ Merid. pts. 2626.6 Longitude $3^{\circ} 52' E.$
 Lat. of C. Sicie= $43. 2 N.$ Merid. pts. 2865.8 Longitude $5. 58 E.$

Diff. of latitude= $2^{\circ} 59'$ Merid. diff. lat.=239.2 Diff. long. $2^{\circ} 6'$
 $= 179$ miles. $= 126$ miles.

To find the Course.

Difference longitude 126 miles . . Logarithm = 2.100371
 Merid. diff. of latitude 239 miles . Logarithm = 2.378398

Course N. $27^{\circ} 47' 53'' E.$ = . . . Log. tang. = 9.721973

To find the Distance.

Course $27^{\circ} 47' 53''$. . . Log. secant = . 10.053254
 Difference latitude= 179 miles Logarithm = . 2.252853

Distance in miles = 202.3 Logarithm = . 2.306107

Hence the true course is N. $27^{\circ} 47' 53'' E.$, or N.N.E. $\frac{1}{2} E.$ nearly,
 and the distance $202\frac{1}{2}$ miles.

Note.—The true course is to be reduced to the magnetic or compass course by Problem V., page 576.

PROBLEM VI.

Given the Latitude and Longitude of the Place sailed from, with the Course and Distance ; to find the Latitude and Longitude of the Place come to.

RULE.

To the logarithmic co-sine of the course,* add the logarithm of the distance ; the sum, abating 10 in the index, will be the logarithm of the difference of latitude ; which being applied to the latitude left by addition or subtraction, according as the latter is increasing or decreasing, the sum, or difference, will be the latitude come to. Now, to the logarithmic tangent of the course, add the logarithm of the meridional

* The course steered, per compass, is to be reduced to the true course by Problem VI., page 577.

difference of latitude; the sum, abating 10 in the index, will be the logarithm of the difference of longitude; which being applied by addition or subtraction to the longitude left, according as the latter is increasing or decreasing, the sum or difference will be the longitude come to.

Example 1.

A ship from Cape Ortegal, in latitude $43^{\circ}47'$ N. and longitude $7^{\circ}49'$ W., sailed N.W. $\frac{1}{2}$ N. 560 miles; required the latitude and longitude of the place come to?

To find the Difference of Latitude.

Course steered = $3\frac{1}{2}$ points	Log. co-sine = 9.888185
Distance sailed 560 miles	Logarithm = 2.748188
	<hr/>
Difference of latitude 432.8 miles .	Logarithm = 2.636373

To find the Latitude come to.

Latitude of Cape Ortegal $43^{\circ}47'$ N.	Meridional parts . 2927.8
Diff. of lat. 432.8 N. = 7.13 N.	
	<hr/>

Latitude come to . . . $51^{\circ} 0'$ N.	Meridional parts . 3568.8
	<hr/>

Meridional difference of latitude =	641.0
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To find the Difference of Longitude, and hence the Longitude come to.

Course steered = $3\frac{1}{2}$ points.	Log. tang. = 9.914173
Meridional difference of latitude = 641 miles.	Logarithm = 2.806858
	<hr/>

Difference of long. = $8^{\circ}46'$ W. = 526 miles.	Logarithm = 2.721031
Long. of C. Ortegal = 7.49 W.	
	<hr/>

Long. come to = . $16^{\circ}35'$ W.

Remarks.—When a ship decreases her latitude; that is, when the difference of latitude made good is of a different name to the latitude sailed from; then, if the difference of latitude, expressed in degrees, be greater than the latitude left, their difference will be the latitude come to; which will be of a contrary denomination to that sailed from; because, in this case, it is evident that the ship must have crossed the Equator.

And, when a ship decreases her longitude; that is, when the dif-

ference of longitude made good is of a contrary name to the longitude sailed from ; then, if the difference of longitude, expressed in degrees, be greater than the longitude left, their difference will be the longitude come to ; which will be of a contrary name to that sailed from ; because, in this case the ship will have crossed the meridian whence the longitude is reckoned.

Again.—When a ship increases her longitude ; that is, when the difference of longitude made good, expressed in degrees, is of the same name with the longitude sailed from, their sum will be the longitude come to ; but, if this sum exceeds 180 degrees, then, its difference to 360 degrees will express the longitude come to, which will be of a contrary denomination to that sailed from ; for, in this case, also, the ship will have crossed the meridian that the longitude was reckoned from :—see Problems, Rules, and Remarks, between pages 211 and 217.

Example 2.

A ship from the Island of Annabona, in latitude $1^{\circ}23'$ S., and longitude $5^{\circ}34'$ E., sailed W. N. W. 546 miles ; required the latitude and longitude of the place at which she arrived ?

To find the Difference of Latitude.

Course steered = 6 points	Log. co-sine =	9.582840
Distance sailed 546 miles	Logarithm =	2.737193
				<hr/>
Difference of latitude 208.9 miles =		Logarithm =	2.320033

To find the Latitude come to.

Latitude sailed from =	. .	1°23' S.	Merid. parts =	. .	83.0
Diff. lat. = 208.9 miles =	. .	3.29 N.			

Latitude come to =	$2^{\circ} 6'$ N.	Merid. parts =	126.0
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Meridional difference of latitude =	209.0
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To find the Difference of Longitude, and hence the Longitude come to.

Course steered =	6 points.	Log. tang. =	10.382776
Meridional difference of lat. =	209 miles.	Logarithm =	2.320146	
<hr/>				

Diff. of long. =	8°25' W. = 504.6 miles.	Logarithm =	2.702922
Long. sailed from	5.34 E.		

Long. come to = $2^{\circ}51'$ W.

Hence, the latitude come to is $2^{\circ}6'$ N. and the longitude $2^{\circ}51'$ W.

Example 3.

A ship from Pitt's Island, in latitude $2^{\circ}54'$ N. and longitude $174^{\circ}30'$ E. sailed S. E. by E. $\frac{1}{2}$ E. 760 miles; required the latitude and longitudes of the place at which she arrived?

To find the Difference of Latitude.

Course steered = $5\frac{1}{2}$ points.	Log. co-sine = 9.673387
Distance sailed 760 miles.	Logarithm = 2.880814
Diff. of lat. = 358.4 miles.	Logarithm = <u>2.554201</u>

To find the Latitude come to.

Latitude sailed from = $2^{\circ}54'$ N.	Merid. parts = 174.1
Diff. of lat. 358.4 miles = 5.58 S.	

Latitude come to =	$3^{\circ}4'$ S.	Merid. parts = <u>184.1</u>
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Meridional difference of latitude =	<u>358.2</u>
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To find the Difference of Longitude, and hence the Longitude come to.

Course steered =	$5\frac{1}{2}$ points.	Log. tangent = 10.272043
Meridional diff. of lat. =	358.2 miles.	Logarithm = <u>2.554126</u>

Diff. of long. made good = $11^{\circ}10'$ E.	= 670 miles.	Log. = 2.826169
Longitude sailed from =	174.30 E.	

Sum = <u>$185^{\circ}40'$ E.</u>

Longitude come to =	$174^{\circ}20'$ W.
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Hence, the latitude come to is $3^{\circ}4'$ S., and the longitude $174^{\circ}20'$ west.

PROBLEM VII.

Given both Latitudes and the Course; to find the Distance Sailed and the Longitude come to.

RULE.

To the logarithmic secant of the course,* add the logarithm of the difference of latitude; the sum, abating 10 in the index, will be the logarithm of the distance.—Then,

* The course steered, per compass, is to be reduced to the true course by Problem VI., page 577.

To the logarithmic tangent of the course, add the logarithm of the meridional difference of latitude; the sum, abating 10 in the index, will be the logarithm of the difference of longitude; which being applied to the longitude left by addition or subtraction, according as it is increasing or decreasing, the sum or difference will be the longitude come to.

Example.

A ship, from a place in latitude $3^{\circ}4'$ S., and longitude $174^{\circ}20'$ W., sailed N. W. by W. $\frac{1}{2}$ W. until she was found, by observation, to be in latitude $2^{\circ}54'$ N.; required the distance sailed, and the longitude at which the ship arrived?

Lat. sailed from =	$3^{\circ} 4' \text{ S.}$	Merid. parts =	184.1 miles.
Latitude come to =	2.54 N.	Merid. parts =	174.1 miles.
<hr/>			
Diff. of latitude =	$5^{\circ}58' = 358 \text{ ms.}$	Merid. diff. lat. =	358.2 miles.

To find the Distance Sailed.

Course =	$5\frac{1}{2}$ points.	Log. secant =	10.326613
Difference of latitude =	358 miles.	Logarithm =	2.553883
<hr/>			
Distance sailed =	759.4 miles.	Logarithm =	2.880496

To find the Difference of Longitude.

Course =	$5\frac{1}{2}$ points.	Log. tangent =	10.272043
Merid. diff. latitude =	358.2	Logarithm =	2.554128
<hr/>			
Difference of longitude =	670.1 miles.	Logarithm =	2.826169
<hr/>			
Longitude sailed from =	$174^{\circ}20' \text{ W.}$		
Difference of longitude made good 670 miles =	11.10 W.		
<hr/>			
		Sum =	$185^{\circ}30' \text{ W.}$
<hr/>			
Longitude come to =	$174^{\circ}30' \text{ E.}$		

Note.—The three last Problems comprehend all the cases that usually occur in the practical part of Mercator's sailing;—for the speculative cases, see pages from 236 to 248, inclusive.

PROBLEM VIII.

*To find the Course, Distance, Difference of Latitude, and Difference of Longitude made good upon compound Courses, and also the Bearing and Distance from a Ship to the Place to which she is bound, viz. :—
To make out a Day's Work at Sea.*

RULE.

Make a Table of any convenient size, and divide it into six columns :—in the first of these place the several courses, taken from the log board (corrected for lee-way, if any, and also for variation), and in the second place their corresponding distances.—The third and fourth columns are to contain the differences of latitude, and, therefore, to be marked N. S. at top ; and the fifth and sixth the departures, or meridian distances, which are to be marked at top, also, with the letters E. W.—Now,

Enter the general Traverse Table, and take out the difference of latitude and departure answering to each *corrected course* and distance, and place them in their respective columns :—then, the difference between the sums of the N. and S. columns will be the whole difference of latitude made good, of the same name with the greater ; and the difference between the sums of the E. and W. columns will be the whole departure made good, of the same name with the greater term.

Remark.—The courses, taken from the log board, are to be corrected for variation, and lee-way, if any, in the following manner, viz.

If the variation be easterly, it is to be allowed to the right hand of the course steered by compass ; but to the left hand if it be westerly.*—And,

If the larboard tacks be aboard, the lee-way is to be allowed to the right hand of the course steered by compass ; but, to the left hand if the starboard tacks be aboard.

To find the Course and Distance made good.

From the logarithm of the departure, the index being increased by 10, subtract the logarithm of the difference of latitude ; the remainder will be the logarithmic tangent of the course.—Then,

To the logarithmic secant of the course, thus found, add the logarithm of the difference of latitude, and the sum, abating 10 in the index, will be the logarithm of the distance.

* See Problem VI., page 577.

To find the Latitude in, by Account, or Dead Reckoning.

If the difference of latitude, and the latitude of the place from which the ship's departure was taken, or the yesterday's latitude, be of the same name; their sum will be the latitude in, by account: but if of contrary names, their difference will be the latitude in, of the same name with the greater term.

To find the Difference of Longitude; and thence the Longitude come to.

To the logarithmic tangent of the course made good, add the logarithm of the meridional difference of latitude (by observation); the sum, abating 10 in the index, will be the logarithm of the difference of longitude.—Now, if the difference of longitude, and the longitude of the place from which the ship's departure was taken, or the yesterday's longitude, be of the same name; their sum will be the longitude in, by account, when it does not exceed 180 degrees; otherwise it is to be taken from 360 degrees, and the remainder will be the longitude in, of a contrary name to that left:—but, if the difference of longitude, and the longitude left be of contrary names, their difference will be the longitude come to, of the same name with the greater term.

To find the Bearing and Distance of the Ship to the Port, or Place to which she is Bound.

From the logarithm of the difference of longitude between the ship and the place to which she is bound, the index being increased by 10, subtract the logarithm of the meridional difference of latitude; the remainder will be the logarithmic tangent of the course. Then,—To the logarithmic secant of the course, thus found, add the logarithm of the difference of latitude, and the sum, rejecting radius, will be the logarithm of the distance.

Note.—The true bearing, or course thus found, may be reduced to the magnetic, or compass course, if necessary, by allowing the value of the variation to the right hand thereof if it be westerly; but, to the left hand, if easterly:—this being the *converse* of reducing the course steered by compass to the true course.*

And this rule comprises the substance of that nautical operation, which is generally termed making out a day's work at sea.

* See Problem V., page 576.

Example 1.

A ship from Cape Espichell, in latitude 38°25' N. and longitude 9°13' W. bound for Porto Santo, in latitude 33°3' N. and longitude 16°17' W., by reason of contrary winds was obliged to sail upon the following courses: viz. (with the larboard tacks aboard), W. by S. 56 miles; N. W. by W. 110 miles; W. N. W. 95 miles (and then with the starboard tacks aboard); S. by E. $\frac{1}{2}$ E. 50 miles; S. by W. $\frac{3}{4}$ W. 103 miles; and S. S. W. 116 miles, when she was found by observation to be in latitude 34°17' N. and longitude 13°42' W.; the lee-way on each of the courses was about half a point; the variation was two points westerly on the three first courses, and 1 $\frac{1}{4}$ point on the three last; required the true course and distance made good; the latitude and longitude at which the ship arrived by account; and the direct course and distance between her true place, by observation, and the port to which she is bound?

TRAVERSE TABLE.					
Corrected Courses.	Dis- tances.	Difference of Latitude.		Departure.	
		N.	S.	E.	W.
S.W.b.W. $\frac{1}{4}$ W.	56	—	26.4	—	49.4
W.byN. $\frac{1}{4}$ N.	110	31.9	—	—	105.3
W. $\frac{1}{2}$ N.	95	9.3	—	—	94.5
S.E. $\frac{1}{4}$ S.	50	—	37.0	33.6	—
S. $\frac{1}{2}$ E.	103	—	102.5	10.1	—
S. $\frac{1}{4}$ E.	116	—	115.9	5.7	—
		41.2	281.8	49.4	249.2
			41.2		49.4
		Diff. lat.	240.6	Departure	199.8

To find the Course made good.

Departure = 199.8 miles. Logarithm = 2.300596
Difference of latitude = . . 240.6 miles. Logarithm = 2.381296
Course made good S. 39°42'25" W. . . . Log. tang. = 9.919300

To find the Distance made good.

Course made good = S. 39°42'25" W. Log. secant = 10.113892
Difference of latitude = 240.6 miles. Logarithm = 2.381296
Distance made good = 312.7 miles. Logarithm = 2.495188

Hence, the course made good is S. $39^{\circ}42'25''$ W. or S. W. $\frac{1}{4}$ S. nearly, and the distance 313 miles nearly.

To find the Latitude and Longitude come to by Account, or Dead Reckoning.

Latitude sailed from $38^{\circ}25' N.$. . . $38^{\circ}25' N.$ M. pts. = 2500.1 ms.

Diff. of lat. made

good = 240.6 ms. = $4^{\circ} 1' S.$

Lat. come to by acc. = $34^{\circ}24' N.$ By ob. = $34^{\circ}17' N.$ M. pts. = 2192.0 ms.

Meridional difference of latitude by observation = . . . 308.1 ms.

Meridional difference of lat. = 308.1 miles. Logarithm = 2.488692

Course made good = . S. $39^{\circ}42'25''$ W. Log. tang. = 9.919300

Diff. of long. made good = $4^{\circ}16' W.$ = 255.8 ms. Log. = 2.407992

Longitude sailed from = $9.13 W.$

Long. come to by acct. = $13^{\circ}29' W.$

To find the Course and Distance from the Ship to her intended Port.

Lat. of ship by ob. = $34^{\circ}17' N.$ Mer. pts. = 2192.0 Long. = $13^{\circ}42' W.$

Lat. of Porto Santo = $33. 3 N.$ Mer. pts. = 2103.1 Long. = $16.17 W.$

Diff. of latitude = $1^{\circ}14'$ Mer. diff. lat. 88.9 Diff. long. $2^{\circ}35'$

= 74 miles.

= 155 miles.

Difference of longitude = . . . 155 miles. Logarithm = 2.190332

Merid. diff. latitude = . . . 88.9 miles. Logarithm = 1.948902

Course = S. $60^{\circ}9'49''$ W. Log. tangent = 10.241430

Course = S. $60^{\circ}9'49''$ W. Log. secant = 10.303185

Difference of latitude 74 miles. . . . Logarithm = 1.869232

Distance = . . . 148.7 miles. . . . Logarithm = 2.172417

Hence,—The course made good is S. $39^{\circ}42'25''$ W. or S.W. $\frac{1}{4}$ S. nearly.

Distance made good = 313 miles.

Latitude come to by account = . . . $34^{\circ}24' N.$

Latitude by observation = $34^{\circ}17' N.$

Longitude come to by account = . . . $13^{\circ}29' W.$

Longitude by observation = $13^{\circ}42' W.$

Porto Santo bears S. $60^{\circ}9'49''$ W. or S.W. by W. $\frac{1}{4}$ W. nearly.

Distant 149 miles.

Note.—If the variation be one point and three-quarters west, the ship must steer W. b. S., by compass.

Example 2.

A ship from Port Royal, Jamaica, in latitude 17°58' N., and longitude 76°53' W., got under weigh for Hayti, St. Domingo, in latitude 18°30' N., and longitude 69°49' W., and sailed upon the following courses ; viz., S. 40 miles, S. E. b. S. 97 miles, N. b. E. 72 miles, S. E. $\frac{1}{4}$ S. 108 miles, N. b. E. $\frac{1}{2}$ E. 114 miles, S. E. 126 miles, N. N. E. 86 miles ; and then by observation was found to be in latitude 16°55' N., and longitude 72°30' W. ; the lee-way on each of those courses was a quarter of a point (the wind being between S. E. b. E. $\frac{1}{4}$ E. and E. b. N. $\frac{1}{4}$ N.), and the variation of the compass half a point easterly ; required the true course and distance made good, the latitude and longitude at which the ship arrived by account, with the direct course and distance between her true place by observation and the port to which she is bound ?

TRAVERSE TABLE.					
Corrected Courses.	Dis- tances.	Difference of Latitude.		Departure.	
		N.	S.	E.	W.
S. $\frac{1}{4}$ W.	40	—	39.6	—	5.9
S.S.E. $\frac{1}{4}$ E.	97	—	87.7	41.5	—
N.b.E. $\frac{1}{4}$ E.	72	69.8	—	17.5	—
S.S.E. $\frac{1}{4}$ E.	108	—	92.6	55.5	—
N.b.E. $\frac{1}{4}$ E.	114	107.3	—	38.4	—
S.E. $\frac{1}{4}$ S.	126	—	101.2	75.1	—
N.N.E. $\frac{1}{4}$ E.	86	77.7	—	36.8	—
		254.8	321.1 254.8	264.8 5.9	5.9
		Diff. of Lat.	66.3	258.9 =	Departure.

To find the Course made good.

Departure = 258.9 miles. Log. = 2.413132

Difference of latitude = . 66.3 miles. Log. = 1.821514

Course = . . . S. 75°38'10" E. Log. tangent = 10.591618

To find the Distance made good.

Course made good =	S. 75°38'10" E.	Log. secant=	10.605409
Difference of latitude =	66.3 miles.	Log. =	1.821514
Distance =	267.3 miles.	Log. =	2.426923

To find the Latitude and Longitude come to by Account, or Dead Reckoning.

Lat. sailed from=	17°58'N.	17°58'N. Mer.pts.=	1096.1 miles.
Diff. of lat. made good	66.3 miles	=	1. 6 S.

Lat. come to by acc.	16°52'N.	By obs. 16°55'N. Mer.pts.=	1030.1 miles.
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Meridional difference of latitude, by observation =	66.0 miles.
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Meridional difference of latitude =	66 miles.	Log. =	1.819544
Course made good =	S. 75°38'10" E.	Log. tangent=	10.591618

Difference of long. made good =	4°18'E.	=257.7 miles.	Log.=2.411162
Longitude sailed from =	76.53 W.		

Long. come to by account =	72°35'W.
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To find the Course and Distance from the Ship to her intended Port.

Lat. of ship by obs.=	16°55'N.	Merid.pts.=	1030.1	Long.=	72°30'W.
Lat. of Hayti =	18.30 N.	Merid.pts.=	1129.8	Long.=	69.49 W.

Diff. of latitude =	1°35'	Mer. diff. lat.=	99.7	Diff. long. 2°41'
	= 95 miles			= 161 miles.

Difference of longitude =	161 miles.	Log. =	2.206826
Meridional difference of latitude =	99.7 miles.	Log. =	1.998695

Course =	N. 58°13'55" E.	Log. tang. =	10.208131
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Course =	N. 58°13'55" E.	Log. secant=	10.278617
Difference of latitude =	95 miles.	Log. =	1.977724

Distance =	180.4 miles.	Log. =	2.256341
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Hence,—The course made good is S. $75^{\circ}38'10''$ E., or E. b. S. $\frac{1}{4}$ S. nearly.

Distance made good = $267\frac{1}{2}$ miles.

Latitude come to, by account = . . . $16^{\circ}52'$ north.

Latitude by observation = $16^{\circ}55'$ north.

Longitude come to, by account = . . . $72^{\circ}35'$ west.

Longitude, by observation = $72^{\circ}30'$ west.

The true course from the ship to Hayti is N. $58^{\circ}13'55''$ E., or
N. E. b. E. $\frac{1}{4}$ E. nearly.

The course, by compass, is N. E. $\frac{3}{4}$ E.

And the distance $180\frac{1}{2}$ miles nearly.

Note.—For the method of making out a day's work by *inspection*, see Problem IX., page 249.

OF THE LOG-BOOK.

A Log-Book is a true and correct register of all the various transactions which happen on board of a ship, whether at sea or in harbour: such as, coming to an anchor, getting under weigh, loosing or furling sails, mooring or unmooring, making or shortening sail, mustering at quarters or by divisions, exercising great guns and small arms, &c. &c. &c. This book should be a faithful transcript of the log-board.

The sea day, like the civil, begins at midnight in the Royal Navy, and ends at the midnight following: it is, however, divided into two parts, each consisting of 12 hours. The first 12 hours, or those contained between midnight and noon, are denoted by A. M., which signifies ante meridiem, or *before mid-day*; and the other 12 hours, or those from noon to midnight, are denoted by P. M., which signifies post meridiem, or *after mid-day*. The reckoning, however, is kept from noon to noon, the same as in the merchant-service.

When a ship is bound to a distant port or place, the bearing and distance of that port or place must be previously computed, by Problem V., page 694. The bearing or true course, thus determined, must be reduced to the compass course, by applying the variation to the right hand thereof if it be westerly, but to the left hand if easterly:—(see Problem V., page 576). If islands, capes, or head-lands intervene, it will be necessary to find the several courses and distances between each successively; making proper allowance for the variation.

At the time of leaving the land, the bearing of some point or place is to be carefully observed, whose latitude and longitude are known; which, together with the estimated distance of the ship from such point

or place, is to be noted down on the log-board. This is called taking a *departure*.

As the distance inferred from estimation is very susceptible of error, particularly in hazy weather, or when that distance is considerable, it will be advisable to make use of the following method in taking a *departure*; viz., Let the bearing of some well-known place be observed, and, when the ship has run a convenient distance, on a direct course, let the bearing of the same well-known place be again observed; then there will be a triangle formed, in which there is one side given: that is, the distance sailed between the times of observation, and all the angles, to find the distance between the ship and the place observed. This may be done by Problem I., Oblique Sailing, page 256; or it may be very readily determined by means of a good chart. In like manner may a departure be taken from a light-house at night.

In making out the first day's work after leaving the land, especial care must be taken, in setting down the bearing and distance of the departure in a traverse table, to make use of the opposite point of the compass to that bearing; and, also, to make due allowance for the variation. Thus, if the object from which the departure was taken bore N. E. b. E., and the variation of the compass be 2 points westerly, then the true course for the traverse Table is S. W. b. S.; abreast of which, in the proper column, is to be placed the estimated or computed distance.

The course steered, is indicated by the compass; the distance sailed, in a given time, is determined by the log-line and the half-minute or quarter-minute glass. In Her Majesty's Royal Navy, the log is hove once in every hour; and so it is on board ships belonging to the East India Company.

The several courses and distances sailed during the interval of 24 hours, or from noon to noon, together with all the remarks and occurrences that are worthy of notice, are generally marked down with chalk on a board, painted black, called the log-board. This board is usually divided into six columns: the first column on the left hand contains the hours from noon to noon, viz., from noon to midnight, and then from midnight to noon; the second and third columns contain the knots and fathoms sailed every hour; the fourth contains the courses steered; the fifth the winds; and in the sixth the various remarks are written,—such as, the state of the weather, the sails set or taken in, the observations for ascertaining the ship's place, the variation of the compass, and whatever else may be deemed necessary. The log-board is transcribed every day at noon (under the direction of the Master), into the log-book, which is divided into columns exactly in the same manner.

The form of the log-book which is now made use of in the Royal Navy, will be shown presently.

The courses steered must be corrected for the variation of the compass, and also for lee-way, if any. If the variation be westerly, it must be allowed to the left hand of the course steered; but if easterly, to the right hand thereof, in order to obtain the true course.—See Problem VI., page 577.

The lee-way is to be allowed to the right hand of the course steered, if the larboard tacks be on board; but to the left hand, if the starboard tacks be on board.

The variation of the compass should be determined twice a day (every morning and evening), if possible. The method of doing this is shown in the several problems contained between pages 565 and 577.

With respect to the lee-way, its nature or effect may be thus explained:—

When a ship is close-hauled, and the wind blowing fresh, that part of the wind which acts upon the hull and rigging, together with a considerable part of the force which is exerted on the sails, tends to drive her immediately from the direction of the wind, or, as it is termed, to lee-ward. But since the bow of a ship exposes less surface to the water than her side, the resistance will be less in the first case than in the second; the velocity, therefore, in the direction of her head, will, in most cases, be greater than in the direction of her side; and the ship's real course will be between those two directions. Hence the angle contained between the line of the ship's apparent course and the line she actually describes through the water, is termed the *angle of lee-way*, or, simply, the *lee-way*.

The *angle of lee-way* that a ship makes may be very readily determined in the following manner; viz., Draw a semi-circle on the taff-rail, with its diameter at right angles to the ship's keel, and its circumference divided into points and quarter-points; then let the angle be observed which is contained between the semidiameter pointing right aft, or parallel to the keel, and that which points in the direction of the wake, and it will be the lee-way required. Or, after heaving the log, if the line (before it is drawn in) be applied to the centre of the semi-circle, the points and quarter-points contained between its direction and the fore and aft radius of the semi-circle will be the lee-way, as before.

Many writers on navigation have given rules for ascertaining the quantity of lee-way which a ship makes, independent of observation. These are as follow; viz.,

1. When a ship is *close-hauled*, has all her sails set, the water smooth,

with a light breeze of wind, she is then supposed to make little or no lee-way.

2. Allow one point when the top-gallant-sails are handed.

3. Allow two points when under close-reefed top-sails.

4. Allow two points and a half when one top-sail is handed.

5. Allow three points and a half when both or the three top-sails are handed.

6. Allow four points when the fore-sail or fore-course is handed.

7. Allow five points when under the main-sail or main-course only.

8. Allow six points when under a balanced mizen.

9. Allow seven points when under bare poles.

As these rules depend entirely upon the quantity of sail set, without any regard to the model of the ship, or to the nature of the way in which she may be trimmed for sailing, it is evident that they are far from being general, and that they are, in reality, little more than mere probable conjectures. But since the accuracy of a ship's reckoning depends, in some measure, upon the truth of the lee-way, it ought to be deduced, at all times, from actual observation, as above directed; and then its value should be carefully noted down, in a separate column, on the log-board; so that all concerned may be thereby enabled to correct the courses steered, in making out their days' works at noon.

In very strong gales, with a contrary wind and a high sea, it is not prudent to attempt working to windward: in such cases, the grand object is, to avoid, as much as possible, losing ground, or being driven back. With this intention, it is customary to lay the ship to, under no more sail than may be barely sufficient to check that violent rolling which she would otherwise acquire, to the endangering of her masts, yards, and rigging. When a ship is brought to, the helm is kept about three parts alee, which brings her head gradually round to the wind. The force of this element having then very little power on the sails, the ship consequently loses her way through the water, which ceasing to act upon the rudder, her head falls off from the wind; the sail which she has set fills, and gives her fresh way through the water, which, acting on the rudder, brings her head again gradually round to the wind; and thus she obtains a kind of vibratory motion, coming up to the wind and falling off from it alternately.

Ships lie-to under different sails, according to circumstances; and one vessel will lie-to considerably better under some particular sail than another. But, in general, a close-reefed main-top-sail is, perhaps, the most eligible sail to lie-to under; because of its being nearly over the centre of motion, and, also, because of its elevated position, which renders it far less susceptible of being becalmed in the trough of the sea than either the courses or storm-stay-sails.

When a ship is lying-to, observe the points of the compass upon which she comes up and falls off, and take the middle point for her apparent course : to which let the variation and the lee-way be applied, and the true course will be obtained. Thus, suppose a ship lying-to under a close-reefed main-top-sail, with her larboard tacks on board, comes up S. S. W., and falls off to S. W. b. W. ; then, allowing the variation to be $1\frac{1}{2}$ point west, and the lee-way to be $2\frac{1}{2}$ points, the course made good is S. W. $\frac{1}{2}$ W. : for the middle point between S. S. W. and S. W. b. W. is S. W. $\frac{1}{2}$ S. ; to which, $1\frac{1}{2}$ point westerly variation being allowed to the left, and $2\frac{1}{2}$ points lee-way to the right, makes the true course S. W. $\frac{1}{2}$ W.

The setting and drift of currents, with the heave and drift of the sea, should be set down as courses and distances upon the log-board : these are to be corrected for variation only.

The computation made from the several corrected courses, and their corresponding distances, is called a *day's work* ; and the ship's place, deduced therefrom, is called her place by account, or dead reckoning.

If the course and distance made by a ship could be correctly ascertained, by means of the compass and the log, nothing more would be necessary in determining her true place at sea ; for the absolute course and distance being known, the latitude and longitude could be readily computed, by Problem VI., page 695. But, in consequence of the irregularities to which the heaving of the log is subject, particularly during the night, with many unforeseen and unavoidable causes, such as sudden squalls, imperfect compasses, unequal care in the helmsman, inaccurate allowances for variation and lee-way, &c. &c., the latitude and longitude of the ship, as inferred from dead reckoning, will very seldom agree with the truth, or with those immediately deduced from celestial observation. In consequence of this discrepancy, several writers on navigation have proposed to apply a conjectural correction to the departure or meridian distance, in order to find the true longitude. Thus, if the course be near the meridian, the error is wholly attributed to the distance, and the departure is to be increased or diminished accordingly ; if it be near a parallel, that is, near the east or west point of the compass, the course only is supposed to be erroneous ; and if the course be towards the middle of the quadrant, viz., near four points, the assumption is that both course and distance are wrong. These corrections, being computed and applied according to the rules given by different authors, will generally place the ship upon different sides of her meridian by account : hence, since the corrections arising from these rules are evidently founded upon a vague kind of guess-work, they ought to be absolutely rejected.

When the latitude by account differs from that by observation, the

log-line and half-minute glass should be carefully examined, and, if found erroneous, the distance sailed, as indicated thereby, should be corrected accordingly, by the Problems given for that purpose, between pages 272 and 276. If the corrected distance, thus found, with the course, does not produce a coincidence in the latitudes by account and observation, the mariner should then consider whether the variation has been properly determined and allowed upon the courses steered by compass; if not, these courses are to be again corrected; but no other alteration whatever should be made in them. If the latitudes by account and observation be still found to disagree, the navigator should next consider whether the ship's place has been affected by a current or by the heave of the sea, and allow for their course and drift to the best of his judgment. By carefully applying these corrections, a new difference of latitude and departure, and a new course and distance, will be obtained; which will, in general, produce an approximation in the latitudes: beyond this, no alteration whatever should be made in the departure with the view of finding the longitude by account.

However, since there are many mariners who, from long-established practice, are not willing to depart from the common system of correcting the dead reckoning by the rules laid down for that purpose in certain Epitomes of Navigation; and since these rules are exceedingly complicated, and admit of a *variety of cases*, the following General Rule is given for the use and guidance of such persons, which reduces those various cases into one very concise method, and thus does away with the necessity of consulting several complex rules before the desired correction can be obtained.

A General Rule for Correcting the Dead Reckoning.

Augment the distance sailed by *two-thirds* of the difference between the latitude by account and that by observation, when the observed latitude is before or *ahead* of that by account; but diminish the distance sailed in the same proportion, when the observed latitude is *astern* or behind that by account. Then,

Enter the general Traverse Table with this corrected distance and the difference of latitude by observation, and find the corresponding departure. Now, with the departure, thus found, in a latitude column, and the middle latitude as a course, find the corresponding distance, and it will be the corrected difference of longitude.

Example 1.

Suppose a ship, from a place in latitude $47^{\circ}49'$ N. and longitude $9^{\circ}29'$ W., sailed S. 43° W. 160 miles, and then finds her latitude by

account to be $45^{\circ}54'N.$, but by observation her $N.$; required the longitude come to by account,

Solution.—The difference between the latitude by observation, is 15 miles; the two-thirds of which being added to the distance sailed, because before or *ahead* of that by account, makes the 130 miles: with this corrected distance and the difference of latitude by observation, viz., $2^{\circ}10'$ or 130 miles, the corrected distance, in the general Traverse Table, is 109.3 miles. Then in a latitude column, and the middle latitude (between that sailed from and that arrived at by observation), viz., the difference of longitude corresponding thereto, in the top or bottom of the page, is 159 miles, or 159 added to the longitude left, shows the longitude arrived to be $12^{\circ}8'$ west.

Example 2.

Suppose a ship from Porto Santo, in latitude $16^{\circ}17'W.$, sailed $N. 47^{\circ}E.$ 210 miles, and the account to be $35^{\circ}26'N.$, but by observation her $35^{\circ}8'N.$; required the longitude come to by account.

Solution.—The difference between the latitude by observation, is 18 miles; the two-thirds of which being subtracted from the distance sailed, the corrected distance is 198 miles: with this corrected distance and the difference of latitude by observation, viz., $2^{\circ}5'$ or 125 miles, the corrected distance, in the general Traverse Table, is 153 miles. Then in a latitude column, and the middle latitude (between that sailed from and that come to by observation), viz., the difference of longitude corresponding thereto, in the top or bottom of the page, is 132 miles, or 132 added to the longitude left, shows the longitude arrived to be $13^{\circ}12'$ west.

Remark.—Although the above general rule of reckoning is the most simple, and, perhaps, the best that have been as yet devised for that purpose, frequently found, on making the land after a long voyage deduced therefrom differed several degrees from the true, notwithstanding the easy and specious fe

that the prudent mariner will do well to be extremely cautious in applying it to practice; nor should he ever place any manner of faith in the longitude so deduced, particularly if he has been any considerable time from the land. From this it is manifest that the navigator should determine the longitude of his ship, as often as possible, both by the lunar observations and by a chronometer; and from the true longitude, thus found, the reckoning of this element is to be carried forward, in the same manner as that of the latitude, from the last observation. A separate account, however, should be kept of the longitude by dead reckoning: such account is not only very satisfactory, but it often proves highly useful as a reference; particularly in comparing the computed velocity and drift of a current with those deduced from actual experiment.

The following is the form of the log-book which is now used in Her Majesty's Royal Navy, and from which we will make out a *practical day's work*.

Log-Book of His Majesty's Ship ———, Wednesday June 4th, 1823.

H.	K.	F.	Courses.	Winds.	No. of Signals.	Remarks and Occurrences.				
1						A.M. Moderate breezes and hazy weather.				
2				S. E.						
3						At 4 ³⁰ ^m employed washing decks.				
4										
5						At 7 ³⁰ ^m unmoored ship, and hove short on the best bower.				
6						At 8 ⁴⁰ ^m weighed and made sail.				
7										
8						At noon light winds and hazy weather.				
9				S. E.						
10										
11										
12										
Course.			Distances.	Lat. by Account.	Latitude by Observ.	Long. by Account.	Long. by Lunar Observ.	Long. by Chron.	Variation at Noon.	Bearings and Dist. at Noon.
					34°20' S.				25°20' W.	Cape of Good Hope E. by S. 15 miles.
1	5	4	N.W. by W.	S. E.		P.M. Moderate breezes, with thick hazy weather.				
2	5	4								
3	6	0	N.W. $\frac{1}{4}$ W.			At 2 ³⁰ ^m shook a reef out of the top-sails, and set the top-mast and top-gallant studding-sails.				
4	6	2								
5	6	2	N.W. $\frac{1}{4}$ W.	S. E.		At 5 ^h beat to quarters, and exercised great guns and small arms.				
6	6	2								
7	7	6				At 8 ^h fresh breezes and cloudy weather.				
8	8	0	N. W.							
9	8	6				At midnight fresh breezes and clear weather.				
10	9	0								
11	9	4								
12	9	6		S. E.						

Log-Book of His Majesty's Ship ———, Thursday, June 5th, 1823.

H.	K.	F.	Courses.	Winds.	No. of Signals.	Remarks and Occurrences.		
1	9	4	N.W.	■		A M. Fresh breezes and clear weather.		
2	9	4						
3	9	4				At 4 ^h fine clear weather; employed washing decks.		
4	9	6						
5	9	6				At 6 ^h 30 ^m set the lower studding-sails.		
6	9	6	N.W. b. N.	S.E.		At 8 ^h 40 ^m in lower studding-sails.		
7	10	0						
8	10	0				At 10 ^h 30 ^m mustered by divisions, and inspected the people's clothing.		
9	9	6						
10	9	6						
11	9	4						
12	9	2				S.E.	At noon fresh breezes and fine clear weather.	
Course.	Distance.	Lat. by Account.	Latitude by Observ.	Long. by Account.	Long. by Lunar Observ.	Long. by Chron.	Variation at Noon.	Bearings and Dist. at Noon.
N. 69 W.	214 miles.	33°6' S.	33°4'30" S.	14°24' E.	14°28' E.	14°24' E.	23°10' W.	St. Helena N. 46°51' W. dist. 1505 ms.

Note.—The departure is taken from the Cape of Good Hope; and as this place bore, at noon, E. b. S. from the ship, distant 15 miles, the compass bearing or course of the ship from the Cape was, therefore, W. b. N. Now, the variation, $2\frac{1}{2}$ points west, being allowed to the left-hand of W. b. N., shows the true bearing or course to be W. b. S. $\frac{1}{4}$ S. The other courses are, in like manner, to be corrected for variation; but since the value of this element is not the same at both noons, it is advisable to allow 25°20', or $2\frac{1}{2}$ points west, on the courses in the first 12 hours, or between noon and midnight, and 23°10', or 2 points west, on the courses in the other 12 hours, or between midnight and noon; then, these corrected courses, with their respective distances, being inserted in a Traverse Table, after the following manner, the difference of latitude and departure corresponding thereto, with the course and distance made good, may be readily determined by calculation, agreeably to the rule given in Problem VIII., page 700; or, perhaps more readily, by the general Traverse Table.—See Problem II., page 108.

TRAVERSE TABLE.					
Corrected Courses.	Distances	Difference of Latitude.		Departure.	
		N.	S.	E.	W.
W.b.S. $\frac{1}{4}$ S.	15	—	3.6	—	14.6
W. $\frac{1}{4}$ N.	11	1.6	—	—	10.9
W.b.N. $\frac{1}{4}$ N.	18	4.4	—	—	17.5
W.b.N. $\frac{1}{2}$ N.	22	6.4	—	—	21.1
W.b.N. $\frac{3}{4}$ N.	37	12.5	—	—	34.8
W.N.W.	48	18.4	—	—	44.3
N.W.b.W.	68	37.8	—	—	56.5
		81.1	3.6	—	199.7
		3.6			0.0
	Diff.of lat.	77.5		Departure =	199.7

To find the Course and Distance made good.

The difference of latitude 77.5, and the departure 199.7, are found to agree nearest abreast of 69 degrees, and under or over 214 miles distance.

Hence the course made good is N. 69° W., or W. b. N. $\frac{3}{4}$ N. nearly, and the distance 214 miles.

To find the Latitude and Longitude come to by Account.

Lat. Cape Good Hope 34°23'40"S. . 34°23'40"S. Mer. pts.=2200.1
Diff. Lat. 77.5 ms. . 1.17.30 N.

Lat. in by acc. = . 33° 6'10"S. By ob.33° 4'30"S.Mr.ps.=2104.9

Meridional difference of latitude, by observation = 95.2
[miles.

Meridional difference of latitude=95.2 miles Log. = . 1.978637
Course made good = N. 69°W. Log. tang.=10.415823

Diff. of long. made good = 4° 8' 0" W.=248ms. Log.=2.394460
Long. of C. of Good Hope =18°32'15" E.

Long. come to by account = 14°24'15" E.

To find the Course and Distance from the Ship to St. Helena.

Lat. of ship by obs. $33^{\circ} 4' S.$ Mer. pts. = 2104.3 Long. = $14^{\circ} 26' E.$
 Lat. of St. Helena $15.55 S.$ Mer. pts. = 967.5 Long. = $5.44 W.$

Diff. of lat. = $17^{\circ} 9'$ Mer. diff. lat. = 1136.8 Diff. long. $20^{\circ} 12'$
 = 1029 miles. = 1212 miles.

Difference of longitude = 1212 miles. Log. = . . 3.083503
 Merid. diff. lat. = . . 1136.8 miles. Log. = . . 3.055684

Course = . . . N. $46^{\circ} 51' W.$ Log. tang. = 10.027819

Course = . . . N. $46^{\circ} 51' W.$ Log. secant = 10.165001
 Diff. of lat. = . . . 1029 miles. Log. = . . 3.012415

Distance = . . . 1505 miles. Log. = . . 3.177416

Hence,—The course made good is N. $69^{\circ} W.$ or W. by N. $\frac{1}{4}$ N. nearly.

Distance made good = . . . 214 miles.

Latitude come to by account = $33^{\circ} 6' 10'' S.$

Latitude by observation = . . $33. 4.30 S.$

Longitude come to by account = $14.24.15 E.$

Longitude by lunar observation = $14.28. 0 E.$

Longitude by chronometer = . $14.24. 0 E.$

Variation at noon = . . . $23.10. 0 W.*$

St. Helena bears N. $46^{\circ} 51' W.$ or N.W. $\frac{1}{4}$ W. nearly, independent of variation.

Distant . . . 1505 miles.

OF THE MEASURE OF A KNOT ON THE LOG LINE.

It has been remarked, page 272, in the introduction to the Problems for correcting the distance sailed on account of any errors that may be discovered in the log line and half-minute glass, that the distance between any two adjacent knots on the log line should bear the same

* The variation of the compass may be very readily determined at noon (sufficiently correctly for nautical purposes,) by the second part of the Rule to Problem IV., page 574, reading *sun* instead of *star* or *planet*.—See Note at bottom of page 575.

proportion to a nautical mile that half a minute does to an hour, viz., *the one hundred and twentieth part*; that a nautical mile contains 6080 feet; and that this number divided by 120, gives the true measure of a knot, viz., 50 feet and 8 inches.—But, since the young navigator may be desirous of being made acquainted with the principles upon which this measure has been determined, the following considerations are, therefore, submitted to his attention; which, besides satisfying him in that particular, may do something towards giving him a just idea of the true figure of the earth; without which idea he can never clearly comprehend the principles upon which the art of navigation is founded.

The earth is a planet, and the next, in the solar system, above Venus.—Our senses assure us of its opacity;—and that it is of a globular or spherical figure will appear evident from the arguments which follow:—

A lunar eclipse is occasioned by the moon's passing through the earth's shadow; and since this shadow, when projected on the lunar disc, is observed to be always circular in every different position of the earth, it necessarily follows that the earth, which casts the shadow, must be spherical, since nothing but a sphere, when turned in various positions with respect to a luminous body, can project a circular shadow.—Again,

A lunar eclipse is observed sooner by those who live eastward than by those who live westward; the difference of time being always proportional to the difference of longitude between the places of observation.

Now, if the earth were an extended plane, as the primitive fathers asserted, the eclipse would happen at the same instant in all places:—but this is so far from being the case, that the inhabitants of Jamaica will not see an eclipse of the moon until about five hours after it takes place at Greenwich;—therefore the figure of the earth must be spherical, or very nearly so.

If the earth were an extended plane, the meridian zenith distance of any one fixed star would be the same in all parts of the world; because the measure of the earth's diameter bears no more proportion to the immeasurable distance of the nearest fixed star than an indivisible point does to the diameter of the earth.—But, since the meridian zenith distance of the same fixed star is found to differ with the latitude, the difference in the zenith distance being always proportional to the intercepted arch of the meridian; and since it is the known property of a curve that the arches are proportional to their correspondent angles, therefore the surface of the earth and sea is of a curvilinear form.—Hence the earth must be of a spherical figure.

The earth has been circumnavigated by many persons, at different periods, who, by sailing in a westerly direction, allowance being made

for promontories, &c. arrived at the place whence they sailed.—Hence, the earth must be either of a cylindrical, or a spherical figure;—but that it is not of a cylindrical figure will appear obvious by considering that the difference of longitude and meridional distance between two places would, on the cylindrical hypothesis, be equal;—whereas, experience and actual observation demonstrate that the very reverse of this takes place:—therefore the earth must be of a spherical form from west to east.

If a ship in north latitude sails southerly, the north polar star will be found gradually to decrease in altitude till the vessel reaches the Equator; at which place the star will be seen immersed in the horizon.—After crossing the Equator, and as the ship advances in the southern hemisphere, the stars in the neighbourhood of the south celestial pole will be seen gradually emerging from the southern horizon, and increasing in altitude, whilst those about the north celestial pole will be entirely lost sight of; being hid below the horizon:—hence the earth is spherical from north to south; but it is also spherical from west to east, as appears from its circumnavigation; therefore the figure of the earth is that of a sphere.

When two distant ships are approaching each other, at sea, the royals and top-gallant sails only of each are visible at first; the lower sails and hulls being concealed by the convex surface of the water:—but as they draw nearer towards each other, the parts that were so concealed by the convexity of the sea's surface, will be seen to rise gradually above the horizon.—Now, if the sea were an extended plane, the hulls or bodies of the ships would be the first parts seen; and because they are the largest, they would, evidently, be seen at the greatest distance; nor would the small sails near the masts' heads be visible until the approach of the ships brought them considerably nearer.

In making the land the most elevated parts are first seen, such as mountains, &c.; then tops of light-houses and steeples, and shortly afterwards the coast, or beach:—this plainly demonstrates that the surface of the sea is convex.

The sun is observed sooner at rising and later at setting by a person at the mast-head of a ship than by one on deck; and so is the moon and all other celestial objects.—These phenomena evidently arise from the spherical figure of the earth; and are, therefore, most convincing and satisfactory proofs of its globularity.

Again.—The continual presence of the sun above the horizon, during the space of several months, in the neighbourhood of one terrestrial pole, while at a place equally distant from the other, he is as long absent, affords another convincing proof that the earth is of a spherical figure.

The spherical figure of the earth may be also inferred from the method of *levelling*, or the art of conveying water from one place to another;—for, in this operation it is always found necessary to make an allowance between the true and the apparent levels on account of the rotundity of the earth; the true level being a curve line which falls below the straight line of apparent level about 8 inches in 1 mile; 32 inches in two miles; 128 inches in 4 miles, &c., the curvature always augmenting in proportion to the square of the distance.—See Problem X., between pages 628 and 630.

Finally,—All the planets are observed to be of a spherical figure; and since the earth is a planet, subject to the same laws, and revolving round the sun in the same manner as the other planets, it must, therefore, by analogy, be also spherical.

The irregularities on the earth's surface, occasioned by mountains and valleys, are very inconsiderable compared with its magnitude; and take off no more from its actual rotundity than the little risings on the coat of an orange do from the rotundity of that fruit:—for the highest eminence or mountain bears a less proportion to the magnitude of the earth than the smallest grain of sand does to an 18-inch globe.—Thus,

The summit of Chimborazo, one of the Andes Mountains, and the highest in the known world, is only 20280 feet above the level of the sea, or not quite 4 miles in perpendicular height.—Now, the radius of the earth is 20902200 feet, and that of an 18-inch globe 9 inches;—hence, by the rule of proportion, as 20902200 feet : 20902200 feet + 20280 feet = 20922480 feet :: 9 inches to 9.0087 inches; from which deduct 9 inches (the radius of the artificial globe,) and the remainder 0.0087 is the relative elevation of Mount Chimborazo on an 18-inch globe; and as this is scarcely *the one hundred and fifteenth part* of an inch, it is, therefore, considerably less than a common grain of sand.—Hence it is evident that the highest mountains and deepest valleys, take little or nothing from the earth's rotundity.

Although when speaking of the earth in general terms, it may be considered as a globe or sphere; yet, in strictness it is not a perfect sphere, but rather an oblate spheroid; which is a solid generated by the revolution of a semi-ellipse about its shorter axis or diameter;—and actual admeasurements, in sundry places, have clearly proved that the polar axis, or diameter, is about 26 miles less than the equatorial diameter.—However, since the earth differs so very little from a globe or sphere, it may, therefore, be very safely considered as being perfectly spherical in all nautical calculations whatever.

The spherical figure of the earth being thus satisfactorily established, its magnitude may be determined by measuring a small portion of a meridian, and observing the zenith distances of one or more stars at

the extreme stations; then, the difference between the zenith distances of the same star gives the correspondent celestial arch.—Now,

As the celestial arch, thus found, is to the measured or intercepted portion of the meridian; so is one degree, to its absolute length in the same measure in which the portion of the meridian was taken.

In this manner the celestial arch of one degree has been found to contain 69.093 English miles; and since the earth's circumference, like that of all other spheres, contains 360 degrees; therefore $360 \text{ degrees} \times 69.093 \text{ miles} = 24873.48$, is the true measure of the earth's circumference in English miles.—Hence, its diameter is $7917\frac{1}{2}$ miles, English measure.

Now, since the nautical arch is, in every respect, equal to the celestial arch, the length of a degree in the one being precisely equal to the length of a degree in the other, each containing 60 geographical miles; and since the measure of a degree of this arch in English miles, is 69.093, or 364815 English feet;—therefore $364815 \text{ feet} \div 60 \text{ miles} = 6080 \text{ feet}$; which, evidently, is the true length of a nautical mile, expressed in English measure.—And, if 6080 feet be divided by 120 (the number of half minutes in an hour,) the quotient 50 feet and 8 inches will be the true measure of a knot.—And, hence the principles upon which the measure of a knot upon the log line has been determined.

But, because it is safest to have the reckoning a-head of the ship, 48 feet, or 8 fathoms are, therefore, commonly allowed between every two adjacent knots on the log line:—and this measure is to correspond to a glass running 30 seconds; or, rather $29\frac{1}{2}$ seconds, so as to make up for any time that may be unavoidably lost in the act of turning the glass. See the Problems between pages 272 and 276 relative to the errors of the log. sine and the half-minute glass.

Remark.—The method of *adjusting* the instruments used at sea will be found in the Articles between pages 316 and 326.

SOLUTION OF MISCELLANEOUS PROBLEMS.

PROBLEM I.

Given the Base and Perpendicular Height of a Plane Triangle; to find its Area.

RULE.

To the logarithm of the base add the logarithm of half the perpendicular height, and the sum will be the logarithm of the area, or superficial content of the triangle.

Example.

Let the base of a plane triangle be 37.6 yards, and its perpendicular height 29.8 yards; required its area, or superficial content?

Base of the triangle = 37.6 yards. Log. = 1.575188
 Perpen. height = 29.8 yards ÷ 2 = 14.9 yards. Log. = 1.173186

Area, or superficial content = . 560.24 . . Log. = 2.748374

PROBLEM II.

*Given two Sides and the contained Angle of a Plane Triangle;
 to find its Area.*

RULE.

To the logarithmic sine of the contained angle, add the logarithms of the containing sides; and the sum (abating 10 in the index) will be the logarithm of twice the area of the triangle.*

Example.

Let the two given sides of a triangle be 109.5 yards and 168.2 yards respectively, and the contained angle 79°16'; required the area, or superficial content of that triangle?

Contained or included angle = . . 79°16' Log. sine = 9.992335
 One of the containing sides = . . 109. 5 Log. = . 2.039414
 The other containing side = . . 168. 2 Log. = . 2.225826

Twice the area of the triangle = 18095. 7 Log. = . 4.257575

Area of the triangle = . . . 9047.85 yards, as required.

Note.—The above Problem will be found exceedingly useful in the practice of land-surveying.

* Or, to the logarithmic sine of the contained angle, add the logarithm of one of the containing sides and the logarithm of *half the other containing side*; the sum of these three logarithms (abating 10 in the index), will be the logarithm of the area of the triangle.

PROBLEM III.

Given the three Sides of a Triangle ; to find its Area, or superficial Content.

RULE.

Add the three sides together, and take half their sum ; subtract each side severally from that half sum, noting the remainders : then,

To the logarithm of the half sum add the logarithms of the three remainders ; now, the sum of these four logarithms, being divided by 2, will give the area of the triangle.

Example.

Let the three sides of a triangle be 433, 312, and 205 yards respectively ; required its area ?

First side =	433	First remainder =	42	Log. =	1.623249
Second side =	312	Second remainder =	163	Log. =	2.212188
Third side =	205	Third remainder =	270	Log. =	2.431364
Sum =	<u>950</u>				
Half sum =	475	Log. =			2.676694
				Divide by 2)	<u>8.943495</u>
Area of the triangle =	29631.08	Log. =			4.471747

PROBLEM IV.

Given the Diameter of a Circle ; to find its Circumference, and conversely

RULE.

To the logarithm of the diameter add the constant logarithm 0.497150, and the sum will be the logarithm of the circumference. And, to the logarithm of the circumference add the constant logarithm 9.502850, and the sum will be the logarithm of the diameter.

Example 1.

The earth's diameter is 7917.5 miles ; required its circumference ?

Diameter of the earth =	7917.5 miles.	Log. =	3.898588
Constant log. =			0.497150
Circumference of the earth =	24873 miles.	Log. =	4.395738

Example 2.

If the circumference of the earth be 25000 miles, what is its diameter?

Circumference of the earth =	. . . 25000 miles.	Log. =	4.397940
Constant log. =		9.502850
<hr/>			
Diameter of the earth, in miles =	7957.7	Log. =	3.900790

Note.—The circumference of a circle whose diameter is unity or 1, is 3.14159265; and, since the circumferences of circles are to each other as their diameters, or radii,—therefore, as the diameter 1, is to its circumference 3.14159265; so is the diameter of any other circle, to its circumference: and hence the above Rule. The constant logarithm is expressed by the logarithm of 3.14159265, or log. arithmetical complement of that number.

PROBLEM V.

Given the Diameter, or the Circumference of the Earth; to find the whole Area of its Surface.

RULE.

Since the *surface* of a sphere or globe whose diameter is unity or 1, is 3.14159265, the logarithm of which is 0.497150, and its arithmetical complement 9.502850; and since the *surfaces* of spheres are to each other as the squares of their diameters; therefore,

To twice the logarithm of the earth's diameter add the constant logarithm 0.497150, and the sum will be the logarithm of the area of the earth's surface, in square miles. Or,

To twice the logarithm of the earth's circumference add the constant logarithm 9.502850, and the sum will be the logarithm of the earth's surface, in square miles.

Example 1.

Required the area or superficial measure, in square miles, of the whole of the earth's surface, allowing its diameter to be 7917.5 English miles?

Diameter of the earth =	. . . 7917.5	Twice its log. =	7.797176
Constant log. =		0.497150

Area, in square miles =	. 196936545.5	Log. =	. . 8.294326
<hr/>			
3 A 2			

Example 2.

Required the area or superficial measure of the whole of the earth's surface, in square miles, allowing its circumference to be 24873 English miles ?

Circumference of the earth =	. 24873	Twice its log. =	8.791476
Constant log. =		9.502850

Area, in square miles =	. 196936545.5	Log. =	. . 8.294326
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PROBLEM VI.

To find the Length of any Arc of a Circle

RULE.

To the logarithm of the degrees in the given arc, considered as a natural number, add the logarithm of the radius of that arc, and the constant logarithm 8.241877 ; the sum will be the logarithm of the length of the arc.

Example.

Required the length of an arc of 45 degrees, the radius being 9 inches?

Length of the arc, in degrees =	45	Log. =	. . 1.653213
Radius of the arc, in inches =	9	Log. =	. . 0.954243
Constant log. =		8.241877

Length of the arc, in inches =	. 7.0686	Log. =	. . 0.849333
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Note.—The circumference of a circle whose diameter is unity or 1, is 3.14159265. And since the circumference of a circle contains 360 degrees ; and that the measure of the longest arc thereof is that which corresponds to the semi-circle :—therefore, as 180° : 3.14159265 :: 1°, to 0.0174533 ; the logarithm of which is 8.241877 ; which is the true constant logarithm for determining the length of the arc of a circle corresponding to any given number of degrees, and parts of a degree.

PROBLEM VII.

Given the Length of an Arc of a Circle, and the number of Degrees contained therein ; to find its Radius.

RULE.

To the logarithm arithmetical complement of the number of degrees contained in the given arc, esteemed as a common number, add the logarithm of its length, in inches, and the constant logarithm 1.758123 ; the sum (abating 10 in the index) will be the length of the radius in inches.

Note.—The constant logarithm made use of in this Rule is the arithmetical complement of that which is used in the preceding Rule.

Example.

Let the length of the arc of a circle be 10.82 inches, and the number of degrees contained therein 40 ; required the length of the radius that would sweep that arc ?

Given number of degrees = . . . 40	Log. arith. comp. = 8.397940
Length of the given arc = 10.82 inches	Log. = . . . 1.034270
Constant logarithm =	1.758123
<hr/>	
Length of the radius, in inches = 15.5	Logarithm = . 1.190333

PROBLEM VIII.

Given the length of an Arc of a Circle, and its Radius ; to find the number of Degrees contained therein.

RULE.

To the logarithm arithmetical complement of the length of the radius of the given arc, add the logarithm of the length of the arc, and the constant logarithm 1.758123 ; the sum, abating 10 in the index, will be the logarithm of the number of degrees contained in the given arc.

Note.—The constant logarithm made use of in this Rule is the arithmetical complement of that which is made use of in Problem VI.

Example.

Let the length of the arc of a circle be 10.82 inches, and its radius 15.5 inches ; required the number of degrees contained therein ?

Radius of the arc = . . 15.5 inches. Log. ar. comp. = 8.809668
 Length of the arc = . . 10.82 inches. Log. = . . 1.034270
 Constant logarithm = 1.758123

Number of degrees in the arc = 40 . . . Logarithm = 1.602061

Remark.—The two last Problems will be found useful in adapting *Pendulums* to given spaces, on board a man-of-war, for showing the actual number of degrees that a ship may ~~put~~^{heel} down.

PROBLEM IX.

To find the Length of a Pendulum for Vibrating Seconds at the Equator, and also at the Poles of the World.

RULE.

Since the circumference of a circle whose diameter is unity or 1, is found by computation to be 3.14159265; and since a pendulum vibrates in the arc of a circle, or cycloid, the radius of which is equal to the length of the pendulum from the centre of oscillation; therefore, if twice the space passed through by a falling body in one second of time, be divided by the square of the above circumference, the quotient will be the length of the pendulum for vibrating seconds. Now, it has been found, taking the *mean* of many experiments, that a heavy body let fall at the equator will descend, by the force of gravity, 16.04436 feet in one second of time:—hence, by logarithms,

16.04436 ft. = 192.53232 inches $\times 2 = 385.06464$ Logarithm = 2.5855336
 Circum. of a circle to diameter 1 = 3.14159265 Tw. its log. = 0.9942998

Length of the pendulum, in inches = 39.0152, Logarithm = 1.5912338; which, therefore, is the correct length of the pendulum for vibrating seconds on all points of the equator.

Now, since the ratio of the earth's equatorial semidiameter to its polar semi-axis (*taking the mean of many results*) is as 305 to 304.212;* and, since *gravity* decreases in proportion to the square of the distance, and *conversely*; therefore it will be,—As the square of the polar semi-axis is to the square of equatorial radius, so is the length of the pendulum at the equator, to the length of the pendulum for vibrating seconds at the poles of the world.—Hence, by logarithms,

* This is given in round or whole numbers, page 337, as 305 to 304.

Polar semi-axis = . 304.212, Twice its log. ar. comp. = 5.0336472
 Equatorial radius = . 305, Twice its logarithm = 4.9685996
 Equatorial pendulum = 39.0152 inches . Logarithm = 1.5912338

Polar pendulum = . . 39.2176 inches . Logarithm = 1.5934806
 which is the correct length of the pendulum for vibrating seconds at the poles of the world.

Remark.—The pendulum which is properly adapted to the equator vibrates 86400 times in 24 hours. Now, if this pendulum be made to oscillate at any given degree of latitude, the number of its oscillations in 24 hours will exceed the above equatorial number, in proportion to the excess of the force of gravity beyond its force at the equator; the intensity of the force at each point being always proportional to the square of the number of oscillations. And the square of the number of oscillations at the equator and the given degree of latitude will be as the correct length of the pendulum for vibrating seconds at each point respectively.

Hence, the length of the pendulum which will vibrate 86400 times in 24 hours at the poles of the world may be readily determined, as thus,—

Since the force of gravity at the equator is 16.04436 feet in one second, and that its intensity is as the square of the distance from the earth's centre; therefore, $16.04436 \times 305^2 + 304.212^2 = 16.12758$ is the force of gravity at the poles of the world.—Then,

As force of gravity at the equator = 16.04436 Log. ar. comp. = 8.7946776
 : Ditto at the poles of the world = 16.12758 Logarithm = 1.2075692
 :: □ Oscillations at the equator = 86400 Twice its log. = 9.8730274

Square of the log. = 9.8752742

Oscillations at the poles = . 86623.75 Logarithm = 4.9376371

Hence it is manifest that, if the equatorial pendulum were placed at the poles of the world, it would oscillate 86623 $\frac{3}{4}$ times in 24 hours, which is 223 $\frac{3}{4}$ times more than the truth.—Now,

As □ oscillations at the equator = 86400 Twice its log. ar. cp. = 0.1269726
 : □ Oscillations at the poles = 86623.75 Twice its logarithm = 9.8752742
 :: Equatorial pendulum = 39.0152 inches Logarithm = 1.5912338

To polar pendulum = . . 39.2176 inches Logarithm = 1.5934806

Hence, the correct length of the pendulum for vibrating seconds at the poles of the world, and which will oscillate 86400 times in 24 hours, is 39.2176 inches ; being exactly the same as that deduced from the direct operation in the solution of Problem IX., as above.

PROBLEM X.

To find the Length of a Pendulum for Vibrating Seconds in any part of the world betwixt the Equator and the Poles.

RULE.

It has been shown in the preceding Problem that the length of a pendulum for vibrating seconds at the equator is 39.0152 inches, and at the poles of the world 39.2176 inches : hence, the excess of the polar pendulum above the equatorial is 0.2024. Now, since this excess, at any point betwixt the equator and the poles, is as the square of the sine of the *geocentric* latitude ; therefore, let the latitude of the given place be reduced by the equation in Table B, vol. ii., page 611* :—then, to twice the log. sine of the reduced latitude, add the constant logarithm 9.306211 ; the sum, abating 20 in the index, will be the logarithm of a natural number in *decimals*, which being added to the equatorial pendulum, the result will be the correct length of the pendulum for vibrating seconds in the given parallel of latitude.

Example.

Required the length of a pendulum for vibrating seconds in the parallel of London ?

Latitude of London = $51^{\circ}30'49''$

Equation in Table B = 11. 0

Geocentric latitude = $51^{\circ}19'49''$ Twice its log. sine = 19.785036

Constant logarithm, the log. of 0.2024 = 9.306211

Natural number = +0.1234 Log. = 9.091247

Equatorial pendulum = 39.0152 inches.

Length of the pendulum at London = 39.1386 inches.

Remark.—The length of the pendulum determined in this manner will be found to coincide with that deduced from actual observation in every part of the world where experiments have been made ; and this striking coincidence affords an incontestable proof that the earth is of an oblate spheroidal figure, respecting which *I once entertained a doubt.*

PROBLEM XI.

To find the Length of a Pendulum for vibrating Half-Seconds.

RULE.

To the arithmetical complement of twice the logarithm of 120 (the number of vibrations in a minute for the half-seconds' pendulum) add, twice the logarithm of 60 (the number of vibrations in a minute for the seconds' pendulum), and the logarithm of the length of the latter pendulum : the sum of these three logarithms (abating 10 in the index) will be the logarithm of the length of the pendulum for vibrating half-seconds.

Example.

Let the length of a pendulum for vibrating seconds be 39.125 inches; required the length of a pendulum that will vibrate half-seconds ?

Vibrations for $\frac{1}{2}$ -secs.' pendulum = 120 Ar.co. of twice its log. = 5.841638

Ditto for the seconds' pendulum = 60 Twice its log. = . 3.556302

Length of the pendulum for secs. = 39.125 inches Log. = 1.592454

Length of half-seconds' pendulum = 9.781 inches Log. = 0.990394

Hence the length of a pendulum for vibrating half seconds, is $9\frac{3}{4}$ inches.

Remark.—Since the lengths of pendulums are as the squares of the times of vibration ; therefore if the length of a pendulum for vibrating seconds in any given latitude be multiplied by the square of one-half or .5, viz., .25 ; the product will be the length of the pendulum for vibrating half-seconds ; as thus :—

Length of the seconds' pendulum, as above = 39.125 Log. = 1.592454

Square of one-half = 0.25 Log. = 9.397940

Length of the half-seconds' pendulum = 9.781 Log. = 0.990394 ;
being exactly the same as above.

PROBLEM XII.

Give the Time of Descent of a Heavy Body ; to find the Height from which it Fell.

RULE.

The space passed through by a falling body is as the square of the time of descent :—hence, since a heavy body falls $16\frac{1}{4}$ feet in one se-

cond of time, it will be, as the square of 1 second, is to $16\frac{1}{2}$ feet; so is the square of the number of seconds which a body takes to descend, to the height, in feet, from which it fell:—or, as thus by logarithms: to twice the logarithm of the time of descent, add the constant logarithm 1.206376; and the sum will be the logarithm of the height or depth in feet.

Example.

A stone let fall from the top of a precipice was 15 seconds in descending to the bottom of a deep cavern; required the number of feet betwixt the precipice and the bottom of the cavern?

Time of descent = 15 seconds; twice its logarithm = . . . 2.352182
Constant logarithm = 1.206376

Height or depth, in feet = . . . 3619 Logarithm = 3.558558

Note.—In this manner the depth of *dry wells* may be determined.

PROBLEM XIII.

Given the Circumference of a Cable, and its Length; to find its Weight.

RULE.

To twice the logarithm of the circumference of the given cable, add the logarithm of its length and the constant logarithm 9.321963: the sum of these three logarithms (abating 10 in the index) will be the weight of the given cable, in pounds, avoirdupois.

Example.

Let the circumference of a cable be 21 inches, and its length 110 fathoms; required its weight?

Circumference or girt of given cable = 21 in. Twice its log. = 2.644438
Length of ditto, in fathoms = . . 110 Log. = . . 2.041393
Constant log. = 9.321963

Weight of the given cable, in pounds = 10181 Log. = . . 4.007794

Remark.—It has been found, by actual experiment, that 1 fathom of a hemp cable which measures 9 inches in circumference weighs 17 lbs. avoirdupois. Now, since cylinders of equal lengths are as the squares of their circumferences,—therefore, as the square of 9 inches (the circumference of the experimented cable), is to the weight of 1 fathom

... of, viz., 17 lbs.; so is the square of the circumference of any other
 ... le, to the weight of 1 fathom of such cable: which, multiplied by
 ... length of the cable, will give its whole weight. The constant
 ... arithm 9.821963 is found by adding the arithmetical complement
 ... twice the logarithm of 9 inches to the logarithm of 17 pounds.

PROBLEM XIV.

Given the Diameter of a Circle; to find its Area, or superficial Content.

RULE.

All circles are to one another, as the squares of their diameters; and
 the area of a circle whose diameter is unity or 1, is .7853982, the
 logarithm of which is 9.895090,—therefore, to twice the logarithm of
 the diameter of the given circle, add the constant logarithm 9.895090;
 and the sum (abating 10 in the index) will be the logarithm of the
 area or superficial content of that circle.

Example.

If the diameter of a circle be 78.41 yards, what is its area or super-
 ficial content?

Diameter of the given circle = 78.41 yards Twice its log. = 3.788744
 Constant log. = 9.895090

Area of the given circle, in yards = 4828.8 Log. = . . 3.683834

PROBLEM XV.

Given the Area or superficial Content of a Circle; to find its Diameter.

RULE.

As this problem is evidently the converse of the last,—therefore, to
 the logarithm of the area of the given circle, add the constant logarithm
 0.104910 (the arithmetical complement of 9.895090): divide the sum
 by 2, and the quotient will be the logarithm of the diameter of the
 given circle.

Example.

Let the area of a circle be 4828.8 yards; required its diameter?

Area or superficial content of given circle = 4828.8 yds. Log. = 3.683834
 Constant log. = 0.104910

Divide by 2) 3.788744

Diameter of the given circle, in yards = 78.41 Log. = 1.894372

PROBLEM XVI.

Given the Diameter of a Circle ; to find the Side of a Square equal in Area to that Circle.

RULE.

To the logarithm of the diameter of the given circle, add the constant logarithm 9.947545 (the logarithm of the square root of .7853982); and the sum (abating 10 in the index) will be the logarithm of the side of a square equal in area or superficial content to that circle.

Example.

If the diameter of a circle be 78.41 yards, what is the side of a square equal in area to that circle?

Diameter of the given circle = . . 78.41 yards Log. = 1.894372
 Constant log. = 9.947545

Side of the required square, in yards = 69.49 Log. = 1.841917

PROBLEM XVII.

Given the Diameter of a Circle ; to find the Side of a Square inscribed in that Circle.

RULE.

To the logarithm of the diameter of the given circle, add the constant logarithm 9.849485 ; and the sum (abating 10 in the index) will be the logarithm of the side of a square inscribed in that circle.

Example.

If the diameter of a circle be 78.41 yards, what is the side of a square inscribed in that circle?

Diameter of the given circle = . . 78.41 yards Log. = 1.894372
 Constant log. = 9.849485

Side of the inscribed square, in yards = 55.44 Log. = 1.743857

PROBLEM XVIII.

*Given the transverse and the conjugate Diameters of an Ellipsis ;
to find its Area.*

RULE.

To the logarithms of the longer and the shorter diameters of the ellipsis, add the constant logarithm 9.895090: the sum (abating 10 in the index) will be the area of that ellipsis.

Example.

Let the transverse diameter of an ellipsis be 616 yards, and its conjugate diameter 445 yards ; required the area or superficial content of that ellipsis ?

Transverse diameter = . . . 616 yards Log. = 2.789581

: Conjugate diameter = . . . 445 yards Log. = 2.648360

Constant log. = 9.895090

Area of the given ellipsis, in yards = 215294 Log. = 5.333031

PROBLEM XIX.

Given the transverse and the conjugate Diameters of an Ellipsis ; to find the Diameter of a Circle equal in Area to that Ellipsis.

RULE.

To the logarithm of the longer diameter, add the logarithm of the shorter diameter ; divide the sum by 2, and the quotient will be the logarithm of the diameter of a circle equal in area to the ellipsis.

Example.

Let the transverse diameter of an ellipsis be 616 yards, and its conjugate diameter 445 yards ; required the diameter of a circle equal in area to that ellipsis ?

Transverse diameter of the given ellipsis = 616 yards Log. = 2.789581

Conjugate diameter of ditto = . . . 445 yards Log. = 2.648360

Divide by 2) 5.437941

Diameter of a circle equal in area = 52.356 yards Log. = 2.718970½

Note.—The rationale of this Rule will appear manifest by considering that the ellipsis is a mean proportional between its inscribed and circumscribed circles.

PROBLEM XX.

Given the transverse and the conjugate Diameters of an Ellipsis ; to find its Periphery or Circumference.

RULE.

Square the two diameters ; add those squares together : take half the sum, and find the logarithm corresponding thereto. Now, the half of this logarithm will be the logarithm of a natural number, which, being added to half the sum of the two diameters, will give the corrected mean diameter.

To the logarithm of the corrected mean diameter, thus found, add the constant logarithm 0.196121 ; and the sum will be the logarithm of the circumference of the ellipsis.

Example.

Let the transverse diameter of an ellipsis be 616 yards, and its conjugate diameter 445 yards ; required its circumference ?

Tr.diam. $616 \times 616 = 379456$, the square.

Conj.do. $445 \times 445 = 198025$, ditto.

Divide by 2) 577481, sum of the squares.

Hf.sum of squares = 288740½ Lg. 5.4605077

Half the logarithm = . . . 2.7302538½ Nt.No. 537.3519

Half the sum of the two diameters = . . . 530.5

Corrected mean diameter = 1067.8519 Lg. 3.028511

Constant log. (the log. of half 3.14159265) = 0.196121

Circumference of the ellipsis, in yards = 1677.38 Log. = 3.224632

PROBLEM XXI.

Given the Diameter of a Sphere or Globe ; to find its Solidity.

RULE.

To thrice the logarithm of the diameter of the given sphere, add the constant logarithm 9.718999 ; and the sum (abating 10 in the index) will be the solid content of such sphere.

Example.

If the diameter of the earth be 7917.5 miles, what is its solidity?

Diameter of the earth = . 7917.5 miles Thrice its log. = 11.695764
Constant log. = 9.718999

Solidity of the earth, in miles = 259874059701.5 Log. = 11.414763

Note.—It has been found that the solidity or solid content of a sphere, whose diameter is unity or 1, is .5235988; and since spheres are to one another, as the cubes of their diameters,—therefore, as the cube of the diameter 1, is to its solidity .5235988; so is the cube of the diameter of any other sphere or globe, to the solidity of such sphere or globe: and hence the above rule. The constant logarithm is expressed by the logarithm of .5235988.

Remark.—For the method of finding the number of square miles contained in the earth's superficies, see Problem V., page 723.

PROBLEM XXII.

Given the Earth's Diameter; to find the Height to which a Person should be raised to see one-third of its Surface.

RULE.

From twice the logarithm of the earth's semidiameter, subtract the logarithm of its one-third: the remainder will be the logarithm of the height to which a person should be raised above the earth's centre, to see one-third of its surface; from which let the earth's radius or semidiameter be taken, and the remainder will be the required height above its surface.

Example.

How high above the earth must a person be raised, that he may see one-third of its surface?

Earth's semidiameter = . 3958.75 miles Twice its log. = 7.195116
One-third of ditto = . . 1319.5833 miles Log. = . 3.120437

Height above the earth's centre = 11876.25 miles Log. = . 4.074679
Deduct the earth's semidiam. = 3958.75 miles

Remainder = 7917.50 miles; which is the true height to which a person should be raised above the earth, to see one-third of its surface.

Note.—Since spherical curves are to each other in the same proportion as their altitudes ; therefore, as one-third of the earth's semidiameter, *measured from the centre or rational horizon*, is to the semidiameter ; so is the semidiameter to the absolute height above the earth's centre or rational horizon : hence the above Rule is manifest.

PROBLEM XXIII.

To find the Height of the Earth's Atmosphere.

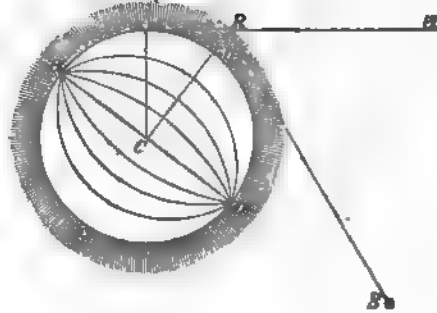
The *atmosphere* signifies the invisible fluid, or body of air, that surrounds the globe on which we live, and to which it is attached by the force of gravitation ; and although it is so exceedingly thin and subtle as to be impalpable, yet it possesses weight, density, and elasticity ; and is susceptible of expansion by heat, and condensation by cold.

The atmosphere is replete with particles of reflective matter, which, by receiving the solar rays and reflecting them in every direction, cause the heavens to appear bright in the day-time.—And were it not for this, only that part of the firmament would be illuminated, at any time, in which the sun is placed. This is a fact that has been verified experimentally ; for, by ascending mountains of *great altitude*, the atmosphere becomes so extremely rare, that at the height of about three miles, or 15840 feet, above the level of the sea, if one turns his back to the sun, the sky will appear to be of a beautiful jet-black colour, in which all the planets and stars of *the first magnitude*, above the observer's horizon, will be visible at noon or mid-day.

As the density and weight of the atmosphere are greatest at the earth's surface, and as these elements are found to diminish gradually from the surface *upwards*, the absolute extent of the circle which its most ethereal parts circumscribe round the globe, must ever remain as a *desideratum* :—but, the altitude at which it is of sufficient density to reflect the rays of light, and which, in effect, constitutes *the height of the atmosphere*, may be very readily and *correctly* deduced from the following principles and considerations, *viz.*,

The atmosphere possesses the power of refracting the rays of light, and hence it is that the sun is visible *every clear day*, about six minutes, at a mean rate, before he actually rises above, and after he sets below, the horizon of an observer.—The sun is seen in the morning when his upper limb is within 18 degrees of the horizon ; and he does not become invisible in the evening, until his upper limb is depressed 18 degrees below the horizon.

Now, in the annexed diagram, let $A B E D$ represent the earth, surrounded by the atmosphere, as indicated by the dotted circular space. Let the point A be the place of an observer upon the earth, and $R H$ his visible horizon; S the place of the sun within 18 degrees of the horizon, either at his rising or setting; and the line $S R$, a ray of light from that luminary falling upon the atmosphere at the point R , and making the angle $S R H$ equal to 18 degrees, the depression of the sun below the horizon.



Now, because the angle $S R H$ is 18 degrees, its supplement, or the angle $S R A$, is 162 degrees.* From the centre C , draw the line $C R$, and it will be perpendicular to the aerial particles of the atmosphere; and, by the laws of optics, it will bisect the supplemental angle $S R A$, making the angle $C R A$ equal to 81 degrees.*—Hence, in the right angled plane triangle $C R A$, the angles and one side are given, to find the hypotenuse $C R$; from which, let the semidiameter $C d$ ($=C A$) be taken, and the remainder, or $d R$ will be the height of the atmosphere. The computation may be performed by *plane trigonometry*, Problem II., page 172; or by *contraction*, as in the following

Example.—The semidiameter of the earth is 3958½ English miles; required the height of the atmosphere?

Angle $C R A = 81$ degrees Logarithmic co-secant $= 10.0053801$
 Earth's semidiameter $C A = 3958.75$ miles Logarithm $= 3.5975581$

Hypotenuse, or side $C R = 4008.10$ miles Logarithm $= 3.6029382$

Difference, or side $d R = 49.35$ miles; which is the correct height of the atmosphere, as required.

Remark.—Although the atmosphere extends to the distance of 49.35, or about 49½ miles above the earth's surface, yet it is seldom of sufficient

* The angle $S R H$, is assumed considerably more than 18 degrees, and the angle $S R A$ less than 162 degrees, so as to render the triangle $C R A$ clearly perceptible to the view of the young computer: for, were the proportions strictly adhered to in the projection, the triangle would be scarcely discernible on so small a scale; and for the same reason the supplemental angle $S R A$ is not fairly bisected.

density to buoy up clouds to a greater altitude than about two miles; except, indeed, those very volatile or fleecy ones, called *cirrus*, which sometimes float at an elevation of five or six miles.—At the height of about three miles the atmosphere becomes so attenuated, or loses so much of its density, as to be incapable of reflecting the heat of the sun, or of communicating its genial influence to the mountainous tracts at that altitude:—hence it is that, in general, at an elevation of about 16000 feet above the level of the sea, there is a total suspension of vegetation:—in that aerial region there is a perpetual congelation; and thus it is that the summits of all very high mountains, even of those within the tropics, are eternally involved in frozen snow, or crowned with impassable glacial masses, which can never be thawed or reduced to a state of fluidity by the action of the solar beams.

PROBLEM XXIV.

Given the Earth's Semidiameter, and the Sun's mean horizontal Parallax; to find the Earth's Distance from the Sun.

RULE.

To the logarithm of the earth's semidiameter, add the logarithmic co-tangent of the sun's mean horizontal parallax; and the sum (abating 10 in the index) will be the logarithm of the sun's mean distance from the earth.

Example.

By the transits of Venus over the sun's disc in the years 1761 and 1769, the sun's mean horizontal parallax appears to be about 8.65 seconds of a degree; now, if the earth's semidiameter be 3958.75 miles, its mean distance from the sun is required?

Semidiameter of the earth = . 3958.75 miles Log. = 8.5975381
 Mean horizontal parallax of the sun = 8".65 Log. co-tang. = 14.3780860

Earth's mean distance from the sun = 94546196 miles Log. = 7.9756441

See the result deduced from the inverse ratio of the parallaxes of the moon and sun, page 34.

PROBLEM XXV.

Given the Sun's mean Distance from the Earth, and his apparent Semidiameter, at a mean Rate; to find the true Measure of his Diameter, in English Miles.

RULE.

To the logarithm of the sun's mean distance from the earth, add the

logarithmic tangent of his semidiameter; and the sum (abating 10 in the index) will be the logarithm of the sun's semidiameter, in English miles; the double of which will be the measure of his whole diameter.

Example.

If the sun's mean distance from the earth be 94546196 English miles, and his mean apparent semidiameter 16'.1".65, the true measure of his diameter is required?

Sun's mean distance from the earth = 94546196 miles Log. = 7.9756441

Sun's apparent semidiameter = . 16'.1".65 Log. tang. = 7.6685950

Sun's true semidiameter = . . . 440797.5 miles Log. = 5.6442391

True measure of the sun's diameter = 881595 English miles.

PROBLEM XXVI.

Given the Diameters of the Earth and the Sun; to find the Ratio of their Magnitudes.

RULE.

Since the magnitudes of all spherical bodies are as the cubes, or triplicate ratio, of their diameters (Euclid, Book XII., Prop. 18),—therefore, from thrice the logarithm of the sun's diameter subtract thrice the logarithm of the earth's diameter, and the remainder will be the logarithm of the ratio of their magnitudes.

Example.

If the earth's diameter be 7917.5 English miles, and that of the sun 881595 such miles, required the ratio of their magnitudes?

Sun's diameter, in English miles = 881595 Thrice its log. = 17.8358076

Earth's diameter, in ditto = . 7917.5 Thrice its log. = 11.6957643

Ratio of the magnitudes of the earth and sun = 1380522 Log. = 6.1400433

Note.—The ratio of the magnitudes of the earth and a planet may be determined in the same manner; as thus:—to find how many times Jupiter is larger than the earth.

Jupiter's mean diameter = 94100 miles Thrice its log. = 14.9207688

Earth's diameter = . . . 7917.5 Thrice its log. = 11.6957643

Ratio of the magnitudes = 1678.82 Log. = . . . 3.2250045

Hence it appears that the planet Jupiter is about 1679 times larger than the globe of earth on which we live !

PROBLEM XXVII.

Given the Circumference of the Earth ; to find the Rate, per Hour, at which the Inhabitants under the Equator are carried, in consequence of the Earth's diurnal Motion round its Axis.

RULE.

To the arithmetical complement of the logarithm of 24 hours, add the logarithm of the earth's circumference, and the logarithm of 1 hour : the sum of these three logarithms (abating 10 in the index) will be the logarithm of the rate per hour at which the inhabitants under the equator are carried by the earth's diurnal motion on its axis.

Example.

Let the circumference of the earth be 24873.5 miles ; required the rate per hour at which the inhabitants under the equator are carried, in consequence of the earth's diurnal motion ?

One day, or 24 hours, Arith. comp. of its log. =	. . .	8.6197888
Earth's circumference =	. 24873.5 miles Log.=	. 4.3957369
Given time, or 1 hour	Log.=	. 0.0000000
Rate per hour, in miles =	. 1036.396	Log.= . 3.0155257

PROBLEM XXVIII.

To find the Rate at which the Inhabitants under any given Parallel of Latitude are carried, in consequence of the Earth's Diurnal Motion on its Axis.

RULE.

The circumference of the earth under the equator is 24873.5 miles ; and since the circumference under any parallel of latitude decreases in proportion to the co-sine of the latitude of such parallel,—therefore, to the logarithm of the earth's circumference, under the equator, add the logarithmic co-sine of the latitude of the given parallel ; and the sum (abating 10 in the index) will be the logarithm of the earth's circumference under that parallel : with which proceed as directed in the last Problem.

Example.

Let the circumference of the earth be 24873.5 miles ; required the

rate per hour at which the inhabitants under the parallel of London are carried by the earth's motion on its axis ?

Circumference of the earth = 24873.5 miles Log. = . . 4.3957369

Latitude of the parallel of London = 51°31' Log. co-sine = 9.7939907

Circum. under given parallel = 15478.45 Log. = . . 4.1897276

One day, or 24 hours, Arith. comp. of its log. = . . . 8.6197888

Rate per hour, in miles, as required = 644.93 Log. = . . 2.8095164

PROBLEM XXIX.

To find the Length of the Tropical or Solar Year.

RULE.

It has been found, by observation, that the sun *apparently* advances in the ecliptic 59'8".33* of a degree every day at a mean rate ; that is, from the time of his leaving any given meridian to the time of his returning to the same meridian. Now, since the ecliptic is a great circle of 360 degrees,—therefore, as the sun's apparent diurnal motion in the ecliptic, is to 1 day, or 24 hours ; so is the great circle of 360 degrees, to the true length of the tropical or solar year ; that is, to the time of the sun's periodical revolution round the ecliptic from any equinoctial or solstitial point to the same point again. Hence, by logarithms,

Example.

The sun's daily motion in the ecliptic is 59'8".33 in every natural day, or 24 hours, at a mean rate ; required the length of the tropical or solar year ?

Sun's app. diur. motion 59'8".33, in secs. = 3548.33 Log. ar. co. 6.4499760

One day, or 24 hours, in seconds = . 86400 Log. = 4.9365137

Ecliptic, or great circle of 360°, in secs. = 1296000 Log. = 6.1126050

Length of the tropical year, in seconds = 31556928 Log. = 7.4990947

Hence the tropical or solar year consists of 365'5'48"48', as required.

* See Explanatory Article 6, page 304.

PROBLEM XXX.

To find the Rate at which the Earth moves in the Ecliptic during the Time of its Annual or Periodical Revolution round the Sun.

RULE.

Since the earth's mean distance from the sun is 94546196 miles (Problem XXIV., page 738), the diameter of the orbit in which it moves round that great luminary is 189092392 miles; and since the diameter of a circle is to its circumference in the ratio of unity or 1, to 3.14159265, the circumference of the earth's orbit is 594051320 miles. Now, as the earth describes this circumference in 365° 5' 48" 48' (last Problem), or 8766 hours nearly, we have the following computation by logarithms:—

As the length of the year, in hours=8766	Log. ar. comp.=6.0571985
Is to the circumf. of the earth's orbit=594051320 miles	Log.=8.7738239
So is 1 hour	Log.=0.0000000

To the earth's hourly motion in its orbit=67768 miles Log.=4.8310224;
which is about 141 times swifter than the ordinary flight of a cannon-ball.

PROBLEM XXXI.

Given the Moon's Mean Distance from the Earth, and her Apparent Semidiameter, at a Mean Rate; to find the True Measure of her Diameter, in English Miles.

RULE.

It is shown in page 9, under the head "Augmentation of the Moon's Semidiameter," that the moon's mean distance from the earth is 236692.35 miles. Now, since her apparent semidiameter is 15' 43" at a mean rate,—therefore, to the logarithm of her mean distance from the earth, add the logarithmic tangent of her apparent semidiameter; and the sum (abating 10 in the index) will be the logarithm of the moon's semidiameter in English miles: the double of which will be the measure of her whole diameter.

Example.

Let the moon's distance from the earth be 236692.35 miles, and her semidiameter 15' 43"; required the true measure of her diameter in English miles?

Moon's mean distance from the earth = 236692.35 miles Log. = 5.3741842

Moon's apparent semidiameter = 15'.43" Log. tang. = 7.6600896

Moon's true semidiameter = . . 1082.1 miles Log. = 3.0342738

True measure of the moon's diameter = 2164.2 English miles.

PROBLEM XXXII.

Given the Diameters of the Earth and the Moon; to find the Ratio of their Magnitudes.

Note.—This is performed by Problem XXVI., page 739.

Example.

If the earth's diameter be 7917.5 English miles, and that of the moon 2164.2 such miles, required the ratio of their magnitudes?

Diameter of the earth = . . 7917.5 Thrice its log. = 11.6957643

Diameter of the moon = . . 2164.2 Thrice its log. = 10.0058922

Ratio of the magnitudes of the earth and moon = 48.96 Log. = 1.6898721

PROBLEM XXXIII.

To find how much larger the Earth appears to the Lunar Inhabitants than the Moon appears to the Terrestrial Inhabitants.

RULE.

Since the distance between the earth and the moon is such as to cause their opposing hemispheres to appear, reciprocally from each other, like flat circles; and since circles are to one another as the squares of their diameters (Euclid, Book XII., Prop. 2), or, which is the same thing, since spherical surfaces are to each other as the squares of their radii,—therefore, from twice the logarithm of the earth's diameter, subtract twice the logarithm of that of the moon; and the remainder will be the logarithm of the number of times that the earth appears larger to the inhabitants of the moon than the moon does to the inhabitants of the earth.

Example.

The diameter of the earth is 7917.5 miles, and that of the moon

2164.2 miles ; required how much larger the earth appears from the moon than the moon does from the earth ?

Diameter of the earth = . . . 7917.5 Twice its log.=7.7971762
 Diameter of the moon = . . . 2164.2 Twice its log.=6.6705948

Number of times the earth is larger than the $\text{moon} = 13.38$ Log.=1.1265814

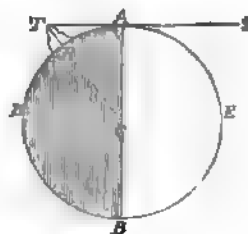
PROBLEM XXXIV.

To find the Height of a Mountain in the Moon.

RULE.

When the moon is viewed through a good telescope, her face appears to be diversified with extensive ranges of remarkably high mountains :—now, although the present Problem may appear as *merely chimerical* to the uninformed in astronomy, yet, there is not a more rational and demonstrable Problem, or proposition, in the whole range of the mathematics, as may be seen in the following operations, viz. :—

In the annexed diagram let A D B E be the disc or face of the moon at the time of the *quadratures* ; for, it is at this time, or when the moon is 90 degrees from the sun, that the altitudes of her mountains are most easily determined.



Let the diameter A C B be the boundary of light and darkness, and R T the mountain called St. Catherine, the summit of which, at the point T, is just beginning to be enlightened by the solar ray S A T. Now, by means of a *micrometer* fitted in a suitable telescope, the distance between the points T and A may be easily measured ; and thus their *ratio* will be known :—this, according to *Hevelius* and other astronomers, is the $\frac{1}{13}$ part of the diameter A B, or the $\frac{1}{13}$ part of the semidiameter A C. Now, we have seen in the preceding Problem that the value of the semidiameter C A, or C R, is 1082.1 miles ; therefore, agreeably to the above *ratio*, $1082.1 \div 13 = 83.24$ miles, is the value of T A : then, in the right angled triangle T A C, the legs are given ; viz. C A = 1082.1 miles, and T A = 83.24 miles ; to find the hypotenuse T C ; from which take away the semidiameter C R (= C A) and the remainder, or R T, will be the height of the mountain. Hence, by right angled plane trigonometry, Problem IV., page 175,—

As the semidiameter C A = . 1082.1 Log. ar. comp. = 6.9657326
 : Radius, or 90° Log. sine = . 10.0000000
 :: the leg T A = 83.24 Logarithm = 1.9203321

To the angle A C T = . . . 4°23'56" Log. tangent = 8.8860647

And,

As radius, or 90° Log. sine ar. comp. = 10.0000000
 : the semidiameter C A = 1082.1 Logarithm = . . 3.0342674
 :: the angle A C T = . 4°23'56" Log. secant = . . 0.0012812

To the hypotenuse C T = 1085.3 Logarithm = . . 3.0355486
 Subtract the semidiam. C A = 1082.1

The remainder, or R T = 3.2 :—hence, the height of the lunar mountain called *St. Catherine* is $3\frac{1}{2}$ miles, or 3 miles, and 352 yards ; and thus it is manifest that the *apparently paradoxical* Problem for determining the height of a mountain in the moon is simply the resolution of a plane right angled triangle.

PROBLEM XXXV.

To find the Rate at which the Moon revolves round her Orbit.

Note.—This is performed by Problem XXX., page 742, as thus :—

Since the moon's mean distance from the earth is 236692.35 miles, the diameter of her orbit must be twice that distance, or 473384.70 miles : hence its circumference is 1487182 miles. And since the moon goes through this circuit, or orbit, in 27°7'43"5', her hourly motion may be determined in the following manner ; viz.,

As one lunation = 27°7'43"5', in secs. = 2360585 Log. ar. co. = 3.6269804
 Is to the circumference of the moon's orbit = 1487182 Log. = 6.1723641
 So is one hour, in seconds = 3600 Log. = 3.5563025

To the moon's hourly motion in her orbit = 2268 miles Log. = 3.3556470

PROBLEM XXXVI.

To find the Mean Distance of a Planet from the Sun.

RULE.

It has been demonstrated, by the celebrated *Kepler*, that if two or

more bodies move round another body as their common centre of motion, the squares of their periodical times will be to each other in the same proportion as the cubes of their mean distances from the central body ; and hence the following rule :—

As the square of the earth's periodical or annual motion round the sun, is to the cube of its mean distance from that luminary ; so is the square of any other planet's periodical revolution round the sun, to the cube of its mean distance therefrom ; the root of which will be the distance sought.

Example.

The earth's periodical or annual motion round the sun is completed in 365 days, 5 hours, 48 minutes, 48 seconds, and that of Venus in 224 days, 16 hours, 49 minutes, 11 seconds. Now, if the earth's mean distance from the sun be 94546196 miles, what is Venus's distance from that luminary ?

Earth's periodical revolution

365° 5' 48" 48', in secs. 31556928 Ar.co. of twice its log.=5. 0018106

Earth's mn.dist.from sun,in miles=94546196 Thrice its log. 23. 9269323

Venus's per.rev.224° 16' 49" 11',in secs. 19414151 Twice log. 14. 5762368

Reject 20 from index; and, to extract the root, divide by 3) 23. 5049797

Venus's mean distance from sun, in miles=68390098 Log.=7. 8349932½

PROBLEM XXXVII.

To find how much more Heat and Light the Planets adjacent to the Sun receive from that Luminary than those which are more remote.

RULE.

Since the effects of heat and light are reciprocally proportional to the squares of the distances from the centre whence they are generated, —therefore, from twice the logarithm of the remote planet's distance from the sun, subtract twice the logarithm of the adjacent planet's distance therefrom ; and the remainder will be the logarithm of the number of times that the planet adjacent to the sun is hotter and more luminous than that which is more remote.

Example.

If the earth's distance from the sun be 94546196 miles, and that of

Venus 68390098 miles, required how much more heat and light the latter planet receives from the sun than the former ?

Earth's mean dist. from sun = 94546196 miles Twice its log. = 15.9512882
 Venus's ditto = 68390098 miles Twice its log. = 15.8699865

Heat & light Venus receives more than the earth = 1.205 Log. 0.0813017

PROBLEM XXXVIII.

Given the Apparent Diameter of a Planet ; to find the Measure of its true Diameter.

RULE.

Find the difference between the earth's and the planet's mean distances from the sun, and it will show the planet's mean distance from the earth ; with which and the planet's apparent semidiameter, compute her true diameter, by Problem XXXI., page 742.

Example.

Let the earth's distance from the sun be 94546196 English miles, that of Venus 68390098 such miles, and her apparent diameter 58".79 ; required the true measure of her diameter, in English miles ?

Earth's distance from the sun = 94546196 miles
 Venus's ditto = 68390098 miles

Venus's mean dist. from earth = 26156098 miles Log. = , 7.4175729
 Venus's apparent semidiameter = 29".395 Log. tang. = 6.1537885

Venus's true semidiameter = . 3727 miles Log. = . 3.5713614

True measure of Venus's diameter = 7454 English miles.

Note.—If the ratio of the magnitudes of the earth and Venus be required, it may be determined by Problem XXVI., page 739 ; as thus,—

Diameter of the earth = 7917.5 miles Thrice its log. = 11.6957643
 Diameter of Venus = . 7454 miles Thrice its log. = 11.6171682

Ratio of the magnitudes of the earth and Venus = 1.198 Log. = 0.0785961

The velocity or rate at which a planet moves round its orbit may be determined by Problem XXX., page 742.

PROBLEM XXXIX.

To find the Time that the Sun takes to turn round its Axis.

RULE.

If the bright face of the sun be carefully observed through a good telescope, large black spots will be found to make their appearance on its eastern limb: from this they gradually advance to the middle of the disc, and thence to the western limb, where they disappear. After being absent for nearly the same period of time that they were visible, they will be observed to appear again on the eastern limb as at first; thus finishing their career in 27 days, 12 hours, and 20 minutes. Call this time the *observed interval*.

Find the number of degrees and parts of a degree that the earth has moved eastward in the ecliptic during that interval, in the following manner; viz.,

As the earth's annual motion

round the sun = $365^{\circ} 5' 48'' 48'''$, in secs. 31556928 Log. ar. co. 2. 5009053

Is to eclip., or great circle of 360° , in secs. 1296000 Log. = 6. 1126050

So is the obs. interv. = $27^{\circ} 12' 20''$, in secs. 2377200 Log. = 6. 3760657

Earth's adv. in ecliptic during obs. interv. = $97628''$ Log. = 4. 9895760

Ditto, in degrees and parts of a degree = $27^{\circ} 7' 8''$

Now, as 360 degrees, augmented by the earth's advance in the ecliptic during the observed interval, thus found, is to the observed interval; so is the great circle of 360 degrees, to the absolute time of the sun's rotatory motion on its axis; thus:—

As $360^{\circ} + 27^{\circ} 7' 8'' = 387^{\circ} 7' 8''$, in secs. = 1393628 Log. ar. co. 3. 8558532

Is to the obs. int. = $27^{\circ} 12' 20''$, in secs. = 2377200 Log. = 6. 3760657

So is the great circle of 360° in secs. = 1296000 Log. = 6. 1126050

To the time of the sun's rotatory motion = 2210670' Log. = 6. 3445239

Ditto, in days and parts of a day = $25^{\circ} 14' 4'' 30'''$; which, therefore, is the true time that the sun takes to turn round *once* upon its axis, as required.

PROBLEM XL.

To find how long it would take the Sun to perform one complete Revolution round the Earth ; on the mistaken hypothesis that the Earth is at rest and the Heavenly Bodies in motion moving round it.

RULE.

It is a universal law in Nature that, if two or more bodies revolve round another body as their common centre of motion, the squares of their periodical times of revolution will be to each other, in the same proportion, as the cubes of their distances from the central body.—See Problem XXXVI, page 745.

Now, let the earth be at rest, and the sun and moon in motion performing periodical revolutions round it:—then, since the distances of those luminaries from the earth, as a central body, are known, and also the periodical revolution of the moon round that central body; the periodical revolution of the sun round the same body may be readily determined agreeably to the above expression by saying,—As the cube of the moon's distance from the earth is to the square of her periodical revolution, so is the cube of the sun's distance from the earth to the square of its periodical revolution; the root of which will be the exact time that it would take the sun to revolve once round the earth.

Since the moon performs one complete revolution round the earth, with respect to the fixed stars, in 27 days, 7 hours, 43 minutes, 5 seconds; which, when the hours &c. are reduced to decimals, is 27.3216 days, and that her mean distance from the earth (page 34) is 237726½ English miles; and, since the sun's mean distance from the earth is 95328955 English miles, its periodical revolution may, therefore, be easily computed by logarithms; as thus,—

Moon's dist. from the earth=237726½	Thrice its log. ar. com.	3.8717702
Moon's periodical revolution=27.3216	Twice its log. =	. 2.8730122
Sun's dist. from the earth=95328955	Thrice its log. =	. 23.9376744
		<hr/>
	Sum =	10.6824568
		<hr/>

Sun's periodical revolution = 219396 days Log. = . . 5.3412284

Hence, it is clearly evident that the sun would require the long period of 219396 days, or 601 years and 31 days, to perform one complete revolution round the globe on which we live! which period would also be the correct length of the solar day; for, during that time the sun

could not *rise in the east and set in the west* more than once. And thus, while the great luminary of heaven would be continually visible, or *never set*, during the space of 300 years, 6 months, 15 days, to the inhabitants of one-half of the earth; it would be invisible; or *never rise*, for an equal length of time to the inhabitants on the opposite half of the earth. One hemisphere would be blessed with the cheerful presence of day-light for an uninterrupted period of more than three centuries; whilst the other would be involved in continual darkness,—in the cheerless gloom of a dreary night,—for more *than ten generations at a time*. But, thanks to Infinite Wisdom! things have been more rationally ordained: and thus the earth, by a silent rotatory motion round its axis from west to east, easily effects that grateful return of *light and dark* once in 24 hours,—in all parts that do not come within the circles of perpetual apparition,—which could not be effected by the sun in a less space of time than six hundred and one years and thirty-one days.

END OF THE FIRST VOLUME.



